

1. (d) Pitch  $\Rightarrow \frac{2.5 \text{ mm}}{5} = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$

$$\text{L.C.} \Rightarrow \frac{\text{Pitch}}{\text{No. of divisions}} = \frac{5 \times 10^{-4}}{100} \text{ m}$$

$$= 5 \times 10^{-6} \text{ m}$$

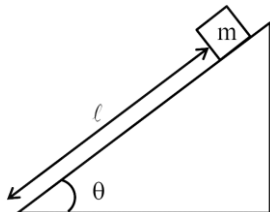
$$= 5 \times 10^{-3} \text{ mm}$$

2. (b) Bulk modulus  $\beta = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$$100 \times \frac{\Delta V}{V} = \frac{-\Delta P}{\beta} \times 100 = -\frac{6.3 \times 10^7}{2.1 \times 10^9} \times 100 = -3\%$$

Percentage decrease = 3%

3. (c)



For Rough incline plane

$$a = g(\sin\theta - \mu\cos\theta)$$

$$t_1 = \sqrt{\frac{2\ell}{g(\sin\theta - \mu\cos\theta)}}$$

For smooth incline plane

$$t_2 = \sqrt{\frac{2\ell}{g\sin\theta}} \text{ \& } t_1 = 1.5t_2$$

$$\sqrt{\frac{2\ell}{g(\sin\theta - \mu\cos\theta)}} = \frac{3}{2} \times \sqrt{\frac{2\ell}{g\sin\theta}}$$

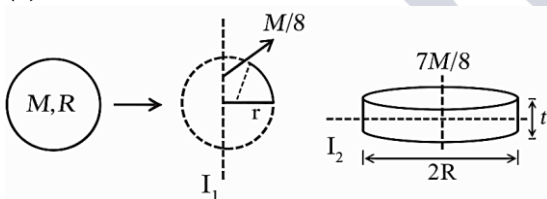
$$(\sin\theta - \mu\cos\theta) = \frac{4}{9}\sin\theta$$

$$\frac{5}{9}\sin\theta = \mu\cos\theta \Rightarrow \frac{5}{9}\tan\theta$$

$$\mu = \frac{5}{9}\tan 45^\circ = \frac{5}{9}$$

4. (c)  $\Delta mc^2 = [2 \times 3 \times 5.6 + 4 \times (7.4) - 6.1 \times 10]$   
 $= 2.2 \text{ MeV}$

5. (a)



$$I_1 = \frac{2}{5} \left( \frac{M}{8} \right) r^2$$

$$I_2 = \frac{7M}{8} \times (2R)^2$$

$$M = \rho \times \frac{4\pi}{3} \times R^3$$

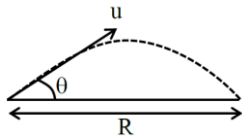
$$\frac{M}{8} = \rho \times \frac{4\pi}{3} \times |r|^3$$

$$r = \frac{R}{2}$$

$$\frac{I_1}{I_2} = \frac{\frac{2}{5} \left(\frac{M}{8}\right) \left(\frac{R}{2}\right)^2}{\frac{7M}{8 \times 4} \times (2R)^2} = \frac{2}{5} \times \frac{1}{4 \times 7} = \frac{1}{70}$$

$$\frac{I_2}{I_1} = \frac{70}{1}$$

6. (b)



$$R_1 = \frac{u^2 \sin 2 \times 15^\circ}{g}$$

$$R_2 = \frac{u^2 \sin 2 \times 30^\circ}{g}$$

$$\frac{R_1}{R_2} = \frac{\sin 30^\circ}{\sin 60^\circ} = \frac{1}{\sqrt{3}} = \frac{1}{x}$$

$$x = \sqrt{3}$$

7. (a)  $h\nu = \phi + e\nu_0$

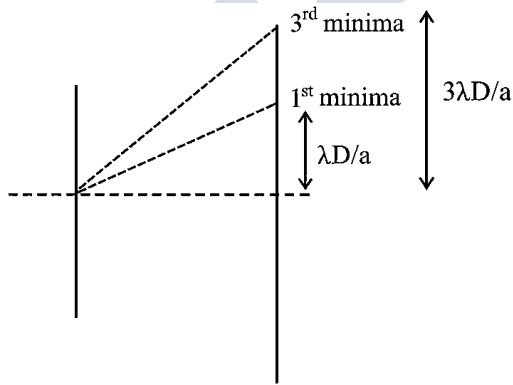
$$\nu_0 = \frac{h\nu}{e} - \frac{\phi}{e} \Rightarrow 0 \Rightarrow \frac{h\nu}{e} = \frac{\phi}{e}$$

$$\begin{pmatrix} \phi_1 : \phi_2 : \phi_3 \\ \nu_1 : \nu_2 : \nu_3 \\ 1 : 1.5 : 2 \end{pmatrix}$$

As metal (1) having minimum value of work function

So  $x_1$  will having maximum kinetic energy

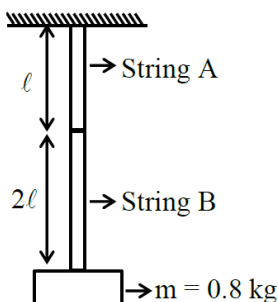
8. (c)



Separation between 3<sup>rd</sup> & 1<sup>st</sup> minima

$$= \frac{2D\lambda}{a}$$

9. (b)



$$\Delta \ell \Rightarrow \Delta \ell_1 + \Delta \ell_2$$

$$\Rightarrow \frac{mg\ell}{AY} + \frac{mg(2\ell)}{A(2Y)} = \frac{2mg\ell}{AY}$$

$$= \frac{2 \times 0.8 \times 10 \times (0.314)}{3.14 \times (2 \times 10^{-4})^2 \times 2 \times 10^{10}} = 2 \text{ mm}$$

