1. **(a)** (I) $[h]=\frac{[E]}{[v]}=\frac{ML^2\,T^{-2}}{T^{-1}}=ML^2\,T^{-1}$

(II) Stopping potential : $E=q\cdot V_0$

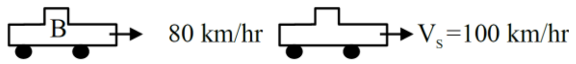
$$[V_0]=[v_0]=\frac{[E]}{[q]}=\frac{ML^2\,T^{-2}}{AT}=ML^2\,T^{-3}\,A^{-1}$$

(III) Work function $(\phi)\rightarrow$ Energy form

$[\phi]=ML^2\,T^{-2}$

(IV) Threshold freq. $[v_0]=T^{-1}$

2. **(c)**



$V=\vec{V}_{SB}$

$V=\vec{V}_S-\vec{V}_B$

$V_B+V=\vec{V}_S$

$\vec{V}_s=V+80$

$\vec{V}_{SA}=5\text{ m/s}=18\text{ km/hr}$

$\vec{V}_S-\vec{V}_A=\vec{V}_{SA}$

$V+80-100=18\text{ km/hr}$

$V-20=18$

$V=38\text{ km/hr}$

3. **(d)** $\vec{a}=\frac{\vec{F}}{m}=\frac{5\hat{i}+10\hat{j}}{0.1}=(50\hat{i}+100\hat{j})$

$$\vec{s}=\vec{u}t+\frac{1}{2}\vec{a}t^2$$

$$\vec{s}=0+\frac{1}{2}[50\hat{i}+100\hat{j}][2]^2$$

$\vec{s}=100\hat{i}+200\hat{j}=\vec{r}=2x\hat{i}+5y\hat{j}$

$2x=100\quad 5y=200$

$x=50\quad y=100$

$$\frac{x}{y}=\frac{5}{4}$$

4. **(a)** $x=24t\;y=43.6t-4.9t^2$

$$v_x=\frac{dx}{dt}=24$$

$$v_y=\frac{dy}{dt}=43.6-9.8t$$

$$\tan\theta=\frac{v_y}{v_x}=\frac{43.6-9.8t}{24}$$

At $t=2\,\frac{43.6-19.6}{24}$

$\tan\theta=1$

$\theta=45^\circ$

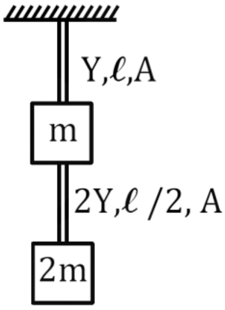
5. **(c)** $g_h=\frac{g}{\left(1+\frac{h}{R}\right)^2}$

$$\frac{g}{9}=\frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$1+\frac{h}{R}=3$$

$h=2R$

6. **(d)**



$\Delta \ell = \frac{T\ell}{YA}$

$\Delta \ell_1 = \frac{3mg\ell}{YA}$

$\Delta \ell_2 = \frac{2mg(\ell/2)}{2YA} = \frac{mg\ell}{2YA}$

$\frac{\Delta \ell_1}{\Delta \ell_2} = 6$

7. **(b)** Using equation of continuity

AV = 10av

$v = \frac{AV}{10a} = \frac{30 \times 50}{10 \times .15} = 1000 \text{ cm/s} = 10 \text{ m/s}$

8. **(c)** Avg. kinetic energy of molecule will be $\frac{f}{2}$KT

But $v_{rms} = \sqrt{\frac{3RT}{M}}$

Molar mass m different so v_{rms} is different

9. **(d)** $\Delta L_1 = L_0 \alpha (T_2 - T_1)$

So final length at $T_2 = L_0 + \Delta L_1$

$\Delta L_2 = (L_0 + \Delta L_1)\alpha(T_3 - T_2)$ and $T_3 - T_2 = T_2 - T_1 = \Delta T$

$\Delta L_2 = L_0 \alpha \Delta T + \Delta L_1 \alpha \Delta T$

$= \Delta L_1 + \Delta L_1 \alpha \Delta T$

$= \Delta L_1 (1 + \alpha \Delta T)$

10. **(a)** $I_{disc} = \frac{mR^2}{4} + mR^2 = \frac{5mR^2}{4}$

∴ For small displacement

$\tau = -mgR\theta$

$\therefore \alpha = -\frac{mgR}{\frac{5mR^2}{4}}\theta$

$\frac{4g}{5R}$

$\therefore \omega = \sqrt{\frac{4g}{5R}}$

$\therefore T = 2\pi \sqrt{\frac{5R}{4g}}$

11. **(a)** Initial freq. $f_1 = \frac{1}{2\pi}\sqrt{\frac{P(E_0)}{I}}$

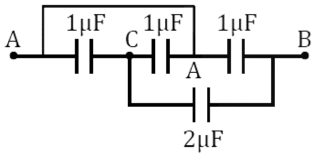
Final $EF = E_0\hat{\imath} + 2E_0(\hat{\jmath} + \hat{k})$

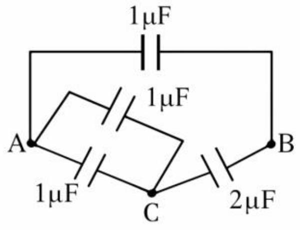
Magnitude $= 3E_0$

Final Freq. $f_2 = \frac{1}{2\pi}\sqrt{\frac{P(3E_0)}{I}}$

% change $= \frac{\Delta f}{f_1} = (\sqrt{3} - 1) \times 180 \simeq 73$

12. (b)





