1. **(b)** $[h] = [ML^2T^{-1}]$

$[c] = [LT^{-1}]$

and $[G] = [M^{-1}\,L^3\,T^{-2}]$

then $\left[\sqrt{\frac{hc}{G}}\right] = \left[\frac{ML^2\,T^{-1}LT^{-1}}{M^{-1}\,L^3\,T^{-2}}\right]^{1/2} = [M]$

$$\left[\sqrt{\frac{Gh}{c^5}}\right] = \left[\frac{M^{-1}\,L^3\,T^{-2}ML^2\,T^{-1}}{L^5\,T^{-5}}\right]^{1/2} = [T]$$

$$\left[\sqrt{\frac{k^2\,L^2c^3}{Gh}}\right] = \left[\frac{k^2\,L^2\,L^3\,T^{-3}}{M^{-1}\,L^3\,T^{-2}ML^2\,T^{-1}}\right]^{1/2} = [k]$$

$$\left[\sqrt{\frac{Gh}{c^3}}\right] = \left[\frac{M^{-1}\,L^3\,T^{-2}ML^2\,T^{-1}}{L^3\,T^{-3}}\right]^{1/2} = [L]$$

2. **(d)** $V = IR$

$$R = \frac{10}{5} = 2\Omega$$

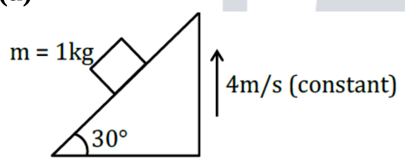
$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$$\frac{\Delta R}{R} = \frac{500 \times 10^{-3}}{10} + \frac{200 \times 10^{-3}}{5}$$

$$\frac{\Delta R}{2} = 0.05 + 0.04$$

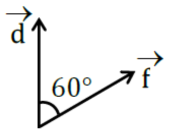
$$\Delta R = 0.09 \times 2 = 0.18\Omega$$

3. **(a)**



$f = mg\sin 30 = 5$ N

work done in 2 sec will be



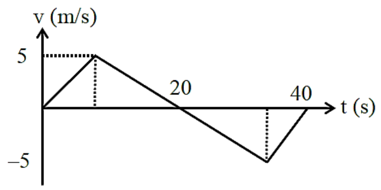
$d = 4 \times 2 = 8$ m

$W = fd\cos 60°$

$W = (5)(8)\cos 60°$

$W = 20$ joule

4. **(c)**



Total distance travel

$$d = \frac{1}{2}(20)(5) + \frac{1}{2}(20) \times 5$$

$d = 100$ m

Average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$

$< v >= \frac{0}{40} = 0$

5. **(b)** Let α be angular acceleration

In first 2 sec

$\theta_1 = \frac{1}{2} \cdot \alpha(2)^2 = 2\alpha$

Angular covered in 4 sec

$\theta' = \frac{1}{2}(\alpha)(4)^2 = 8\alpha$

Angle in next 2 sec

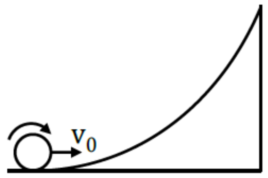
$\theta_2 = \theta' - \theta_1 = 8\alpha - 2\alpha$

$\theta_2 = 6\alpha$

$\frac{\theta_2}{\theta_1} = \frac{6\alpha}{2\alpha} = 3$

6. **(d)** $h = \frac{7v_0^2}{10\,g}$

Applying work energy theorem



$-mgh = 0 - \frac{1}{2}mv^2 - \frac{1}{2}I\omega^2$

$-mg\left(\frac{7v_0^2}{10\,g}\right) = -\frac{1}{2}mv_0^2 - \frac{1}{2}\frac{Iv_0^2}{R^2}$

$\frac{7mv_0^2}{10} = \frac{mv_0^2}{2} + \frac{Iv_0^2}{2R^2}$

$\frac{mv_0^2}{5} = \frac{Iv_0^2}{2R^2}$

$I = \frac{2mR^2}{5}$

7. **(d)** $U_1 = \frac{-GMm}{R_e}$

$U_2 = \frac{-GMm}{R_e + 2R_e} = -\frac{GMm}{3R_e}$

$\Delta U = U_2 - U_1 = \frac{-GMm}{3R_e} + \frac{GMm}{R_e}$

$\Delta U = \frac{2}{3}\frac{GMm}{R_e} = \frac{2}{3}\left[\frac{GM}{R_e^2}\right](mR_e)$

$\Delta U = \frac{2}{3}mgR_e$

8. **(b)** Volume conservation

$(8)\left[\frac{4}{3}\pi r^3\right] = \frac{4}{3}\pi R^3$

$R = 2r$

$U_i = (8)(4\pi r^2)(s); s = \text{surface tension}$

$U_i = 32\pi r^2\ s$

$U_f = (4\pi R^2)(s)$

$U_f = 4\pi(2r)^2(\ s) = 16\pi r^2\ s$

Energy released $= U_i - U_f$

Energy released $= 16\pi r^2 s$

9. **(c)** $PT^3 = \text{constant (C)}$

$\frac{nRT}{V} \cdot T^3 = C$

$V = \frac{nR}{C} \cdot T^4$

Volume expansion coeff.