10. (b)  $n = n_1 + n_2$

$$\therefore PV = nRT \Rightarrow n = \frac{PV}{RT}$$

$$\frac{P(V)}{T_{\text{Final}}} = \frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}$$

$$\frac{PV}{RT_f} = \frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2}$$

$$T_f = \frac{PV}{P_1 V_1/T_1 + P_2 V_2/T_2}$$

11. (b) For  $V = V_0$ ,  $P = \frac{P_0}{2}$

$$\frac{P_0}{2} \times V_0 = 2 \times RT_A$$

$$T_A = \frac{P_0 V_0}{4R}$$

for  $V = 3 V_0$ ,  $P = \frac{9P_0}{10}$

$$\frac{9P_0}{10} \times 3 V_0 = 2 \times RT_B$$

$$T_B = \frac{27P_0 V_0}{20R}$$

So,  $T_B - T_A$

$$= \frac{11P_0 V_0}{10R}$$

12. (a) Since maximum current through voltmeter remains same

So,  $\frac{20}{x} = \frac{30}{x+R}$

$$20x + 20R = 30x$$

$$20R = 10x$$

$$R = \frac{x}{2}$$

This extra resistance must be connected in series.

13. (a)  $Y = \overline{A} \cdot B = A + \overline{B}$

$$A = 1101$$

$$B = 1010$$

$$\overline{B} = 0101$$

$$A + \overline{B} = 1101$$

14. (b) For A  $u = -20 \text{ cm}$ ,  $f = -10 \text{ cm}$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \therefore v = \frac{u \times f}{u - f} = -20 \text{ cm}$$

For B  $u = -30 \text{ cm}$ ,  $f = -10 \text{ cm}$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$v = \frac{uf}{u - f} = -15 \text{ cm}$$

Length of image =  $20 - 15 = 5 \text{ cm}$

15. (a) Capacitance =  $C = \frac{\epsilon_0 A}{x}$

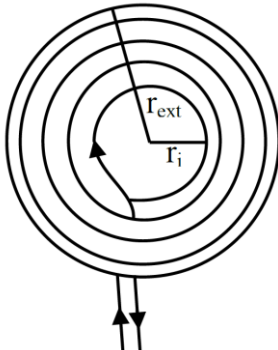
Energy stored =  $U = \frac{1}{2} C V^2$

$$U = \frac{1}{2} \frac{\epsilon_0 A}{x} V^2$$

$$\frac{dU}{dt} = \frac{1}{2} \epsilon_0 A v^2 \left( -\frac{1}{x^2} \right) \frac{dx}{dt}$$

$$\frac{dU}{dt} \propto \frac{1}{x^2}$$

16. (a)



$$dM = (dI)\pi r^2 = (IdN)\pi r^2$$

$$M = \int_i^f I \left( \frac{N}{r_{ex} - r_i} \right) dr \pi r^2$$

$$M = \frac{I\pi N}{r_{ex} - r_i} \left[ \frac{r^3}{3} \right]_{r_i}^{r_{ex}}$$

$$M = \frac{I\pi N}{3(r_{ex} - r_i)} (r_{ex}^3 - r_i^3)$$

$$M = \frac{20 \times 10^{-3} \times 3.14 \times 200 [6^3 - 3^3] 10^{-6}}{3[3] \times 10^{-2}}$$

$$M = \frac{2373.84 \times 10^{-6}}{9 \times 10^{-2}}$$

$$M = 263.76 \times 10^{-4}$$

$$M = 2.64 \times 10^{-2} \text{ Am}^2$$

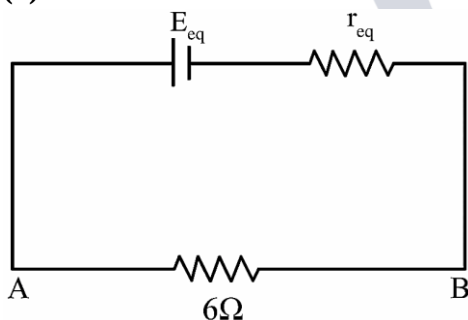
17. (a)  $\tau = RC$

$$\tau = (110) \times 20 \times 10^{-6} \text{ sec}$$

$$\tau = 2200 \times 10^{-6} \text{ sec}$$

$$\tau = 2.2 \times 10^{-3} \text{ sec}$$

18. (b) This circuit can be



$$\text{Here : } E_{eq} = \frac{\frac{27}{3} + \frac{27}{3} + \frac{27}{3} + \frac{27}{3}}{\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$$

$$E_{eq} = 27 \text{ V}$$

$$r_{eq} = \frac{3}{4} \Omega$$

$$i_{AB} = \frac{27}{6 + \frac{3}{4}} = 4 \text{ A}$$

$$V_{AB} = 4 \times 6 = 24 \text{ Volt}$$

19. (b)  $\theta = 5 \times 10^{-7} \text{ radian}$

$$\lambda = 500 \times 10^{-9} \text{ m}$$

$$\theta = \frac{1.22\lambda}{R}$$

$$R = \frac{1.22\lambda}{\theta} = 1.22 \text{ m} = 122 \text{ cm}$$

20. (a) Is the light is propagating along the positive z -axis and its is reflected from plane then correct phase for reflected light is  $kx - kz - \omega t$  and for electric field as the light incident on Brewster angle so the reflected light should be perfectly plane polarized that's why polarization direction should be y -axis so electric field of reflected light contain no zcomponent.

