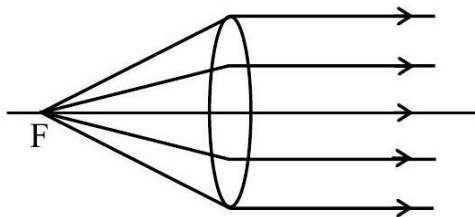
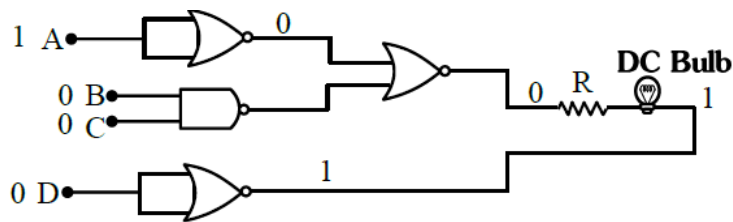


1. (d) Light emerges parallel

∴ planor wavefront



2. (b) Bulb will glow if bulb have potential drop on it. One end of bulb must be at high (a) and other must be at low (0).



3. (b) $[uG] = [(M^1 L^{-1} T^{-2})(M^{-1} L^3 T^{-2})]$

$$[uG] = [M^0 L^2 T^{-4}]$$

$$[\sqrt{uG}] = [L^1 T^{-2}]$$

4. (a)

Capillary rise

$$h = \frac{2 T \cos \theta}{\rho g r}$$

If $\theta = 0^\circ$ then rise is non-zero

∴ Statement-1 is incorrect.

5. (a) $eV_0 = h\nu - \phi$

$$V_0 = \frac{h}{e} \nu - \frac{\phi}{e}$$

M_2 material has higher work function, so statement-(II) is incorrect.

6. (a) $\vec{R} = (2\vec{Q} + 2\vec{P}) + (2\vec{Q} - 2\vec{P})$

$$\vec{R} = 4\vec{Q}$$

Angle between $\vec{Q}$ and $\vec{R}$ is zero

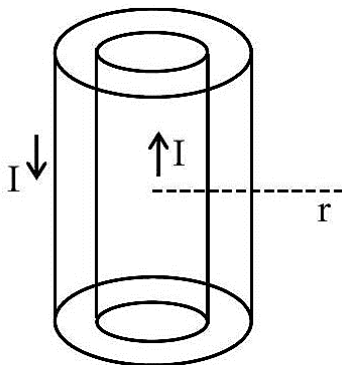
7. (a)

$$F_e = \frac{kQ_1Q_2}{r^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{r^2}$$

$$F_g = \frac{Gm_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-27}}{r^2}$$

$$\frac{F_e}{F_g} \cong 0.23 \times 10^{40} \cong 2.3 \times 10^{39}$$

8. (c)



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_{\text{enc}} = 0$$

$\therefore B = 0$ outside the cable

9. (d) $F = \frac{k(Ze)(e)}{r^2} = \frac{mv^2}{r}$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2} \frac{k(Ze)(e)}{r}$$

$$PE = -\frac{k(Ze)(e)}{r}$$

$$TE = \frac{k(Ze)(e)}{2r} - \frac{k(Ze)(e)}{r} = \frac{-k(Ze)(e)}{2r}$$

$$TE = \frac{PE}{2}$$

$$2TE = PE$$

10. (c) Assuming mean free path constant. $f \propto v \propto \sqrt{T}$

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{400}}$$

$$f_2 = \sqrt{\frac{4}{3}} = f_1 = \frac{2}{\sqrt{3}} Z$$

11. (c) $I_{\text{sphere}} = \frac{2}{3}MR^2 = Mk_1^2$

$$I_{\text{cylinder}} = \frac{1}{12}M(4R^2) + \frac{1}{4}MR^2 + M(2R)^2$$

$$= \frac{67}{12}MR^2 = Mk_2^2$$

$$\frac{k_1}{k_2} = \sqrt{\frac{2}{3} \cdot \frac{12}{67}} = \sqrt{\frac{8}{67}}$$

12. (a) $\phi = M_i = B A$

$$\Rightarrow M_i = \frac{\mu_0 i}{2b} \pi a^2$$

$$\therefore M = \frac{\mu_0 \pi a^2}{2b}$$

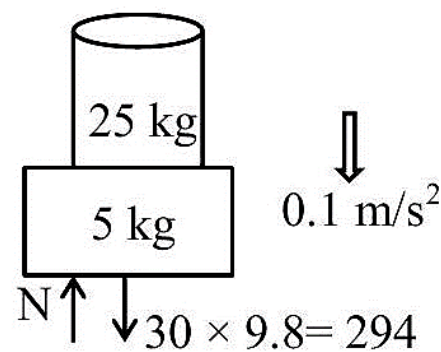
13. (a)

$$KE = \frac{1}{2} m v^2 = \frac{G M m}{2a}$$

$$PE = -2KE$$

$$TE = -KE$$

14. (c) Taking $g = 9.8 \text{ m/s}^2$



$$294 - N = 30 \times 0.1$$

$$N = 291$$

15. (c) $T_1 = 2\pi \sqrt{\frac{\ell}{GM}} (2R)^2$

$$T_2 = 2\pi \sqrt{\frac{\ell}{GM}} (3R)^2$$

$$\therefore \frac{T_1}{T_2} = \frac{2}{3}$$

16. (a) Work done by gravity is independent of path. It depends only on vertical displacement so work done in both cases will be same.

17. (c) Sum of number by considering significant digit $\text{sum} = 4.6 + 4.6 + 4.6 + 4.6 = 18.4$

$$\text{Arithmetic Mean} = \frac{\text{sum}}{4} = \frac{18.4}{4} = 4.6$$

18. (a) $\Delta U = 0$ (Cyclic process)

$\Delta Q = W = \text{area of } P - V \text{ curve.}$

$$= \pi \times (140 \times 10^3 \text{ Pa}) \times (140 \times 10^{-6} \text{ m}^3)$$

$$\Delta Q = 61.6 \text{ J}$$

19. (d) Here R_2, R_3, R_4 are in parallel

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$R_{234} = 2\Omega$$

R_{234} is in series with R_1 so

$$R_{eq} = R_{234} + R_1 = 2 + 10 = 12\Omega$$

$$i = \frac{12}{12} = 1\text{Amp}$$

20. (d) Displacement current is same as conduction current in capacitor.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 4 \times 10^3 \times 12 \times 10^{-6}} = 3.317\Omega$$

$$I = \frac{V}{X_C} = \frac{40}{3.317} = 12 \text{ A}$$

$$21. (10) \beta = \frac{\lambda D}{d} = \frac{5 \times 10^{-7} \times 2}{3 \times 10^{-4}} = \frac{10 \times 10^{-3}}{3} \text{ m For } 3^{\text{rd}} \text{ maxima } y_3 = 3\beta = 10 \times 10^{-3} \text{ m} = 10 \text{ mm}$$

