

PHYSICS

SECTION-A

1. Match the LIST-I with LIST-II

List-I		List-II	
(A)	Planck's constant	(I)	$ML^2 T^{-2}$
(B)	Stopping potential	(II)	T^{-1}
(C)	Work function	(III)	$ML^2 T^{-1}$
(D)	Threshold frequency	(IV)	$ML^2 T^{-3} A^{-1}$

Choose the correct answer from the options given below :

- (a) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(b) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(c) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(d) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)
- Two cars A and B are moving in the same direction along a straight line with speeds 100 km/h and 80 km/h, respectively such that car A is moving ahead of car B. A person in car B throws a stone with a speed v so that it hits the car A with a speed of 5 m/s. The value of v is ____ km/h.
(a) 18 (b) 28
(c) 38 (d) 48
 - At $t = 0$, a body of mass 100 g starts moving under the influence of a force $(5\hat{i} + 10\hat{j})$ N. After 2 s its position is $(2x\hat{i} + 5y\hat{j})$ m. The ratio $x : y$ is ____ .
(a) 1:2 (b) 2:5
(c) 5:2 (d) 5:4
 - If x and y coordinates of a projectile as a function of time (t) are given as $24t$ and $43.6t - 4.9t^2$, respectively, then the angle (in degrees) made by the projectile with horizontal when $t = 2$ s is ____ .
(a) 60 (b) 45
(c) 30 (d) 75
 - The height in terms of radius of the earth (R), at which the acceleration due to gravity becomes $\frac{g}{9}$, where g is acceleration due to gravity on earth's surface, is ____ .
(a) $\sqrt{3}R$ (b) $2\sqrt{2}R$
(c) $2R$ (d) $\frac{4}{9}R$
 - A metal string A is suspended from a rigid support and its free end is attached to a block of mass M . Second block having mass $2M$ is suspended at the bottom of the first block using a string B. The area of cross sections of strings A and B are same. The ratio of lengths of strings of A to B is 2 and the ratio of their Young's moduli (Y_A/Y_B) is 0.5. The ratio of elongations in A to B is ____ .
(a) 1 (b) 4
(c) 8 (d) 6
 - A water spray gun is attached to a hose of cross-sectional area 30 cm^2 . The gun comprises of 10 perforations each of cross-sectional area of 15 mm^2 . If the water flows in the hose with the speed of 50 cm/s , calculate the speed at which the water flows out from each perforation. (Neglect any edge effects)
(a) 100 m/s (b) 10 m/s
(c) 1000 m/s (d) $15 \times 10^2 \text{ m/s}$
 - Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R
Assertion A: If the average kinetic energy of H_2 and O_2 molecules, kept in two different sized containers are

same, then their temperatures will be same.

Reason R: The r.m.s. speed of H_2 and O_2 molecules are same at same temperature.

Choose the correct answer from the options given below

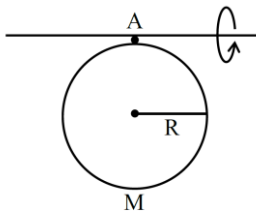
- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

9. The temperature of a metal strip having coefficient of linear expansion α is increased from T_1 to T_2 resulting in increase of its length by ΔL_1 . The temperature is further increased from T_2 to T_3 such that the increase in its length is ΔL_2 .

Given $T_3 + T_1 = 2 T_2$ and $T_2 - T_1 = \Delta T$, the value of ΔL_2 is ____ .

- (a) $\Delta L_1 [1 + 2\alpha^2 (\Delta T)^2]$
- (b) $\Delta L_1 [1 + \alpha^2 (\Delta T)^2]$
- (c) $\Delta L_1 [1 + 2\alpha \Delta T]$
- (d) $\Delta L_1 [1 + \alpha \Delta T]$

10. A uniform disc of radius R and mass M is free to oscillate about the axis A as shown in the figure. For small oscillations the time period is ____ . (g is acceleration due to gravity)

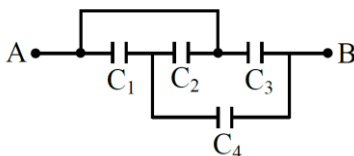


- (a) $2\pi \sqrt{\frac{5R}{4g}}$
- (b) $2\pi \sqrt{\frac{2R}{3g}}$
- (c) $2\pi \sqrt{\frac{3R}{2g}}$
- (d) $2\pi \sqrt{\frac{3R}{g}}$

11. A rigid dipole undergoes a simple harmonic motion about its centre in the presence of an electric field $\vec{E}_1 = E_0 \hat{x}$. If another electric field $\vec{E}_2 = 2E_0(\hat{y} + \hat{z})$ is introduced to the system, what will be the percentage change in the frequency of the oscillation (approximate) ?