21. (35)  $\vec{F}_{\text{Net}} = 5\hat{i} + 2\hat{j} + 2\hat{k}$

$$\vec{S} = 15\hat{i} - 20\hat{j}$$

$$W = \vec{F} \cdot \vec{S} = 75 - 40 = 35 \text{ J}$$

22. (264) W.D. =  $2 \text{ T} [4\pi R_2^2 - 4\pi R_1^2]$

$$= 2 \times 3.5 \times 10^{-2} \times 4 \times \frac{22}{7} [2^2 - 1^2] \times 10^{-4}$$

$$= 2 \times 132 \times 10^{-6}$$

$$= 264 \times 10^{-6} \text{ J} = \alpha \times 10^{-6} \text{ J}$$

$$\alpha = 264$$

23. (44)  $V^2 = \omega^2 A^2 - \omega^2 x^2$

Comparing  $\omega = 1$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi = \frac{x}{7}$$

$$x = 14\pi$$

$$x = 43.98 \approx 44$$

24. (200)  $\vec{a} = \frac{\vec{F}}{m} = t\hat{i} + 3t^2\hat{j}$

$$\vec{v} = \int_0^2 \vec{a} \cdot dt = \left[ \frac{t^2}{2} \hat{i} + \frac{3t^3}{3} \hat{j} \right]_0^2$$

$$= (2\hat{i} + 8\hat{j}) \text{ m/s}$$

$$\vec{F}_{t=2} = (4\hat{i} + 24\hat{j}) \text{ N}$$

$$P = \vec{F} \cdot \vec{v} = 8 + 192 = 200 \text{ W}$$

25. (100)  $\cos\phi = 0.5 = \frac{1}{2}$

$$\phi = 60^\circ$$

$$\tan\phi = \frac{|x_L - x_C|}{R}$$

$$\tan 60^\circ = \sqrt{3} = \frac{x_L - x_C}{R}$$

$$|x_L - x_C| = R\sqrt{3} = 100\sqrt{3} = \sqrt{3}\alpha$$

$$\alpha = 100$$

26. (b) moles =  $\frac{V \text{ lit}}{22.7 \text{ lit}} \Rightarrow \frac{1.4187 \text{ lit}}{22.7 \text{ lit}} \Rightarrow 0.0633$

$$\text{no. of molecules} = \text{moles} \times 6.023 \times 10^{23}$$

$$\Rightarrow 0.0633 \times 6.032 \times 10^{23}$$

$$= 3.812 \times 10^{22}$$

27. (b)  $\bar{V}_1 = R_H(z)^2 \left[ \frac{1}{2^2} - \frac{1}{3^2} \right] \Rightarrow \text{I}^{\text{st}} \text{ line of Balmer series}$

$$\bar{V}_2 = R_H(z)^2 \left[ \frac{1}{4^2} - \frac{1}{5^2} \right] \Rightarrow \text{I}^{\text{s}} \text{ line of Brackett series}$$

$$\frac{\bar{V}_1}{\bar{V}_2} = \frac{500}{81} \Rightarrow 5:0.81$$

28. (d)  $\text{Cr} 1s^2 2s^2 2p^6 3s^2 3p^6 4s^1 3d^5$

$\Rightarrow 19^{\text{th}} e^-$  belongs to 4s subshell

$$n = 4; \ell = 0; m = 0; m_s = +\frac{1}{2} \text{ or } -\frac{1}{2}$$

29. (d) From first law of thermodynamics

$$\Delta U = q + w$$

constant volume  $w = 0$

$$\Delta U = q$$

$$\text{Also } C_p = C_v + R$$

$C_p$  = heat capacity at constant pressure

$C_v$  = Heat capacity at constant volume.

$R$  = constant

30. (d)  $A_2(g) + B_2(g) \rightleftharpoons (g) K_{eq} = 2.7 \times 10^{-5}$

$$\frac{1}{3} A_2(g) + \frac{1}{3} B_2(g) \rightleftharpoons \frac{1}{3} C(g) K_{eq}^1 = (K_{eq})^{1/3}$$

$$K_{eq}^1 = (2.7 \times 10^{-5})^{1/3}$$

$$= 3 \times 10^{-2}$$

31. (b)  $\text{Meq of } KMnO_4 = \text{meq of } FeC_2O_4 + \text{meq of } Fe_2(C_2O_4) + \text{meq of } FeSO_4$

$$\text{moles} \times 5 = 1 \times 3 + 1 \times 6 + 1 \times 1$$

$$\text{moles} = \frac{10}{5} = 2$$

32. (d)  $I^{\text{st}}$  order of  $R\mu^H$

$$a - x = ae^{-K_1 t}$$

$$\frac{x}{a} = 1 - e^{-K_1 t}$$

33. (a)  $X = \text{Br}$ ; Atomic mass = 80

Period 4

$\text{Br}_2$  exist as liquid.

34. (a) Oxygen can have covalency 4

In  $\text{SO}_2$ ; O.S. of O = -2

In  $\text{OF}_2$ ; O.S. of O = +2

Anamolous behaviour of oxygen is due to high EN and small size.

35. (c)(A) Mo(VI) & W(VI) more stable than Cr(VI)

(B) In lanthanoids +3 oxidation state is more stable hance +4 ions act as oxidising agents and +2 ions act as reducing agents.