$\gamma = \frac{1}{V}\frac{dV}{dT}$

$\gamma = \frac{C}{nRT^4} \cdot \frac{nR}{C} \cdot 4\,T^3 = \frac{4}{T}$

10. **(a)** (A) $\sin^2\omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$

it represents SHM motion having period $= \frac{\pi}{\omega}$

(B) $\sin^3 2\omega t = \frac{3}{4}\sin 2\omega t - \frac{1}{4}\sin 6\omega t$

common period $\frac{\pi}{\omega}$ but not SHM.

(C) $\sin\omega t + \cos\pi\omega t \rightarrow$ no common period hence it is a nonperiodic function

(D) $\cos\omega t + \cos 2\omega t$

common period $\frac{2\pi}{\omega}$ but it do not represent SHM.

$A \rightarrow III, B \rightarrow I, C \rightarrow IV, D \rightarrow II$

11. **(a)** $e = \int_0^L B(\omega r)dr = B_0\omega\int_0^L e^{-\lambda r} \cdot rdr$

$= B_0\omega\left[r\int e^{-\lambda r}dr - \int 1 \cdot \frac{e^{-\lambda r}}{-\lambda}dr\right]_0^L$

$= B_0\omega\left[\frac{re^{-\lambda r}}{-\lambda} - \frac{e^{-\lambda r}}{\lambda^2}\right]_0^L$

$= B_0\omega\left[\left(0 + \frac{1}{\lambda^2}\right) - \left(\frac{Le^{-\lambda L}}{\lambda} + \frac{e^{-\lambda L}}{\lambda^2}\right)\right]$

$e = B_0\omega\left[\frac{1}{\lambda^2} - e^{-\lambda L}\left(\frac{1}{\lambda^2} + \frac{L}{\lambda}\right)\right]$

12. **(a)** $i = \frac{2}{6+2} = \frac{1}{4}$ A

$v_C = v_{2\Omega} = i \times 2 = \frac{1}{2}\text{ V} = 0.5\text{ V}$

13. **(a)** $\Delta K = W_e + W_m = W_e + 0$

$= \int_0^L qEdx = qE_0\int_0^L e^{-\lambda x}dx$

$= \frac{qE_0}{\lambda}(1 - e^{-\lambda L})$

14. **(b)** Assertion and reason both are correct but not related.

15. **(a)** $1 \rightarrow$ convex lens

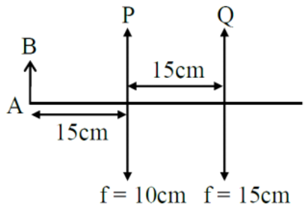
$2 \rightarrow$ concave lens

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{-f_2} : \frac{1}{f_1} - \frac{1}{f_2}$

for $f_1 > f_2 \therefore f < 0$

$\therefore$ concave lens.

16. (b)



for P $\frac{1}{V}-\frac{1}{-15}=\frac{1}{10}$

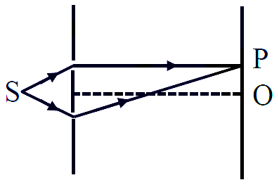
$\therefore$ v = +30 cm = +15 cm right of Q .

for Q $\frac{1}{V'}-\frac{1}{15}=\frac{1}{15}$

$V'=7.5$ cm

$$m=m_1\, m_2=\frac{30}{-15}\times\frac{7.5}{15}=-1$$

17. (b)



At P $y=\frac{d}{2}$

$$\Delta Q=\frac{2\pi}{\lambda}\frac{yd}{D}=\frac{2\pi}{\lambda}\frac{d}{2}\frac{d}{D}=\frac{\pi d^2}{\lambda D}$$

$$\Delta Q=\frac{\pi\times 25\lambda^2}{\lambda\times 10\ d}=\frac{5\pi\lambda}{2\times 5\lambda}=\frac{\pi}{2}$$

$$I=I_0\cos^2\left(\frac{\Delta Q}{2}\right)=I_0\cos^2\left(\frac{\pi}{2\times 2}\right)=\frac{I_0}{2}$$

18. (c) $\lambda_0=\frac{h}{mv_0}$

$$\lambda=\frac{h}{mv}=\frac{h}{m(0.8v_0)}=\frac{1}{0.8}\frac{h}{mv_0}=1.25\lambda_0$$

$\therefore \alpha=1.25$

19. (b) $\Delta m=\left(6\,m_p+6\,m_n\right)-m_c$

$=6(1.00727+1.00866)-12$

$=0.09558\text{amu}=89.03\text{MeV}/C^2$

20. (b) $N_P X_P=N_n X_n$

$N_P<N_n \therefore x_P>x_n$ in depletion region

21. (33) $U=\frac{1}{2}y(\text{ strain })^2\times$ volume

$$=\frac{1}{2}\times 1.1\times 10^{11}\times\left(\frac{3\times 10^{-3}}{3}\right)^2\times 600\times 10^{-6}$$

U = 33 J

22. (922) $x\times 4200\times 50=(1000-x)\times 4200\times 50+(1000-x)\times 2256\times 10^3$

$x=1000-x+(1000-x)\times 10.74$

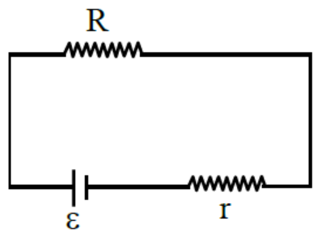
$x=921.52$

23. (200) For resonance

$$\omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{1.6\times 40\times 10^{-6}}}=\frac{10^3}{8}=125$$