22. (5)

$$R = \frac{\rho \ell}{A} \Rightarrow \frac{2 \times 10^{-6} \times \ell}{10^{-5}} = 1 \Rightarrow \ell = 5$$

$$mg = Bi\ell$$

$$B = \frac{mg}{i\ell} = \frac{5}{2 \times 5} = 0.5 = 5 \times 10^{-1} \text{ Tesla}$$

$$23. (4) E = \frac{E_0}{3} \Rightarrow V = \frac{V_0}{3}$$

$$\frac{V_0}{3} = V_0 e^{-\frac{t}{\tau}}$$

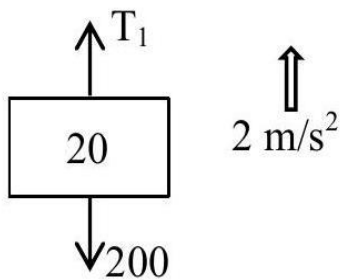
$$t = \tau \ln 3$$

$$6.6 \times 10^{-6} = R(1.5 \times 10^{-6})(1.1)$$

$$R = \frac{6}{1.5} = 4\Omega$$

24. (240)

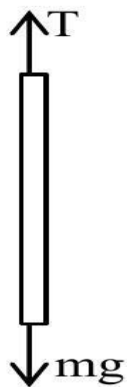
FBD of M_1 :



$$T_1 - 200 = (4 + 6 + 10) \times 2$$

$$\therefore T_1 = 240$$

25. (600)



$$T = mg$$

$$\sigma = \frac{T}{A} = \frac{mg}{A}$$

$$\frac{(\sigma A \ell)g}{A}$$

$$\Rightarrow \ell = \frac{\sigma}{\rho g} = \frac{1.2 \times 10^8 \times 3}{6 \times 10^4 \times 10} = 600$$

26. (19)

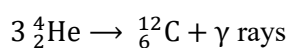
$$S_n = \frac{1}{2}a(2n - 1) = \frac{19a}{2}$$

$$S_{n-1} = \frac{1}{2}a(2n - 3) = \frac{17a}{2}$$

$$\frac{S_{n-1}}{S_n} = \frac{17}{19} = 1 - \frac{2}{x} \Rightarrow x = 19$$

27. (727)

Reaction:



$$\text{Mass defect} = \Delta m = (3 m_{\text{He}} - m_{\text{C}})$$

$$= (3 \times 4.002603 - 12) = 0.007809 \text{u}$$

Energy released

$$= 931 \Delta m \text{MeV} = 7.27 \text{MeV} = 727 \times 10^{-2} \text{MeV}$$

$$\mathbf{28. (50)} \quad X_L = \omega L = 100 \times 1 = 100 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 20 \times 10^{-6}} = 500 \Omega$$

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\sqrt{(100 - 500)^2 + 300^2}$$

$$Z = 500 \Omega$$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{50}{500} = 0.1 \text{ A}$$

rms voltage across capacitor

$$V_{\text{rms}} = X_C i_{\text{rms}}$$

$$= 500 \times 0.1 = 50 \text{ V}$$

$$\mathbf{29. (5)} \quad i = K\theta$$

$$\frac{2}{G + R} = K\theta$$

$$\Rightarrow \frac{1}{\theta} = \frac{(G + R)K}{2} = R \left(\frac{K}{2} \right) + \frac{KG}{2}$$

$$\text{Slope} = \frac{K}{2} = \frac{1}{4} \Rightarrow K = 0.5 = 5 \times 10^{-1} \text{ A}$$

30. (86) In parallel combination: Potential difference is same across all

$$\text{Energy} = \frac{1}{2} (C_1 + C_2 + C_3) V^2$$

$$= \frac{1}{2} (25 + 30 + 45) \times (100)^2 \times 10^{-6} = 0.5 = E$$

In series combination: Charge is same on all.

$$\frac{1}{C_{\text{equ}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{25} + \frac{1}{30} + \frac{1}{45}$$

$$\frac{1}{C_{\text{equ}}} = \frac{(18 + 15 + 10)}{450} = \frac{43}{450} \Rightarrow C_{\text{equ}} = \frac{450}{43}$$

$$\text{Energy} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$= \frac{Q^2}{2} \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]$$

$$\frac{(V \times C_{\text{equ}})^2}{2} \times \frac{1}{C_{\text{equ}}} = \frac{V^2 C_{\text{equ}}}{2}$$

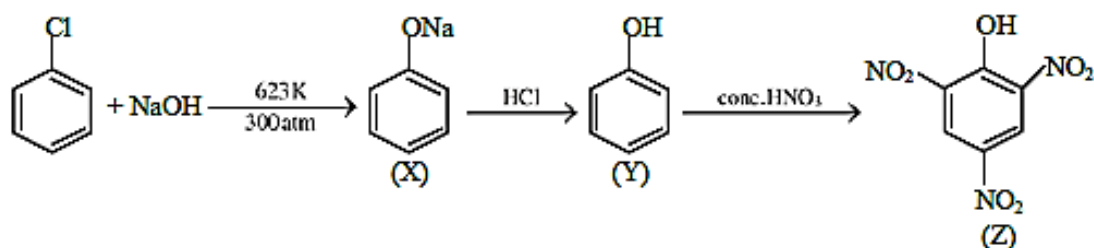
$$\frac{(100)^2}{2} \times \frac{450}{43} \times 10^{-6}$$

$$\Rightarrow \frac{4.5}{86} = \frac{9}{x} E = \frac{9}{x} \times 0.5 \Rightarrow x = 86$$

31. (d)

32. (a) When solid added no effect on equilibrium.

33. (c)



34. (b) Enthalpy of neutralization of SA & SB is always -57 kJ/mol because strong monoacid gives one mole of H^+ and strong mono base gives one mole of OH^- which form one mole of water.

35. (d)



(No. of e^-) 10 10 10 10

(Ionic radius) $\text{O}^{-2} > \text{F}^{-} > \text{Na}^{+} > \text{Mg}^{+2}$

Z_{eff} $\text{O}^{-2} < \text{F}^{-} < \text{Na}^{+} < \text{Mg}^{+2}$

36. (b) Compounds having same number of carbon atoms follow the boiling point order as:

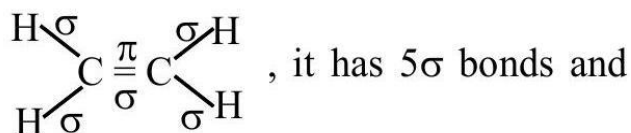
(Boiling point) $_{\text{Hydrogen bonding}} > (\text{Boiling point})_{\text{high polarity}} >$

(Boiling point) $_{\text{low polarity}} > (\text{Boiling point})_{\text{non polar}}$

37. (d) Statement I: Number of d & f electrons, increases down the group and due to poor shielding of d&f e^- , stability of lower oxidation states increases down the group

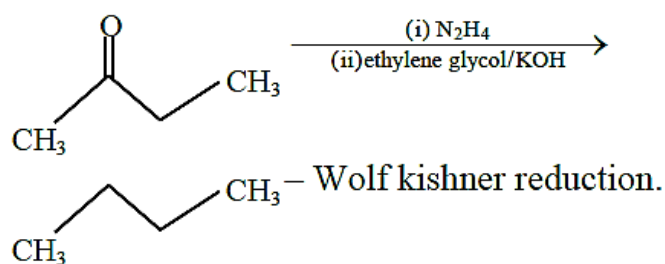
Statement II: The atomic size of aluminium is greater than that of gallium.

