1. (a)

As $\lambda_{red} > \lambda_{yellow} > \lambda_{violet}$

Light ray with longer wavelength bends less.

2. (c) $KE_{max} = h\nu - \phi_0 = eV$

3. (a) $F_C = \frac{mv^2}{r}$

$$\frac{Kq_1q_2}{r^2} = \frac{mv^2}{r}$$

$$mv^2r^2 = Kq_1q_2r$$

$$\frac{L^2}{m} = Kq_1q_2r$$

$$L \propto \sqrt{r}$$

4. (c)

$$i_g = \frac{10}{400 + 100} = 20 \times 10^{-3}\ A$$

For ammeter

Let shunt resistance = S

$$i_gR = (i - i_g)S$$

$$20 \times 10^{-3} \times 100 = 10\ S$$

$$S = 20 \times 10^{-2}\Omega$$

5. (a) Static charge is developed due to air friction. This can result in combustion. So, metallic chains is used to discharge excess charge.

6. (c)

n = number of molecule per unit volume

d = diameter of the molecule

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

(By Theory)

7. (a)

$\vec{P} = \cos(kt)\hat{i} - \sin(kt)\hat{j}; |\vec{P}| = 1$

$\because \vec{P} = m\vec{v}$

$\therefore \hat{P} = \hat{V}$

$\Rightarrow \hat{v} = \cos(kt)\hat{i} - \sin(kt)\hat{j}$

$\hat{a} = \dfrac{-k\sin(kt)\hat{i} - k\cos(kt)\hat{j}}{k}$

$\Rightarrow \hat{a} = -\sin kt\hat{i} - \cos kt\hat{j}$

$\because \hat{F} = \hat{a} = -\sin kt\hat{i} - \cos kt\hat{j}$

$\cos\theta = \dfrac{\hat{F}\cdot\hat{P}}{|\hat{F}||\hat{P}|} = -\dfrac{\sin kt\cos t + \sin kt\cos t}{1\times 1} = 0$

$\Rightarrow \theta = \dfrac{\pi}{2}$

8. (c)

$\vec{F}_1 = q\vec{E}$ (Theory)

$\vec{F}_2 = q(\vec{V}\times\vec{B})$

9. (b) Given: $R = 9$ m,

120 revolution in 3 min

$\omega = \dfrac{120\text{ Re v.}}{3\text{ min .}} = \dfrac{120\times 2\pi\text{rad}}{3\times 60\text{sec}} = \dfrac{4\pi}{3}\text{rad/s}$

$a_{\text{centripetal}} = \omega^2 R = \left(\dfrac{4\pi}{3}\right)^2 \times 9 = 16\pi^2\ \text{m/s}^2$

10. (c) Voltage across inductor $V_L = IX_L$

$31.4 = I[L\omega]$
$31.4 = I[L(2\pi f)]$
$31.4 = I[10\times 10^{-3}(2\times 3.14)\times 50$
$\Rightarrow I = 10\text{ A}$

11. (b)

$.\because [V] = [b]$

$\therefore$ Dimension of $b = [L^3]$

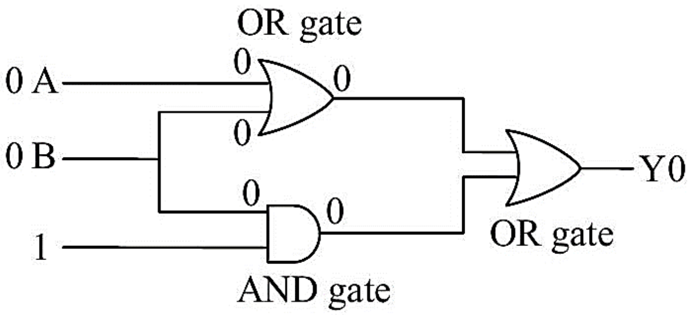
$\&[P] = \left[\dfrac{a}{V^2}\right]$

$[a] = [PV^2] = [ML^{-1}T^{-2}][L^6]$

Dimension of $a = [ML^5T^{-2}]$

$\therefore ab^{-1} = \dfrac{[ML^5T^{-2}]}{[L^3]} = [ML^2T^{-2}]$

12. (b)



13. (c) $P = \text{costant} \Rightarrow FV = \text{constant}$

$\Rightarrow m\frac{dV}{dt}V = \text{constant}$

$\int_0^V VdV = (C)\int_0^t dt$

$\left(\frac{V^2}{2}\right) = Ct$

$V \propto t^{1/2}$

$\frac{ds}{dt} \propto t^{1/2}$

$\int_0^S ds = K\int_0^t t^{1/2}dt$

$S = K \times \frac{2}{3}t^{3/2}$

$S \propto t^{3/2}$

$\therefore$ displacement is proportional to $(t)^{3/2}$

14. (b) Infrared is the least energetic thus having biggest wavelength (λ)& gamma rays are most energetic thus having smallest wavelength (λ).

15. (c)

$P \propto T^3$

$PT^{-3} = \text{constant}$

$\because \frac{PV}{T} = nR = \text{constant from ideal gas equation}$

$(P)\ (PV)^{-3} = \text{constant}$

$P^{-2}\ V^{-3} = \text{constant}\ldots\ldots\ldots\ldots (1)$

$\because$ Process equation for adiabatic process is

$PV^y = \text{constant}\ldots\ldots\ldots.. (2)$

Comparing equation (1) and (2)

$\frac{C_P}{C_V} = y = \frac{3}{2}$

16. (c)

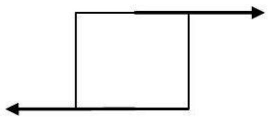
(A) stress $= \frac{F_{\text{restoring}}}{A}$

If $A = 1$

Stress $= F_{\text{restoring}}$

(A)-(III)

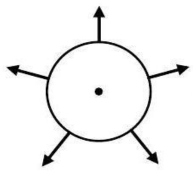
(B)



$\Rightarrow$ Shear stress

(B)-(IV)

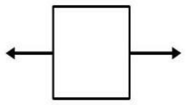
(C)



$\Rightarrow$ Volumetric stress

(C)-(I)

(D)



$\Rightarrow$ Longitudinal stress

(D)-(II)

17. (c) $20\text{VSD} = 19\text{MSD}$

$1\text{VSD} = \frac{19}{20}\text{MSD}$

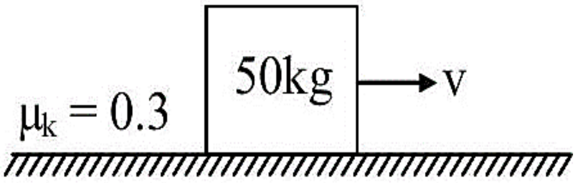
$\text{L.C.} = 1\text{MSD} - 1\text{VSD}$