- (a) 73%
- (b) 63%
- (c) 83%
- (d) 53%

12. From the circuit given below, the capacitance between terminals A and B shown in the circuit is ____ μF . (take $C_1 = C_2 = C_3 = 1\mu F$ and $C_4 = 2\mu F$.)



- (a) 2
- (b) 7/2
- (c) 7/3
- (d) 5/2

13. Given below are two statements: one is labelled as

Assertion A and the other is labelled as Reason R

Assertion A: In electrostatics, a conductor does not store any net charge inside.

Reason R : Inside the capacitor (with no dielectric medium), the free charge carriers, if placed between the plates of capacitor, experience force and drift.

Choose the correct answer from the options given below.

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is NOT the correct explanation of A.

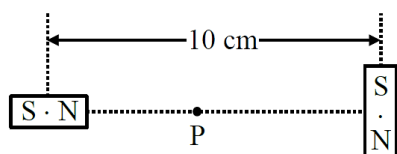
- (c) A is true but R is false
(d) A is false but R is true
14. A solenoid has a core made of material with relative permeability 400. The magnetic field produced in the interior of solenoid is 1.0 T. The magnetic intensity in SI units is $\alpha \times 10^5$. The value of α is ____ . (Free space permeability $\mu_0 = 4\pi \times 10^{-7}$ SI units).
- (a) $\frac{25}{\pi}$ (b) $\frac{1}{16\pi}$
(c) $\frac{1}{\pi}$ (d) $\frac{1}{4\pi}$
15. A magnetic field vector in an electromagnetic wave is represented by, $\vec{B} = B_0 \sin \left(2\pi vt - \frac{2\pi x}{\lambda} \right) \hat{j}$. Its associated electric field vector is ____ .
- (a) $\vec{E} = -v\lambda B_0 \sin \left(2\pi vt - \frac{2\pi x}{\lambda} \right) \hat{k}$
(b) $\vec{E} = -v\lambda B_0 \sin \left(2\pi vt - \frac{2\pi x}{\lambda} \right) \hat{i}$
(c) $\vec{E} = v\lambda B_0 \sin \left(2\pi vt - \frac{2\pi x}{\lambda} \right) \hat{k}$
(d) $\vec{E} = v\lambda B_0 \sin \left(2\pi vt - \frac{2\pi x}{\lambda} \right) \hat{i}$
16. A convex lens is made from glass material having refractive index of 1.4 with same radius of curvature on both sides. The ratio of its focal length and radius of curvature is ____ .
- (a) 0.5 (b) 2.5
(c) 0.8 (d) 1.25
17. An unpolarized light of certain intensity passes through a combination of two polarizers whose transmission axes are at 30° and 90° , respectively, with respect to the horizontal axis. A third polarizer with its transmission axis at 60° with the horizontal axis is placed between the two existing polarizers. The ratio of the output intensities with and without the third polarizer is ____ .
- (a) $\frac{3}{4}$ (b) $\frac{4}{3}$
(c) $\frac{9}{4}$ (d) $\frac{4}{9}$
18. In Rutherford's alpha-particle scattering experiment, only a few alpha particles rebound back because
- (A) The size of gold nucleus is very small as compared to the size of gold atom.
(B) Alpha particle and gold nucleus have equal charge.
(C) The impact parameter is minimum for a few alpha particles.
(D) A few alpha particles have very high kinetic energy.
(E) Only a few alpha particles undergo head-on collision with the nuclei.
- Choose the correct answer from the options given below :
- (a) (A), (B) Only (b) (B), (E) Only
(c) (C), (D) Only (d) (A), (C), (E) Only
19. The de Broglie wavelength associated with an electron accelerated through a potential difference V is λ_e and the de Broglie wavelength associated with a proton accelerated through the same potential difference is λ_p . If their corresponding masses are m_e and m_p , respectively, then the ratio of their de Broglie wavelengths $\left(\frac{\lambda_e}{\lambda_p} \right)$ is ____ .
- (a) $\sqrt{\frac{m_p}{m_e}}$ (b) $\sqrt{\frac{m_e}{m_p}}$
(c) $\frac{m_p}{m_e}$ (d) $\left(\frac{m_p}{m_e} \right)^2$
20. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R
- Assertion A :** A diode under reverse-biased condition provides very small current which is nearly independent of voltage until a critical limit at which the current increases drastically.
- Reason R :** Below the critical voltage limit, only majority charge carriers flow which increases drastically above critical voltage.