$C_{AB} = 2\mu F$

13. (b)

14. (b) $H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r} = \frac{1}{400 \times 4\pi \times 10^{-7}}$

$H = \frac{1}{16\pi} \times 10^5$

15. (a) $\hat{E} \times \hat{B} = \hat{C}$

$\hat{B} = \hat{j}$

$\hat{C} = \hat{i}$

$\therefore \hat{E} = -\hat{k}$

$\frac{E_0}{B_0} = C = v\lambda$

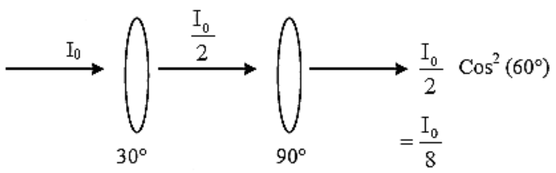
$E_0 = B \times v\lambda$

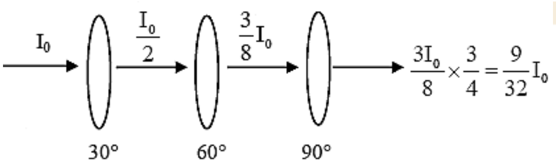
16. (d) $\frac{1}{f} = (1.4 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$

$\frac{1}{f} = \frac{0.8}{R}$

$\frac{f}{R} = 1.25$

17. (c)





$\frac{9I_0}{32} \times \frac{8}{I_0} = \frac{9}{4}$

18. (d)

19. (a) $\lambda = \frac{h}{\sqrt{2mqV}} \propto \frac{1}{\sqrt{m}}$

So $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$

$\sqrt{\frac{m_e}{m_p}}$

20. (c)

21. (300) Maximum current from diode

$$i=\frac{P}{V}=\frac{0.5}{10}=0.05\ \mathrm{A}$$

Now let resistance is R

$V_B=i(R+R_{\text{diode}})$

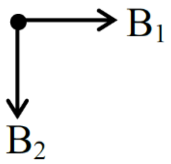
$25=0.05\times R+10$

$$R=\frac{15}{0.05}=300\Omega$$

22. (8) $R_{\max}=\frac{v^2}{g}=6.4$

$\mathbf{v}=\sqrt{64}=8\ \mathrm{m/s}$

23. (12) $B_{net}=\sqrt{B_1^2+B_2^2}=\frac{\mu_0}{4\pi}\frac{M}{(r)^3}\sqrt{5}$



$$=10^{-7}\times 3\sqrt{5}\times\sqrt{5}\times\frac{8}{10^{-3}}$$

$=120\times 10^{-4}=12\mathrm{mT}$

24. (314)

Sol. $\tau=|\vec{M}\times\vec{B}|$

$=|NI\vec{A}\times\vec{B}|$

$$\tau=125\times 1\times\pi\left(\frac{2}{100}\right)^2\times 0.4\times\sin 30^\circ$$

$=100\pi\times 10^{-4}\ \mathrm{N-m}$

So $\alpha=100\pi=314$

25. (5625) $\Delta y=(\mu-1)t\frac{D}{d}=7\frac{\lambda D}{d}$

$(\mu-1)t=7\lambda$

$(1.56-1)t=7\times 450\ \mathrm{m}$

$t=5625\ \mathrm{nm}$

26. (c) (A) Mole of atoms in C_6H_{12}

$=2\times 18$

$=36$

(B) Moles of atoms of sucrose $=\frac{604}{342}\times 45$

(C) $n_{H_2}=\frac{90.8}{22.7}=4$ moles

Mole fo atoms $=4\times 2=8$

(B) > (A) > (C)

27. (b) (A) → same

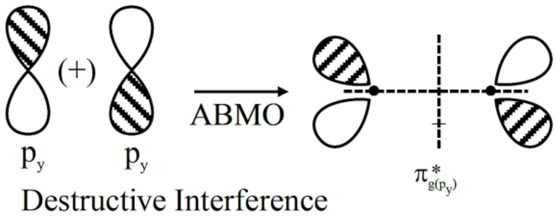
(B) → same

(C) → same

(D) → $r=\frac{90\times(1)^2}{3}=\frac{90}{3}$

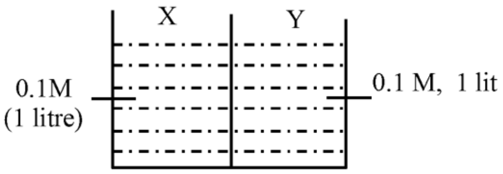
(E) → same

28. (c)



Destructive Interference

29. (c)



30. (b) In experiment 1, 28.4 ml of NaOH is required for complete neutralization, therefore for 14.2 ml , half equivalence point will be achieved to form acidic buffer.