$0.1 \text{ mm} = 1\text{MSD} - \frac{19}{20}\text{MSD}$

$0.1 = \frac{1}{20}\text{MSD}$

$1\text{MSD} = 2 \text{ mm}$

18. (b)



$F_k = \mu_k N = 0.3 \times 50 \times 9.8 = 147$ N

19. (d)

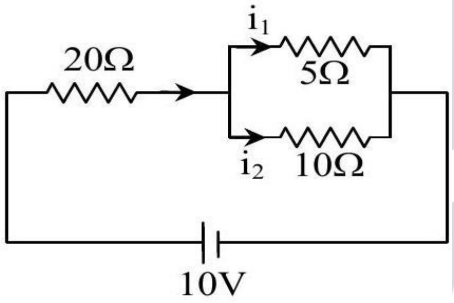
$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}\left(\frac{M_2}{M_1}\right)^{1/2}$$

$$\frac{6}{24} = \frac{(r_1)^{3/2}}{(4.2 \times 10^4)^{3/2}}\left(\frac{M}{M/4}\right)^{1/2}$$

$r_1 = 1.05 \times 10^4$ km

20. (b)



$$\frac{i_1}{i_2} = \frac{10}{5} = \frac{2}{1}$$

$$\frac{P_1}{P_2} = \frac{i_1^2 R_1}{i_2^2 R_2} = \left(\frac{2}{1}\right)^2 \times \frac{5}{10} = \frac{2}{1}$$

21. (500)

$\mu_0 ni = B$ n = number of turns per unit length

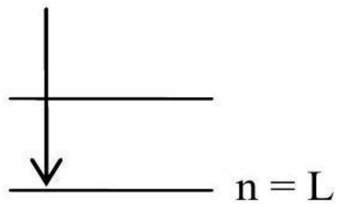
$$\mu_0\left(\frac{m}{\ell}\right)i = B$$

$$m = \frac{B \cdot \ell}{\mu_0 i} = \frac{6.28 \times 10^{-3} \times 0.5}{12.56 \times 10^{-7} \times 5}$$

$m = 500$

22. (6588)

Lyman Series



Shortest, $\frac{hc}{\lambda} = -13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\lambda \downarrow E \uparrow; \frac{hc}{\lambda_0} = -13.6(a)$

Balmer Series :

_______n = 3

_________n = 2

$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

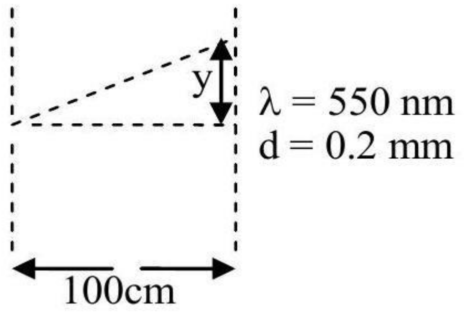
$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{hc}{\lambda_1} = -13.6 \times \left(\frac{5}{36}\right)$$

$$\Rightarrow \frac{-13.6\lambda_0}{\lambda_1} = -13.6 \times \frac{5}{36}$$

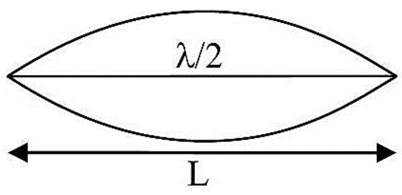
$$\lambda_1 = \frac{\lambda_0 \times 36}{5} = \frac{915 \times 36}{5} = 6588$$

23. (275)



$$y = \frac{\lambda D}{d} = \frac{550 \times 10^{-9} \times 100 \times 10^{-2}}{0.2 \times 10^{-3}} = 275$$

24. (60)



$f_0 = 400$ Hz; $v = \sqrt{\frac{T}{\mu}} =$ constant

$\frac{\lambda}{2} = L;\ v = f_0\lambda$

$\frac{v}{2f_0} = L \Rightarrow v = 2Lf$

$L' = \frac{v}{2f'} = \frac{2Lf_0}{2f'}$

$= \frac{Lf_0}{f'} = \frac{90 \times 400}{600} = 60$

25. (2)

$$\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2} = \frac{\left(\frac{1}{2}\right)\left(\frac{2}{3}mR^2\right)\omega^2}{\left(\frac{1}{2}\right)\left(\frac{2}{3}mR^2\right)\omega^2 + \frac{1}{2}\ m(R\omega)^2}$$

$$= \frac{\frac{2}{3}}{\frac{2}{3}+1} = \frac{2}{5}$$

$x = 2$

26. (1000 N)

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$\frac{F_1}{\pi(7)^2} = \frac{10}{\pi \times (0.7)^2}$

$F_1 = 1000$ N

27. (16)

$E_P = \frac{2KP}{r^3} = E$

$E_R = \frac{KP}{(2r)^3} = \frac{E}{16}$

$x = 16$

28. (16)

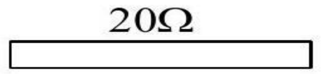
$H_{max} = \frac{u^2 \sin^2\theta}{2\ g}$

$\frac{H_{1max}}{H_{2max}} = \frac{u_1^2}{u_2^2}$

$$\frac{64}{H_{2max}} = \frac{u^2}{(u/2)^2}$$

$H_{2\ max} = 16\ m$

29. (5)



$\Rightarrow$ 10 equal part

Each part has resistance $= 2\Omega$

2 parts are connected in parallel so, $R = 1\Omega$

Now, there will be 5 parts each of resistance 1Ω, they are connected in series.

$R_{eq} = 5R, R_{eq} = 5\Omega$

30. (4)

$I = 3t + 8$

$\varepsilon = 12mV$

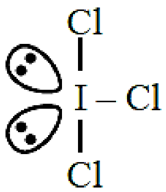
$$|\varepsilon| = L\left|\frac{dI}{dt}\right|$$

$12 = L \times 3$

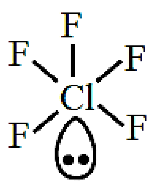
$L = 4mH$

31. (c)

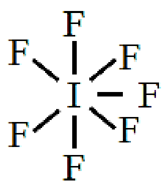
A. I – Cl (iv) linear

B. 

(I) T-shape

C. 

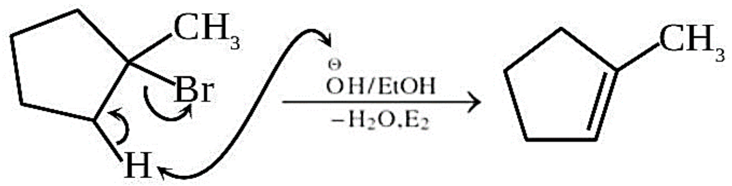
(II) Square pyramidal

D. 