(C)  $\text{Cm} - [\text{Rn}] 5f^7 6d^1 7s^2 = 8$  unpaired electrons

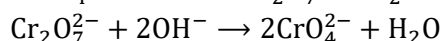
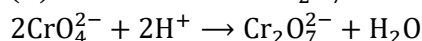
$\text{Am} - [\text{Rn}] 5f^7 6s^2 = 7$  unpaired electrons

(D) Actinoid contraction is more than lanthanoid contraction

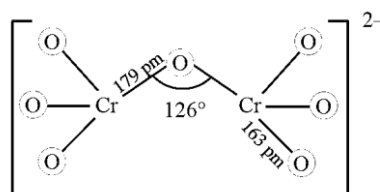
36. (b)(A)  $\text{MnO}_4^- + \text{Fe}^{+2} + \text{H}^+ \rightarrow \text{Fe}^{+3} + \text{Mn}^{+2} + \text{H}_2\text{O}$

(B)  $\text{Na}_2\text{Cr}_2\text{O}_7$  generally not used as a primary standard in volumetric analysis because it is deliquescent.

(C) In acidic medium  $\text{Cr}_2\text{O}_7^{2-}$  exists and it get converted into  $\text{CrO}_4^{2-}$  in basic medium.



(D)  $\text{K}_2\text{Cr}_2\text{O}_7$  is a good oxidising agent and bond angle of  $\text{Cr} - \text{O} - \text{Cr}$  bond in  $\text{Cr}_2\text{O}_7^{2-}$  is  $126^\circ$



Dichromate ion

37. (d)  $[\text{Cr}(\text{H}_2\text{O})_6]^{2+} \rightarrow 4 \text{ unpaired } e^-; 4.90 \text{ BM(III)}$

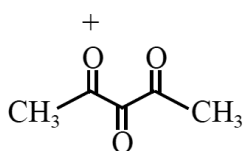
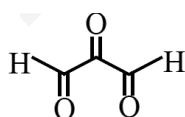
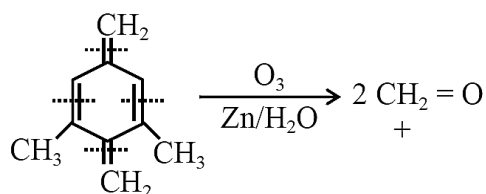
$[\text{Co}(\text{H}_2\text{O})_6]^{2+} \rightarrow 0 \text{ unpaired } e^-; 3.87 \text{ BM(I)}$

$[\text{Cu}(\text{H}_2\text{O})_6]^{2+} \rightarrow 1 \text{ unpaired } e^-; 1.73 \text{ BM(IV)}$

$[\text{Mn}(\text{H}_2\text{O})_6]^{2+} \rightarrow 5 \text{ unpaired } e^-; 5.92 \text{ BM(II)}$

38. (c)

39. (d)

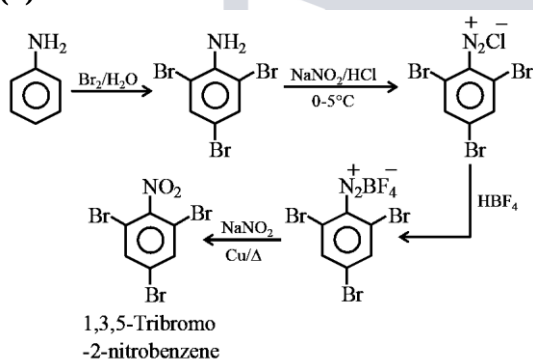


40. (b)

41. (a) II, III, IV, V, VII

Compounds which are more acidic than  $\text{H}_2\text{O}$  can soluble in  $\text{NaOH}$ .

42. (b)

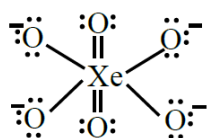


43. (d) Maltose and lactose both have hemiacetal linkage. So both are reducing sugars.

44. (a) Arginine (R) and Lysine (K) are essential amino acid. Aspartic acid (D) and Glutamic acid (E) are non-essential amino acid.

45. (d) Phenolphthalein indicator turns colourless in excess  $\text{NaOH}$ .

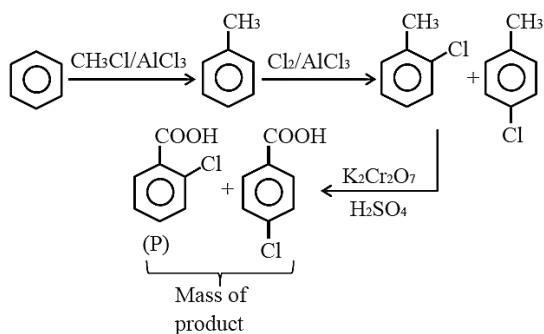
46. (6)