38. (d) ethylene is



1 π bond.

39. (b)



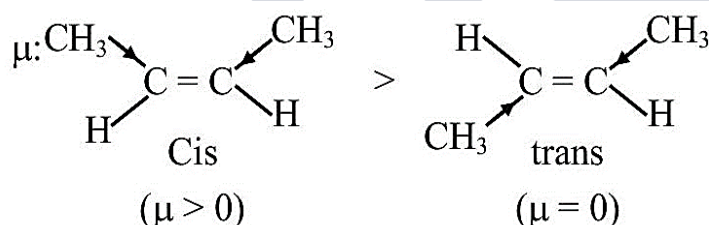
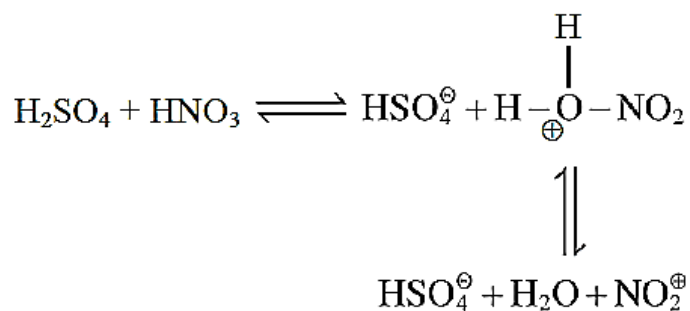
40. (a) In the cathode reaction manganese (Mn) is reduced from the +4-oxidation state to the +3 state.

41. (a)

	Number of unpaired e	$\mu = \sqrt{n(n+2)}$ B.M.
$[\text{Co}(\text{H}_2\text{O})_6]^{2+}$	3	3.87
$[\text{Fe}(\text{H}_2\text{O})_6]^{2+}$	4	4.89
$[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$	5	5.92
$[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$	4	4.89

Least paramagnetic behaviour = $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$

42. (c) Dipole moment is a vector quantity and for compound net dipole moment is the vector sum of all dipoles hence dipole moment of cis form is greater than trans form.


43. (b) In nitration of benzene concentrated H_2SO_4 and HNO_3 is used as reagent which generates electrophile NO_2^+ in following steps:


Lewis acids can promote the formation of electrophiles not Lewis base

44. (c) Experimental order $\text{Br}^- < \text{F}^- < \text{H}_2\text{O} < \text{NH}_3$

45. (d) Ninhydrin test is a test of amino acids. Egg albumin contains protein which is a natural polymer of amino acids which will show positive ninhydrin test

46. (b) Mn shows highest oxidation state (Mn^{+7}) in 3d series metals.

47. (d) only $C_{12}H_{22}O_{11}$ has 42.1% carbon, 6.4% hydrogen & 51.5 percent oxygen.

48. (d) Phenol is a highly activated compound which can undergo bromination directly with Bromine without any Lewis acid.

49. (b)

$$\Lambda_C^{+2} = 57 \text{ S cm}^2 \text{ mol}^{-1}$$

$$\Lambda_A^{+2} = 73 \text{ S cm}^2 \text{ mol}^{-1}$$

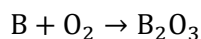
$$\Lambda_{\text{Solution}} = \lambda_C^{+2} + \Lambda_A^{-2}$$

$$= 57 + 73 = 130$$

50. (d) More abundant isotope = B^{11}

[Number of neutrons = 6]

$$x = 6$$



Oxidation state of B in $B_2O_3 = +3$

$$\text{So, } y = 3$$

$$\text{Hence } x + y = 9$$

51. (22)

$$V = 2.18 \times 10^6 \times \frac{Z}{n}$$

$$= 21.8 \times 10^5 \times \frac{1}{1} \approx 22 \times 10^5 \text{ (nearest)}$$

52. (6)

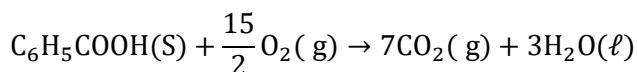
Fe^{+3} will give green coloured bead when heated at point B.

Number of unpaired e^- in $Fe^{+3} = 5$

$$\mu = 5.92$$

Nearest integer = 6

53. (150)



$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -321.30 - \frac{1}{2} \frac{R}{100} \times 300$$

$$= (-321.30 - 150R) \text{ kJ}$$

54. (14) Picric acid is prepared by treating phenol first with concentrated sulphuric acid which converts it to phenol-2,4-disulphonic acid and then with concentrated nitric acid to get 2, 4, 6 trinitrophenol.

55. (5) Strong oxidising agent = Co^{+3}

No. of unpaired e^- in $\text{Co}^{+3} [3d^6] = 4$

$$\text{Hence } \mu = \sqrt{n(n+2)} = \sqrt{24} \text{ BM}$$

Nearest integer = 5

56. (3)

$$r = K[A]^x[B]^y$$

$$\text{(I)} \quad 6 \times 10^{-3} = K[0.1]^x[0.1]^y$$

$$\text{(IV)} \quad 2.4 \times 10^{-2} = K[0.4]^x[0.1]^y$$

$$\text{(IV)/(I)}$$

$$4 = (4)^x$$

$$x = 1$$

$$r = K[A]^x[B]^y$$

$$\text{(III)} \quad 2.88 \times 10^{-1} = K[0.3]^x[0.4]^y$$

$$\text{(II)} \quad 7.2 \times 10^{-2} = K[0.3]^x[0.2]^y$$

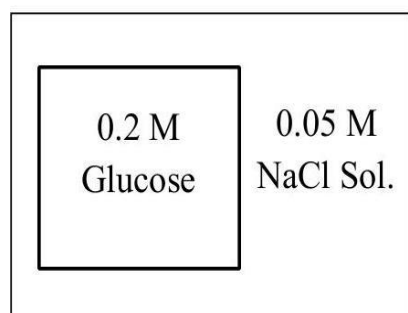
$$\text{(III)/(II)}$$

$$4 = 2^y$$

$$y = 2$$

$$\text{Overall order} = x + y = 1 + 2 = 3$$

57. (25)