$X_L=\omega L=125\times 1.6=200\Omega$

24. (1)



$i=\frac{\varepsilon}{R+r}$

$0.25=\frac{\varepsilon}{5+\mathrm{r}}$

$0.5=\frac{\varepsilon}{2+\mathrm{r}}$

on solving, $\mathrm{r}=1\Omega$

25.(44) $\phi=\mathrm{B}.\pi\mathrm{r}^2$

$\boldsymbol{\varepsilon}=\frac{\mathrm{d}\phi}{\mathrm{dt}}=\pi\mathrm{r}^2\frac{\mathrm{dB}}{\mathrm{dt}}$

$\mathrm{i}=\frac{\varepsilon}{\mathrm{R}}=\frac{\pi\mathrm{r}^2}{\mathrm{R}}\cdot\left(\frac{\mathrm{dB}}{\mathrm{dt}}\right)=\frac{\pi}{2}\left(\frac{1}{5}\right)^2(4\mathrm{t}+2)$

for $\mathrm{t}=3$

$i=\frac{11}{7}\times\frac{1}{25}\times 14$

$i=\frac{44}{50}$

26.(c) Mass of $H_2SO_4=\frac{50\times1.3}{98}\times\frac{50}{100}=\frac{32.5}{98}$

$\mathrm{Zn(s)}+\mathrm{H_2SO_4(aq)}\rightarrow\mathrm{ZnSO_4(aq)}+.\mathrm{H_2(g)}$

$\frac{20}{65}$ $\frac{32.5}{98}$

Moles of H_2 formed $=\frac{20}{65}$

Volume of $H_2=\frac{20}{65}\times 22.7=6.9$ L.

27. (b) (A) If two orbitals are having same value of " $n+\ell$ " then the orbital having lower value of ' n ' will have lower energy.

(B) Energies of the orbitals in the same subshell decreases with increase in atomic number.

(C) Size of $2p_x<3p_x$

(D) Orbital Radial node (n- $\ell-1$)

Orbital	Radial node
5f	1
6s	5
4d	1
5p	3
5d	2

28.(a) According to NCERT

Bond length $=\mathrm{r_A}+\mathrm{r_B}$

Total length of molecule $=2(\mathrm{r_A}+\mathrm{r_B})$

29. (b) $\Delta_rG=\Delta_rH-T.\Delta_rS$

$\Delta_rH=(2\times42-80-8)=-4$ kJ/mol

$\Delta_r S=(400-250-140)=+10$ J/K.mol

$\Delta_rG=-4000-600\times(10)$

$=-10{,}000$ J/mol $=-10$ kJ/mol

30. (b) Generally molar heat capacity of liquids is greater than solids because liquids have greater degrees of freedom & thus can store more energy with lesser temperature rise.

Also, for monoatomic gases molar heat capacity is low as only translational degrees of freedom are present.

∴ order is :

$Br_2(\ell) > Cu(s) > He(g)$

31. (b) $A(g) \rightleftharpoons B(g) + C(g)$

$t = 0 \quad a \quad 0 \quad 0$

$t_{eq} \quad a - x \quad x \quad x$

$$K_p = \frac{(x)(x)}{(a-x)} \times \left(\frac{P}{a+x}\right)$$

$$K_P = \frac{x^2 P}{a^2 - x^2}$$

32. (b) Cell Reaction :

$Zn(s) + 2Ag^{\oplus}(aq) \longrightarrow 2Ag(s) + Zn^{+2}(aq)$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.059}{n}\log(Q)$$

$$1.6 = 1.56 - \frac{0.059}{2}\log\frac{1}{(Ag^+)^2}$$

$$0.04 = +0.059\log[Ag^+]$$

$$\log[Ag^+] = \frac{4}{5.9}$$

33. (a) Statement I :

Al_2O_3, Cr_2O_3 : Amphoteric

CO, N_2O : Neutral

Na_2O, V_2O_3 : Basic

Cl_2O_7, Mn_2O_7 : Acidic

Statement II :

Most basic oxide : Na_2O

Most acidic oxide : Cl_2O_7

34. (d) Statement I :

$Al + NaOH(\text{excess}) \rightarrow Na[Al(OH)_4] + H_2(g)$

Statement II :

$ClO_3^{\ominus}$ is sp^3 and pyramidal

$ICl_4^{\ominus}$ is sp^3d^2 and square planar

$IBr_2^{\ominus}$ is sp^3d and linear.

35. (a) Statement I :

More are the number of unpaired electrons, stronger is metal-metal boding thus higher enthalpy of atomization.

Statement II :

$[Fe(H_2O)_6]^{3+}$ is octahedral complex.

Thus, $\left[t_{2g}\left(d_{xy} = d_{yz} = d_{zx}\right) < e_g\left(d_{x^2-y^2} = d_{z^2}\right)\right]$

$[NiCl_4]^{2-}$ is tetrahedral complex.

Thus, $\left[e\left(d_{x^2-y^2} = d_{z^2}\right) < t_2\left(d_{xy} = d_{yz} = d_{zx}\right)\right]$

36. (c) (A) $[Fe(C_2O_4)_3]^{3-}$ has oxalate ion as ligand which leads to chelation thus maximum stability among given complexes.