$\sigma = 6$

Lone pairs = 0

47. (78)



$$\text{Moles of P} = \frac{11.2}{22.4} = 0.5 \text{ mole}$$

$$M_{C_7H_5O_2Cl} = 156.5 \text{ gm}$$

$$W_{P(C_7H_5O_2Cl)} = \frac{156.5}{2} = 78.25 \approx 78 \text{ grams}$$

$$48. (5) n_{Br} = \frac{3.36}{(108+80)}$$

r = no. of Br atoms in organic compound

$$n_{\text{organic compound}} = \frac{3.36}{(108 + 80) \times x} = \frac{2}{M}$$

$$M = \frac{2 \times 188 \times x}{3.36}$$

$$M = \frac{1}{2} \times x$$

(x is integer)

$$W_C = \frac{112 \times x \times 26.7}{100} = 30 \times x$$

$$n_C = \frac{30 \times x}{12} = 2.5 \times x$$

$$\text{take } x = 2, n_C = 5$$

(Empirical formula is  $C_5H_4Br$ )

$$49. (756) \text{ On mixing final volume} = 255 \text{ ml}$$

$$[NH_4OH] = \frac{5 \times 0.1}{255}, [NH_4Cl] = \frac{250 \times 0.1}{255}$$

$$p^{OH} = p_b + \log \frac{[NH_4Cl]}{[NH_4OH]}$$

$$p^{OH} = 4.74 + \log \frac{250 \times 0.1}{5 \times 0.1}$$

$$p^H = 14 - p^{OH} = 14 - 6.44$$

$$50. (5) \frac{P^\circ - P_s}{P_s} = i \cdot \text{molality} \times \frac{(M.\text{solvent})}{1000}$$

$$\Delta T_b = i \cdot K_b \cdot \text{molality} \Rightarrow \text{molality} = \frac{0.5}{0.3}$$

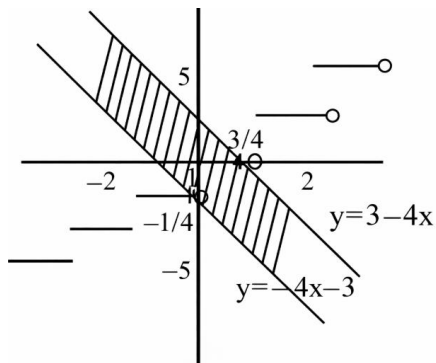
$$(\text{Molecular Mass}) = \frac{600}{75} \text{ g}$$

$$\text{Moles} = \frac{40}{600/75} = 5$$

$$51. (a) -1 \leq \frac{4x+2[x]}{3} \leq 1$$

$$-3 - 4x \leq 2[x] \leq 3 - 4x$$





$$x \in \left[ \frac{-1}{4}, \frac{3}{4} \right]$$

$$12(\alpha + \beta) = 6$$

52. (b)  $|a| + |b| = |a + b|$

$$\Rightarrow a \cdot b \geq 0$$

$$\Rightarrow x(x^2 - 9) \geq 0$$

$$x \in [-3, 0] \cup [3, \infty]$$

$$\alpha^2 + \beta^2 + \gamma^2 = 18$$

53. (a)  $|z + 2| = |z - 2|$

$$\Rightarrow z = 0 + iy \quad y \in \mathbb{R}$$

$$\arg\left(\frac{3 + iy}{iy - i}\right) = \frac{\pi}{4}$$

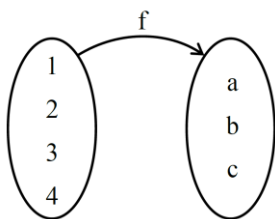
$$\Rightarrow \arg\left(\frac{y}{y - 1} + \frac{3i}{1 - y}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{y}{y - 1} = \frac{3}{1 - y} > 0$$

$$\Rightarrow z = -3i$$

$$|z|^2 = 9$$

54. (b)



No. of into function

$$= {}^3C_1 2^4 - {}^3C_2 1 + 0$$

$$= 3 \times 16 - 3 \times 1$$

$$= 45$$

55. (d)  $a, b, c, d \in \{0, 1, 2, 3, 4\}$

$$a + d = 4 \quad ad - bc = 3$$

Case-I  $a = 0 \quad d = 4$

$$0 - bc = 3$$

No solution

Case-II  $a = 1 \quad d = 3$

$$3 - bc = 3$$

$$bc = 0 \Rightarrow 9 \text{ ways}$$

Case-III  $a = 2 \quad d = 2$

$$4 - bc = 3$$

$$bc = 1 \Rightarrow 1 \text{ ways}$$

Case-IV  $a = 3 \quad d = 1$

$bc = 0 \Rightarrow 9 \text{ way}$

Case-V  $a = 4 \quad d = 0 \Rightarrow \text{No Solution}$

Total 19 matrix

56. (d)  $|A| = -4$

$$\text{As } (\text{adj} A)^{-1} = \frac{A}{|A|} = \frac{-A}{4}$$

$$B = 2(\text{adj} A)^{-1} = \frac{-A}{2}$$

$$|B| = \left| \frac{-A}{2} \right|$$

$$= \frac{-1}{8} |A| = \frac{1}{2}$$

$$\text{adj}(\text{adj} B) = |B|^{n-2} B = |B| B$$

$$= \frac{B}{2} = \frac{-A}{4}$$

$$\text{Sum of element} = \frac{-12}{4} = -3$$

57. (a)  $a = \frac{10}{3} : S_{30} = (T_{30})^3$

$$\frac{30}{2} \left[ 2 \times \frac{10}{3} + 29d \right] = \left[ \frac{10}{3} + 29d \right]^3$$

$$15 \left( \frac{20}{3} + 29d \right) = \left( \frac{10}{3} + 29d \right)^3$$

$$15 \left( \frac{10}{3} + \frac{10}{3} + 29d \right) = \left( \frac{10}{3} + 29d \right)^3$$

$$\text{By observation } \frac{10}{3} + 29d = 5$$

$$29d = 5 - \frac{10}{3} = \frac{5}{3}$$

$$d = \frac{5}{87}$$

58. (d) No. of ways to arrange 7 students = 7 !