$$0.05\text{M} \quad 0.05\text{M} \quad 0.05\text{M}$$

$$\text{Total } C_1 = 0.05 + 0.05 = 0.1\text{M}(\text{NaCl})$$

$$C_2 = 0.2\text{M}(\text{glucose})$$

$$\pi = (C_2 - C_1)RT$$

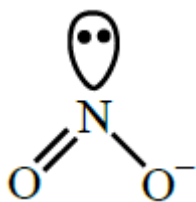
$$= (0.2 - 0.1) \times 0.083 \times 300$$

$$= 2.49\text{bar}$$

$$= 24.9 \times 10^{-1} \text{ bar}$$

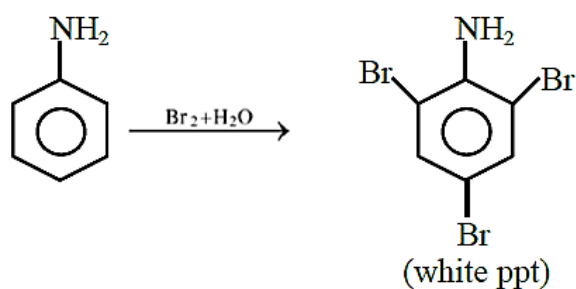
58. (2) In Sandmeyer reaction only bromobenzene & chlorobenzene are prepared

59. (8)



Number of valence e^- around N-atom = 8

60. (80)



93 g of aniline produces 330 g of 2,4,6 – tribromoaniline. Hence 9.3 g of aniline should produce 33 g of 2 , 4, 6-tribromoaniline. Hence percentage yield $\frac{26.4 \times 100}{33} = 80\%$

61. (c)

$$\frac{x+6}{3} = \frac{y}{2} = \frac{z+1}{1} = \lambda \dots\dots (1)$$

$$x = 3\lambda - 6, y = 2\lambda, z = \lambda - 1$$

$$\frac{x-7}{4} = \frac{y-9}{3} = \frac{z-4}{2} = \mu \dots\dots (2)$$

$$x = 4\mu + 7, y = 3\mu + 9, z = 2\mu + 4$$

$$3\lambda - 6 = 4\mu + 7 \Rightarrow 3\lambda - 4\mu = 13 \dots\dots (3) \times 2$$

$$2\lambda = 3\mu + 9 \Rightarrow 2\lambda - 3\mu = 9 \dots\dots (4) \times 3$$

$$6\lambda - 8\mu = 26$$

$$6\lambda - 9\mu = 27$$

$$\begin{array}{r} - + - \\ \hline \mu = -1 \end{array}$$

$$\Rightarrow 3\lambda - 4(-1) = 13$$

$$3\lambda = 9$$

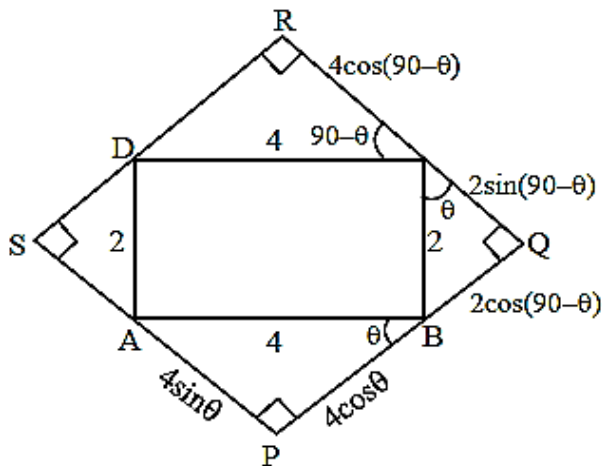
$$\lambda = 3$$

int. point (3,6,2); (7,8,9)

$$d^2 = 16 + 4 + 49 = 69$$

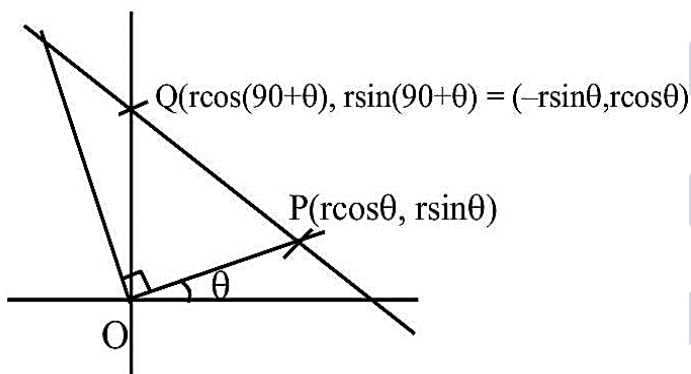
$$\text{Ans. } d^2 + 6 = 69 + 6 = 75$$

62. (a)



$$\begin{aligned}
 \text{Area} &= (4\cos\theta + 2\sin\theta)(2\cos\theta + 4\sin\theta) \\
 &= 8\cos^2\theta + 16\sin\theta\cos\theta + 4\sin\theta\cos\theta + 8\sin^2\theta \\
 &= 8 + 20\sin\theta\cos\theta \\
 &= 8 + 10\sin 2\theta \\
 \text{Max Area} &= 8 + 10 = 18 (\sin 2\theta = 1) \theta = 45^\circ \\
 (a+b)^2 &= (4\cos\theta + 2\sin\theta + 2\cos\theta + 4\sin\theta)^2 \\
 &= (6\cos\theta + 6\sin\theta)^2 \\
 &= 36(\sin\theta + \cos\theta)^2 \\
 &= 36(\sqrt{2})^2 \\
 &= 72
 \end{aligned}$$

63. (c)



$$3x + 4y = 12$$

$$3(r\cos\theta) + 4(r\sin\theta) = 12$$

$$r(3\cos\theta + 4\sin\theta) = 12 \dots\dots (1)$$

$$3(-r\sin\theta) + 4(r\cos\theta) = 12$$

$$r(-3\sin\theta + 4\cos\theta) = 12 \dots\dots (2)$$

$$\left(\frac{12}{r}\right)^2 + \left(\frac{12}{r}\right)^2 = (3\cos\theta + 4\sin\theta)^2 + (-3\sin\theta + 4\cos\theta)^2$$

$$2\left(\frac{12}{r}\right)^2 = 9 + 16$$

$$\frac{2 \times 144}{r^2} = 25 \Rightarrow 288 = 25r^2$$

$$\Rightarrow \frac{288}{25} = r^2$$

$$\Rightarrow \sqrt{2} \left(\frac{12}{5} \right) = r$$

$$\ell = OP^2 + PQ^2 + QO^2$$

$$\ell = r^2 + r^2 + r^2(\cos \theta + \sin \theta)^2 + r^2(\sin \theta + \cos \theta)^2$$