Choose the correct answer from the options given below.

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is NOT the correct explanation of A.
- (c) A is true but R is false
- (d) A is false but R is true

SECTION-B

21. A diode has Zener voltage of 10 V and maximum power dissipation of 0.5 W, then the minimum resistance to be used in series with this diode for safety when it is connected to a 25 V power supply is $_____\Omega$.
22. A gun mounted on the ground fires bullets in all directions with same speed. The farthest distance the bullets could reach is 6.4 m. The speed of the bullets from the gun is $_____\text{m/s}$. (take $g = 10 \text{ m/s}^2$)
23. Two identical small bar magnets each of dipole moment $3\sqrt{5} \text{ J/T}$ are placed at a center to center separation of 10 cm, with their axes perpendicular to each other as shown in figure. The value of magnetic field at the point P midway between the magnets is $\alpha \times 10^{-3} \text{ T}$. The value of α is $______$. ($\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$)



24. A circular coil of radius 2 cm and 125 turns carries a current of 1 A. The coil is placed in a uniform magnetic field of magnitude 0.4 T. The axis of the coil makes an angle of 30° with the direction of the magnetic field. The torque acting on the coil is $\alpha \times 10^{-4} \text{ N.m}$. The value of α is $______$. ($\pi = 3.14$)
25. In a double slit experiment, when one of the slits is covered by a transparent mica sheet of refractive index 1.56, the central fringe shifts to the position of 7th bright fringe, obtained with both slits uncovered. If the light source wavelength is 450 nm, the thickness of mica sheet is $\alpha \times 10^{-9} \text{ m}$. The value of α is $______$.

CHEMISTRY

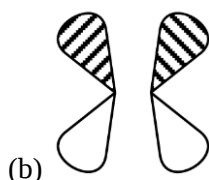
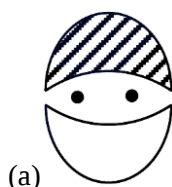
SECTION-A

26. The correct order of total number of atoms of present in
- (A) 2 moles of cyclohexane
 - (B) 684 g of sucrose
 - (C) 90.8 L of dihydrogen at STP
- (a) (C) > (A) > (B)
 - (b) (C) > (B) > (A)
 - (c) (B) > (C) > (A)
 - (d) (B) > (A) > (C)
27. The species having identical radii according to the Bohr's theory are :
- (A) H (first orbit)
 - (B) He^+ (first orbit)
 - (C) He^+ (second orbit)
 - (D) Li^{2+} (first orbit)
 - (E) Be^{3+} (Second orbit)

Choose the correct answer from the options given below :

- (a) (A) and (C) Only
- (b) (A) and (E) Only
- (c) (B) and (E) Only
- (d) (C) and (D) Only

28. Which of the following pictorial diagram most correctly represents the π^* (π - antibonding) molecular orbital between two atoms if the internuclear axis is taken to be in the z -direction ($\vec{z}$ -axis $\rightarrow$) ?



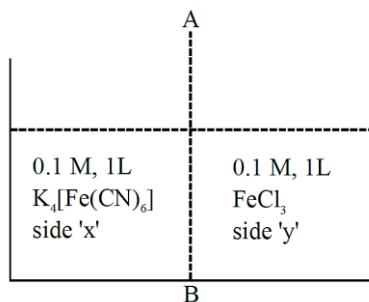


(c)



(d)

29. At 27°C , 0.1 M , 1 L $\text{K}_4[\text{Fe}(\text{CN})_6]$ aqueous solution and 0.1 M , 1 L FeCl_3 aqueous solution are placed in a container separated by a semi permeable membrane AB. Assume complete dissociation of both the solutes. Which of the following statement is correct?



