1. **(c)** Scale reading = main scale + Least count × V.S.R .

$= 0 + \frac{1}{10} \times 7 = 0.7$ mm

$= 0.07$ cm

2. **(a)** $R = \frac{V}{I} = \frac{w}{qI} = \frac{ML^2\,T^{-2}}{I^2\,T} = ML^2\,T^{-3}I^{-2}$

$U = \frac{1}{2}LI^2 \rightarrow ML^2\,T^{-2} = LA^2$

$L = ML^2\,T^{-2}\,A^{-2}$

$Q = CV$

$C = \frac{IT \times q}{ML^2\,T^{-2}} = \frac{A^2\,T^2}{ML^2\,T^{-2}}$

$C = M^{-1}\,L^{-2}\,A^{-2}\,T^4$

$LC = ML^2\,T^{-2}\,A^{-2} \times M^{-1}\,L^{-1}\,A^2\,T^4$

$LC = T^2$

$\frac{R}{\sqrt{LC}} = \frac{ML^2\,T^{-3}\,A^{-2}}{T} = (ML^2\,T^{-4}\,A^{-2})$

3. **(b)** $g_{above} = g - \frac{2hg}{R} = 10 - \frac{2\,h}{R}$

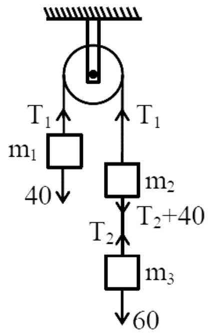
$g_{below} = g - \frac{hg}{R}$

$\Delta g = g_{above} - g_{below} = g - \frac{2hg}{R} - g + \frac{hg}{R}$

$\Delta g = \frac{hg}{R}$

$\frac{\Delta g}{g} = \frac{h}{R} = \frac{16 \times 100}{6400} = 0.25\%$

4. **(a)**



$60 - T_2 = 6a$

$T_2 + 40 - T_1 = 4a$

$60 = 14a$

$a = \frac{60}{14} = \frac{30}{7}$

$T_1 = 40 + 4 \times \frac{30}{7} = \frac{280 + 120}{7}$

$T_1 = \frac{400}{7}$

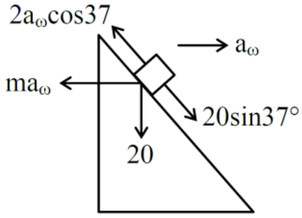
$60 - T_2 = 66 \times \frac{30}{7}$

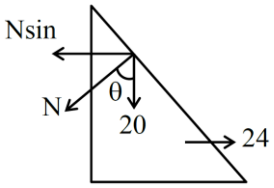
$60 - \frac{180}{7} = T_2$

$\frac{420 - 180}{7} = T_2 = \frac{240}{7}$

$\frac{T_1}{T_2} = \frac{5}{3}$

5. **(a)**





$20\sin37° - 2a_\omega\cos37° = 2a_{rel}$

$20\cos37° - N + 2a_\omega\sin37° = 0$

$24 - N\sin37° = 10 \times a_\omega$

$24 - (20\cos37° + 2a_\omega\sin37°) \times \sin37° = 10a_\omega$

$24 - \left(16 + 2a_\omega \times \frac{3}{5}\right)\frac{3}{5} = 10a_\omega$

$24 - \frac{48}{5} - \frac{18}{25}a_\omega = 10a_\omega$

$\frac{72}{5} = \left(10 - \frac{18}{25}\right)a_\omega$

$\frac{72}{5} = a_\omega = \frac{180}{116} = \frac{90}{58}$

$a_\omega = \frac{45}{29}$

$20\sin37° - 2 \times \frac{45}{29} \times \frac{4}{5} \times \cos37° = 2a_{rel}$

$12 - \frac{72}{29} = 2a_{rel}$

$\frac{348 - 72}{29} = 2a_{rel}$

$\frac{276}{29 \times 2} = a_{rel}\,; \frac{138}{29} = a_{rel}$

$t = \sqrt{\frac{2 \times 8.8 \times 29}{138}}$

$t = \sqrt{\frac{2 \times 8.8 \times 29}{10 \times 138}}$

$t = \sqrt{3.69}$

$t \approx 2\text{sec}$

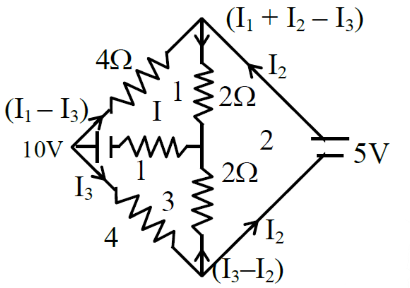
After rounding off

6. **(d)** Young's modulus is the property of material not depends on dimensions of material.

7. **(b)** Both statement are correct but between does not explain the statement-1

8. **(c)** Statement A, B and D are correct

9. (a)



Loop-1 :

$-4(i_1 - i_3) - 2(i_1 + i_2 - i_3) - i_1 + 10 = 0$

$-7i_1 - 2i_2 + 6i_3 + 10 = 0 \qquad \ldots(1)$

Loop-2 :

$-5 - 2(i_3 - i_2) + 2(i_1 + i_2 - i_3) = 0$

$2i_1 + 4i_2 - 4i_3 - 5 = 0 \qquad \ldots(2)$

Loop-3 :

$-i_1 + 10 - 4i_3 - 2(i_3 - i_2) = 0$

$2i_2 - 6i_3 - i_1 + 10 = 0 \qquad \ldots(3)$

On solving equation- (i) & (ii) & (iii)