$$= 2r^2 + r^2(1 + \sin 2\theta + 1 - 2\sin 2\theta)$$

$$= 2r^2 + 2r^2$$

$$= 4r^2$$

$$= 4 \left(\frac{288}{25} \right) = \frac{1152}{25} = 46.08$$

$$[\ell] = 46$$

64. (b)

$$\frac{dy}{dx} + 2y = \sin 2x, y(0) = \frac{3}{4}$$

$$I.F = e^{\int 2dx} = e^{2x}$$

$$y \cdot e^{2x} = \int e^{2x} \sin 2x dx$$

$$y \cdot e^{2x} = \frac{e^{2x}(2\sin 2x - 2\cos 2x)}{4 + 4} + C$$

$$x = 0, y = \frac{3}{4} \Rightarrow \frac{3}{4} \cdot 1 = \frac{1(0 - 2)}{8} + C$$

$$\frac{3}{4} = -\frac{1}{4} + C$$

$$1 = C$$

$$y = \frac{2\sin 2x - 2\cos 2x}{8} + 1 \cdot e^{-2x}$$

$$x = \frac{\pi}{8}, y = \frac{1}{8} \left(2\sin \frac{\pi}{4} - 2\cos \frac{\pi}{4} \right) + e^{-2\left(\frac{\pi}{8}\right)}$$

$$y = 0 + e^{-\frac{\pi}{4}}$$

65. (d)

$$f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x) \quad x \in \left[0, \frac{\pi}{2}\right]$$

$$f'(x) = \cos x + 3 - \frac{2}{\pi}(2x + 1) > 0 \quad f(x) \uparrow$$

$$f'(x) = -\sin x + 0 - \frac{\pi}{2}(2)$$

$$= -\sin x - \frac{4}{\pi} < 0 \quad f'(x) \downarrow$$

$$0 < x < \frac{\pi}{2}$$

$$\Rightarrow -\frac{2}{\pi} \left(0 < 2x < \pi \right)$$

$$-\frac{2}{\pi} > -\frac{2}{\pi}(2x+1) > -\frac{2}{\pi}(\pi+1)$$

$$3 - \frac{2}{\pi} > 3 - \frac{2}{\pi}(2x+1) > 3 - \frac{2}{\pi}(\pi+1)$$

66. (c) $11x + y + \lambda z = -5$

$$2x + 3y + 5z = 3$$

$$8x - 19y - 39z = \mu$$

for infinite sol.

$$D = \begin{vmatrix} 11 & 1 & \lambda \\ 2 & 3 & 5 \\ 8 & -19 & -39 \end{vmatrix} = 0$$

$$\Rightarrow 11(-117 + 95) - 1(-78 - 40) + \lambda(-38 - 24)$$

$$\Rightarrow 11(-22) + 118 - \lambda(62) = 0$$

$$\Rightarrow 62\lambda = 118 - 242$$

$$\Rightarrow \lambda = \frac{-124}{62} = -2 \quad D_1 = \begin{vmatrix} -5 & 1 & -2 \\ 3 & 3 & 5 \\ \mu & -19 & -39 \end{vmatrix} = 0$$

$$\Rightarrow -5(-117 + 95) - 1(-117 - 5\mu) - 2(-57 - 3\mu) = 0$$

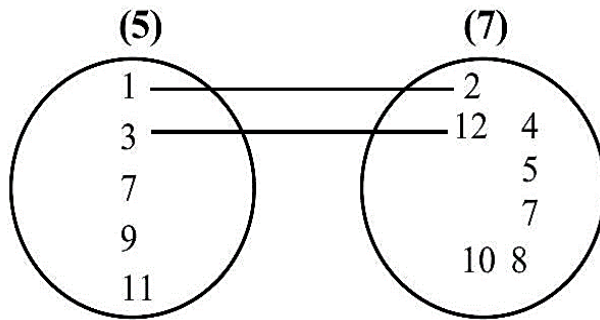
$$\Rightarrow -5(-22) + 117 + 5\mu + 114 + 6\mu = 0$$

$$\Rightarrow 11\mu = -110 - 231 = -341$$

$$\Rightarrow \mu = -31$$

$$\lambda^4 - \mu = (-2)^4 - (-31) = 16 + 31 = 47$$

67. (d)



$$A = \{1, 3, 7, 9, 11\}$$

$$B = \{2, 4, 5, 7, 8, 10, 12\}$$

$$f(a) + f(c) = 14$$

$$(i) 2 + 12$$

$$(ii) 4 + 10$$

$$2 \times (2 \times 5 \times 4 \times 3) = 240$$

$$68. (d) f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$$

is continuous at $x = 0$

$$\lim_{x \rightarrow 0} \frac{3x - \frac{(3x)^3}{6} + \dots + \alpha \left(x - \frac{x^3}{6} \dots \right) - \beta \left(1 - \frac{(3x)^2}{2} \dots \right)}{x^3} = f(0)$$

$$\lim_{x \rightarrow 0} \frac{-\beta + x(3 + \alpha) + \frac{9\beta x^2}{2} + \left(\frac{-27}{6} - \frac{\alpha}{6} \right) x^3 \dots}{x^3} = f(0)$$

for exist

$$\beta = 0, 3 + \alpha = 0, -\frac{27}{6} - \frac{\alpha}{6} = f(0)$$

$$\alpha = -3, -\frac{27}{6} - \frac{(-3)}{6} = f(0)$$

$$f(0) = \frac{-27 + 3}{6} = -4$$

$$69. (a) I = \int_0^{\pi/4} \frac{136 \sin x}{3 \sin x + 5 \cos x} dx$$

$$136 \sin x = A(3 \sin x + 5 \cos x) + B(3 \cos x - 5 \sin x)$$

$$136 = 3A - 5B$$

$$0 = 5A + 3B$$

$$3B = -5A \Rightarrow B = -\frac{5}{3}A$$

$$136 = 3A - 5\left(-\frac{5}{3}A\right)$$

$$136 = 3A + \frac{25}{3}A$$

$$136 = \frac{34A}{3}$$

$$\Rightarrow A = \frac{136 \times 3}{34} = 12$$

$$B = \frac{-5}{3}(12) = -20 \quad I = \int_0^{\pi/4} \frac{A(3\sin x + 5\cos x)}{3\sin x + 5\cos x} + \int_0^{\pi/4} \frac{B(3\cos x - 5\sin x)}{3\sin x + 5\cos x}$$

$$= A(x)_0^{\pi/4} + B[\ln(3\sin x + 5\cos x)]_0^{\pi/4}$$

$$= 12\left(\frac{\pi}{4}\right) - 20\ln\left(\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{2}}\right) - \ln(0 + 5)$$