- (a) Blue color is formed on both sides.
 (b) Ionic solutes in aqueous solution can pass through semi-permeable membrane.
 (c) Solution on side 'y' is hypotonic.
 (d) To cause the reverse flow of solvent during osmosis, external pressure (any value) should be applied to side 'x'.
30. 20 mL of a solution of acetic acid required 28.4 mL of 0.1 M NaOH for its neutralization. A solution (X) was prepared by mixing 20 mL of the above acetic acid and 14.2 mL of 0.1 M NaOH solution. What is the pH of the solution (X)? (pK_a value of acetic acid is 4.75).
- (a) 7.0 (b) 4.45
 (c) 3.5 (d) 4.82
31. Match the LIST-I with LIST-II

List-I Reaction		List-II Mechanism	
(A)	Williamson Synthesis	(I)	Electrophilic addition
(B)	Friedel Craft Reaction	(II)	Free radical substitution
(C)	Brominating of vinyl benzene	(III)	Nucleophilic substitution
(D)	Chlorination of toluene in light	(IV)	Electrophilic substitution

Choose the correct answer from the options given below:

- (a) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
 (b) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
 (c) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
 (d) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
32. The 1^{st} ionization enthalpy for Mg is $+737\text{ kJ/mol}$. The most probable estimated value of the 2^{nd} ionization enthalpy of Mg is ____.
- (a) -906 kJ/mol
 (b) -856 kJ/mol
 (c) $+1450\text{ kJ/mol}$
 (d) $+590\text{ kJ/mol}$

33. The electronegativity of a group 13 element 'E' is same as that of Ge (on Pauling scale and up to one decimal point). The CORRECT statements about E^{3+} are

- (A) It can act as a reducing agent.
- (B) It can act as an oxidizing agent.
- (C) E^{3+} is more stable than E^+ .
- (D) The standard electrode potential value for E^{3+}/E is positive.

Choose the correct answer from the options given below

- (a) (A) and (C) Only
- (b) (B) and (C) Only
- (c) (B) and (D) Only
- (d) (A) and (D) Only

34. Pairs of elements with the same number of electrons in their respective 4f orbital are [Atomic number, Eu-63, Gd-64, Dy-66, Ho-67, Tm-69, Yb-70, Lu-71, Hf-72]

- (A) (Eu and Gd)
- (B) (Dy and Ho)
- (C) (Yb and Hf)
- (D) (Lu and Tm)

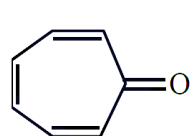
Choose the correct answer from the options given below:

- (a) (B) and (C) Only
- (b) (A) and (B) Only
- (c) (A) and (D) Only
- (d) (A) and (C) Only

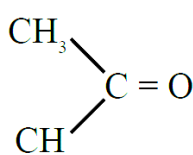
35. Consider the metal complexes $[\text{Ni}(\text{en})_3]^{2+}$ (A), $[\text{NiCl}_4]^{2-}$ (B) and $[\text{Ni}(\text{NH}_3)_6]^{2+}$ (C). Choose the CORRECT option by considering the number of unpaired electron present in (A), (B) and (C) respectively and the order of frequency of absorption.

- (a) 2, 2, 2 and (A) > (C) > (B)
- (b) 0, 2, 0 and (A) > (C) > (B)
- (c) 2, 2, 0 and (B) > (C) > (A)
- (d) 2, 2, 2 and (C) > (A) > (B)

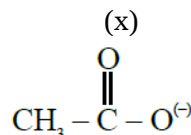
36. Consider the following molecules/species:



(x)



(y)



(z)

The correct order of carbon - oxygen double bond length is:

- (a) (x) > (y) > (z)
- (b) (y) > (z) > (x)
- (c) (z) > (x) > (y)
- (d) (x) > (z) > (y)

37. Consider $|x|$ is the difference in oxidation states of Mn in highest manganese fluoride and highest manganese oxide. The ions with $|x|$ number of unpaired electrons from the following are:

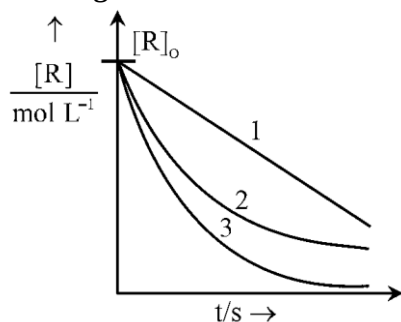
- (A) Sc^{3+}
- (B) Zn^{2+}
- (C) V^{2+}
- (D) Fe^{2+}
- (E) Co^{2+}

Choose the correct answer from the options given below:

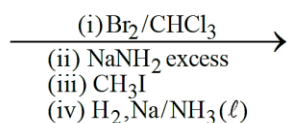
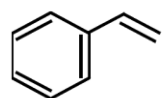
- (a) (A) and (B) Only
- (b) (C), (D) and (E) Only
- (c) (C) and (E) Only
- (d) (B) and (E) Onl

38. Consider the given graph showing variation of reactant concentration with time.

Three different reactions were started with identical initial concentration of reactants. Which of the following statement is correct?



- (a) The order of all the three reactions is same.
- (b) The rate constant of reaction 3 is larger than the rate constant of reaction 2 if the order of reaction is same for both.
- (b) The SI unit of rate constant of reaction 1 is s^{-1} .
- (c) Thermal decomposition of HI on gold surface is an example of reaction 2 .

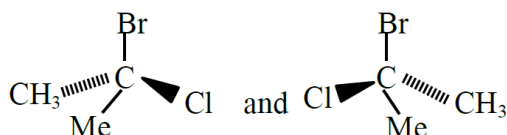


39.

Compound (X) is subjected to the sequence of reactions as shown above. Molar mass of the major product (Y) formed is ___ gmol^{-1} . (Given molar mass in gmol^{-1} C: 12, H: 1, O: 16)

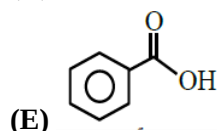
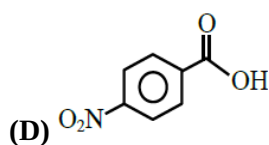
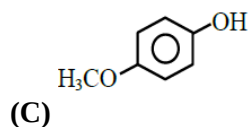
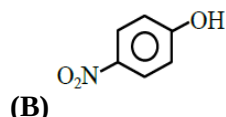
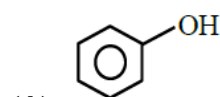
- (a) 90
- (b) 118
- (c) 160
- (d) 125

40. The following structures are



- (a) enantiomer
- (b) Identical molecules
- (c) Diastereomers
- (d) meso compound

41. The descending order of acidity among the following compounds is

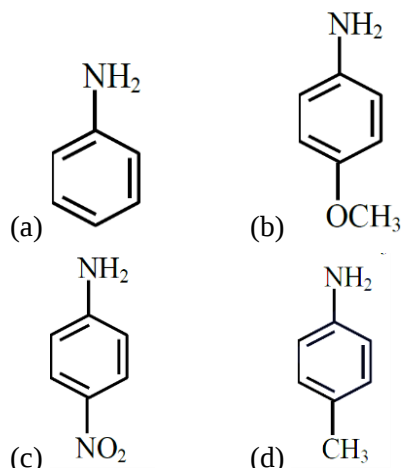


Choose the correct answer from the option given below options

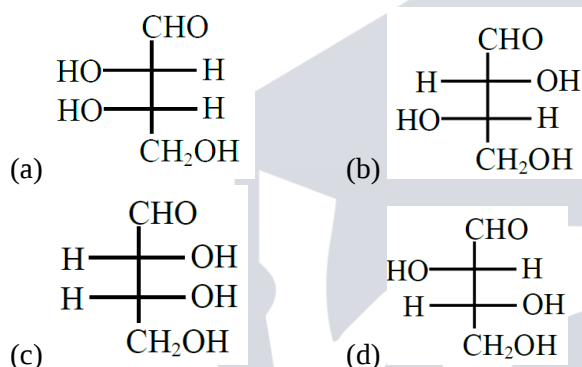
- (a) (B) > (D) > (E) > (A) > (C)

- (b) (D) > (B) > (E) > (A) > (C)
 (c) (C) > (A) > (B) > (D) > (E)
 (d) (D) > (E) > (B) > (A) > (C)