Cases where all girls are together

$$B_1 B_2 B_3 B_4 G_1 G_2 G_3 = 5! 3!$$

$$\text{Ans.} = 7! - 5! 3!$$

$$= 7! - 6!$$

$$= 6 \times 6 !$$

$$= 6 \times 720$$

$$= 4320$$

59. (b)  ${}^3C_3 + {}^4C_3 + {}^5C_3 + \dots + {}^{99}C_3 + {}^{100}C_3 \quad K^3 = \left( 43n + \frac{101}{4} \right) {}^{100}C_3$

$$\Rightarrow {}^4C_4 + {}^4C_3 + {}^5C_3 + \dots + {}^{99}C_3 + {}^{100}C_3 \quad K^3 = \left( 43n + \frac{101}{4} \right) {}^{100}C_3$$

$$= {}^{100}C_4 + {}^{100}C_3 \quad K^3 = \left( 43n + \frac{101}{4} \right) {}^{100}C_3$$

$$\frac{{}^{100}C_4}{{}^{100}C_3} + K^3 = 43n + \frac{101}{4}$$

$$\frac{97}{4} + K^3 = 43n + \frac{101}{4}$$

$$K^3 = 43n + 1$$

$$n = 5$$

$$K^3 = 216$$

$$K = 6 = P$$

$$P + n = 11$$

60. (d) 8, 13, b, 17, 21, 51, a, 67, 103

$$\text{mean} = 40 = \frac{280 + a + b}{9}$$

$$a + b = 80$$

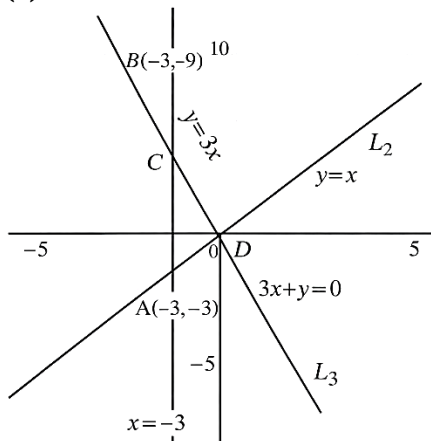
mean deviation about median

$$= \frac{13 + 8 + 4 + 30 + 46 + 82 + (a - 21) + (21 - b)}{9} = 26$$

$$a - b = 51$$

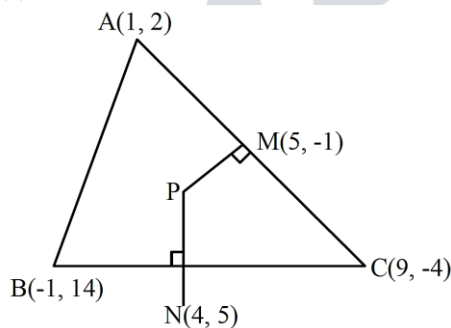
$$\Rightarrow 2a = 131$$

61. (a)



$$\frac{BC^2}{AC^2} = \left(\frac{OB}{OA}\right)^2 = \frac{9 + 81}{9 + 9} = 5$$

62. (c)



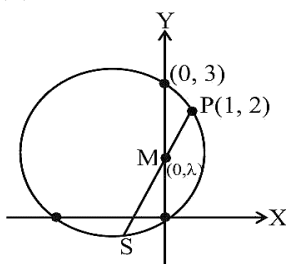
$$\text{equation of PM : } 4x - 3y = 23$$

$$\text{equation of PM : } 5x - 9y = -25$$

$$P\left(\frac{94}{7}, \frac{215}{21}\right)$$

$$21(\alpha + \beta) = 497$$

63. (b)



chord PS bisected by y-axis

at point M(0, 2)

$$\therefore S(-1, 2)$$

Which satisfy equation of circle

$$\Rightarrow (-1)^2 + (2\lambda - 2)^2 - 1 - 3(2\lambda - 2) = 0$$

$$\Rightarrow \lambda = 1 \text{ \& } \lambda = \frac{5}{2}$$

Therefore two such chord PS & PR will exist

$$\text{When } \lambda = 1 \Rightarrow S(-1, 0)$$

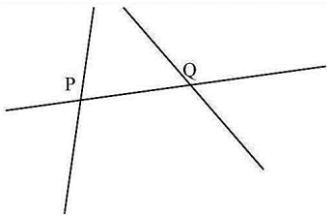
$$\text{When } \lambda = \frac{5}{2} \Rightarrow R(-1, 3)$$

$$\therefore \text{Midpoint of } R \text{ \& } S \text{ will be } \left(-1, \frac{3}{2}\right)$$

$$\therefore \alpha = -1, \beta = \frac{3}{2}$$

$$\therefore 6(\alpha + \beta) = 3$$

64. (b)



$$P(2\lambda, 3\lambda, 3\lambda - 1) \text{ and } Q(-\mu - 1, \mu + 2, 4\mu)$$

$$\text{D.R of PQ } 2\lambda + \mu + 1, 3\lambda - \mu - 2, 3\lambda - 4\mu - 1, \frac{2\lambda + \mu + 1}{1} = \frac{3\lambda - \mu - 2}{-1} = \frac{3\lambda - 4\mu - 1}{2}$$