$i_1 = 2.5\ \text{A}$

$i_2 = 1.875\ \text{A}$

$i_3 = 1.875\ \text{A}$

10. (c) $V = U + at$

$$V = V_0 + \left(\frac{2eE_0}{m}\right)t$$

$$\because a = \frac{(2E_0)e}{m}$$

$$\lambda = \frac{h}{mV}$$

$$\lambda = \frac{h}{m\left[V_0 + \frac{2eE_0 t}{m}\right]}$$

Put $\lambda_0 = \frac{h}{4mV_0} \Rightarrow h = 4\lambda_0 mV_0$

$$\lambda = \frac{4\,m\lambda_0\,V_0}{m\left[V_0 + \frac{2eE_0}{m}t\right]}$$

$$\lambda = \frac{4\lambda_0}{1 + \frac{2eE_0}{mV_0}t}$$

11. (c) Given : $\frac{r_i}{r_f} = \frac{16}{4} = \frac{n_i^2}{n_f^2}$

$\frac{n_i}{n_f} = \frac{4}{2}$l

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = 1.0973 \times 10^7\left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right)$$

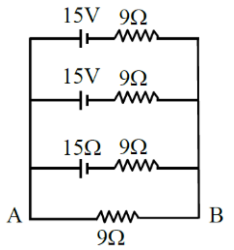
$\lambda = 4.86 \times 10^{-7}\ \text{m} \approx 486\ \text{nm}$

12. (b) $I_d = C\frac{dV}{dt}$

$4 = (6 \times 10^{-6})(\alpha \times 10^6)$

$$\alpha = \frac{2}{3} \approx 0.67$$

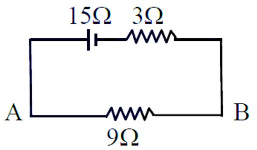
13. **(b)** Solving each branch as series



$$\varepsilon_{eq} = \frac{\frac{15}{9}+\frac{15}{9}+\frac{15}{9}}{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}} = 15 \text{ volt}$$

and $\frac{1}{r_{eq}} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$

$r_{eq} = 3\Omega$



$$I_{AB} = \frac{15}{12} = 1.25 \text{ A}$$

14. **(b)** Since Fringe width in a medium becomes

$\beta' = \frac{\beta}{\mu}$ (Here $\beta = \frac{\lambda D}{d}$)

$$\beta' = \frac{\beta}{\mu} = \frac{2.4}{1.2} = 2\mu \text{ m}$$

15. **(b)** $\mu = \frac{\sin\left[\frac{\delta_{\min}+A}{2}\right]}{\sin\frac{A}{2}}$

$$\mu = \frac{C}{V} = \frac{3\times10^8}{2.12\times10^8} = \sqrt{2}$$

$$\sqrt{2} = \frac{\sin\left[\frac{\delta_{\min}+60^\circ}{2}\right]}{\sin 30^\circ}$$

$$\sin\left[\frac{\delta_{\min}+60^\circ}{2}\right] = \frac{1}{\sqrt{2}}$$

$\delta_{\min} + 60^\circ = 90^\circ \Rightarrow \delta_{\min} = 30^\circ$

16. **(c)** $V_s = \frac{1}{e}\left(\frac{hc}{\lambda} - \phi\right)$

$$\phi = \frac{hc}{\lambda} - ev_s$$

$$= \left(\frac{6.62\times10^{-34}\times3\times10^{8}}{331\times10^{-9}}\right) - (1.6\times10^{-19}\times0.2)$$

$= 5.68\times10^{-19}$ J

17. **(a)** $M = \frac{L}{F_0}\frac{D}{F_e}$

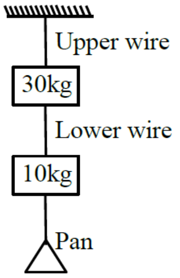
$$\frac{1}{F_0} = (n-1)\left(\frac{2}{R}\right)$$

But when cut

$$\frac{1}{F_0^1} = (n-1)\left(\frac{1}{R}\right) = \frac{1}{2\,F_0}$$

So, $\frac{L}{F_0} = \frac{L'}{F_0^1} = 2\text{ L}$

18. **(b)**



$T_1 = mg$
$T_2 = (10 + m)g$
$T_3 = (30 + 10 + m)g = g(40 + m)g$
Stress in upper wire
$= \dfrac{(40+m)g}{0.008\ cm^2}$
$= \dfrac{(40+m) \times 10}{8 \times 10^{-7}}$
Stress in lower wire =
$= \dfrac{(10+m)g}{0.004\ cm^2}$
$= \dfrac{(10+m) \times 10}{4 \times 10^{-7}}$
Upper wire breaks :
$\dfrac{(40+m) \times 10}{8 \times 10^{-7}} = 12 \times 10^8$
So, maximum mass, without breaking any wire is 38 kg

19. **(c)** As we know that current in RLC circuit is given by

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (X_L - X_C)^2]^{1/2}}$$

$$I = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

As circuit is RC

$$I_0 = \frac{V_0}{(R^2 + X_C^2)^{1/2}}$$

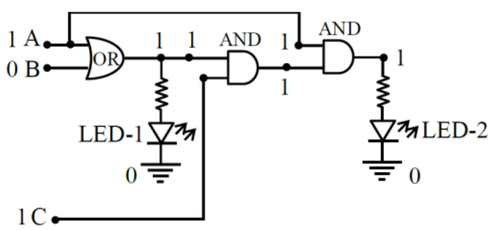
$$\frac{I_0}{3} = \frac{V_0}{[R^2 + (4X_C)^2]^{1/2}}$$

$$R^2 + (X_C)^2 = \frac{(R^2 + 16X_C^2)}{9}$$

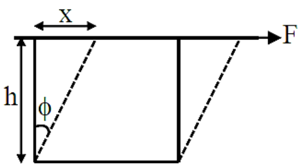
$$\frac{8R^2}{9} = \frac{7X_C^2}{9}$$

$$\frac{R}{X_C} = \sqrt{\frac{7}{8}}$$

20. **(c)**



21. (2)