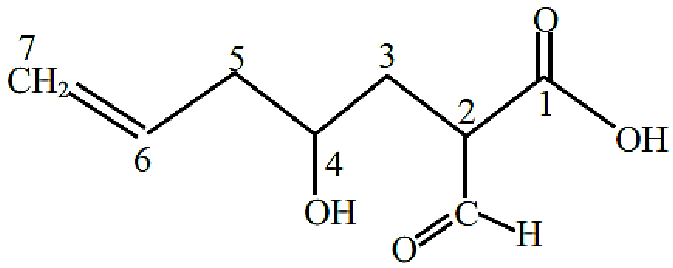
(III) Pentagonal bipyramidal

32. (a) Fe^{+2} ions undergoes hydrolysis, therefore while preparing aqueous solution of ferrous sulphate and ammonium sulphate in water dilute sulphuric acid is added to prevent hydrolysis of ferrous sulphate.

33. (c)

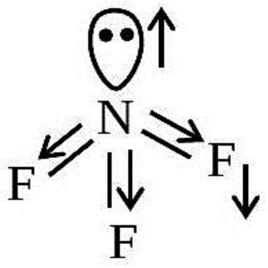


34. (c)

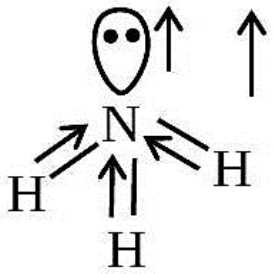


2-formly-4-hydroxyhept-6-enoic acid

35. (a)



Resultant dipole moment $= 0.80 \times 10^{-30}$Cm

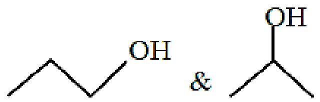


Resultant dipole moment $= 4.90 \times 10^{-30}$ cm

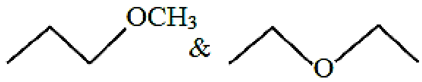
36. (a) $BaCl_2$, NaCl are soluble but on adding HCl(g) to $BaCl_2$, NaCl solutions, Sodium or Barium chlorides may precipitate out, as a consequence of the law of mass action.

37. (b) Tetrahedral complex does not show geometrical isomerism.

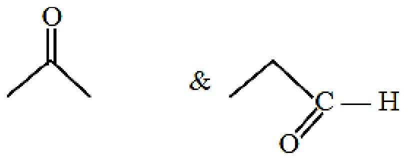
38. (d)



$\Rightarrow$ Position isomers



$\Rightarrow$ Metamers



$\Rightarrow$ Functional isomers

neopentane & isopentane $\Rightarrow$ Chain isomers

39. (d)

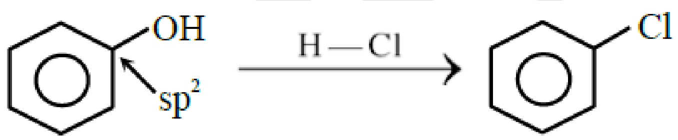
$W = ZIt$

$W = ZQ$

$Q = \frac{W}{Z}$

$W = ZQ = $ (electrochemical equivalent)

40. (b)



Not Possible

41. (a)

$r_{Na} > r_{Na^+}$

So, Statement (I) is correct but size of anions are greater than size of neutral atoms.

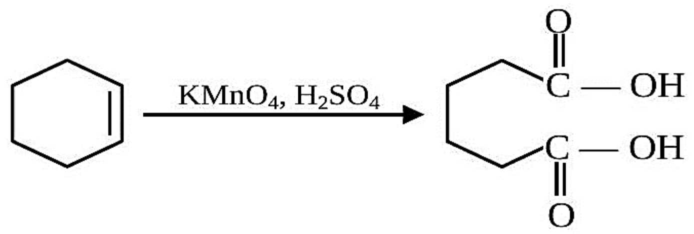
So statement (II) is incorrect.

42. (a)

$$CH_3-CH_2-OH \xrightarrow[KMnO_4]{\text{Joner reagent }(CrO_3 + H^{\oplus})} CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-OH$$

$$CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-OH \xrightarrow[\text{NaOH, CaI, }\Delta]{\text{Soda lime process}} CH_4 + Na_2CO_3$$

43. (d)



44. (c) $M \mid M^{+2} \parallel X/X^{2-}$

$E^{o}_{cell} = E^{o}_{M/M^{+2}} + E^{o}_{X/X^{-2}}$

$= -0.46 + 0.34 = -0.12\ V$

As E^{o}_{cell} is negative so anode becomes cathode and cathode become anode. Spontaneous reaction will be $M^{+2} + X^{2-} \longrightarrow M + X$

45. (b)

$$C_nH_{2n+2} + \frac{3n+1}{2}O_2 \longrightarrow nCO_2 + (n+1)H_2O$$

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

4gm 11gm

0.25 mole 0.25 mole

0.25 $mole_{CH_4}$ gives 0.25 mole (or 11gm) CO_2

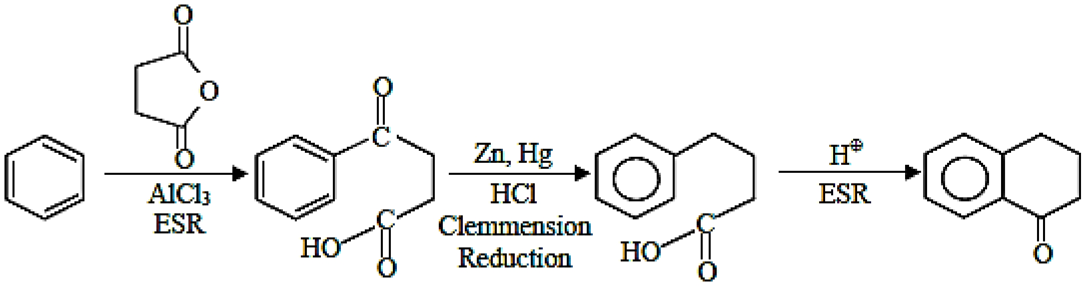
46. (a)

$TiCl_4 \Rightarrow Ti^{+4}$ $e^0t_2^0$
$MnO_4^- \Rightarrow Mn^{+7}$ $e^0t_2^0$
$FeO_4^{2-} \Rightarrow Fe^{+6}$ $e^2t_2^0$
$FeCl_4^{2-} \Rightarrow Fe^{+2}$ $e^3t_2^3$
$CoCl_4^{2-} \Rightarrow Co^{+2}$ $e^4t_2^3$

47. (c) The ions Ti^{+2}, $V^{+2}Cr^{+2}$ are strong reducing agents and will liberate hydrogen from a dilute acid, eg.