31. (c)

32. (c) Since successive ionization energy increases thus $IE_2(Mg) > IE_1(Mg)$

33. (c) EN of ' $T\ell$ ' is 1.8 that is same as 'Ge'

' E ' $= T\ell$

$T\ell^{+3}$ acts as oxidizing agent

Stability order : $T\ell^{+3} < T\ell^{+1}$ (due to inert pair effect)

The standard electrode potential value $E^\circ(T\ell^{3+}/T\ell)$

$= +1.26$ V

34. (d) Gd $-$ $[Xe]4f^7 5\,d^1 6\,s^2$

Eu $-$ $[Xe]4f^7 6\,s^2$

Dy $-$ $[Xe]4f^{10} 6\,s^2$

Ho $-$ $[Xe]4f^{11} 6\,s^2$

Hf $-$ $[Xe]4f^{14} 6\,s^2 5\,d^2$

Yb $-$ $[Xe]4f^{14} 6\,s^2$

Lu $-$ $[Xe]4f^{14} 5\,d^1 6\,s^2$

Tm $-$ $[Xe]4f^{13} 6\,s^2$

35. (a) (A) $Ni^{2+} \Rightarrow 3\,d^8$, SFL, $t_{2\,g}^{2,2,2} e_g^{1,1} \Rightarrow$ unpaired $e^- = 2$

(B) $Ni^{2+} \Rightarrow 3\,d^8$, WFL , tetrahedral, $e^{2,2} t_2^{2,1,1}$

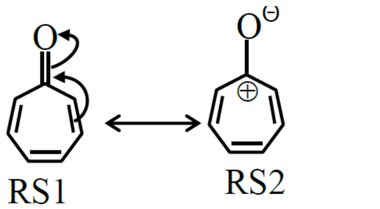
$\Rightarrow$ unpaired $e^- = 2$

(C) $Ni^{2+} \Rightarrow 3\,d^8$, SFL , $t_{2\,g}^{2,2,2} e_g^{1,1} \Rightarrow$ unpaired $e^- = 2$

As $\Delta_o > \Delta_t$ and Ligand strength : en $> NH_3 > Cl^-$

Strength of ligand increases, Δ increases, hence more is the frequency required of absorption.

36. (d)



Quasi aromatic

$C-O$ bond length is maximum in compound (b) because RS2 of this compound has aromatic character.

37. (c) Oxide of Mn in its highest O.S. $= Mn_2O_7[Mn^{+7}]$

Fluoride of Mn in its highest O.S. $= MnF_4[Mn^{+4}]$

$\Rightarrow$ Difference in O.S. $= |x| = 3$

$Sc^{+3} \Rightarrow [Ar]$; 0 unpaired e^-

$Zn^{2+} \Rightarrow [Ar]3\,d^{10}$; 0 unpaired electrons

$Fe^{+2} \Rightarrow [Ar]3\,d^6$; 4 unpaired electrons

$Co^{+2} \Rightarrow [Ar]3\,d^7$; 3 unpaired electrons

$V^{+2} \Rightarrow [Ar]3\,d^3$; 3 unpaired electrons

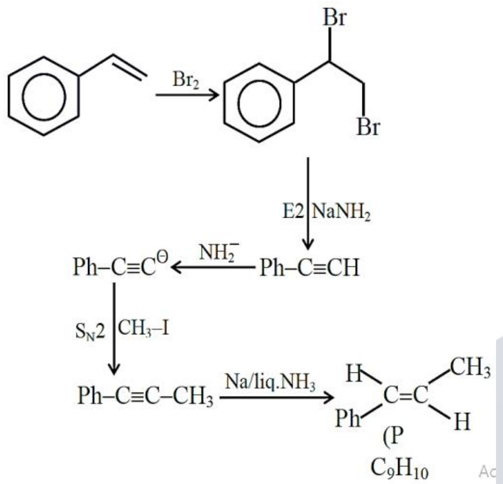
38. (b) $\Rightarrow$ Curve I^{st} represent zero order kinetics

$[R_t] = [R]_0 - Kt$

$\Rightarrow K_3 > K_2$

$\Rightarrow$ Reaction-1 is zero order having unit of rate constant same as rate of reaction i.e., molarity / second.

39. (b)

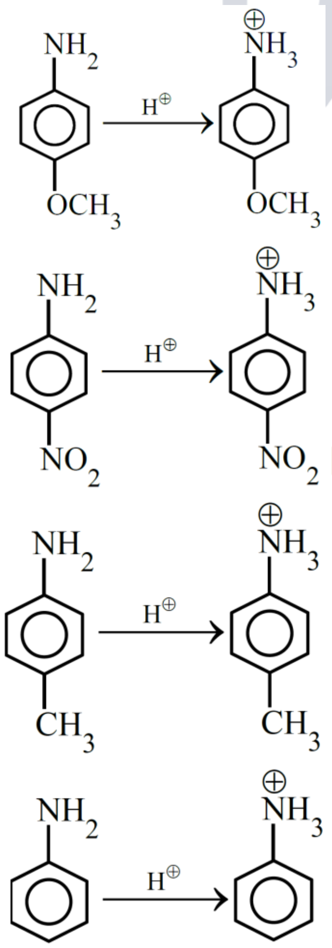


Molar mass of Y = 118gm

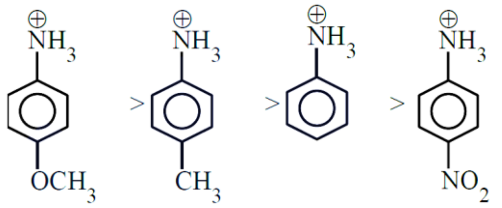
40. (b) No chiral center (2 methyl groups)

41. (d) Benzoic acid is more acidic than phenol. Electron withdrawing group in increases acidicity.