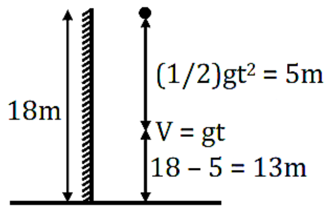


$$\eta=\frac{F/A}{\phi}=\frac{Fh}{Ax}$$

$$x=\frac{Fh}{An}=\frac{10\times5\times10^{-2}}{25\times10^{-4}\times10^{5}}=\frac{1}{500}\ m$$

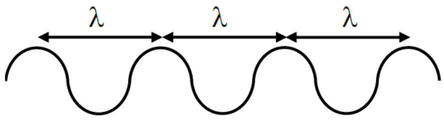
$\therefore x=2\ mm$

22. (13)



At any time, t after it is released velocity $V=gt$
Acceleration of the ball remains constant $=g$
$\therefore$ Both will be equal at $t=1sec$
$\therefore$ Position of ball from ground will be $(18-5)$ m

23. (349)



Least distance between successive crest $=\lambda$

$y=A\sin(\omega t+kx+\phi),\omega=2\pi f\&k=\frac{2\pi}{\lambda}$

$\&y=A\sin(36t+0.018x+\pi/4)$

$\Rightarrow k=0.018\ cm^{-1}\ \Rightarrow\frac{2\pi}{\lambda}=0.018$

$\lambda=\frac{2\pi}{0.018}=\frac{2\times3.14}{18\times10^{-3}}\ cm=348.88\ cm$

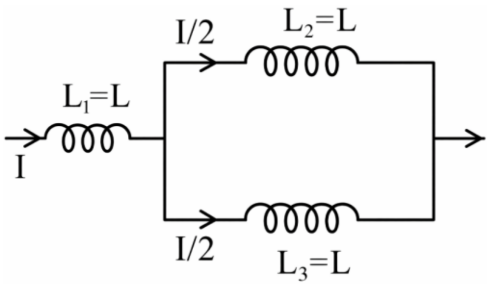
≈ 349 cm (nearest integer)

24. (12) $\vec{F}_B\propto(\vec{V}\times\vec{B})\ \Rightarrow\ \vec{F}_B\perp\vec{B}$

$\vec{a}\perp\vec{B}\rightarrow\vec{a}\cdot\vec{B}=0$

$12-x=0\ \Rightarrow x=12$

25. (6)



Magnetic energy of inductor is $U=\frac{1}{2}Li^2$

So $U_t=\frac{1}{2}LI^2+\frac{1}{2}L\left(\frac{I}{2}\right)^2+\frac{1}{2}L\left(\frac{I}{2}\right)^2=\frac{3}{4}LI^2$

$U_\ell=\frac{1}{2}L\left(\frac{I}{2}\right)^2\Rightarrow\frac{U_t}{U_\ell}=6$

26. **(b)** $Ag_2CO_3(s) \xrightarrow{\Delta} 2Ag(s) + CO_2(g) + \frac{1}{2}O_2(g)$

2.76g

$\frac{1}{100}$ mol

Moles of Ag(s) $= \frac{2}{100}$ mol

Mass of residue $= 2.16$ g

27. **(d)** (A) $n = 3, l = 2, m = +1; (n + l) = 5$

(B) $n = 4, l = 0, m = 0; (n + l) = 4$

(C) $n = 6, l = 1, m = 0; (n + l) = 7$

(D) $n = 5, l = 1, m = +1; (n + l) = 6$

(E) $n = 2, l = 1, m = +1; (n + l) = 3$

More the value of $(n + l)$, more will be the energy of that particular orbital.

$E < B < A < D < C$

28. **(b)**(A) Heisenberg uncertainty principle is applicable to all the moving microscopic particles.

(B) $2p_x$ has less value of n thus has lesser size than $3p_x$.

(C) $E_{2s}(H) \neq E_{2s}(Li)$

(D) Cr: $-[Ar]3d^5 4s^1$

Correct statement are A, B and D.

29. **(d)** Mass ratio = mol ratio × GMM ratio

$$\frac{10}{90} = \frac{n}{N} \times \frac{60}{18}$$

$$\frac{n}{N} = \frac{1}{30}$$

$$x_{H_2O} = \frac{W}{n + N} = \frac{30}{31} = 0.967$$

30. **(a)** Molarity

$M_3A_2(s)$	$\rightleftharpoons$ $3M^{+2}(aq.)$	$+ 2A^{-3}(aq.)$
$\frac{x}{y}$	$-$	$-$
$-$	$3x/y$	$2x/y$

$K_{sp} = [M^{+2}]^3[A^{-3}]^2$

$K_{sp} = [3x/y]^3[2x/y]^2$

Ratio of $\frac{[A^{-3}]}{K_{sp}} = \frac{2x/y}{\left[\frac{3x}{y}\right]^3\left[\frac{2x}{y}\right]^2}$

$$= \frac{1}{54}\frac{y^4}{x^4}$$

31. **(c)** (A) $H^+_{(aq)} + OH^-_{(aq)} \rightarrow H_2O(\ell)$

$t = 0$ 2.5mmol 4mmol

$t = \infty$ 0 1.5mmol

an final solution, $[OH^-] = \frac{1.5\text{mmol}}{35\text{ml}} = \frac{1.5}{35}$

$$pOH = -\log\frac{1.5}{35}$$

$$pH = 14 + \log\left(\frac{1.5}{35}\right)$$

(B) $H^+_{(aq)} + OH^-_{(aq)} \rightarrow H_2O(\ell)$

$t = 0$ 0.2mmol 0.2mmol

final pH = 7

(C) $H^+_{(aq)} + OH^-_{(aq)} \rightarrow H_2O(\ell)$

$t = 0$ 2mmol 1mmol

$t = \infty \quad 1 \text{ mmol} \quad 0$

Final pH < 7

So, order of pH values :