$2Cr^{+2}_{(aq.)} + 2H^{+}_{(aq.)} \longrightarrow 2Cr^{+3}_{(aq.)} + H_2\,(g)$

48. (b)



(A) (B)

49. (a)

A. size order $T\ell > In > Al > Ga > B$

B. Electronegativity order $B > Al < Ga < In < T\ell$
D. B, Al are more stable in +3 oxidation state

So, only C, E statements are correct.

50. (a) Coagulation of egg give primary structure of protein, which is known as denaturation of protein
51. (6535)

$6C$ (graphite) $+3H_2(g) \rightarrow C_6H_6(\ell); \Delta H = 48.5\ \text{kJ/mol}$

C (graphite) $+O_2(g) \rightarrow CO_2(g); \Delta H = -393.5\ \text{kJ/mol}\ H_2^{(g)} + \frac{1}{2}(g) \longrightarrow H_2O(\ell); \Delta H = -286\ \text{kJ/mol}$

equation $-(1) \times 1 + (2) \times 6 + (c) \times 3$

$-48.5 - 6 \times 393.5 - 3 \times 286$

$= -3267.5$ kJ for 1 mol

$= -6535$ kJ for 2 mol

52. (6)

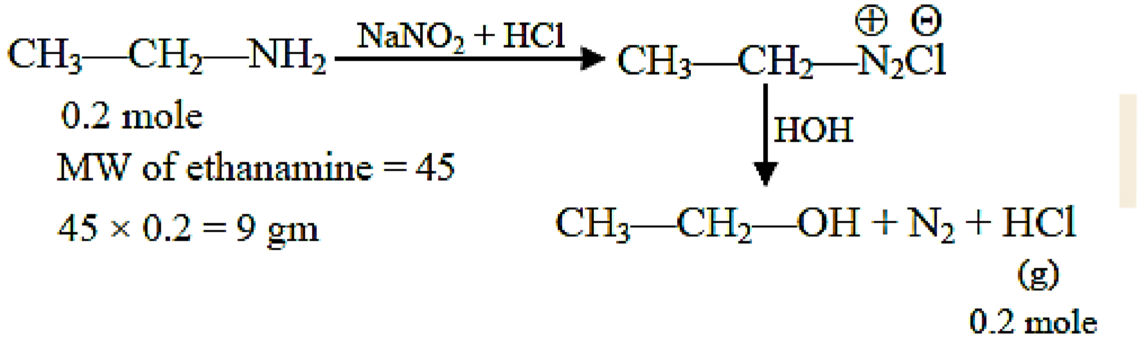
$4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow$

$\underset{A}{8Na_2CrO_4} + \underset{B}{2Fe_2O_3} + 8CO_2$

Spin only magnetic moment

For Na_2CrO_4 $\mu_B = 0$
For Fe_2O_3 $\mu_B = 5.9$
sum $= 5.9$

53. (9)



54. (6)

$n = 4$

ℓ	m_ℓ
0	0
1	$-1, 0, +1$
2	$-2, -1, 0, +1, +2, +3$

So number of orbital associated with $n = 4, |m_\ell| = 1$ are 6

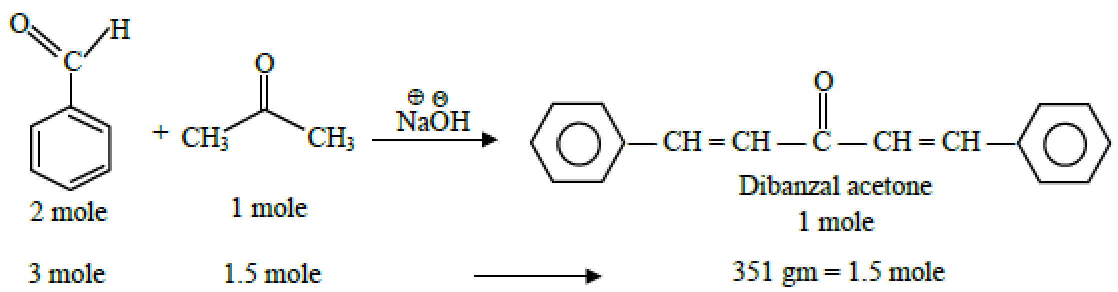
Now each orbital contain one e^- with $m_s = -\frac{1}{2}$

55. (50)

R_f of $A = \frac{5}{12.5}$ R_f of $C = \frac{10}{12.5}$

Ratio $= \frac{R_{f(A)}}{R_{f(C)}} = \frac{1}{2} = 0.5$ or 50×10^{-2}

56. (318) Claisen Schmidt reaction



mw of benzaldehyde = 106

$106 \times 3 = 318$gm. Benzaldehyde is required to give 1.5 mole (or 351gm) product

57. (315)

$2A(g) + B(g) \longrightarrow C(g)$

r_1 1.5 atm 0.7 atm

r_2 0.5 atm 0.2 atm 0.5 atm

$\because r = K[P_A]^2[P_B]$

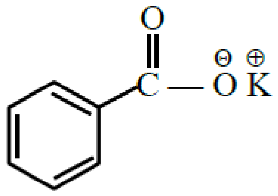
$r_1 = K[1.5]^2[0.7]$

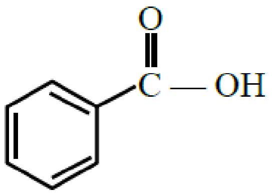
$r_2 = K[0.5]^2[0.2]$

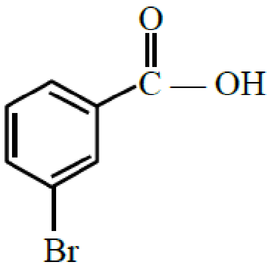
$\frac{r_1}{r_2} = 9 \times \frac{7}{2} = 31.5 = 315 \times 10^{-1}$

Ans. 315

58. (4)

A = 

B = 

C = 

π bonds = 4

59. (19)

$$\begin{aligned}&\text{Mass of } CH_3COOH = V \times d\\ &= 5\text{ml} \times 1.2 \text{ g/ml}\\ &= 6\text{gm}\end{aligned}$$

$$n_{CH_3COOH} = \frac{6}{60} = 0.1 \text{ mol}$$

$$m_{CH_3COOH} \approx M_{CH_3COOH} = \frac{0.1}{1} = 0.1\text{M}$$

$$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$$

C

$C - C\alpha \; C\alpha \; C\alpha$

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

$$1 - \alpha \approx 1 \Rightarrow K_a = C^2$$

$$\alpha = \sqrt{\frac{Ka}{C}} = \sqrt{\frac{6.25 \times 10^{-5}}{0.1}} = 25 \times 10^{-3}$$

$$\begin{aligned}&\text{V.f. (i) } = 1 + \alpha(n-1) = 1 + \alpha(2-1) = 1 + \alpha\\ &= 1 + 25 \times 10^{-3} = 1.025\end{aligned}$$

$$\begin{aligned}&\Delta T_f = 1\, K_f m\\ &= (1.025)(1.86)(0.1)\\ &= 0.19\\ &= 19 \times 10^{-2}\end{aligned}$$