$$= 3\pi - 20\ln 4\sqrt{2} + 20\ln 5$$

$$= 3\pi - 20 \times \frac{5}{2} \ln 2 + 20\ln 5$$

$$= 3\pi - 50\ln 2 + 20\ln 5$$

70. (c) $ax^2 + bx + c = 0$

$$a, b, c \in \{1, 2, 3, 4, 5, 6, 7, 8\}$$

Repeated roots $D = 0$

$$\Rightarrow b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$$

$$\text{Prob} = \frac{8}{8 \times 8 \times 8} = \frac{1}{64}$$

$$\Rightarrow (a, b, c)$$

$$(1, 2, 1); (2, 4, 2); (1, 4, 4); (4, 4, 1); (3, 6, 3);$$

$$(2, 8, 8); (8, 8, 2); (4, 8, 4)$$

8 case

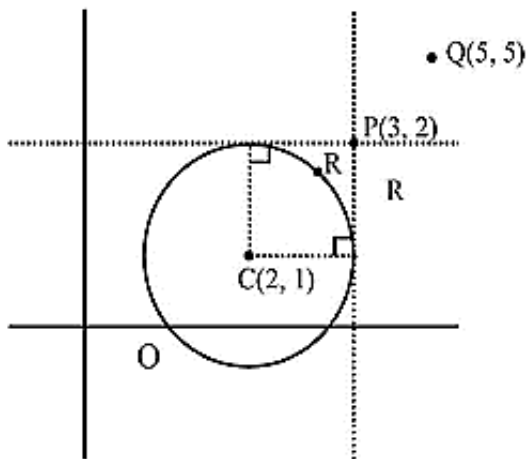
71. (a) $|A| = 3, |B| = 2$

$$|A^T A (\text{adj}(2A))^{-1} (\text{adj}(4B)) (\text{adj}(AB))^{-1} A A^T|$$

$$= 3 \times 3 \times |(\text{adj}(2A))^{-1}| \times |\text{adj}(4B)| \times |(\text{adj}(AB))^{-1}| \times 3 \times 3$$

$$\begin{aligned}
 & \downarrow \quad \quad \downarrow \quad \quad \downarrow \\
 & \frac{1}{|\text{adj}(2A)|} \cdot 2^{12} \times 2^2 \cdot \frac{1}{|\text{adj}(AB)|} \\
 & = \frac{1}{2^6 |\text{adj} A|} = \frac{1}{|\text{adj} B \cdot \text{adj} A|} \\
 & = \frac{1}{2^6 \cdot 3^2} = \frac{1}{2^2 \cdot 3^2} \\
 & = 3^4 \cdot \frac{1}{2^6 \cdot 3^2} \cdot 2^{12} \cdot 2^2 \cdot \frac{1}{2^2 \cdot 3^2} = 64
 \end{aligned}$$

72. (d)



Coordinates of the centre will be (2,1)

Equation of circle will be

$$(x - 2)^2 + (y - 1)^2 = 1$$

$$QC = \sqrt{(5 - 2)^2 + (5 - 1)^2}$$

$$QC = 5$$

shortest distance

$$= RQ = CQ - CR$$

$$= 5 - 1$$

$$= 4$$

73. (b) Centre of the circle = $\left(\frac{3}{2}, 1\right)$

$$\text{Equation of diameter} = 2x + 3y - k = 0$$

$$2\left(\frac{3}{2}\right) + 3(1) - k = 0$$

$$\Rightarrow k = 6$$

Now, Equation of ellipse becomes

$$x^2 + 9y^2 = 36$$

$$\frac{x^2}{6^2} + \frac{y^2}{2^2} = 1$$

$$\text{length of LR} = \frac{2b^2}{a} = \frac{2 \cdot 2^2}{6} = \frac{8}{6} = \frac{4}{3} = \frac{m}{n}$$

$$\therefore 2m + n = 2(4) + 3 = 11$$

74. (c) STATEMENT I:

$$(|z_1| + |z_2|) \left| \frac{z_1}{|z_1|} + \frac{z_2}{|z_2|} \right|$$

$$\text{Since } \left| \frac{z_1}{|z_1|} + \frac{z_2}{|z_2|} \right| \leq \left| \frac{z_1}{|z_1|} \right| + \left| \frac{z_2}{|z_2|} \right|$$

$$\left| \frac{z_1}{|z_1|} + \frac{z_2}{|z_2|} \right| \leq \frac{|z_1|}{|z_1|} + \frac{|z_2|}{|z_2|}$$

$$\left| \frac{z_1}{|z_1|} + \frac{z_2}{|z_2|} \right| \leq 2(|z_1| + |z_2|) \left(\left| \frac{z_1}{|z_1|} + \frac{z_2}{|z_2|} \right| \right) \leq 2(|z_1| + |z_2|)$$

$\therefore$ statement I is correct

For Statement II :

$$\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$$

$$\frac{a^2}{|y-z|^2} = \frac{b^2}{|z-x|^2} = \frac{c^2}{|x-y|^2} = \lambda$$

$$a^2 = \lambda(|y-z|^2) = \lambda(y-z)(\bar{y}-\bar{z})$$

$$b^2 = \lambda(z-x)(\bar{z}-\bar{x}) \text{ and } c^2 = \lambda(x-y)(\bar{x}-\bar{y})$$

$$\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = \lambda(\bar{y}-\bar{z} + \bar{z}-\bar{x} + \bar{x}-\bar{y}) = 0$$

Statement II is false

75. (a)

$$4 \left(\frac{1 - \tan^2 \theta/2}{1 + \tan^2 \theta/2} \right) - 3 \left(\frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \right) = 1$$

$$\text{let } \tan \frac{\theta}{2} = t$$

$$\frac{4 - 4t^2 - 6t}{1 + t^2} = 1$$

$$4 - 4t^2 - 6t = 1 + t^2$$

$$\Rightarrow 5t^2 + 6t - 3 = 0$$

$$\Rightarrow t = \frac{-6 \pm \sqrt{36 - 4(5)(-3)}}{2(5)}$$

$$= \frac{-6 \pm \sqrt{96}}{10}$$

$$= \frac{-6 \pm 4\sqrt{6}}{10}$$

$$t = \frac{-3 + 2\sqrt{6}}{5}$$

$$\cos \theta = \frac{1 - t^2}{1 + t^2} = \frac{1 - \left(\frac{2\sqrt{6} - 3}{5}\right)^2}{1 + \left(\frac{2\sqrt{6} - 3}{5}\right)^2} = \frac{1 - \left(\frac{24 + 9 - 12\sqrt{6}}{25}\right)}{1 + \left(\frac{24 + 9 - 12\sqrt{6}}{25}\right)}$$

$$= \frac{25 - 33 + 12\sqrt{6}}{25 + 33 - 12\sqrt{6}} = \frac{12\sqrt{6} - 8}{58 - 12\sqrt{6}} = \frac{6\sqrt{6} - 4}{29 - 6\sqrt{6}} \times \frac{29 + 6\sqrt{6}}{29 + 6\sqrt{6}}$$

$$= \frac{100 + 150\sqrt{6}}{625} = \frac{4 + 6\sqrt{6}}{25} \times \frac{4 - 6\sqrt{6}}{4 - 6\sqrt{6}}$$

$$= \frac{-200}{25(4 - 6\sqrt{6})} = \frac{-8}{4 - 6\sqrt{6}} = \frac{4}{3\sqrt{6} - 2}$$