(C) < (B) < (A)

32. (a)

$A(g) \rightarrow B(g) + C(g)$

$t = 0 \quad P_i \quad - \quad -$

$t = t \quad P_i - x \quad x \quad x$

As given $(P_i - x) + x + x = P_t$

$x = (P_t - P_i)$

$$K = \frac{1}{t}\ell n\frac{P_i}{P_i - x} = \frac{1}{t}\ell n\frac{P_i}{P_i - (P_t - P_i)}$$

$$K = \frac{1}{t}\ln\frac{P_i}{2P_i - P_t}$$

33. (a) Statement-I

$$\left.\begin{aligned}(EN)_F &= 4.0\\(EN)_O &= 3.5\\(EN)_N &= 3.0\end{aligned}\right\}\text{ on pauling scale}$$

Statement-II

Oxidation state of oxygen in OF_2

$OF_2 \Rightarrow x - 1 \times 2 = 0$

$x = +2$

Oxidation state of oxygen in Na_2O

$Na_2O \Rightarrow +1 \times 2 + x = 0$

$x = -2$

Both the statement I and statement II are correct.

34. (d) (A) In the case of nitrogen, all oxidation states from +1 to +4 tend to disproportionate in acid solution. For example.

$3HNO_2 \rightarrow HNO_3 + H_2O + 2NO$

(B) Nitrogen form $p_\pi - p_\pi$ bonds with itself and other elements with small size and high electronegativity, $d_\pi - p_\pi$ is not possible with itself as it has no d-orbital.

(C) The single N − N bond is weaker than the single

P − P bond because of high intereelecronic repulsion of the non-bonding electrons (lone pair), owing to the small bond length.

(D) Nitrogen has lowest density in its group.

(E) The maximum covalency of nitrogen is four.

35. (b) The physical and chemical characteristics of interstitial compounds are :

(a) They have high melting points, higher than those of pure metals.

(b) They are very hard, some borides approach diamond in hardness.

(c) They retain metallic conductivity

(d) They are chemically inert and usually nonstoichiometric.

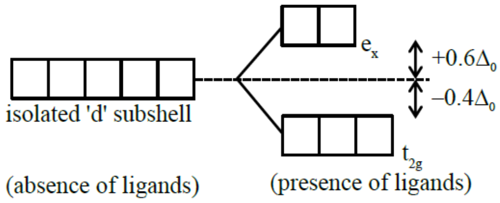
36. (b) (A) The metal-carbon bonds in metal carbonyls possess both σ and π-character.

(B) The metal to ligand bonding creates a synergic effect which strengthens the bond between CO and the metal i.e. metal carbon bond becomes strong.

(C) The metal-carbon σ is formed by the donation of lone pair of electrons on the carbonyl carbon into a vacant orbital of metal.

(D) The metal-carbon π bond is formed by the donation of electrons from filled d-orbial of metal into vacant π^* orbital of CO.

37. (a) Statement-I



Statement-II

The degeneracy of the d-orbitals is removed due to ligand electron-metal electron repulsions according to crystal field theory.

Both the statement I and II are correct.

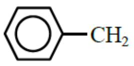
38. (c) Statement-I

$$\overset{\ominus}{C}H_3 > CH_3-\overset{\ominus}{C}H_2 > (CH_3)_2\overset{\ominus}{C}H > (CH_3)_3\overset{\ominus}{C}$$

It is correct order of anion stability and it is based on inductive effect.

Statement-II

$CH_2=CH-\overset{\ominus}{C}H_2$ &
Allyl anion



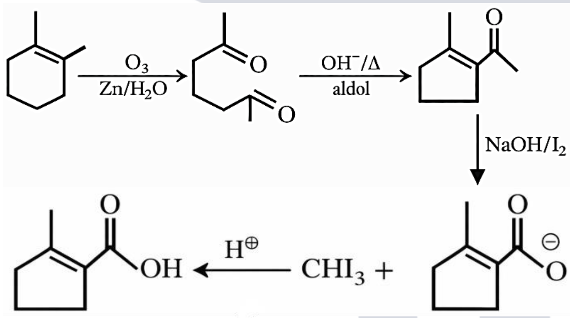
Benzyl carbanion

Both are stabilized by resonance

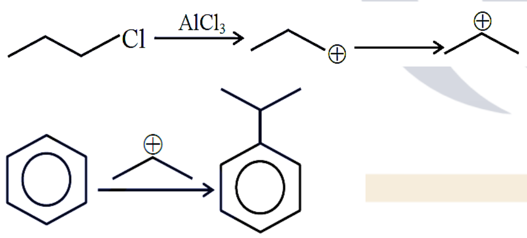
Statement-I : Correct

Statement-II : Incorrect

39. (d)

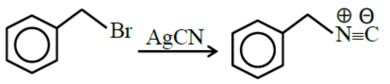


40. (b)



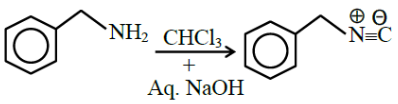
Major product

41. (a)

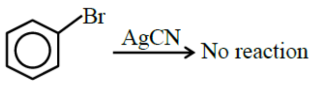


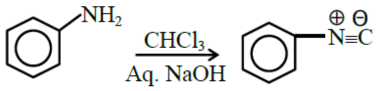
Benzyl Isocyanide

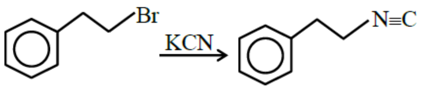
(A)



(B)

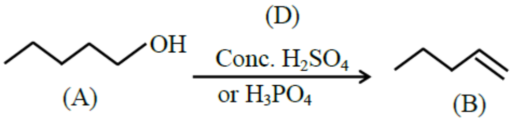
(C) 

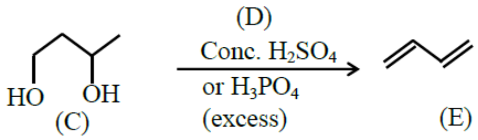
(D) 

(E) 

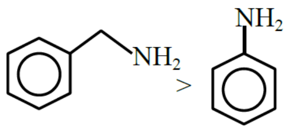
(A) & (B) gives correct product.