42. The strongest conjugate acid will result from



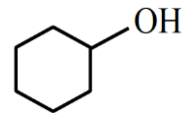
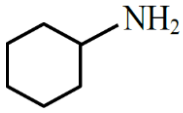
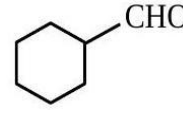
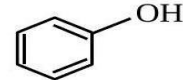
43. A D-aldosterone on oxidation with concentrated HNO_3 resulted in optically inactive dicarboxylic acid. The structure of the D-aldotetrose is :



44. Among Fe^{3+} , Pb^{2+} , Cu^{2+} and Mn^{2+} , identify the one that gets precipitated out while passing H_2S in presence of NH_4OH as group reagent. The highest possible oxidation state of the corresponding metal is

- (a) +3 (b) +4
 (c) +2 (d) +7

45. Match the LIST-I with LIST-II

List-I Compound		List-II Test	
(A)		(I)	Heinsberg's reagent test
(B)		(II)	Phthalein dye test
(C)		(III)	Lucas test
(D)		(IV)	Tolle's test

Choose the correct answer from the options given below :

- (a) (A)-(III), (B)-(I), (C)-(IV) , (D)-(II)

- (b) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
 (c) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
 (d) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

SECTION-B

46. If 3.365 g of ethanol (l) burnt completely in a bomb calorimeter at 298.15 K, the heat produced is 99.472 kJ. The $|\Delta H_f^\circ|$ of ethanol at 298.15 K is $___ \times 10^2$ kJ mol⁻¹. (Nearest integer)

Given :

Standard enthalpy of combustion of graphite = -393.5 kJ mol⁻¹

Standard enthalpy of formation of water (l) = -285.8 kJ mol⁻¹

Molar mass in g mol⁻¹ of C, H, O are 12, 1 and 16 respectively.

47. For the following reaction at 50°C and 2 atm pressure,
 $2\text{N}_2\text{O}_5(\text{g}) \rightleftharpoons 2\text{N}_2\text{O}_4(\text{g}) + \text{O}_2(\text{g})$ N_2O_5 is 50% dissociated
 The magnitude of standard free energy change at this temperature is x.
 $x = ___ \text{J mol}^{-1}$ [Nearest integer]

Given : $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$, $\log 2 = 0.30$, $\log 3 = 0.48$, $\ln 10 = 2.303$, $^\circ\text{C} + 273 = \text{K}$

48. An electrochemical cell consists of the following two redox couples,

$\text{M}^{x+}(\text{aq})/\text{M}(\text{s}) [E_{\text{red}}^\ominus = +0.15 \text{ V}]$ and $\text{Fe}^{3+}(\text{aq})/\text{Fe}(\text{s}) [E_{\text{red}}^\ominus = -0.036 \text{ V}]$

The cell EMF (E_{cell}) is recorded to be 0.2057 V.

If the reaction quotient of the electrochemical reaction is found to be 10^{-2} , then the value of x is $___$ (Nearest integer)

[Given : M is a p-block metal and $\frac{2.303RT}{F} = 0.059 \text{ V}$]

49. For a first order reaction $\text{A} \rightarrow \text{B}$

t/min	[A]/M
0	0.6500
x	0.0650
20	0.00065

$x = ___ \text{ min.}$ (Nearest integer)

50. In Sulphur estimation, 2.0×10^{-3} mol of an organic compound (X) (molar mass 76 g mol⁻¹) gave 0.4813 g of barium sulphate (molar mass 233 g mol⁻¹). The percentage of sulphur in the compound (X) is $___ \times 10^{-1}\%$ (Nearest integer)

MATHEMATICS

SECTION-A

51. For the function $f: [1, \infty) \rightarrow [1, \infty)$ defined by $f(x) = (x-1)^4 + 1$, among the two statements :

(I) The set $S = \{x \in [1, \infty): f(x) = f^1(x)\}$ contains exactly two elements, and