42. (c)

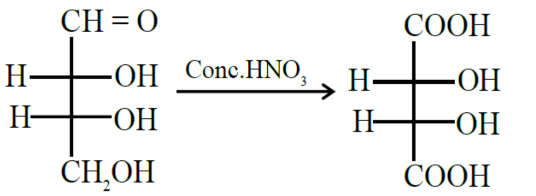


Stability order of conjugated acid :



For stronger conjugate acid, we need weaker base.

43. (c)



D-Aldotetrose

Optically inactive
Dicarboxylic acid

44. (d) Group IV reagent : $NH_4OH + H_2\ S$

Mn^{+2} will be precipitated out and hence highest possible oxidation state of Mn is +7

45. (a) Tolle's test is used for test for aldehydes.

Heinsberg's reagent is used to test for amines.

Lucas test is used to test for alcohols.

Phthalein dye test is used for test of phenols.

46. (282) $\Delta U^{o}_{\text{combustion}} = \frac{-q}{\text{moles}} = \frac{-99.472}{3.365/46} \Rightarrow 1359.8\text{KJ/mole}$

$C_2H_5OH(\ell) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(\ell); \Delta H^{o}_{r} = \Delta H^{o}_{C}$

$\Delta H^{o}_{C} = \Delta U^{o} + \Delta n_g RT$

$= -1359.8 + \frac{(-1) \times 8.314 \times 298.15}{1000}$

$= -1362.27\text{KJ/mole}$

$-1362.27 = [2 \times (-393.5) + 3 \times (-285.8)] -$

$\Delta H^{o}_{fC_2H_5OH(\ell)}$

$-1362.27 = -787 - 857.4 - \Delta H^{o}_{f}[C_2H_5OH(\ell)]$

$\Delta H^{o}_{f\,[C_2H_5OH(\ell)]} = -2.82 \times 10^2\text{KJ/mole}$

$\simeq -3 \times 10^2$

47. (2474) $2\ N_2O_5(q) \rightleftharpoons 2\ N_2O_4(g) + O_2(g)$

$t = 0$	1	–	–
$t_{\text{eqm.}}$	$1 - 2x$	$2\ x$	x
	$1 - 0.5$	0.5	0.25
	0.5	0.5	0.25
	$\frac{0.5}{1.25} \times 2$	$\frac{0.5}{1.25} \times 2$	$\frac{0.25}{1.25} \times 2$

$k_p = \frac{P^2_{N_2O_4} \cdot P_{O_2}}{P^2_{N_2O_5}} = 0.4$

$\Delta G^{\circ} = -2.303RT\log\ K_p$

$= -2.303 \times 8.314 \times 323\log(4/10)$

$= 2473.81\ \text{J/mole}$

48. (2) $E_{\text{cell}} = E^{\circ}_{\text{cell}} - \frac{0.059}{3x}\log 10^{-2}$

$0.2057 = 0.186 - \frac{0.059}{3x}(-2)$

$3x = \frac{0.059 \times 2}{0.0197}$

$x = 1.99 \simeq 2$

49. (7) $K = \frac{1}{t}\ln\left[\frac{A_0}{A_t}\right]$

$K = \frac{1}{20}\ln\left[\frac{0.6500}{0.00065}\right] \Rightarrow \frac{3\ln 10}{20}\ \text{min}^{-1}$

Also, $x = \frac{1}{K}\ln\left[\frac{A_0}{A_x}\right] \Rightarrow \frac{20}{3\ln 10}\ln\left(\frac{0.6500}{0.0650}\right)$

$= \frac{20}{3}\ \text{min} \Rightarrow 6.7\ \text{min}$

50. (435) % of S $= \frac{\text{Mass of S}}{\text{Mass of BaSO}_4} \times \frac{m}{w} \times 100$

m = gm of $BaSO_4$ ppt

w = gm of Organic compound

$w = 2 \times 10^{-3} \times 76 = 0.152\text{gm}$

% of S $= \frac{32}{233} \times \frac{0.4813}{0.152} \times 100 = 43.487$

In form of 10^{-1}

$= 43.487 \times 10^{-1} = 434.87 \approx 435$

51. (a) $f(x) = (x-1)^4 + 1$

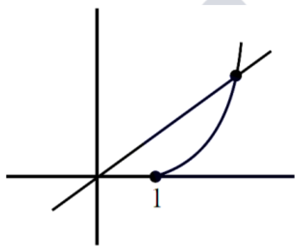
$f'(x) = 4(x-1)^3; f'(x) \geq 0$

$\therefore$ f(x) is increasing

$\therefore (x-1)^4 + 1 = x$

$(x-1)[(x-1)^3 - 1] = 0$

x = 1, x = 2 are two solutions



Now

$f^{-1}(x) = (x-1)^{1/4} + 1$

$f^{-1}(x+1) = x^{1/4} + 1$

$f^{-1}(x+1) = f(x)$

$x^{1/4} + 1 = (x-1)^4 + 1$

$x = (x-1)^{16}$

Above equation has one solution using graph.