42. (d)

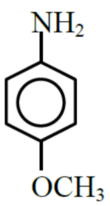




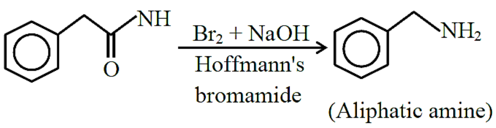
43. (c)

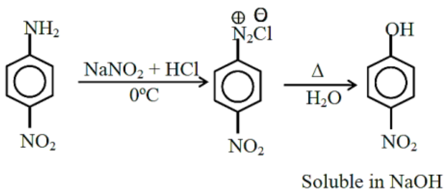
(A) 

(Basic strength order)

(B) 

Cannot be synthesized by Gabriel phthalimide synthesis

(C) 

(D) 

Only (B) and (C) are incorrect hence Ans. is (c)

44. (d)

45. (b) $Na_2Cr_2O_7$ is hygroscopic in nature and is not used as primary standard.

Phenolphthalein is weak acid.

46. (4)

Species	**Hybridization**
(1) BrF_5	$sp^3\,d^2$
(2) $XeF_5^{\ominus}$	$sp^3\,d^3$
(3) $BF_4^{\ominus}$	sp^3

(4) $ICl_4^{\ominus}$ $sp^3\ d^2$
(5) XeF_4 $sp^3\ d^2$
(6) SF_4 $sp^3\ d$
(7) NH_4^+ sp^3
(8) ClF_3 $sp^3\ d$
(9) XeF_2 $sp^3\ d$
(10) $ICl_2^{\ominus}$ $sp^3\ d$
Number of species having sp^3d hybridisation are 4.

47. **(42)** Moles of $BaSO_4$ formed $\xrightarrow[231gm]{0.6gm} = 0.0026$

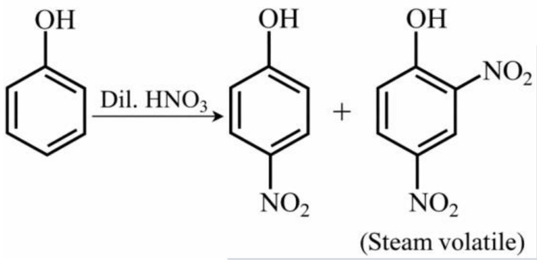
Moles of $BaSO_4$ = Moles of S

Mass of sulphur $(W_s) = \xrightarrow[231]{0.6} \times 32$

% of sulphur (S) $= \frac{0.6\times32}{231\times0.2} \times 100$

$= \frac{1920}{46.2} = 41.558 \approx 42$

48. **(175)**



49. **(230)** Using formula

$\log K_{P_2} - \log K_{P_1} = \frac{\Delta H}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$3.5 - 2.5 = \frac{\Delta H}{2.303R}(0.06 - 0.05)$

$\frac{\Delta H}{R} = \frac{2.303}{0.01}$

$\frac{\Delta H}{R} = 230.3 \approx 230$

50. **(46)** $t_{1/2} = \frac{\ell n2}{K} \Rightarrow K_1 = \frac{0.693}{6.93} \Rightarrow 0.1\ \text{min}^{-1}$

$t_{99\%} = \frac{1}{K}\ln\left[\frac{A_0}{A_t}\right]$

$= \frac{1}{0.1}\ln\left[\frac{100}{1}\right] \Rightarrow \frac{2\ln 10}{0.1}$

$= \frac{2\times2.303}{0.1}$

$\Rightarrow 46.06\text{min} \approx 46$

51. **(b)** $\beta - \alpha = \sqrt{11}$ ……(1)

and $\beta^2 - \alpha^2 = 3i\sqrt{11}$

$\Rightarrow (\beta+\alpha)(\beta-\alpha) = 3i\sqrt{11}$

$\Rightarrow \beta + \alpha = 3i$ ……(2)

from (1) and (2)

$\beta = \frac{\sqrt{11}+3i}{2}, \alpha = \frac{3i-\sqrt{11}}{2}$

$\Rightarrow \alpha\beta = \frac{-9-11}{4} \Rightarrow \alpha\beta = -5$

Now $(\beta^3 - \alpha^3)^2 = (\beta-\alpha)^2(\beta^2+\alpha^2+\alpha\beta)^2$

$= (\beta-\alpha)^2((\beta+\alpha)^2 - \alpha\beta)^2$

$= (11)(-9+5)^2 = 176$

52. (c) $S_n = 3n^2 + 5n$

$T_n = S_n - S_{n-1}$

$\Rightarrow T_n = (3n^2 + 5n) - (3(n-1)^2) + 5(n-1))$

$\Rightarrow T_n = 6n + 2$

$\sum_{n=1}^{10} T_n^2 = \sum_{n=1}^{10} (6n+2)^2$

$= 36\sum_{n=1}^{10} n^2 + 24\sum_{n=1}^{10} n + \sum_{n=1}^{10} 4$

$= 15220$

53. (d) Let $A = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$

Now $A = \begin{pmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ ...(1)

Now $\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ ...(2)

$A^T \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - A^T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

$\Rightarrow A^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ ...(3)

Now $A\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - A\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$

$A\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ ...(4)

$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & \beta_2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$