(B) Stability : $[Cu(NH_3)_4]^{2+} < [Cu(en)_2]^{2+}$; due to higher CFSE and chelation.

(C) $K_4[Fe(CN)_6]$

$\Rightarrow Fe^{2+} = [Ar]3d^6$

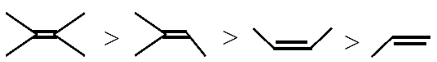
In presence of SFL the electronic configuration of $Fe^{+2}[t_{2g}^6, e_g^0]$, thus d^2sp^3 hybridized.

(D) $[Fe(NO_2)_3Cl_3]^{3-}$ has NO_2^- as ambidentate ligand thus can show linakge isomerism.

(E) NO_2^- and SCN^- both are ambidentate ligands.

37. (c)

38. (a)



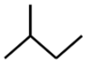
αH=12 > αH=9 > αH=6 > αH=3

Stability of alkene $\propto$ Number of α hydrogen.

39. (c)

(A) (1°C = 2)

(B) (1°C = 4)

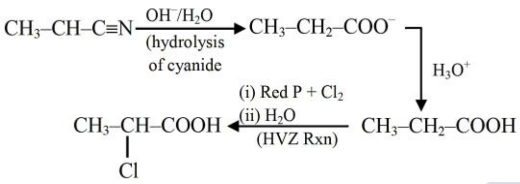


(1°C = 3)

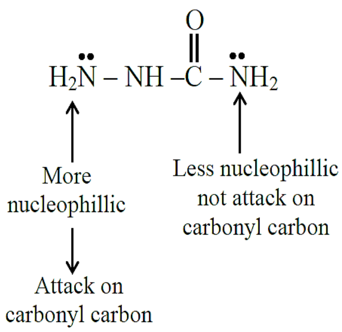
(C) Mol. mass = 72

(B) (1°C = 2)

40. (a)

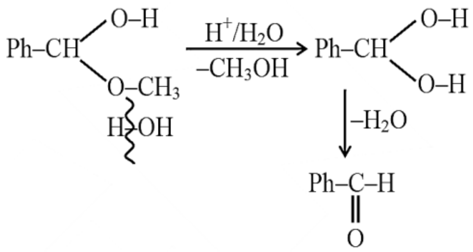


41. (d) Statement-I



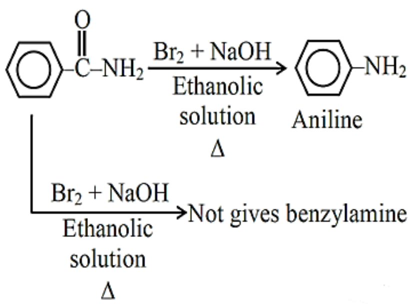
Hence statement-I is incorrect.

Statement-II

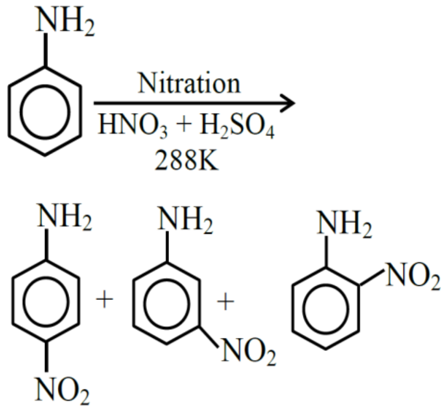


Statement-II is correct.

42. (d) Statement-I



Hence Statement-I is incorrect
Statement-II



(P) (m) (o)
$\simeq 51\%$ $\simeq 47\%$ $\simeq 2\%$
Statement-II is correct

43.(c) On changing pH tertiary strutcure of protein disrupted and amino acids intact only by peptide linkage.

44.(a) α-D-Glucose and β-D-Glucose are anomers. At C-3, C-4 and C-5 are identical configuration in glucose and fructose.

45. (d) SO_2 gas acts as reducing agent.

$$K_2Cr_2O_7 + SO_2 \xrightarrow{H_2SO_4} \underset{\text{Green solution}}{Cr_2(SO_4)_3} + SO_4^{2-}$$

46.(15) $d^3 \rightarrow [t_{2g}^3, e_g^0]$, 3 unpaired electrons

d^4(low spin) $\rightarrow [t_{2g}^4, e_g^0]$, 2 unpaired electrons

d^5 (high spin) $\rightarrow [t_{2g}^3, e_g^2]$, 5 unpaired electrons

d^6 (high spin) $\rightarrow [t_{2g}^4, e_g^2]$, 4 unpaired electrons

d^7 (low spin) $\rightarrow [t_{2g}^6, e_g^1]$, 1 unpaired electrons

47. (30)

$$RMgI \xrightarrow[H_2O]{\text{Ice cold}} \underset{\boxed{\text{Gas}}}{R\text{–}H} + Mg\begin{smallmatrix} I \\ OH \end{smallmatrix}$$

Vol. $= 1.4 dm^3/gm$ at STP
Hence according to this mol. Mass of gas is $= 16$
Hence liberated gas is $= CH_4$ (Methane)

$$CH_4 + I_2 \xrightarrow{HIO_3} \underset{\boxed{X}}{CH_3\text{–}I} + HI$$

$$\underset{\boxed{X}}{CH_3 + I} \xrightarrow[\text{(Wurtz reaction)}]{\text{Na/Dry ether}} \underset{\boxed{Y}}{CH_3 - CH_3}$$