52. (d) $z^2 + 4z + 16 = 0$

$\Rightarrow (z+2)^2 = -12$

$\Rightarrow z = -2 \pm 2\sqrt{3}i$

$\Rightarrow S = \{-2 + 2\sqrt{3}i, -2 - 2\sqrt{3}i\}$

$\therefore \sum_{z \in S} |z + \sqrt{3}i|^2$

$= |-2 + 2\sqrt{3}i + \sqrt{3}i|^2 + |-2 - 2\sqrt{3}i + \sqrt{3}i|^2$

$= |-2 + 3\sqrt{3}i|^2 + |-2 - \sqrt{3}i|^2$

$= 38$

53. (b) $D = \lambda - 5$

$D_1 = \lambda + \mu - 18$

$D_2 = 4\lambda - 2\mu + 6$

$D_3 = \mu - 13$

Here $x = \frac{D_1}{D}; y = \frac{D_2}{D}; z = \frac{D_3}{D}$

For infinitely many solutions

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow \lambda = 5; \mu = 13$

$\therefore \lambda + \mu = 18$

54. (c) Roots are $1 \pm i\sqrt{2}$ & 1

$-a =$ sum of roots

$a = -3$

$-c = 1(1+\sqrt{2}i)(1-i\sqrt{2})$

$c = -3$

$I = \int_{-1}^{1}(x^3 - 3x^2 + bx - 3)dx$

$= 2\int_0^1(-3x^2 - 3)dx$

$= 2(-x^3 - 3x)_0^1 = -8$

55. (b) $f(x) = (\lambda+2)x^2 - 3\lambda x + 4\lambda$

C-1 $af(0) > 0$

$(\lambda+2)4\lambda > 0$

$\Rightarrow \lambda < -2$ or $\lambda > 0$

C-2 $-\frac{b}{2a} > 0$

$\frac{3\lambda}{2(\lambda+2)} > 0$

$\Rightarrow \lambda < -2$ or $\lambda > 0$

C-3 $D \geq 0$

$(-3\lambda)^2 - 4(\lambda+2) \times 4\lambda \geq 0$

$\lambda(7\lambda + 32) \leq 0$

$\Rightarrow \lambda \in \left[\frac{-32}{7}, 0\right]$

Intersection of $C-1, C-2$ and $C-3$

$\Rightarrow \lambda \in \left[\frac{-32}{7}, -2\right)$

$\lambda \in [-4.57, -2)$

$\Rightarrow \lambda = -4, -3$

$\therefore$ Number of values of $\lambda = 2$

56. (c) $\begin{vmatrix} 1-\alpha & 2 & 7 \\ 4 & -2-\alpha & 8 \\ 3 & 8 & -7-\alpha \end{vmatrix} = 0$

$\Rightarrow (1-\alpha)[(\alpha+2)(\alpha+7) - 64] - 2[-28 - 4\alpha - 24] + 7[32 + 6 + 3\alpha] = 0$

$\Rightarrow \alpha^3 + 8\alpha^2 - 88\alpha - 320 = 0$

$(\alpha - 8)(\alpha^2 + 16\alpha + 40) = 0$

$\alpha = 8, \alpha = -8 \pm 2\sqrt{6}$

So $p = -8$

$\Rightarrow (x-8)^2 + (y-16)^2 = 320 = 8^2 + 16^2$

put $y = 0 \Rightarrow x = 16, 0$

put $x = 0 \Rightarrow y = 32, 0$

$\Rightarrow (16,0), (0,32), (0,0)$

$\Rightarrow$ 3 points of intersection

57. (c) $\alpha = \frac{1/4}{1-\frac{1}{2}} = \frac{1}{2}$

$$\beta = \frac{1/3}{1-\frac{1}{3}} = \frac{1}{2}$$

$$(0.2)^{\log_{\sqrt{5}}\alpha} = \alpha^{\log_{\sqrt{5}}\frac{1}{5}} = \left(\frac{1}{2}\right)^{-2} = 4$$

$$(0.04)^{\log_5\beta} = 5^{-2\log_5\beta} = \left(\frac{1}{2}\right)^{-2} = 4$$

$\Rightarrow 4 + 4 = 8$

58. (d) $\sum_{i=1}^{10}(x_i + 2)^2 = 180$

$\sum_{i=1}^{10} x_i^2 + 4\sum_{i=1}^{10} x_i + \sum_{i=1}^{10} 4 = 180$

$\sum_{i=1}^{10} x_i^2 + 4\sum_{i=1}^{10} x_i = 180 - 40 \quad \ldots.(1)$

Also $\sum_{i=1}^{10} (x_i - 1)^2 = 90$

$$\sum_{i=1}^{10} x_i^2 - 2\sum_{i=1}^{10} x_i + \sum_{i=1}^{10} 1 = 90$$

$\sum_{i=1}^{10} x_i^2 - 2\sum_{i=1}^{10} x_i = 90 - 10 \quad \ldots..(2)$

$$\sum_{i=1}^{10} x_i^2 + 4\sum_{i=1}^{10} x_i = 140$$

$$\sum_{i=1}^{10} x_i^2 - 2\sum_{i=1}^{10} x_i = 80$$

$$\sum_{i=1}^{10} x_i^2 = 100 \text{ and } \sum_{i=1}^{10} x_i = 10$$

$$\sigma^2 = \frac{\sum_{i=1}^{10} x_i^2}{\mathrm{N}} - \left(\frac{\sum_{i=1}^{10} x_i}{N}\right)^2$$

$$= \frac{100}{10} - \left(\frac{10}{10}\right)^2$$

$\sigma^2 = 10 - 1 = 9$

$\Rightarrow \sigma = 3$

59. (a) General term in above expression is

$$T_{r+1} = {}^{18}C_r(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$$

$$= \left(-\frac{1}{3}\right)^r {}^{18}C_r 9^{18-r} \cdot x^{18-\frac{3r}{2}}$$

For term independent of x, $18 - \frac{3r}{2} = 0 \Rightarrow r = 12$

$\therefore$ Coefficient of term independent of x

$$= \left(-\frac{1}{3}\right)^{12} {}^{18}C_{12} 9^{18-12}$$

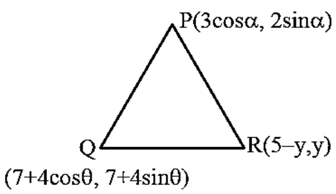
$= 18564$

$= 221k$ (given)