$|A| = 2(\beta_2 - 3) - 1(1-9) + 1(1 - 3\beta_2) = 1$

$\Rightarrow 2\beta_2 - 6 + 8 + 1 - 3\beta_2 = 1$

$\Rightarrow \beta_2 = 2$

So $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$

$A^2 = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 5 & 6 \\ 13 & 8 & 10 \\ 10 & 6 & 7 \end{pmatrix}$

$A^2 + A = \begin{pmatrix} 10 & 6 & 7 \\ 14 & 10 & 13 \\ 13 & 7 & 8 \end{pmatrix} \Rightarrow \det(A^2 + A) = 8$

$\Rightarrow \operatorname{Det}(\operatorname{adj}(A^2 + A)) = 8^2 = 64$

54. (a) $D = \begin{vmatrix} 1 & 2 & t \\ 6 & 1 & 5t \\ 3 & t^2 & f(t) \end{vmatrix} = 0$

$\Rightarrow 1(f(t) - 5t^3) - 2(6f(t) - 15t) + t(6t^2 - 3) = 0$

$f(t) = \dfrac{t^3 + 12t}{11}$

$f'(t) = \dfrac{1}{11}(3t^2 + 12) > 0 \; \forall t \in R$

$\Rightarrow$ f(t) is strictly increasing on R

55. (b) Let $\sum_{n=1}^{10} \frac{528}{n(n+1)(n+2)} = \lambda$

Let $T_n = \frac{1}{n(n+1)(n+2)}$

$T_n = \dfrac{1}{2}\left\{\dfrac{(n+2) - n}{n(n+1)(n+2)}\right\}$

$T_n = \dfrac{1}{2}\left\{\dfrac{1}{n(n+1)} - \dfrac{1}{(n+1)(n+2)}\right\}$

$T_1 = \dfrac{1}{2}\left\{\dfrac{1}{1.2} - \dfrac{1}{2.3}\right\}$

$T_2 = \dfrac{1}{2}\left\{\dfrac{1}{2.3} - \dfrac{1}{3.4}\right\}$

$T_{10} = \dfrac{1}{2}\left\{\dfrac{1}{10.11} - \dfrac{1}{11.12}\right\}$

$S_n = \dfrac{1}{2}\left[\dfrac{1}{2} - \dfrac{1}{132}\right]$

$S_n = \dfrac{1}{2}\left[\dfrac{66 - 1}{132}\right] = \dfrac{65}{264}$

So, $\lambda = \frac{65}{264} \times 528$

$\lambda = 130$

56. (c) $x^2 - 2x - 5 = 0$

$\tan A + \tan B = 2; \tan A \tan B = -5$

$\therefore \tan(A + B) = \dfrac{2}{1 - (-5)} = \dfrac{1}{3}$

$\Rightarrow \cos(A + B) = \dfrac{3}{\sqrt{10}}$

$\therefore 20\left(\sin^2\left(\dfrac{A + B}{2}\right)\right) = \dfrac{10}{2}(1 - \cos(A + B))$

$= 10\left(1 - \dfrac{3}{\sqrt{10}}\right) = (10 - 3\sqrt{10})$

57. (b) $P(I): \frac{1}{2} P(A/I) \to \frac{1}{5}$

$P(II): \dfrac{1}{2} P(A/II) \to \dfrac{2}{7}$

$\therefore$ Reqd. Prob $= \dfrac{\frac{1}{2}\times\frac{2}{7}}{\frac{1}{2}\times\frac{1}{5}+\frac{1}{2}\times\frac{2}{7}} = \dfrac{\frac{2}{7}\times 5}{\frac{17}{7}} = \dfrac{10}{17}$

58. (c) M.D. $= \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$

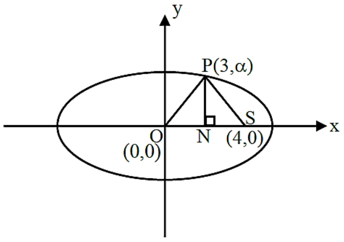
$\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{40 + 42 + 18 + 20 + 24 + 90}{26}$

$= \dfrac{234}{26} = 9$

$$\text{M.D.} = \frac{8(4)+6(2)+2(0)+2(1)+2(3)+6(6)}{26}$$

$$= \frac{84}{26} = \frac{44}{13}$$

59. (c)



$a > b$

Focus $(ae, 0) = (4,0)$

$\therefore ae = 4$

$a\left(\frac{4}{5}\right) = 4 \Rightarrow a = 5$

$\because b^2 = a^2(1 - e^2)$

$b^2 = 25\left(1 - \frac{16}{25}\right)$

$\therefore E: \frac{x^2}{25} + \frac{y^2}{9} = 1$

$\because$ P lies on the ellipse E

$\therefore \frac{9}{25} + \frac{\alpha^2}{9} = 1$

$\therefore \alpha = \pm\frac{12}{5}$

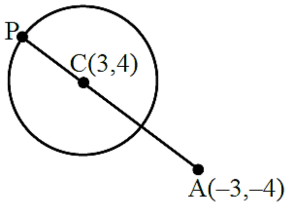
$\therefore$ Area of $\Delta_{POS} = \frac{1}{2}(OS)(PN)$

$= \frac{1}{2} \times 4 \times \frac{12}{5} = \frac{24}{5}$

60. (c) Centre : $C(3,4)$&$r = 2$

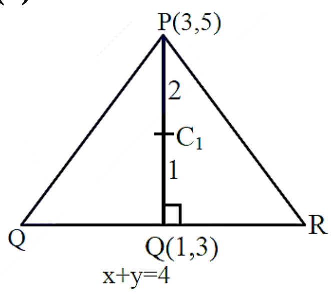
Parabola $(x+3)^2 = -(y+4)$

Vertex is $A(-3,-4)$



$AP_{max} = AC + r = \sqrt{36+64} + 2 = 12$

61. (d)