Molar mass $= 30$

48. (62) Osmotic pressure, $\pi = \rho gh$
$= 1000 \times 10 \times 80 \times 10^{-3}$
$= 800$ Pa ...(1)
Let molar mass of hemoglobin $=$ Mg/mol
conc. of haemoglobin $= \frac{20/M}{10^{-3}}\left(\frac{mol}{m^3}\right)$
$\pi = CRT$
(S.I. units)
$\Rightarrow \pi = \left[\frac{20}{10^{-3}M}\right] \times 8.3 \times 300$

$= 800$ ...[From (1)]

Solving we get :

$M = \frac{20 \times 8.3 \times 300}{0.8}$ g/mol

$= 62.25$ kg mol^{-1}

49. (15) Let molarity of MX solution $=$ yM

$\Rightarrow \Lambda_m = c(\ell/A) \times \frac{1000}{y}$

$\Rightarrow 123.5 = 1.9 \times 10^{-3} \times 1.3 \times \frac{1000}{y}$

$\Rightarrow y = 0.02M$

Let molar mass of MX $=$ Mgmol^{-1}

0.2 mol MX is dissolve in 1 L solution

(0.02×75) gm MX is dissolved in 1000 g solution

$\%w/w = x\% = \frac{0.02 \times 75}{1000} \times 100$

$= 0.15\%$

50. (90) From the plot, $t_{1/2} \propto [\ A]_0$

$\propto [A_0]^{1-N}$

$1 - N = 1$

$N = 0$ (order)

$t_{1/2} = \frac{A_0}{2\ K}$

$240 = \frac{4\times10^{-3}}{2\ K}$ ...(1)

$x = \frac{1.5\times10^{-3}}{2\ K}$ ...(2)

Divide (1) / (2)

$\frac{240}{x} = \frac{4}{1.5}$

$x = 90$

51. (c) Let $\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$

$a + ar = 1$

$ar^2 + ar^3 = 4$

$\Rightarrow ar^2(1 + r) = 4$

$\Rightarrow r = 2, a = \frac{1}{3}$ (reject as $p \in z$)

or $r = -2, a = -1$

Now $|P + q| = |a(ar) + ar^2 - ar^3|$

$= |a^2r(1 + r^4)|$

$|1 \times -2 \times 17| = 34$

52. (d) $Z^2 + 4Z - (1 + 12i) = 0$

$\Rightarrow Z = \frac{-4 \pm \sqrt{16 + 4(1 + 12i)}}{2}$

$\Rightarrow Z = -2 \pm \sqrt{5 + 12i}$

$\Rightarrow Z = -2 \pm (3 + 2i)$

$\Rightarrow Z = 1 + 2i, -5 - 2i$

$\therefore |Z_1|^2 + |Z_2|^2$

$= 5 + 29$

$= 34$

53. (a) $f(n) = \begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}$

$\sum_{n=1}^{k} f(n) = \begin{vmatrix} \frac{k(k+1)}{2} & -1 & -5 \\ -2\frac{k(k+1)(2k+1)}{6} & 3(2k+1) & 2k+1 \\ -3\frac{k^2(k+1)^2}{4} & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix} = 98$

$\Rightarrow \frac{k(k+1)(2k+1)}{2} \cdot \frac{7}{3} = 98$

$\Rightarrow k(k+1)(2k+1) = 84$

$\Rightarrow k = 3$

54. (c) Let $M = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

then $M\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow a_1 = 1,\ b_1 = 2, c_1 = 3$

$M\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \Rightarrow a_2 = 0,\ b_2 = 1, c_2 = 2$

$\&M\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow a_3 = -1,\ b_3 = 1, c_3 = 1$

Now $M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 11 \end{pmatrix}$

$\Rightarrow x + 0y - z = 1$

$2x + y + z = 7$

$3x + 2y + z = 11$

on solving $x = 2, y = 2, z = 1$

$\therefore x + y + z = 5$

55. (c) $T_r = \frac{r}{1+4r^4} = \frac{r}{4r^4+4r^2+1-4r^2} = \frac{r}{(2r^2+1)^2-(2r)^2}$

$= \frac{r}{(2r^2+2r+1)(2r^2-2r+1)}$

$S_{10} = \frac{1}{4}\left(\frac{1}{1} - \frac{1}{221}\right) = \frac{55}{221} = \frac{m}{n}$

$\therefore m + n = 276$

56. (d) 59 $A_1\ A_2\ A_3 \ldots A_{39}$159

$d = \frac{159-59}{39+1} = \frac{5}{2}$

Now, $A_{25} = 59 + 25\,d = 121.5$

$A_{28} = 59 + 28d = 129$

$A_{31} = 59 + 31d = 136.5$

$A_{36} = 59 + 36d = 149$

Mean of

$A_{25}, A_{28}, A_{31}, A_{36} = \frac{121.5 + 129 + 136.5 + 149}{4}$

$= \frac{536}{4} = 134$

57. (c) $T_{r+1} = {}^{10}C_r(2x^2)^{10-r}(1/x)^r$

$= {}^{10}C_r 2^{10-r} x^{20-2r-r}$

$= {}^{10}C_r 2^{10-r} x^{20-3r}$

$\Rightarrow 20 - 3r = 2$

$r = 6$

So required coefficient is ${}^{10}C_6 2^4$

58. (b) $P(\text{ Win }) = P(A \cap \text{ Win }) + P(B \cap \text{ Win })$

$= P(\text{ A}) \cdot P\left(\frac{\text{Win}}{\text{A}}\right) + P(\text{ B}) \cdot P\left(\frac{\text{Win}}{\text{B}}\right)$

$= (0.6)(0.8) + (0.4)(0.7)$

$= 0.48 + 0.28 = 0.76$

59.(a) (But in this questions not mentioned that balls are distinct/alike)?