60. (6) $H_2, CO_2, BF_3, CH_4, SiF_4, BeF_2$ are symm. molecule so dipole moment is zero

61. (c)

62. (c)

$C \equiv x^2 + y^2 + gx + gy = 0 \ldots\ldots\ldots\ldots (1)$

$2x + 2yy' + g + gy' = 0$

$g = -\left(\frac{2x + 2yy'}{1 + y'}\right)$

Put in (1)

$x^2 + y^2 - \left(\frac{2x + 2yy'}{1 + y'}\right)(x + y) = 0$

$(x^2 - y^2 - 2xy)y' = x^2 - y^2 + 2xy$

63. (d)

$S_1 : x^2 + y^2 \le 25 \ldots\ldots (1)$

$S_2 : \text{lm of } \frac{z + (1 - \sqrt{3}i)}{(1 - \sqrt{3}i)} \ge 0$

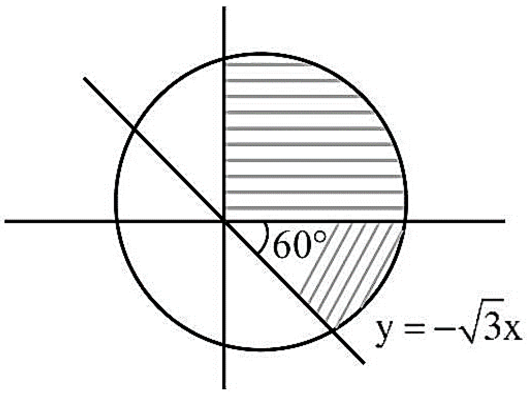
$\text{lm of } \left(\frac{x + iy}{1 - \sqrt{3}i} + 1\right) \ge 0$

$\text{lm of } \left(\frac{(x + iy)(1 + \sqrt{3}i)}{4}\right) \ge 0$

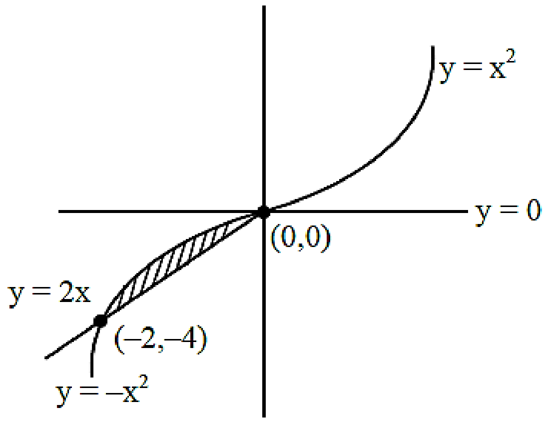
$\Rightarrow \sqrt{3}x + y \ge 0 \ldots\ldots\ldots\ldots (2)$

$S_3 : x \ge 0 \ldots\ldots\ldots\ldots\ldots\ldots (3)$

$\text{Area} = \frac{5}{12}(\pi(5)^2)$



64. (d)



$A=\int_{-2}^{0}-x^2-2x=\frac{4}{3}$

65. (c)

B B H J O

$\boxed{B}$ ______ $4!=24$

$\boxed{H}$ ______ $\frac{4!}{2!}=12$

$\boxed{J}$ ______ $\frac{4!}{2!}=12$

O B B H J
O B B J H $\to 50^{th}$ rank

66. (b)

Let's assume $\vec{v}=\vec{a}+\vec{b}+\hat{i}$

$=5\hat{i}+3\hat{j}+\hat{k}$

and $\vec{c}+\hat{i}=\vec{p}$

So,

$\vec{p}\times\vec{v}=\vec{a}\times\vec{p}$

$\vec{p}\times\vec{v}+\vec{p}\times\vec{a}=\vec{0}$

$\vec{p}\times(\vec{v}+\vec{a})=\vec{0}$

$\Rightarrow\vec{p}=\lambda(\vec{v}+\vec{a})$

$\vec{c}+i=\lambda(7\hat{i}+8\hat{j})$

$\bar{a}\cdot\bar{c}+\bar{a}\cdot\hat{i}=\lambda\bar{a}\cdot(7\hat{i}+8\hat{j})$

$-29+2=\lambda(14+40)$

$\lambda = -\frac{1}{2}$

$\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k}) + \hat{i} \cdot (-2\hat{i} + \hat{j} + \hat{k}) = \lambda(7\hat{i} + 8\hat{j}) \cdot (-2\hat{i} + \hat{j} + \hat{k})$

$= -\frac{1}{2}(-14 + 8) + 2 = 5$

67. (c) $|\vec{c} - \vec{a}| = |\vec{c}|^2 + |\vec{a}|^2 - 2\bar{a}.\bar{c}$

$= |\vec{c}|^2 + 4 - 0$

$\because \vec{a} = \vec{b} \times \vec{c}$

$|\vec{a}| = |\vec{b} \times \vec{c}|$

$2 = 3|\vec{\mathbf{c}}|\sin\alpha$

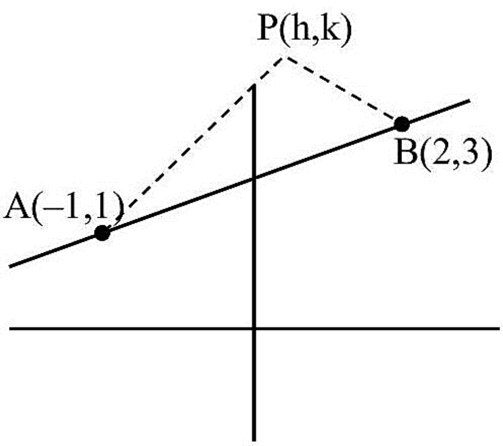
$|\vec{c}| = \frac{2}{3}\operatorname{cosec}\alpha \ \alpha \in \left[0, \frac{\pi}{3}\right]$

$|\vec{c}|_{\min} = \frac{2}{3} \times \frac{2}{\sqrt{3}}$

$\operatorname{cosec}\alpha \in \left[\frac{2}{\sqrt{3}}, \infty\right)$

$\Rightarrow 27|\vec{c} - \vec{a}|^2_{\min} = 27\left(\frac{16}{27} + 4\right) = 124$

68. (a)



$\frac{1}{2}\begin{vmatrix} h & k & 1 \\ -1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 10$