76. (d)

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \dots + \frac{1}{\sqrt{99}+\sqrt{100}} = m \frac{\sqrt{1}-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} \dots \frac{\sqrt{99}-\sqrt{100}}{-1} = m \sqrt{100} - 1 = m \Rightarrow m = 9$$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{99 \cdot 100} = n$$

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} \dots \frac{1}{99} - \frac{1}{100} = n$$

$$1 - \frac{1}{100} = n$$

$$\frac{99}{100} = n$$

$$(m, n) = \left(9, \frac{99}{100}\right)$$

$$\Rightarrow 11(9) - 100\left(\frac{99}{100}\right)$$

$$= 99 - 99 = 0$$

Ans. option (d) $11x - 100y = 0$

77. (d) $f(x) = x^5 + 2x^3 + 3x + 1$

$$f'(x) = 5x^4 + 6x^2 + 3$$

$$f'(a) = 5 + 6 + 3 = 14$$

$$g(f(x)) = x$$

$$g'(f(x))f'(x) = 1$$

$$\text{for } f(x) = 7$$

$$\Rightarrow x^5 + 2x^3 + 3x + 1 = 7$$

$$\Rightarrow x = 1$$

$$g'(7)f'(a) = 1 \Rightarrow g'(7) = \frac{1}{f'(a)} = \frac{1}{14}$$

$$x = 1, f(x) = 7 \Rightarrow g(7) = 1$$

$$\frac{g(7)}{g'(7)} = \frac{1}{1/14} = 14$$

$$78. (a) A(1, -1, 2)$$

$$B(5, 7, -6)$$

$$C(3, 4, -10)$$

$$D(-1, -4, -2)$$

$$\text{Area} = \frac{1}{2} |\vec{AC} \times \vec{BD}| = \frac{1}{2} |(2\hat{i} + 5\hat{j} - 12\hat{k}) \times (6\hat{i} + 11\hat{j} - 4\hat{k})|$$

$$= \frac{1}{2} |112\hat{i} - 64\hat{j} - 8\hat{k}|$$

$$= 4 |14\hat{i} - 8\hat{j} - \hat{k}|$$

$$= 4\sqrt{196 + 64 + 1}$$

$$= 4\sqrt{261}$$

$$= 12\sqrt{29}$$

$$79. (a)$$

$$\int_{-\pi}^{\pi} \frac{2y(1 + \sin y)}{1 + \cos^2 y} dy$$

$$= \int_{-\pi}^{\pi} \frac{2y}{1 + \cos^2 y} dy + \int_{-\pi}^{\pi} \frac{2y \sin y}{1 + \cos^2 y} dy$$

$$(\text{Odd})$$

$$(\text{Even})$$

$$= 0 + 2.2 \int_0^{\pi} y \left(\frac{\sin y}{1 + \cos^2 y} \right) dy$$

$$I = 4 \int_0^{\pi} \frac{y \sin y}{1 + \cos^2 y} dy$$

$$I = 4 \int_0^{\pi} \frac{(\pi - y) \sin y}{1 + \cos^2 y} dy$$

$$2I = 4 \int_0^{\pi} \frac{\pi \sin y}{1 + \cos^2 y} dy$$

$$I = 2\pi \int_0^{\pi} \frac{\sin y}{1 + \cos^2 y} dy$$

$$= 2\pi (-\tan^{-1}(\cos y))_0^{\pi}$$

$$= -2\pi \left[\left(-\frac{\pi}{4}\right) - \left(\frac{\pi}{4}\right) \right]$$

$$= -2\pi \left[-\frac{2\pi}{4} \right] = \pi^2$$

80. (d) $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4-z$

$$\frac{x-2}{(-3)} = \frac{y-\frac{2}{3}}{\left(\frac{4\lambda+1}{3}\right)} = \frac{z-4}{(-1)}$$

$$\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$$

$$\frac{x+3}{3\mu} = \frac{y-\frac{1}{2}}{(-3)} = \frac{z-5}{(-7)}$$

Right angle $\Rightarrow (-3)(3\mu) + \left(\frac{4\lambda+1}{3}\right)(-3) + (-1)(-7) = 0$

$$-9\mu - 4\lambda - 1 + 7 = 0$$

$$4\lambda + 9\mu = 6$$

81. (56) X = denotes number of defective

x	0	1	2	3
P(x)	$\frac{7}{15}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$
x_1^2	0	1	4	9
$P_i x_1^2$	0	$\frac{5}{12}$	$\frac{20}{12}$	$\frac{9}{12}$
$p_i x_i$	0	$\frac{5}{12}$	$\frac{10}{12}$	$\frac{3}{12}$

$$\mu = \Sigma p_i x_i = \frac{18}{12}$$

$$\Sigma p_i x_i^2 = \frac{34}{12}$$

$$\sigma^2 = \Sigma p_i x_i^2 - (\mu)^2$$

$$= \frac{34}{12} - \left(\frac{18}{12}\right)^2 = \frac{17}{6} - \frac{9}{4}$$

$$\frac{34 - 27}{12} = \frac{7}{12}$$

$$96\sigma^2 = 96 \times \frac{7}{12} = 56$$

82. (54)

$$(1 + 2x - 3x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$$

$$\text{General term } m \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$$

$$= {}^9C_r \cdot \frac{3^{9-2r}}{2^{9-r}} (-1)^r \cdot x^{18-3r}$$

$$\text{Put } r = 6 \text{ to get coeff. of } x^0 = {}^9C_6 \cdot \frac{1}{6^3} \cdot x^0 = \frac{7}{18}x^0$$

$$\text{Put } r = 7 \text{ to get coeff. of } x^{-3} = {}^9C_7 \cdot \frac{3^{-5}}{2^2} (-1)^7 \cdot x^{-3}$$

$$= -{}^9C_7 \cdot \frac{1}{3^5 \cdot 2^2} \cdot x^{-3} = \frac{-1}{27}x^{-3}$$

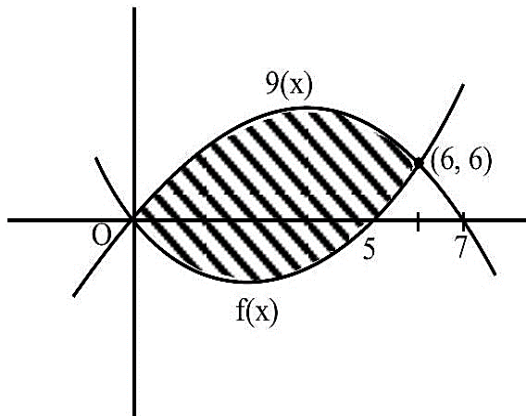
$$(1 + 2x - 3x^3) \left(\frac{7}{18}x^0 - \frac{1}{27}x^{-3}\right)$$