Blue - 5 Yellow - 6 Red - 4

Reqd. No. of ways

$= {}^5C_2 \times {}^6C_2 \times {}^4C_4 + {}^5C_2 \times {}^6C_3 \times {}^4C_3 + {}^5C_2 \times {}^6C_4 \times {}^4C_2$

$+ {}^5C_3 \times {}^6C_2 \times {}^4C_3 + {}^5C_3 \times {}^6C_3 \times {}^4C_2 + {}^5C_4 \times {}^6C_2 \times {}^4C_2$

$= 10 \times 15 \times 1 + 10 \times 20 \times 4 + 10 \times 15 \times 6 +$

$10 \times 15 \times 4 + 10 \times 20 \times 6 + 5 \times 15 \times 6$

$= 150 + 800 + 900 + 600 + 1200 + 450$

$= 4100$

60. (a) $\bar{x} = \frac{0.\,{}^nC_0 + 0.\,{}^nC_1 + 2.\,{}^nC_2 + \cdots + n(n-1) \cdot nCC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + nC_n}$

$= \frac{\sum_{r=0}^{n} r(r-1)^n C_r}{2^n}$

$= \frac{\sum_{r=1}^{n} n(r-1)^{n-1} C_{r-1}}{2^n} = \frac{\sum_{r=2}^{n} n(n-1)^{n-2} C_{r-2}}{2^n}$

$= \frac{n(n-1)2^{n-2}}{2^n} = \frac{n(n-1)}{4}$

$\Rightarrow \frac{n(n-1)}{4} = 60 \Rightarrow n^2 - n - 240 = 0$

$\Rightarrow n = 16, -15$ (Reject)

$N = \sum f_i = {}^{16}C_0 + {}^{16}C_1 + \cdots .. + {}^{16}C_{16} = 2^{16}$

$N = 2^{16}$ (Even)

Median $= \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2}$ term

$\frac{N}{2} = 2^{15} = 32768$

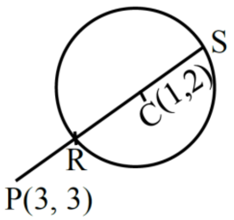
x_i	f_i
0	${}^{16}C_0$
0	${}^{16}C_1$
2	${}^{16}C_2$
6	${}^{16}C_3$
⋮	⋮
⋮	⋮
⋮	⋮
240	${}^{16}C_{16}$

x_i	f_i	CF
0	${}^{16}C_0$	1
0	${}^{16}C_1$	16
2	${}^{16}C_2$	137
6	${}^{16}C_3$	697
12	${}^{16}C_4$	2517
20	${}^{16}C_5$	6885
30	${}^{16}C_6$	14893
24	${}^{16}C_7$	26333

56	$^{16}C_8$	$39203 \rightarrow \frac{N}{2} \& \left(\frac{N}{2}+1\right)$ lies
72	$^{16}C_9$	$\frac{n(n-1)2^{n-2}}{2^n}$
	$\vdots$	$\vdots$
	$\vdots$	$\vdots$
	$\vdots$	$\vdots$

$\Rightarrow$ median $= \frac{56+56}{2} = 56$

61. (b) Parabola $y = x^2 - 6x + 12 = (x-3)^2 + 3y - 3 = (x-3)^2 \Rightarrow$ Vertex P (3,3)



$x^2 + y^2 - 2x - 4y + 3 = 0$

$C(1,2); r = \sqrt{2}$

For max. of $(PR + PS)^2$

RS should be diameter

$(PR + PS)^2_{max} = (PC - r + PC + r)^2$

$= (2PC)^2 = 4(PC)^2$

$= 4 \times 5 = 20$

62. (b) Let $B(2t^2, 4t) \Rightarrow C\left(\frac{2}{t^2}, \frac{-4}{t}\right); A(-2,0)$

given $m_{AB} = \frac{4t}{2t^2+2} = \frac{3}{5}$

$20t = 6t^2 + 6$

$3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$ but $t > 1$ (given)

$\Rightarrow P(18,12), Q\left(\frac{2}{9}, \frac{-4}{3}\right)$

$$A = \left|\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 18 & 12 & 1 \\ \frac{2}{9} & \frac{-4}{3} & 1 \end{vmatrix}\right| = \frac{80}{3}$$

$\Rightarrow 6\,A = 160$

63. (c) $S_1(3,5) \& S_2(3,-4)$

$\because S_1 S_2 = 2ae$

$\Rightarrow 9 = 2ae \quad ......(1)$

and $\because 6e^2 - 11e + 3 = 0$

$\Rightarrow e = \frac{3}{2}, e = \frac{1}{3} < 1$ (rejected)