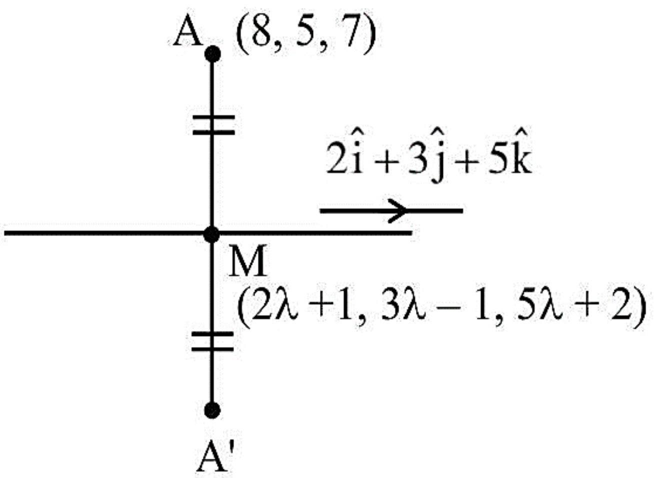
$-2x + 3y = 25$

$-\frac{6}{5}x + \frac{9}{5}y = 15$

$a = -\frac{6}{5}, b = \frac{9}{5}$

$5a = -6, 2b = \frac{18}{5}$

69. (c)



$\overrightarrow{AM}\cdot(2\hat{i}+3\hat{j}+5\hat{k}) = 0$

$(2\lambda-7)(b)+(3\lambda-6)(c)+(5\lambda-5)(5) = 0$

$38\lambda = 57$

$\lambda = \frac{3}{2}$

$M\left(4, \frac{7}{2}, \frac{19}{2}\right)$

$A'(0,2,12)$

70. (c)

$$T_{r+1} = {}^{12}C_r\left(\frac{3^{1/5}}{x}\right)^{12-r}\left(\frac{2x}{5^{1/3}}\right)^r$$

$$T_{r+1} = \frac{{}^{12}C_r(3)^{\frac{12-r}{5}}(2)^r(x)^{2r-12}}{(5)^{r/3}}$$

$r = 6$

$$T_7 = \frac{{}^{12}C_6(3)^{6/5}(2)^6}{5^2} = \left(\frac{9\times11\times7}{25}\right)2^8\cdot3^{1/5}$$

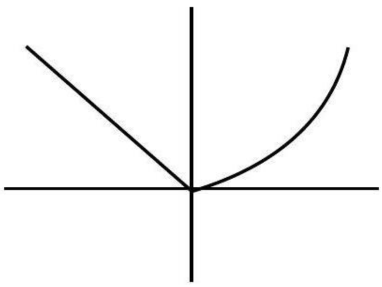
$25\alpha = 693$

71. (a)

$f(g(x)) = |g(x)-1|$

$$fog\begin{bmatrix}|e^x-1| & x\geq0\\ |x+1-1| & x\leq0\end{bmatrix}$$

fog $\begin{bmatrix} e^x - 1 & x \geq 0 \\ -x & x \leq 0 \end{bmatrix}$



72. (a) $S_1 : x^2 + y^2 - 2x - 2y + 1 = 0$

$S_2 : x^2 + y^2 + 2x - 3 = 0$

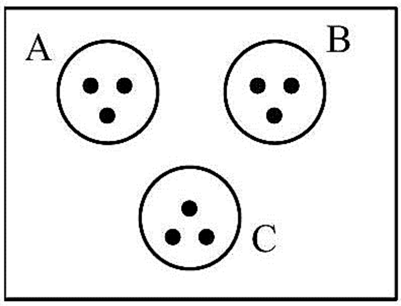
Common chord $= S_1 - S_2 = 0$

$-4x - 2y + 4 = 0$

$2x + y = 2 \Rightarrow P(0,2)$

$d^2_{(c,p)} = (1-0)^2 + (2-1)^2 = 2$

73. (a)



$\frac{9!}{(3!3!3!)3!} \times 3\,!$

74. (d)

$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 5 \\ 1 & 2 & m \end{vmatrix} = 0 \Rightarrow m = 2$

$D_3 = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 5 & 17 \\ 1 & 2 & n \end{vmatrix} = 0 \Rightarrow n = 7$

75. (b)

$D > 0$

$b^2 > 4ac$

$b = 3 : (a, c) = (1,1)(1,2)(2,1)$

$b = 4 : (a, c) = (1,1)(1,2)(2,1)(1,3)(3,1)$

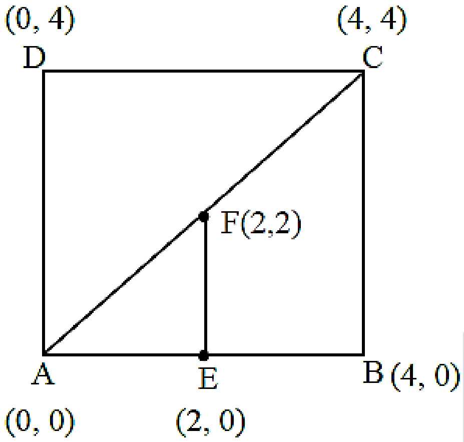
$b = 5: (a, c) = (1,1)(1,2)(2,1)(1,3)(3,1)(1,4)(4,1)\ (1,5)(5,1)(1,6)(6,1)(2,3)(3,2)(2,2)$

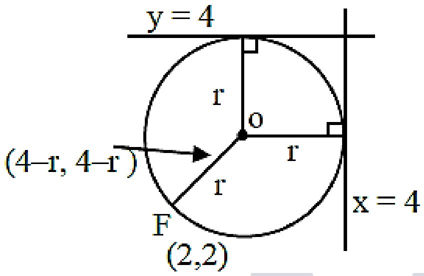
$b = 6: (a, c) = (1,1)(1,2)(2,1)(1,3)(3,1)(1,4)(4,1)\ (1,5)(5,1)(1,6)(6,1)(2,3)(3,2)(2,4)(4,2)(2,2)$

fav. cases = 38

Prob.: $\frac{38}{6\times6\times6}$

76. (b)





$OF^2 = r^2$

$(2-r)^2 + (2-r)^2 = r^2$

$r^2 - 8r + 8 = 0$

77. (d)