(II) The set $S = \{x \in [1, \infty): f(x) = f^1(x+1)\}$ is an empty set,

(a) only (I) is TRUE

(b) only (II) is TRUE

(c) both (I) and (II) are TRUE

(d) neither (I) nor (II) is TRUE

52. Let $S = \{z \in \mathbb{C}: z^2 + 4z + 16 = 0\}$. Then $\sum_{z \in S} |z + \sqrt{3}i|^2$ is equal to :

(a) 42

(b) 23

(c) 27

(d) 38

53. If the system of equations :

$$x + y + z = 5$$

$$x + 2y + 3z = 9$$

$$x + 3y + \lambda z = \mu$$

has infinitely many solutions, then the value of $\lambda + \mu$ is :

- (a) 16 (b) 18
(c) 19 (d) 21

54. If $\alpha = 1$ and $\beta = 1 + i\sqrt{2}$, where $i = \sqrt{-1}$ are two roots of the equation $x^3 + ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$, then $\int_{-1}^1 (x^3 + ax^2 + bx + c)dx$ is equal to :

- (a) -2 (b) -4
(c) -8 (d) -10

55. If the quadratic equation $(\lambda + 2)x^2 - 3\lambda x + 4\lambda = 0$, $\lambda \neq -2$, has two positive roots, then the number of possible integral values of λ is :

- (a) 1 (b) 2
(c) 3 (d) 4

56. Let $A = \begin{bmatrix} 1 & 2 & 7 \\ 4 & -2 & 8 \\ 3 & 8 & -7 \end{bmatrix}$ and $\det(A - \alpha I) = 0$, where α is a real number. If the largest possible value of α is p , then the circle $(x - p)^2 + (y - 2p)^2 = 320$, intersects the co-ordinate axes at

- (a) 1 point (b) 2 points
(c) 3 points (d) 4 points

57. Let $\alpha = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \infty$ and $\beta = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty$. Then the value of $(0.2)^{\log_{\sqrt{5}}(\alpha)} + (0.04)^{\log_5(\beta)}$ is equal to :

- (a) 4 (b) 5
(c) 8 (d) 25

58. For 10 observations $x_1, x_2, \dots, x_{10}$, if $\sum_{i=1}^{10} (x_i + 2)^2 = 180$ and $\sum_{i=1}^{10} (x_i - 1)^2 = 90$, then their standard deviation is :

- (a) 2 (b) $\sqrt{3}$
(c) $2\sqrt{2}$ (d) 3

59. In the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x > 0$, if the term independent of x is $(221)k$, then k is equal to:

- (a) 84 (b) 78
(c) 168 (d) 198

60. Let $P(3\cos\alpha, 2\sin\alpha)$, $\alpha \neq 0$, be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, Q be a point on the circle $x^2 + y^2 - 14x - 14y + 82 = 0$ and R be a point on the line $x + y = 5$ such that the centroid of the triangle PQR is $\left(2 + \cos\alpha, 3 + \frac{2}{3}\sin\alpha\right)$. Then the sum of the ordinates of all possible points R is:

- (a) 6 (b) 2
(c) 4 (d) 8

61. Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be a hyperbola such that the distance between its foci is 6 and the distance between its directrices is $\frac{8}{3}$. If the line $x = \alpha$ intersects the hyperbola H at the points A and B such that the area of the triangle AOB is $4\sqrt{15}$, where O is the origin, then α^2 equals

- (a) 12 (b) 16
(c) 24 (d) 25

62. $\max_{0 \leq x \leq \pi} \left(16\sin\left(\frac{x}{2}\right)\cos^3\left(\frac{x}{2}\right)\right)$ is equal to :

- (a) $\frac{3\sqrt{3}}{2}$ (b) $3\sqrt{3}$
(c) $4\sqrt{3}$ (d) $6\sqrt{3}$

63. The shortest distance between the lines $\vec{r} = \left(\frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k}\right) + \lambda(2\hat{i} - 5\hat{j} + 6\hat{k})$ and $\vec{r} = \left(-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{k}\right) + \mu(\hat{j} - \hat{k})$, $\lambda, \mu \in \mathbb{R}$, is :

- (a) $\sqrt{5}$ (b) 3
(c) $2\sqrt{3}$ (d) $\sqrt{15}$

64. If $(2\alpha + 1, \alpha^2 - 3\alpha, \frac{\alpha-1}{2})$ is the image of $(\alpha, 2\alpha, 1)$ in the line $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z}{1}$, then the possible value(s) of α is(are) :

- (a) Only 3
(b) Only 3 and -1
(c) Only 3, $\frac{1}{4}$ and -1
(d) Only 3 and $\frac{1}{4}$