From (1); $a = 3$

$\Rightarrow LR = 2 \cdot \frac{b^2}{a} = 2a(e^2 - 1) = 2.3\left(\frac{9}{4} - 1\right) = \frac{15}{2}$

64.(c) Let the point P is $(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$

$PR = 6 \Rightarrow PR^2 = 36$

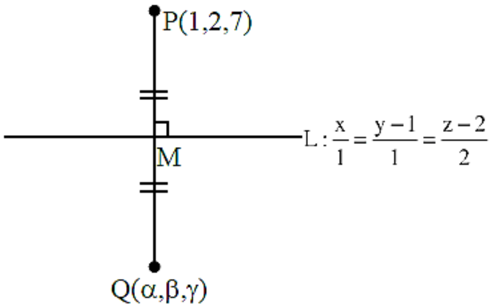
$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$

$(4\lambda - 2)^2 + (\lambda + 1)^2 + (\lambda - 2)^2 = 9$

$\lambda^2 - \lambda = 0 \Rightarrow \lambda = 0,1$

$\lambda = 0 \Rightarrow \mathrm{P}(-3,4,-1)$
$\lambda = 1 \Rightarrow P(5,6,1)$
R(1,2,3)
G(1,4,1)
$= \alpha + \beta + \gamma = 6$

65. (c) $PQ \perp L \Rightarrow (\alpha - 1) + (\beta - 2) + 2(\gamma - 7) = 0$
$\Rightarrow \alpha + \beta + 2\gamma = 17 \qquad \ldots(1)$



M is midpoint of PQ which will satisfy L

$$\frac{\alpha+1}{2} = \frac{\frac{\beta+2}{2} - 1}{1} = \frac{\frac{\gamma+7}{2} - 2}{2}$$

$$\Rightarrow \frac{\alpha+1}{2} = \frac{\beta}{2} = \frac{\gamma+3}{4}$$

$\Rightarrow \alpha + 1 = \beta \qquad \ldots(2)$
and $2\beta = \gamma + 3$
$\Rightarrow \alpha = 3, \beta = 4, \gamma = 5 \qquad \ldots(3)$
Distance from (a, 2, 5) is $= \sqrt{(a-3)^2 + 4 + 0} = 4$
$(\mathrm{a}-3)^2 + 4 = 16 \Rightarrow \mathrm{a}^2 - 6\mathrm{a} - 3 = 0 \Rightarrow$ sum of values of a $= 6$

66. (b) $\overrightarrow{OR} = \frac{\overrightarrow{OP}}{5} = \frac{\vec{a}}{5}$

$\overrightarrow{RM} = \frac{\overrightarrow{OQ}}{5} = \frac{\vec{b}}{5}$

$\overrightarrow{OM} - \overrightarrow{OR} = \frac{\vec{b}}{5}$

$\overrightarrow{OM} = \frac{\vec{a}+\vec{b}}{5}$

$\overrightarrow{PM} = \overrightarrow{OM} - \overrightarrow{OP}$

$\frac{\vec{a}+\vec{b}}{5} - \vec{a}$

$= \frac{\vec{b}-4\vec{a}}{5}$

67. (b) $f(x) = \lim_{y\to0} \frac{1-\cos(xy)}{(xy)^2} \cdot \frac{\tan xy}{xy} \frac{(xy)^3}{y^3}$

$\mathrm{f(x)} = \frac{x^3}{2}$

$\mathrm{f(x)} = \sin x$

$\frac{x^3}{2} = \sin x$

3 points of intersection

68. **(b)** $f(a) = \frac{1}{2}\left(2(2^a + 2^{-a}) + (3^a + 3^{-a})\right)$

$a = \frac{1}{2}(2 \times 2 + 2) = 3$

$I = \int_{\ln(3-1)}^{\ln 3} \frac{e^{2x}}{e^{4x} - 1} dx$

put $e^{2x} = t$

$$I=\frac{1}{2}\int_4^9\frac{dt}{t^2-1}$$

$$I=\frac{1}{4}\left(\ln\left(\frac{t-1}{t+1}\right)\right)_4^9$$

$$I=\frac{1}{4}\ln\left(\frac{4}{3}\right)$$

69.(a) $f(1)=\int_1^1 f(t)dt+(1-1)(\ln 1-1)+e$

$f(1)=e$

$f(x)=\int_1^x f(t)dt+(1-x)(\ln x-1)+e$

diff. w.r.t. x

$f'(x)=f(x)+\frac{1}{x}-\ln x$

I.F. $=e^{\int -1dx}=e^{-x}$

$f(x)e^{-x}=\int e^{-x}\left(\frac{1}{x}-\ell nx\right)dx$

$f(x)e^{-x}=e^{-x}\ln x+C$

put $x=1, y=e$

$C=1$

$f(x)e^{-x}=e^{-x}\mid nx+1$

$f(x)=e^{x}+\ln x$

$f(e)=e^{e}+1$

70. (b) $f''(x)=g''(x)$

Integrate

$f'(x)=g'(x)+C_1$

Put $x=1$

$f'(1)=g'(1)+C_1$

$4=2+C_1$

$C_1=2$

$\Rightarrow f'(x)=g'(x)+2$

$f(x)=g(x)+2x+C_2$

Put $x=2$

$f(2)=g(2)+4+C_2$

$3=9+4+C_2$

$C_2=-10$

$f(x)=g(x)+2x-10$

Put $x=25$

$f(x)-g(25)=40$

71. (18)

A / B	2	3	8
1	3	4	9
4	6	7	12
7	9	10	15

These are possible sum of a_i+b_j.