$\Rightarrow k = 84$

60. (d) Centroid :

$$\left(\frac{3\cos\alpha + 7 + 4\cos\theta + 5 - y}{3}, \frac{2\sin\alpha + 7 + 4\sin\theta + y}{3}\right) = \left(\cos\alpha + 2, \frac{2}{3}\sin\alpha + 3\right)$$



On comparison

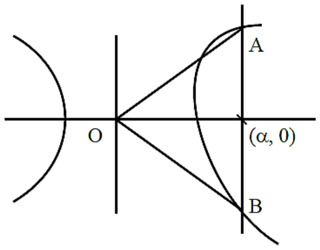
$\cos\alpha = \frac{y-6}{4}$ & $\sin\alpha = \frac{2-y}{4}$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow (y-6)^2 + (y-2)^2 = 16$

$y^2 - 8y + 12 = 0$

$\Rightarrow$ Sum of ordinates $= 8$

61. (b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Given:

$2ae = 6 \Rightarrow ae = 3$

$\frac{2a}{e} = \frac{8}{3} \Rightarrow \frac{a}{e} = \frac{4}{3}$

$a^2 = 4$ $e = \frac{3}{2}$ $b^2 = 5$

$\frac{x^2}{4} - \frac{y^2}{5} = 1$ solve with $x = \alpha$

$y = \pm\sqrt{\frac{5(\alpha^2-4)}{4}}$

$A\left(\alpha, \sqrt{\frac{5(\alpha^2-4)}{4}}\right)$ $B\left(\alpha, -\sqrt{\frac{5(\alpha^2-4)}{4}}\right)$

Ar. of $\Delta OAB = \frac{1}{2}|\alpha|\sqrt{5(\alpha^2-4)} = 4\sqrt{15}$

$\Rightarrow 16 \times 15 = \frac{1}{4}\alpha^2(5)(\alpha^2-4)$

$\Rightarrow \alpha^2 = 16$

62. (b) $E = 16\sin\frac{x}{2}\cos^3\frac{x}{2}$

$E = 4\sin x[1 + \cos x]$

$\frac{dE}{dx} = 4[\cos x + \cos 2x]$

$= 8\cos\frac{3x}{2}\cos\frac{x}{2} = 0$

$\Rightarrow \cos\frac{3x}{2} = 0$ or $\cos\frac{x}{2} = 0$

$\Rightarrow x = \left\{\frac{\pi}{3}, \pi\right\}$ are critical points of the function

$E(0) = 0$

$E(\pi) = 0$

$E\left(\frac{\pi}{3}\right) = 3\sqrt{3}$

$\therefore$ maximum value of $E = 3\sqrt{3}$

63.(b) $\vec{a}_1 = \frac{1}{3}\hat{\imath} + 2\hat{\jmath} + \frac{8}{3}\hat{k}$

$\vec{a}_2 = \frac{-2}{3}\hat{\imath} - \frac{1}{3}\hat{k}$

$\vec{b}_1 = 2\hat{\imath} - 5\hat{\jmath} + 6\hat{k}, \vec{b}_2 = \hat{\jmath} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -5 & 6 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$$

$\vec{a}_1 - \vec{a}_2 = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

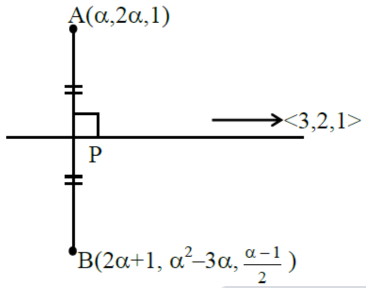
$$\text{S.D} = \left| \frac{(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\text{S.D} = \left| \frac{(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) \cdot (-\hat{\imath} + 2\hat{\jmath} + 2\hat{k})}{3} \right|$$

$= \frac{9}{3} = 3$

64. (a) P is mid point of AB and lies on the given line

$\Rightarrow \text{P}\left(\frac{3\alpha+1}{2}, \frac{\alpha^2-\alpha}{2}, \frac{\alpha+1}{4}\right)$



Now put coordinates of P in the line

$$\Rightarrow \frac{\frac{3\alpha+1}{2} - 2}{3} = \frac{\frac{\alpha^2-\alpha}{2} - 1}{2} = \frac{\alpha+1}{4}$$

$$\Rightarrow \frac{\alpha-1}{2} = \frac{\alpha+1}{4} = \frac{\alpha^2-\alpha-2}{4}$$

$\Rightarrow \alpha = 3$

Now for $\alpha = 3$, clearly AB is perpendicular to $3\hat{\imath} + 2\hat{\jmath} + k$

for $\alpha = 3$ point B is image of point A

65. (a) $|\hat{u} \times \hat{v}| = \frac{\sqrt{3}}{2}$

$|\hat{u}||\hat{v}|\sin\theta = \frac{\sqrt{3}}{2} \therefore \theta = \frac{\pi}{3}$

and $\hat{u} \cdot \hat{v} = |\hat{u}||\hat{v}|\cos\frac{\pi}{3} = \frac{1}{2}$

$\vec{A} = \lambda\hat{u} + \hat{v} + \hat{u} \times \hat{v}$ …(1)