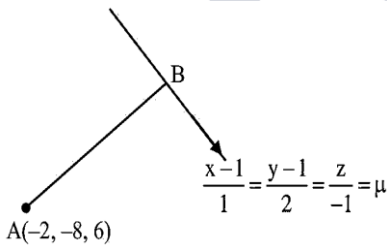
$$\text{Solving } \lambda = \frac{1}{5} \text{ and } \mu = \frac{-8}{15}$$

$$P\left(\frac{2}{5}, \frac{3}{5}, \frac{-2}{5}\right) \text{ \& } Q\left(\frac{-7}{15}, \frac{22}{15}, \frac{-32}{15}\right)$$

$$PQ^2 = \left(\frac{13}{15}\right)^2 + \left(\frac{13}{15}\right)^2 + \left(\frac{26}{15}\right)^2$$

$$\alpha^2 = \frac{1014}{225} \therefore 225\alpha^2 = 1014$$

65. (b)



$$\text{DR of AB is } 1, -1, 2$$

$$\therefore \text{Line AB is } \frac{x+2}{1} = \frac{y+8}{-1} = \frac{z-6}{2} = \lambda$$

$$B \text{ is } (\lambda - 2, -\lambda - 8, 2\lambda + 6)$$

$$B \text{ is } (\mu + 1, 2\mu + 1, -\mu)$$

$$\text{Solving point B is } (-3, -7, 4)$$

$$AB = \sqrt{(-1)^2 + (1)^2 + (2)^2} = \sqrt{6}$$

$$AB^2 = 6$$

66. (c)  $y = \tan^{-1}\left(\frac{\frac{3}{4}\tan x}{1 + \frac{3}{4}\tan x}\right) + 2\tan^{-1}\left(\frac{x}{1 + \sqrt{1-x^2}}\right)$

$$\text{For } 2\tan^{-1}\left(\frac{x}{1 + \sqrt{1-x^2}}\right)$$

$$\text{put } x = \sin\theta$$

$$\therefore 2\tan^{-1}\left(\frac{\sin\theta}{1 + \cos\theta}\right) = \theta = \sin^{-1}x$$

Hence

$$y = \tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}(\tan x) + \sin^{-1}x$$

$$\frac{dy}{dx} = -1 + \frac{1}{\sqrt{1-x^2}}$$

$$x = \frac{\sqrt{3}}{2}$$

$$\frac{dy}{dx} = -1 + 2 = 1$$

67. (c) Let degree of polynomial be 'n'

$$n = n - 1 + n - 2 \Rightarrow n = 3$$

$$f(x) = ax^3 + bx^2 + cx$$

$$(ax^3 + bx^2 + cx) = (3ax^2 + 2bx + c)(6ax + 2b)$$

$$x^3: a = 18a^2 \Rightarrow a = \frac{1}{18}$$

$$x^2: b = 6ab + 12ab$$

$$x: c = 4b^2 + 6ac \Rightarrow \frac{2c}{3} = 4b^2 \Rightarrow b^2 = \frac{c}{6}$$

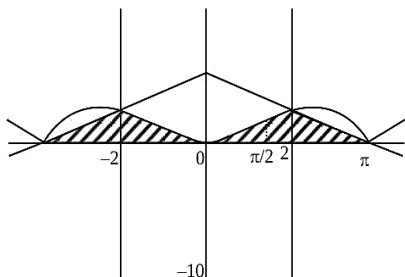
$$\text{const: } bc = 0 \Rightarrow b = c = 0$$

$$\int_0^2 f(x)dx + f'(2) + f''(2)$$

$$\int_0^2 ax^3dx + 12a + 12a = 4a + 24a = 28a = \frac{28}{18} = \frac{14}{9}$$

$$36\left(\frac{14}{9}\right) = 56$$

68. (b)



$$\text{Area; } 2\left(\int_0^{\pi/2} x \sin x dx + \int_{\pi/2}^{\pi} (\pi - x) dx\right)$$

$$= 2(-x \cos x + \sin x)_0^{\pi/2} + 2\left(\frac{(\pi - x)^2}{2}\right)_0^{\pi/2}$$

$$= 2 + \frac{\pi^2}{4}$$

69. (b)  $\int_{-2}^2 (|\sin x| + [x \sin x]) dx$

$$\Rightarrow 2 \int_0^2 (|\sin x| + [x \sin x]) dx$$

$$\Rightarrow 2 \left[ \int_0^2 \sin x dx + \int_0^2 [x \sin x] dx \right]$$

$$2 \left( [-\cos x]_0^2 + \int_0^\alpha 0 dx + \int_\alpha^2 2 dx \right)$$

$$2(3 - \cos 2 - 2 - \alpha)$$

$$2(3 - \cos 2) - 2\alpha$$

where  $\alpha$  is Root of  $x \sin x = 1$

Now comparing  $\beta = -2\alpha$

$$\alpha = -\frac{\beta}{2}$$

$$-\frac{\beta}{2} \sin\left(\frac{-\beta}{2}\right) = 1$$

$$\beta \sin\left(\frac{\beta}{2}\right) = 2$$

70. (c)  $\frac{dy}{y^2-y+1} = (x^2 + x + 1)dx$

$$\frac{dy}{\left(y - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = (x^2 + x + 1)dx$$

$$\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{y - \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \frac{x^3}{3} + \frac{x^2}{2} + x + C$$

$$\frac{2}{\sqrt{3}\sqrt{3}} = \tan^{-1}\left(\frac{2y-1}{\sqrt{3}}\right) = \frac{x^3}{3} + \frac{x^2}{2} + x + C$$

$$y(0) = \frac{1}{2} \Rightarrow 0 = 0 + 0 + 0 + C \Rightarrow C = 0$$

$$\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2y-1}{\sqrt{3}}\right) = \frac{x^3}{3} + \frac{x^2}{2} + x$$

put  $x = 1$

$$\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2y-1}{\sqrt{3}}\right) = \frac{1}{3} + \frac{1}{2} + 1 = \frac{2+3+6}{6} = \frac{11}{6}$$

$$\tan^{-1}\left(\frac{2y-1}{\sqrt{3}}\right) = \frac{11\sqrt{3}}{12}$$

$$\frac{2y-1}{\sqrt{3}} = \tan\left(\frac{11\sqrt{3}}{12}\right)$$

$$\Rightarrow 2y(1) - 1 = \sqrt{3} \tan\left(\frac{11\sqrt{3}}{12}\right)$$

71. (9)  $x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8$

Exactly 4 heads in first six Tosses.