65. Let $\hat{u}$ and $\hat{v}$ be unit vectors inclined at an acute angle such that $|\hat{u} \times \hat{v}| = \frac{\sqrt{3}}{2}$. If

$\vec{A} = \lambda \hat{u} + \hat{v} + (\hat{u} \times \hat{v})$. Then λ is equal to :

- (a) $\frac{4}{3}(\vec{A} \cdot \hat{u}) - \frac{2}{3}(\vec{A} \cdot \hat{v})$
(b) $\frac{2}{3}(\vec{A} \cdot \hat{u}) - \frac{1}{3}(\vec{A} \cdot \hat{v})$
(c) $\frac{4}{3}(\vec{A} \cdot \hat{u}) + \frac{2}{3}(\vec{A} \cdot \hat{v})$
(d) $(\vec{A} \cdot \hat{u}) - \frac{1}{2}(\vec{A} \cdot \hat{v})$

66. Let for some $\alpha \in \mathbb{R}$, $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying $f(x+y) = f(x) + 2y^2 + y + \alpha xy$ for all $x, y \in \mathbb{R}$. If $f(0) = -1$ and $f(1) = 2$, then the value of $\sum_{n=1}^5 (\alpha + f(n))$ is :

- (a) 110 (b) 140
(c) 150 (d) 170

67. Let $A = \{(a, b, c) : a, b, c \text{ are non-negative integers and } a + b + 2c = 22\}$.

The $n(A)$ is equal to :

- (a) 121 (b) 124
(c) 144 (d) 169

68. The area of the region bounded by the curves $x + 3y^2 = 0$ and $x + 4y^2 = 1$ is equal to :

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

69. Let $y = y(x)$ be the solution of the differential equation :

$$\frac{dy}{dx} + \left(\frac{6x^2 + (3x^2 + 2x^3 + 4)e^{-2x}}{(x^3 + 2)(2 + e^{-2x})} \right) y = 2 + e^{-2x}, x \in (-1, 2), \text{ satisfying } y(0) = \frac{3}{2}.$$

If $y(1) = \alpha(2 + e^{-2})$, then α is equal to :

- (a) $\frac{13}{8}$ (b) $\frac{6}{13}$
(c) $\frac{12}{13}$ (d) $\frac{13}{12}$

70. The integral $\int_0^1 \cot^{-1}(1 + x + x^2) dx$ is equal to :

- (a) $2\tan^{-1}2 + \frac{1}{2}\log_e\left(\frac{5}{4}\right) + \frac{\pi}{2}$
(b) $2\tan^{-1}2 + \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}$
(c) $2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) + \frac{\pi}{2}$
(d) $2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}$

SECTION-B

71. From a month of 31 days, 3 different dates are selected at random. If the probability that these dates are in an increasing A.P. is equal to $\frac{a}{b}$, where $a, b \in \mathbb{N}$ and $\gcd(a, b) = 1$, then $a + b$ is equal to ____ .

72. Let $f(x) = \begin{cases} e^{x-1} & , x < 0 \\ x^2 - 5x + 6 & , x \geq 0 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$. If the number of points where g is not continuous and is not differentiable are α and β respectively, then $\alpha + \beta$ is equal to ____
73. Let A, B be points on the two half-lines $x - \sqrt{3}|y| = \alpha, \alpha > 0$ at a distance of α from their point of intersection P. The line segment AB meets the angle bisector of the given half-lines at the point Q. If $PQ = \frac{9}{2}$ and R is the radius of the circumcircle of $\triangle PAB$, then $\frac{\alpha^2}{R}$ is equal to ____.
74. Let A, B and C be the vertices of a variable right angled triangle inscribed in the parabola $y^2 = 16x$. Let the vertex B containing the right angle be (4,8) and the locus of the centroid of $\triangle ABC$ be a conic C_0 . Then three times the length of latus rectum of C_0 is ____.
75. Let f be a twice differentiable function such that $f(x) = \int_0^x \tan(t-x)dt - \int_0^x f(t)\tan t dt, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then $f''\left(\frac{\pi}{6}\right) + 12f'\left(-\frac{\pi}{6}\right) + f\left(\frac{\pi}{6}\right)$ is equal to ____.