$$\frac{7}{18} + \frac{3}{27} = \frac{7}{18} + \frac{1}{9} = \frac{7+2}{18} = \frac{9}{18} = \frac{1}{2}$$

$$\therefore 108 \cdot \frac{1}{2} = 54$$

83. (72)

$$y = x^2 - 5x \text{ and } y = 7x - x^2$$



$$\int_0^6 (g(x) - f(x)) dx$$

$$\int_0^6 ((7x - x^2) - (x^2 - 5x)) dx$$

$$\int_0^6 (12x - 2x^2) dx = \left[12 \frac{x^2}{2} - \frac{2x^3}{3} \right]_0^6$$

$$\Rightarrow 6(6)^2 - \frac{2}{3}(6)^3$$

$$= 216 - 144 = 72 \text{ unit}^2$$

84. (125)

$$(x^1 + x^2 + \dots + x^6)^4$$

$$x^4 \cdot \left(\frac{1 - x^6}{1 - x} \right)^4$$

$$x^4 \cdot (1 - x^6)^4 \cdot (1 - x)^{-4}$$

$$x^4 [1 - 4x^6 + 6x^{12} - \dots] [(1 - x)^{-4}]$$

$$(x^4 - 4x^{10} + 6x^{16} - \dots)(1 - x)^{-4}$$

$$(x^4 - 4x^{10} + 6x^{16})(1 + {}^{15}C_{12}x^{12} + {}^9C_6x^6 - \dots)$$

$$({}^{15}C_{12} - 4 \cdot {}^9C_6 + 6)x^{16}$$

$$({}^{15}C_3 - 4 \cdot {}^9C_6 + 6)$$

$$= 35 \times 13 - 6 \times 8 \times 7 + 6$$

$$= 455 - 336 + 6$$

$$= 125$$

85. (18)

$$|2a - 1| = 3[a] + 2\{a\}$$

$$|2a - 1| = [a] + 2a$$

$$\text{Case-1 : } a > \frac{1}{2}$$

$$2a - 1 = [a] + 2a$$

$$[a] = -1 \therefore a \in [-1, 0) \text{ Reject}$$

$$\text{Case-2 : } a < \frac{1}{2}$$

$$-2a + 1 = [a] + 2a$$

$$a = I + f$$

$$-2(I + f) + 1 = I + 2I + 2f$$

$$I = 0, f = \frac{1}{4} \therefore a = \frac{1}{4}$$

$$\text{Hence } a = \frac{1}{4}$$

$$72 \sum_{a \in S} a = 72 \times \frac{1}{4} = 18$$

86. (24)

$$\lim_{t \rightarrow x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$$

$$\lim_{t \rightarrow x} \frac{2t \cdot f(x) - x^2 f'(x)}{1} = 1$$

$$2x \cdot f(x) - x^2 f'(x) = 1$$

$$\frac{dy}{dx} - \frac{2}{x} \cdot y = \frac{-1}{x^2}$$

$$\text{I.f.} = e^{\int -\frac{2}{x} dx} = \frac{1}{x^2}$$

$$\therefore \frac{y}{x^2} = \int -\frac{1}{x^4} dx + C$$

$$\frac{y}{x^2} = \frac{1}{3x^3} + C$$

$$\text{Put } f(a) = 1$$

$$C = \frac{2}{3}$$

$$y = \frac{1}{3x} + \frac{2x^2}{3}$$

$$y = \frac{2x^3 + 1}{3x}$$

$$f(2) = \frac{17}{6}$$

$$f(3) = \frac{55}{9}$$

$$2f(2) + 3f(3) = \frac{17}{3} + \frac{55}{3} = \frac{72}{3} = 24$$

87. (910)

$d \rightarrow$ common diff.

$$A_k = -kd[2a + (2k - 1)d]$$

$$A_3 = -153$$

$$\Rightarrow 153 = 13d[2a + 5d]$$

$$51 = d[2a + 5d] \dots \dots \dots (1)$$

$$A_5 = -435$$

$$435 = 5d[2a + 9d]$$

$$87 = d[2a + 9d]$$

$$(2) - (1)$$

$$36 = 4d^2$$

$$d = 3, a = 1$$

$$a_{17} - A_7 = 49 - [-7.3[2 + 39]] = 910$$

88. (30)

$$(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$$

$$(2\vec{b} + 4\vec{a}) \times \vec{c} = 0$$

$$\vec{c} = \lambda(4\vec{a} + 2\vec{b}) = \lambda(8\hat{i} - 14\hat{j} + 30\hat{k})$$

$$\vec{a} \cdot \vec{c} = 130$$

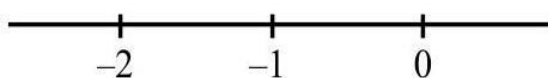
$$8\lambda + 42\lambda + 210\lambda = 130$$

$$\lambda = \frac{1}{2}$$

$$\vec{c} = 4\hat{i} - 7\hat{j} + 15\hat{k}$$

$$\vec{b} \cdot \vec{c} = 8 + 7 + 15 = 30$$

89. (3)



Case-1

$$x \geq 0$$

$$x^2 + 2x - 5x - 5 - 1 = 0$$

$$x^2 - 3x - 6 = 0$$

$$x = \frac{3 \pm \sqrt{9 + 24}}{2} = \frac{3 \pm \sqrt{33}}{2}$$

One positive root

Case-2

$$-1 \leq x < 0$$

$$-x^2 - 2x - 5x - 5 - 1 = 0$$

$$x^2 + 7x + 6 = 0$$

$$(x + 6)(x + 1) = 0$$

$$x = -1$$

one root in range

Case-3

$$-2 \leq x < -1$$

$$x^2 - 2x + 5x + 5 - 1 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

No root in range

Case-4

$$x < -2$$

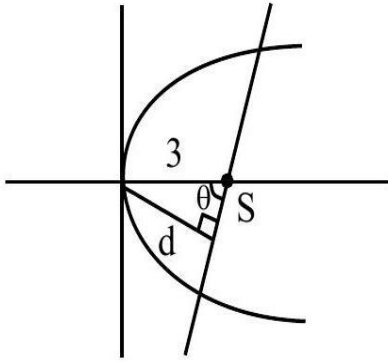
$$x^2 + 7x + 4 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 16}}{2} = \frac{7 \pm \sqrt{33}}{2}$$

one root in range

Total number of distinct roots are 3

90. (108)



$$\ell = 4a \operatorname{cosec}^2 \theta$$

$$\ell = 12 \times \frac{9}{d^2}$$

$$\ell d^2 = 108$$