$Q: \frac{x-3}{1} = \frac{y-5}{1} = -\frac{(3+5-4)}{1+1}$

$Q(1, 3)$

So orthocenter is $\left(\frac{2+3}{3}, \frac{6+5}{3}\right)$

$\left(\frac{5}{3}, \frac{11}{3}\right)$

So $9(\alpha+\beta) = 3(16) = 48$

62. (c) $P = 12\cos x - 3\cos^2 x$

$P = -3(\cos^2 x - 4\cos x)$

$P = -3((\cos x - 2)^2 - 2)$

put $\cos x = -1 \Rightarrow P = -15$

put $\cos x = 1 \Rightarrow P = 9$

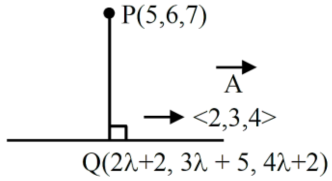
$-15 \leq P \leq 9$

Sum of all integers $= -(10 + 11 + \cdots + 15) = -75$

63. (c) $\overrightarrow{PQ} \cdot \vec{A} = 0$

$\Rightarrow (2\lambda - 3)2 + (3\lambda - 1)3 + (4\lambda - 5)4 = 0$

$\Rightarrow \lambda = 1$



Point $Q = (4,8,6)$

$PQ^2 = 6$

64. (a) $\vec{r} \times \vec{a} - \vec{b} \times \vec{a} = \vec{0}$

$(\vec{r} - \vec{b}) \times \vec{a} = \vec{0}$

$\vec{r} - \vec{b} = \lambda\vec{a}$

$\vec{r} = \vec{b} + \lambda\vec{a}$

$\vec{r} \cdot \vec{a} = 0 \Rightarrow \vec{a} \cdot \vec{b} + \lambda|\vec{a}|^2 = 0$

$$\lambda = -\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} = -\frac{(1-2)}{9} = \frac{1}{9}$$

$$\vec{r} = \vec{b} + \frac{\vec{a}}{9}$$

$$|3\vec{r}|^2 = 9|\vec{r}|^2 = 9\left(b^2 + \frac{a^2}{81} + \frac{2(\vec{a} \cdot \vec{b})}{9}\right) = 44$$

65. (c) Equating both the equations

$(\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(a\hat{\imath} - \hat{\jmath}) = (4\hat{\imath} - \hat{k}) + \mu(2\hat{\imath} + a\hat{k})$

$1 + a\lambda = 4 + 2\mu$

$1 - \lambda = 0 \Rightarrow \lambda = 1$

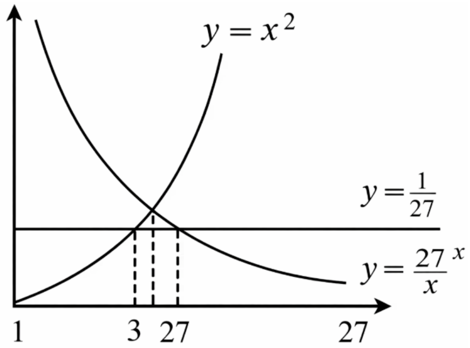
$-1 = -1 + a\mu$

$a\mu = 0 \Rightarrow \mu = 0$

$a = 3$

point of intersection $= \hat{\imath} + \hat{\jmath} - \hat{k} + 3\hat{\imath} - \hat{\jmath} = 4\hat{\imath} - \hat{k}$

66. (b)



Area $= \int_1^3 (x^2 - 1)dx + \int_3^{27} \left(\frac{27}{x} - 1\right) dx$

$= \left[\frac{x^3}{3} - x\right]_1^3 + [27\ell nx - x]_3^{27}$

$= \frac{26}{3} - 2 + 27\ln 9 - 24$

$= 27\ln 9 - \frac{26 \times 2}{3}$

$= 54\ln 3 - \frac{52}{3}$

67. (c) $\lim_{x\to 0} \frac{1-\cos\alpha x\cdot\cos((\alpha+1)x)\cos((\alpha+2)x)}{(\alpha+1)^2x^2}$

By using L.H. Rule we get

$\Rightarrow \frac{1}{2}(\alpha+1)^2[\alpha^2 + (\alpha+1)^2 + (\alpha+2)^2] = 2$

$\alpha^2 + (\alpha+1)^2 + (\alpha+2)^2 = 4(\alpha+1)^2$

$\alpha^2 + (\alpha+2)^2 = 3(\alpha+1)^2$

$\Rightarrow \alpha^2 + 2\alpha - 1 = 0$

Product $= -1$

68. (b) Put $x = 2t \Rightarrow dx = 2dt$

$I = \int_0^\infty \frac{\ell n2t}{4t^2+4}(2dt) = \frac{1}{2}\int_0^\infty \frac{\ell n2 + \ell nt}{t^2+1}dt$

$= \frac{1}{2}\int_0^\infty \frac{\ell n2}{t^2+1}dt + \frac{1}{2}\int_0^\infty \frac{\ell nt}{t^2+1}dt$

$\left[\frac{\ln 2}{2}\tan^{-1}t\right]_0^\infty + I_1$

$= \frac{\ell n2}{2}\cdot\frac{\pi}{2} + I_1$

$I_2 = \frac{1}{2}\int_0^\infty \frac{\ell nt}{t^2+1}dt\ t = \frac{1}{u} \Rightarrow dt = -\frac{du}{u^2}$

$= \frac{1}{2}\int_\infty^0 \frac{\ell n(1/u)}{\frac{1}{u^2}+1}\left(-\frac{du}{u^2}\right)$