Dot with $\hat{u}$

$\vec{A} \cdot \hat{u} = \lambda(1) + \hat{u}.\hat{v} + \hat{u} \cdot (\hat{u} \times \hat{v})$

$\vec{A} \cdot \hat{u} = \lambda + \frac{1}{2}$

$\Rightarrow 2\vec{A} \cdot \hat{u} = 2\lambda + 1$ …(2)

Dot equation (1) with $\hat{v}$

$\vec{A} \cdot \hat{v} = \lambda(\hat{u} \cdot \hat{v}) + \hat{v} \cdot \hat{v} + \hat{v} \cdot (\hat{u} \times \hat{v})$

$\vec{A} \cdot \hat{v} = \frac{\lambda}{2} + 1$

$\vec{A} \cdot \hat{v} - \frac{\lambda}{2} = 1$ …(3)

From (2) and (3)

$2\vec{A} \cdot \hat{u} - 2\lambda = \vec{A} \cdot \hat{v} - \frac{\lambda}{2}$

$\therefore \lambda = \frac{4}{3}\vec{A} \cdot \hat{u} - \frac{2}{3}\vec{A} \cdot \hat{v}$

66. (b) $f(x+y) = f(x) + 2y^2 + y + \alpha xy$ …..(1)

Put $x = 0$ in eq.(1)

$\Rightarrow f(y) = -1 + 2y^2 + y$

Now put $x = y = 1$ in eq. (1)

$\Rightarrow f(2) = f(1) + 3 + \alpha$

$\Rightarrow 9 = 2 + 3 + \alpha \Rightarrow \alpha = 4$

Now $\sum_{n=1}^{5}(\alpha + f(n)) = \sum_{n=1}^{5}(2y^2 + y + 3)$

$= \dfrac{2 \times 5 \times 6 \times 11}{6} + \dfrac{5 \times 6}{2} + 3 \times 5 = 140$

67. (c) $c = 0 \Rightarrow a + b = 22 \rightarrow {}^{22+2-1}C_{2-1} = {}^{23}C_1$

$c = 1 \Rightarrow a + b = 20 \rightarrow {}^{20+2-1}C_{2-1} = {}^{21}C_1$

$\vdots \quad \vdots \quad \vdots$

$c = 10 \Rightarrow a + b = 2 \rightarrow {}^{2+2-1}C_{2-1} = {}^{3}C_1$

$c = 11 \Rightarrow a + b = 0 \rightarrow {}^{0+2-1}C_{2-1} = {}^{1}C_1$

Number of non-negative integral solutions

${}^{1}C_1 + {}^{3}C_1 + {}^{5}C_1 + \cdots + {}^{23}C_1$

$1 + 3 + 5 \ldots\ldots + 23$

$= 144$

68. (c) $x = -3y^2$ and $x = 1 - 4y^2$

$A = \int_{-1}^{1}\left((1 - 4y^2) + 3y^2\right)dy$

$A = \int_{-1}^{1}(1 - y^2)dy = 2\int_{0}^{1}(1 - y^2)dy$

$A = \dfrac{4}{3}$

69. (d) I.F. $= e^{\int \frac{6x^2 + e^{-2x}(3x^2 + 2x^3 + 4)}{(x^3+2)(2+e^{-2x})}dx}$

$= e^{\int \frac{6x^2 + e^{-2x}(3x^2 - 2x^3 - 4) + (4x^3 + 8)e^{-2x}}{(x^3+2)(2+e^{-2x})}dx}$

Let $(x^3 + 2)(2 + e^{-2x}) = t$

$\left(6x^2 + e^{-2x}(3x^2 - 2x^3 - 4)\right)dx = dt$

I.F. $= e^{\ln(x^3+2)(2+e^{-2x}) - \int \frac{4e^{-2x}dx}{2+e^{-2x}}}$

$= e^{\ell n|(x^3+2)(2+e^{-2x})| - 2\ell n|2+e^{-2x}|}$

$= e^{\ln\left|\frac{x^3+2}{2+e^{-2x}}\right|} = \dfrac{x^3 + 2}{2 + e^{-2x}}$

$\Rightarrow y \cdot \left(\dfrac{x^3 + 2}{2 + e^{-2x}}\right) = \int \left(\dfrac{x^3 + 2}{2 + e^{-2x}}\right)(e^{-2x} + 2)dx + C$

$y \cdot \left(\frac{x^3+2}{2+e^{-2x}}\right) = \frac{x^4}{4} + 2x + C$ at $y(0) = \frac{3}{2} \Rightarrow C = 1$

So $y \cdot \left(\frac{x^3+2}{2+e^{-2x}}\right) = \frac{x^4}{4} + 2x + 1$

at $x = 1$

$y \cdot \left(\dfrac{3}{2 + e^{-2}}\right) = \dfrac{1}{4} + 2 + 1$

$\Rightarrow y = \dfrac{13}{12}(e^{-2} + 2)$

So $\alpha = \frac{13}{12}$

70. (d) $= \int_0^1 \tan^{-1}\left(\frac{1}{1+x+x^2}\right)dx$

$= \displaystyle\int_0^1 \tan^{-1}\left(\frac{(x + 1) - x}{1 + x(x + 1)}\right)dx$