$(x_1 x_2 x_3 x_4 x_5 x_6)$

Exactly 3 heads in last 5 Tosses.

i.e.  $(x_4 x_5 x_6 x_7 x_8)$

Common tosses  $(x_4 x_5 x_6)$

Let  $k$  be number of heads on  $(x_4 x_5 x_6)$

$k = 1 \therefore$  ways to overlap  $= {}^3C_1$

${}^3C_3 \times {}^3C_1 \times {}^2C_2 = 3$  ways

$k = 2$

${}^3C_2 \times {}^3C_2 \times {}^2C_1 = 18$

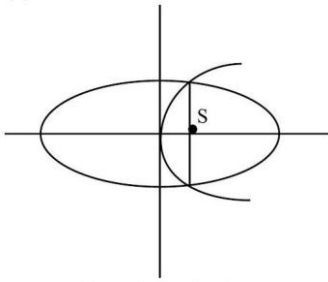
$k = 3$

${}^3C_1 \times 1 = 3$

Total ways  $24 \left(\frac{1}{2}\right)^8$

$$96 \times \frac{24}{28} = \frac{6 \times 24}{16} = 9$$

72. (3)



Focus of parabola ( k, 0 )

Focus of Ellipse (ae, 0)

$$k = ae \quad \dots(i)$$

$$\text{Also } 4k = 2 \frac{b^2}{a} \quad \dots(ii)$$

$$4ae = \frac{2b^2}{a}$$

$$\Rightarrow 4a^2e = 2b^2$$

$$4a^2e = 2a^2(1 - e^2)$$

$$2e = 1 - e^2$$

$$e^2 + 2e + 1 = 2$$

$$(e + 1)^2 = 2$$

$$e + 1 = \sqrt{2} \therefore e = \sqrt{2} - 1$$

$$e^2 + 2\sqrt{2} = 3$$

$$73. (2) \text{ Consider } E = \frac{\sin\theta}{\cos 3\theta} \Rightarrow E = \frac{2\sin\theta\cos\theta}{2\cos 3\theta\cos\theta}$$

$$\Rightarrow E = \frac{\sin 2\theta}{2\cos 3\theta\cos\theta} \Rightarrow E = \frac{\sin(3\theta - \theta)}{2\cos 3\theta\cos\theta}$$

$$\Rightarrow E = \frac{1}{2} [\tan 3\theta - \tan\theta]$$

$$A = \frac{1}{2} [\tan 9^\circ - \tan 3^\circ + \tan 27^\circ - \tan 9^\circ + \tan 81^\circ - \tan 27^\circ]$$

$$\therefore A = \frac{1}{2} [\tan 81^\circ - \tan 3^\circ]$$

$$\therefore \frac{B}{A} = 2$$

$$74. (3) |a_k|^2 = 1 + \tan^2 \theta_k = \sec^2 \theta_k$$

$$|b_k|^2 = 1 + \cot^2 \theta_k = \operatorname{cosec}^2 \theta_k$$

$$\text{Since, } \cot\theta - \tan\theta = 2\cot 2\theta$$

$$\therefore -\operatorname{cosec}^2 \theta - \sec^2 \theta = -4\operatorname{cosec}^2 2\theta$$

$$\sec^2 \theta = 4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta$$

$$\Sigma \sec^2 \theta_k = 4\Sigma \operatorname{cosec}^2 2\theta_k - \Sigma \operatorname{cosec}^2 \theta_k$$

Now

$$\Sigma \operatorname{cosec}^2 2\theta_k = \Sigma \operatorname{cosec}^2 \theta_k$$

since

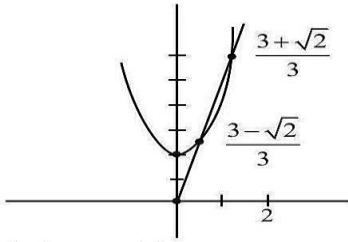
$$\Sigma_{k=1}^n \operatorname{cosec}^2 \frac{2^k \pi}{2^n + 1} = \Sigma_{k=1}^n \operatorname{cosec}^2 \frac{2^{k-1} \pi}{2^n + 1}$$

$$\text{because } \operatorname{cosec} \frac{2^n \pi}{2^n + 1} = \operatorname{cosec} \frac{\pi}{2^n + 1}$$

$$\therefore \Sigma \sec^2 \theta_k = 3\Sigma \operatorname{cosec}^2 \theta_k$$

$$\frac{\Sigma \sec^2 \theta_k}{\Sigma \operatorname{cosec}^2 \theta_k} = 3$$

$$75. (3) 2 + 3x^2 = 6x \Rightarrow 3x^2 - 6x + 2 = 0$$



$$x = \frac{6 \pm \sqrt{12}}{6} = \frac{3 \pm \sqrt{2}}{3}$$

$|x - 1| \cos \left| x^2 - \frac{1}{4} \right|$  is differentiable at  $x = 1$

Total number of points where  $f(x)$  is nonDifferentiable is 3