$I = \int_0^1 1.(1-x^{10})^{20}dx$

$x^{10} = t$

$x = t^{1/10}$

$dx = \frac{1}{10}(t)^{-9/10}dt$

$I = \int_0^1 (1-t)^{20}\frac{1}{10}(t)^{-9/10}dt$

$I = \frac{1}{10}\int_0^1 t^{-9/10}(1-t)^{20}dt$

$a = \frac{1}{10}$ $b = \frac{1}{10}$ $c = 21$

78. (d) Equating co-factor fo A_{21}

$$(2\alpha^2 - 3\alpha) = \alpha$$
$$\alpha = 0,2 \text{ (accept)}$$

Now, $2\alpha^2 - \alpha\beta = 3\alpha$

$\alpha = 2$ $\beta = 1$

$|AB| = |A\text{cof}(A)| = |A|^3$

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix} = 6 - 2(9) + 3(6) = 6$$

79. (d)

$$y = \frac{2\cos\theta + 2\cos^2\theta - 1}{4\cos^3\theta - 3\cos\theta + 8\cos^2\theta - 4 + 5\cos\theta + 2}$$

$$y = \frac{(2\cos^2\theta + 2\cos\theta - 1)}{(2\cos^2\theta + 2\cos\theta - 1)(2\cos\theta + 2)}$$

$$y = \frac{1}{2}\left(\frac{1}{1 + \cos\theta}\right)$$

$$\Rightarrow \theta = \frac{\pi}{2}\ y = \frac{1}{2}$$

$$y' = \frac{1}{2}\left(\frac{-1}{(1 + \cos\theta)^2} \times (-\sin\theta)\right)$$

$$\Rightarrow \theta = \frac{\pi}{2}\ y = \frac{1}{2}$$

$$y'' = \frac{1}{2}\left[\frac{\cos\theta(1 + \cos\theta)^2 - \sin\theta(b)(1 + \cos\theta)(-\sin\theta)}{(1 + \cos\theta)^4}\right]$$

$$\Rightarrow \theta = \frac{\pi}{2}\ y = 1$$

80. (a)

$$k = 4\left(4^x + \frac{1}{4^x}\right) + \left(4^{2x} + \frac{1}{4^{2x}}\right)$$

≥ 2 ≥ 2

$k \geq 10$

81. (5)

$$\frac{1}{3} + k + \frac{1}{6} + \frac{1}{4} = 1 \Rightarrow k = \frac{1}{4}$$

$\mu = \frac{\alpha}{3} + \frac{1}{4} - \frac{3}{4}$

$\mu = \frac{\alpha}{3} - \frac{1}{2}$

$\sigma = \sqrt{\left(\alpha^2 \frac{1}{3} + \frac{1}{4} + 9\frac{1}{4}\right) - \left(\frac{\alpha}{3} - \frac{1}{2}\right)^2}$

$\sigma = \sqrt{\frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4}}$

$\sigma = \mu + 2$

$\sigma^2 = (\mu + 2)^2 \Rightarrow \frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4} = \frac{\alpha^2}{9} + \frac{9}{4} + \alpha$

$\frac{\alpha^2}{9} - \frac{2\alpha}{3} = 0$

$\alpha = 0$, (reject) or $\alpha = 6$

($\because x = 0$ is already given)

$\Rightarrow \sigma + \mu = 2\mu + 2$

$= 5$

82. (18)

$\text{IF} = e^{\int \frac{2x}{(1+x^2)^2} dx} = e^{\frac{-1}{1+x^2}}$

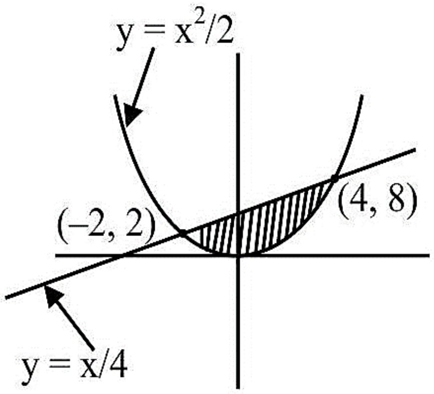
$y \cdot e^{\frac{-1}{1+x^2}} = \int x \cdot e^{\frac{1}{1+x^2}} \cdot e^{\frac{-1}{1+x^2}} dx$

$y \cdot e^{\frac{-1}{1+x^2}} = \frac{x^2}{2} + c$

$(0,0) \Rightarrow C = 0$

$y(x) = \frac{x^2}{2} e^{\frac{1}{1+x^2}}$

$f(x) = \frac{x^2}{2}$

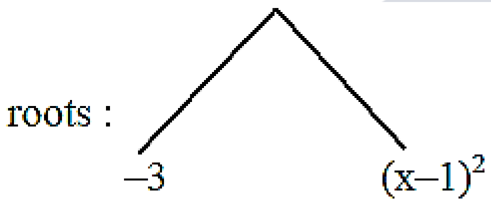


$A = \int_{-2}^{4} (x+4) - \frac{x^2}{2} dx = 18$

83. (2)

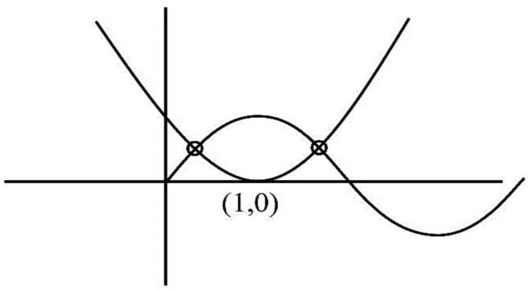
$\sin^2 x - (x^2 - 2x - 2)\sin x - 3(x-1)^2 = 0$

$\sin^2 x - (x-1)^2)\sin x - 3(x-1)^2 = 0$

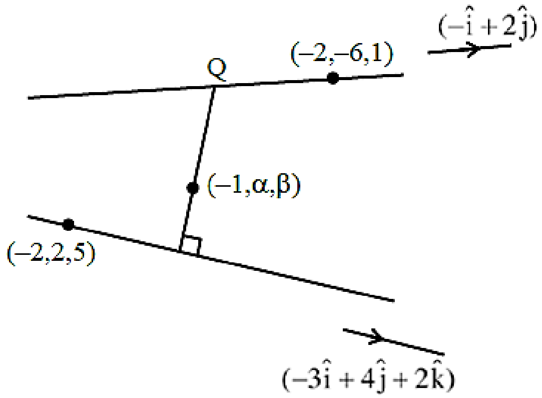


$\sin x = -3$ (reject) or $(x-1)^2$

$\sin x = (x-1)^2$



84. (25)



$P(-3\lambda-2, 4\lambda+2, 2\lambda+5)$

$Q(-\mu-2, 2\mu-6, 1)$

DRS of PQ $= (3\lambda-\mu, 2\mu-4\lambda-8, -2\lambda-4)$

$$\text{DRS of PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 2 \end{vmatrix}$$

$= (4\hat{i}+2\hat{j}+2\hat{k})$

OR

$(2,1,1)$

$$\frac{3\lambda-\mu}{2} = \frac{2\mu-4\lambda-8}{1} = \frac{-2\lambda-4}{1}$$
$\Rightarrow \mu = \lambda + 2 \& 7\lambda = \mu - 8$
$\lambda = -1\ \mu = 1$