$= \displaystyle\int_0^1 (\tan^{-1}(x + 1) - \tan^{-1}x)dx$

$$= \left(x\tan^{-1}(x+1) - \frac{1}{2}\ln|1+(1+x)^2| + \tan^{-1}(x+1)\right)_0^1$$
$$-\left(x\tan^{-1}x - \frac{1}{2}\ln|1+x^2|\right)_0^1$$
$$= \left(2\tan^{-1}2 - \frac{\pi}{4} - \frac{1}{2}\ln\frac{5}{2}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\ln 2\right)$$
$$= 2\tan^{-1}2 - \frac{1}{2}\ln\left(\frac{5}{4}\right) - \frac{\pi}{2}$$

71. (944) Total numbers of ways of selecting

3 numbers $= {}^{31}C_3 = \frac{31\times30\times29}{3\times2\times1} = 4495$

a, b, c are in A.P.

$\Rightarrow$ either a&c both odd numbers

or both even numbers

$= {}^{16}C_2 + {}^{15}C_2$

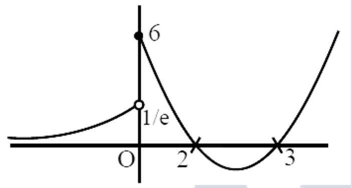
$= 120 + 105$

$= 225$

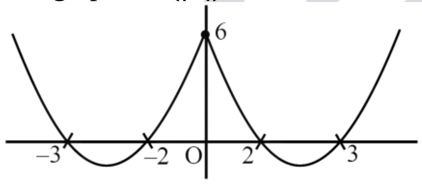
Probability $= \frac{225}{4495} = \frac{45}{899}$

$= \frac{a}{b} \Rightarrow a + b = 944$

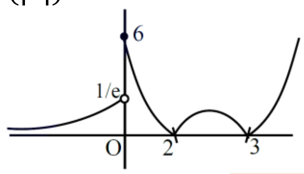
72. (4) Graph of f(x)



Now graph of f(|x|)



f(|x|) is continuous function and it is non diff. at $x = 0$ Graph of |f(x)|



|f(x)| is discontinuous at $x = 0$

|f(x)| is non-diff. at $x = 0,2,3$

$g(x) = f(|x|) + |f(x)|$

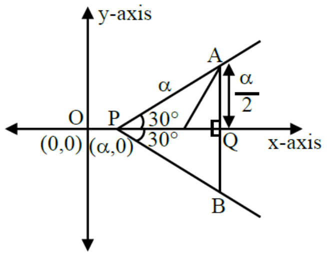
g(x) will be discontinuous at $x = 0$

g(x) will be non diff at $x = 0,2,3$

$\alpha = 1, \beta = 3$

$\alpha + \beta = 1 + 3 = 4$

73. (9) $\triangle$ PAB is equilateral



$$PQ = \frac{\sqrt{3}\alpha}{2} = \frac{9}{2} \therefore \alpha = \frac{9}{\sqrt{3}}$$

$$\alpha = 3\sqrt{3}$$

$$\frac{\alpha}{2r} = \cos 30^\circ$$

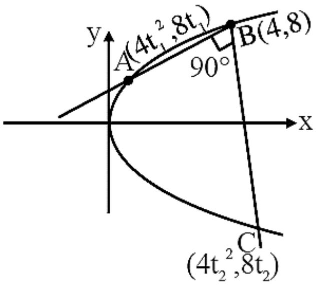
$$\alpha = \frac{\sqrt{3}}{2} \cdot 2R$$

$$\alpha = \sqrt{3}R$$

$$R = 3$$

$$\frac{\alpha^2}{R} = \frac{(3\sqrt{3})^2}{3} = 9$$

74. (16)



Variable point A & C are shown in the figure

$\because m_{AB} \cdot m_{BC} = -1$

$\Rightarrow t_1 + t_2 + t_1 t_2 = -5 \ldots (1)$

Suppose locus of centroid of $\triangle$ ABC is (h, k)

$\therefore 3h = 4 + 4t_1{}^2 + 4t_2{}^2 \,\&\, 3k = 8 + 8t_1 + 8t_2$

by eliminating $t_1 \& t_2$ using equation (1) also

we get $h = \frac{9}{48}k^2 + \frac{40}{3}$

$\therefore$ locus of centroid of parabola is

$$x = \frac{9}{48}y^2 + \frac{40}{3}$$

$\therefore\ \ell(\text{ L.R. }) = \frac{48}{9}$

$\therefore 3\ell(\text{ L.R. }) = \frac{48}{3} = 16$

75. (5) $f(x) = \int_0^x tan(x - t - x)dt - \int_0^x f(t)tantdt$

$$f(x) = -\int_0^x tantdt - \int_0^x f(t)tantdt$$

$$f'(x) = -tanx - f(x)tanx$$

$$\frac{dy}{y+1} = -tan\,xdx$$

$$y + 1 = cosx.c$$

$$f(0) = 0 \Rightarrow 1 = c$$

$$y = cosx - 1 = f(x)$$

$$f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - 1$$

$f'(x) = -sinx$

$f'\left(\frac{-\pi}{6}\right) = \frac{1}{2}$

$f''(x) = -cosx$

$f''\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$f''\left(\frac{\pi}{6}\right) + 12f'\left(\frac{-\pi}{6}\right) + f\left(\frac{\pi}{6}\right) = 5$