Add $\Rightarrow I_1 = 0$

$I = \frac{\pi\ell n2}{4}$

69. (b) $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$, put $x = y = 0$

$f(0) = \frac{2f(0)}{3}$

$\Rightarrow f(0) = 0 \quad \ldots\ldots(1)$

$f'\left(\frac{x+y}{3}\right)\cdot\frac{1}{3} = \frac{1}{3}f'(x)$

put x = 0

$f'\left(\frac{y}{3}\right)\frac{1}{3}=\frac{1}{3}\times 3$

$f'\left(\frac{y}{3}\right)=3$

put y = 3x

$f'(x)=3$

integrate both sides

$f(x)=3x+c$

from eq. (1)

$f(0)=0$

$\Rightarrow f(x)=3x$

Now

$g(x)=3+e^{x}\cdot 3x$

$g'(x)=3[e^{x}+x\cdot e^{x}]$

$=3e^{x}(x+1)$

$g'(x)=0$, at $x=-1$

$(g(x))_{\min}=3+e^{-1}(-3)$

$=3\left[1-\frac{1}{e}\right]=\frac{3(e-1)}{e}$

70. (c) $=\int_{\pi/6}^{\pi/3}\frac{4}{\cos^4 x}dx-\int_{\pi/6}^{\pi/3}\frac{\operatorname{cosec}^2 x}{\cos^4 x}dx$

$$=\int_{\pi/6}^{\pi/3}\frac{4}{\cos^4 x}dx-\left[-\frac{\cot x}{\cos^4 x}-\int_{\pi/6}^{\pi/3}(-\cot x)\frac{-4}{\cos^5 x}(-\sin x)dx\right]$$

$$=\left.\frac{\cot x}{\cos^4 x}\right|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$=\frac{1/\sqrt{3}}{(1/2)^4}-\frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^4}$$

$$=\frac{16}{\sqrt{3}}-\frac{\sqrt{3}\cdot 16}{9}=\frac{16}{\sqrt{3}}-\frac{16}{3\sqrt{3}}=\frac{16}{\sqrt{3}}\left(1-\frac{1}{3}\right)$$

$$\frac{32}{3\sqrt{3}}=\frac{32\sqrt{3}}{9}$$

71. (72) $f(2)+f(3)=5$

C-I : $f(2)=4, f(3)=1$

so, $f(1)=3,5,6\rightarrow 3$ choices

$f(4)\rightarrow 3$ choices

$f(5)\rightarrow 2$ choices

$f(6)\rightarrow 1$ choices

so total function for C-I : $3\times 3\times 2\times 1=18$

Similarly,

C-II : $f(2)=1, f(3)=4\rightarrow 18$ function

C-III : $f(2)=3, f(3)=2\rightarrow 18$ function

C-IV : $f(2)=2, f(3)=2\rightarrow 18$ function

72. (126) Maximum number of games will be 9

A must win 5 gamese so,

Number of ways $={}^{9}C_{5}=126$

73. (21) $T_{r+1}={}^{n}C_{r}\left(\frac{1}{x^3}\right)^{n-r}(-x^4)^r$

$T_{r+1}={}^{n}C_{r}x^{7r-3n}(-1)^r$

for coeff. of $x^7\Rightarrow 7r_1-3n=7$

$\Rightarrow r_1=\frac{7+3n}{7}$

coeff. of $x^7 \Rightarrow {}^nC_{r_1}(-1)^{r_1}$

for coeff. of $x^{14} \Rightarrow 7r_2 - 3n = 14$

$\Rightarrow r_2 = \frac{14+3n}{7} = r_1 + 1$

coeff. of $x^{14} \Rightarrow {}^nC_{r_1+1}(-1)^{r_1+1}$

$\Rightarrow {}^nC_{r_1}(-1)^{r_1} + {}^nC_{r_1+1}(-1)^{r_1+1} = 0$

$\Rightarrow {}^nC_{r_1} = {}^nC_{r_1+1} \Rightarrow r_1 + r_1 + 1 = n$

$\Rightarrow 2r_1 + 1 = n \Rightarrow 2\left(\frac{7+3n}{7}\right) + 1 = n$

74. (2048) $\alpha = \frac{\pi}{4} + \sum_{p=1}^{11} \tan^{-1}\left(\frac{2^{p-1}}{1+2^{2p-1}}\right)$

$= \frac{\pi}{4} + \sum_{p=1}^{11} \tan^{-1}\left(\frac{2^p - 2^{p-1}}{1 + 2^p 2^{p-1}}\right)$

$= \frac{\pi}{4} + \sum_{p=1}^{11}\left(\tan^{-1}(2^p) - \tan^{-1}(2^{p-1})\right)$

$= \frac{\pi}{4} + \tan^{-1}(2^{11}) - \tan^{-1}(2^0) = 2^{11}$

75. (11) $\sin\frac{y}{x}\frac{dy}{dx} = \frac{y}{x}\sin\frac{y}{x} - 1$

put $y = tx \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$\sin t\left(t + x\frac{dt}{dx}\right) = t\sin t - 1$

$x\sin t\frac{dt}{dx} + 1 = 0$

$\sin t dt + \frac{dx}{x} = 0$

$\Rightarrow -\cos t + \ell n x = C$

$\Rightarrow -\cos\left(\frac{y}{x}\right) + \ell n x = C$

$y(1) = \frac{\pi}{2} \Rightarrow C = 0$

$\Rightarrow \cos\left(\frac{y}{x}\right) = \ln x$

$\operatorname{cosec}\left(\frac{y(e^{12})}{e^{12}}\right) = 12 \Rightarrow \alpha = 12$

$x^2 + y^2 - 2px + 2py + 14 = 0$

$r = \sqrt{p^2 + p^2 - 14}$

$\Rightarrow r \le 6 \Rightarrow r^2 \le 36$

$2p^2 - 14 \le 36$

$p^2 \le 25$

$p \in [-5,5]$

number of integral value of $p = 11$