$Q: (-3, -4, 1)$

$$L_{PQ} = \frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$$

$$(-1, \alpha, \beta) \Rightarrow 1 = \frac{\alpha+4}{1} = \frac{\beta-1}{1}$$

$\Rightarrow \alpha = -3, \beta = 2$

$(\alpha-\beta)^2 = 25$

85. (76)

$$S = 1 + \frac{x}{2\sqrt{3}} + \frac{x^2}{18} + \frac{x^3}{36\sqrt{3}} + \frac{x^4}{180} + \cdots \infty$$

Put $\frac{x}{\sqrt{3}} = t$, where $x = \sqrt{3}-\sqrt{2}$

$$S = 1 + \frac{t}{2} + \frac{t^2}{6} + \frac{t^3}{12} + \frac{t^4}{20} + \cdots$$

$$S = 1 + t\left(1-\frac{1}{2}\right) + t^2\left(\frac{1}{2}-\frac{1}{3}\right) + t^3\left(\frac{1}{3}-\frac{1}{4}\right) + t^4\left(\frac{1}{4}-\frac{1}{5}\right)$$

$$S = \left(1 + t + \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^3}{4} + \cdots\right) - \left(\frac{t}{2} + \frac{t^2}{3} + \frac{t^3}{4} + \frac{t^4}{5} + \cdots\right)$$

$$S = \left(t + \frac{t^2}{2} + \cdots\right) - \frac{1}{t}\left(t + \frac{t^2}{2} + \frac{t^3}{3} + \cdots\right) + 2$$

$$S = 2 + \left(1-\frac{1}{t}\right)(-\log(1-t)) = \left(\frac{1}{t}-1\right)\log(1-t) + 2$$

$$S = 2 + \left(\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} - 1\right)\log\left(1 - \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)$$

$$S = 2 + \left(\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)\log e\frac{\sqrt{2}}{\sqrt{3}}$$

$$S = 2 + \frac{(\sqrt{6}+2)}{2}\log e\frac{2}{3} = 2 + \left(\sqrt{\frac{3}{2}} + 1\right)\log e\frac{2}{3}$$

$a = 2, b = 3$

$11a + 18b = 11 \times 2 + 18 \times 3 = 76$

86. (170)

$$2x^2 + x - 2 = 0 \begin{array}{l} \nearrow a \\ \searrow b \end{array}$$

$$2x^2 - x - 2 = 0 \begin{array}{l} \nearrow \frac{1}{a} \\ \searrow \frac{1}{b} \end{array}$$

$$\lim_{x\to\frac{1}{a}} 16 \cdot \frac{\left(1 - \cos 2\left(x - \frac{1}{a}\right)\left(x - \frac{1}{b}\right)\right)}{4\left(x - \frac{1}{b}\right)^2} \times \frac{4\left(x - \frac{1}{b}\right)^2}{a^2\left(x - \frac{1}{a}\right)^2}$$

$$= 16 \times \frac{2}{a^2}\left(\frac{1}{a} - \frac{1}{b}\right)^2$$

$$= \frac{32}{a^2}\left(\frac{17}{4}\right) = \frac{17.8}{a^2} = \frac{17 \times 8 \times 16}{(-1+\sqrt{117})^2}$$

$$= \frac{136.16}{18.2\sqrt{7}} \times \frac{18+2\sqrt{7}}{18+2\sqrt{7}}$$

$$= \frac{136}{256}(18+2\sqrt{7}) \cdot 16$$

$$= 153 + 17\sqrt{17} = \alpha + \beta\sqrt{17}$$

$$\alpha + \beta = 153 + 17 = 170$$

87. (1)

$$f(t) = \int_0^\pi \frac{2x}{1-\cos^2 t\sin^2 x}dx \ldots\ldots\ldots\ldots (1)$$

$$= 2\int_0^\pi \frac{(\pi - x)dx}{1 - \cos^2 t\sin^2 x}$$

$$2f(t) = 2\int_0^\pi \frac{\pi}{1-\cos^2 t\sin^2 x}dx \ldots\ldots\ldots\ldots (2)$$

$f(t)=\int_0^{\pi}\frac{\pi}{1-\cos^2 t\sin^2 x}dx$

divide & by $\cos^2$ x

$f(t)=\pi\int_0^{\pi}\frac{\sec^2 xdx}{\sec^2 x-\cos^2 t\tan^2 x}$

$f(t)=2\pi\int_0^{\pi/2}\frac{\sec^2 xdx}{\sec^2 x-\cos^2 t\tan^2 x}$

$\tan x=z$

$\sec^2 xdx=dz$

$f(t)=2\pi\int_0^{\infty}\frac{dz}{1+\sin^2 t\cdot z^2}$

$=\frac{\pi^2}{\sin t}$

Then $\int_0^{\pi/2}\frac{\pi^2}{f(t)}dt$

$=\int_0^{\pi/2}\sin tdt$

$=1$

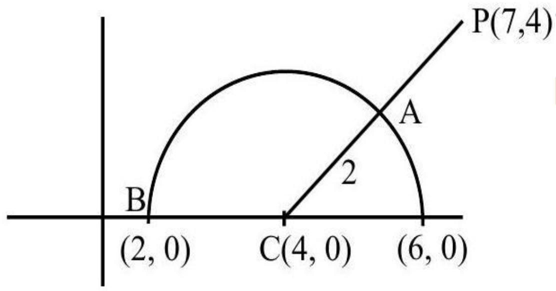
88. (1600)

$(x-7)^2+(y-4)^2$

$y=\sqrt{8x-x^2-12}$

$y^2=-(x-4)^2+16-12$

$(x-4)^2+y^2=4$



$m=9$

$M=4$

$M^2-m^2=41^2-9^2=1600$

89. (10)

$y^2=4(x-9)$

slope of tangent $= \frac{-1}{2}$

Point of contact P $\left(9 + \frac{1}{\left(-\frac{1}{2}\right)^2}, \frac{2\times1}{\frac{-1}{2}}\right)$

$P(13, -4)$

center of circle $C(7,4)$

distance CP $= \sqrt{(13-7)^2 + (-4-4)^2}$

$= 10$

90. (3)

Case I : $x \geq -5$

$$x^2 + 5x + 2x + 12 = 0$$
$$x^2 + 7x + 12 = 0$$
$$x = -3, -4$$

Case II: $-7 < x < -5$

$$-x^2 - 5x + 2x + 14 - 2 = 0$$
$$-x^2 - 3x + 12 = 0$$

$$x = \frac{-3 \pm \sqrt{9+48}}{2}$$

$$= \frac{-3 \pm \sqrt{57}}{2}$$

$x = \frac{-3-\sqrt{57}}{2}, \frac{-3+\sqrt{57}}{2}$ (rejected)

Case III: $x \leq -7$

$$-x^2 - 5x - 2x - 14 - 2 = 0$$

$$x^2 + 7x + 16 = 0$$

$$D = 49 - 64 < 0$$

No solutions

No. of solutions $= 3$