Now b_1+a_2 divides a_1+b_2

So,

a_1+b_2	a_2+b_1	No. of pairs
15	3,15	2

12	3,4,6,12	4
10	10	1
9	3,9	6
7	7	1
6	3,6	2
4	4	1
3	3	1
		18

Number of relations $= 18$

72. **(7)** $\tan 45^{\circ} = \left|\frac{m+1}{1-m}\right|$

$\Rightarrow \pm 1 = \dfrac{m+1}{1-m}$

$\Rightarrow$ m $= 0$ & one line is perpendicular to x axis.

$L_1 : x = -1,\ L_2 : y = -1$

Now taking mirror image of $x + 1 = 0$ in $x + 2y = 1$

$A' : \dfrac{x+1}{1} = \dfrac{y+1}{2} = \dfrac{-2(-1-2-1)}{1^2+2^2}$

$x = \dfrac{3}{5}, y = \dfrac{11}{5}\quad A'\left(\dfrac{3}{5}, \dfrac{11}{5}\right)$

line $= 3x - 4y + 7 = 0$

Now taking mirror image of $y + 1 = 0$ in $x + 2y = 1$

$A'' : \dfrac{x+1}{1} = \dfrac{y+1}{2} = \dfrac{-2(-1-2-1)}{1^2+2^2}$

$x = \dfrac{3}{5}, y = \dfrac{11}{5}$

$A''\left(\frac{3}{5}, \frac{11}{5}\right)$

line : $(y+1) = \dfrac{\left(\frac{11}{5}+1\right)}{\left(\frac{3}{5}-3\right)}(x-3)$

$4x + 3y = 9$

comparing $4x + 3y = 9$ with $ax + by = 9$&

$a = 4, b = 3$

by line

$-3x + 4y = 7$

comparing with

$cx + dy = 7$

$\therefore c = -3, d = 4$

So, $ad + bc$

$= 4 \times 4 + 3 \times -3$

$= 16 - 9 = 7$

73. **(19)** $\cos\theta \cos\frac{5\theta}{2} = \cos 7\theta \cos\frac{7\theta}{2}$

$\cos\dfrac{7\theta}{2} + \cos\dfrac{3\theta}{2} = \cos\dfrac{21\theta}{2} + \cos\dfrac{7\theta}{2}$

$\cos\dfrac{21\theta}{2} - \cos\dfrac{3\theta}{2} = 0$

$-2\sin 6\theta \sin\dfrac{9\theta}{2} = 0$

$\sin 6\theta = 0$

$\theta = 0, \pm\frac{\pi}{6}, \pm\frac{2\pi}{6}, \ldots \pm\frac{5\pi}{6}, \pm\pi$ (13 solutions)

$\sin\dfrac{9\theta}{2} = 0$

$\theta = \pm\frac{2\pi}{9}, \pm\frac{4\pi}{9}, \pm\frac{8\pi}{9}$ (6 more solutions)

Total = 19

74. (16) $f(x) + 3f\left(\frac{\pi}{2} - x\right) = \sin x$ …..(1)

Put $x \to \frac{\pi}{2} - x$

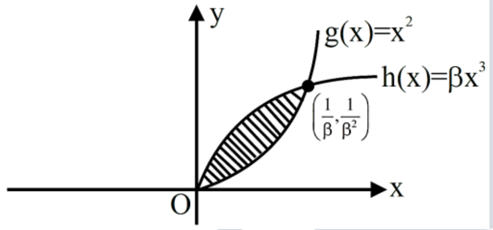
$\Rightarrow f\left(\frac{\pi}{2} - x\right) + 3f(x) = \cos x$ …(2)

From (1) & (2)

$f(x) = \dfrac{1}{8}(3\cos x - \sin x)$

$f_{max} = \dfrac{\sqrt{10}}{8}$

$y = g(x)$ & $y = h(x)$ intersect as shown in the figure



$\therefore$ Area bounded $= \Delta = \left|\int_0^{\frac{1}{\beta}}(\beta x^3 - x^2)dx\right|$

$= \frac{1}{12\beta^3} = \alpha^2$ (given)

$\Rightarrow 30\beta^3 = 16$

75. (48) $\frac{dy}{dx} + y\tan x = \frac{\sec^3 x}{\sqrt{\tan x}}$

$IF = e^{\int \tan x dx} = \sec x$

$y\sec x = \int \dfrac{\sec^4 x}{\sqrt{\tan x}} dx$

$y\sec x = \int \dfrac{(1+\tan^2 x)}{\sqrt{\tan x}} \sec^2 x dx$

$\tan x = t$

$y\sec x = 2\sqrt{\tan x} + \dfrac{2}{5}(\tan x)^{5/2} + c$

$y\left(\frac{\pi}{4}\right) = \frac{6\sqrt{2}}{5}$

$\dfrac{6\sqrt{2}}{5} \times \sqrt{2} = 2 + \dfrac{2}{5} + c \Rightarrow \dfrac{12}{5} = \dfrac{12}{5} + c$

$\Rightarrow c = 0$

$y(\pi/3) = \dfrac{8}{5} \cdot 3^{1/4} \Rightarrow \alpha = 2 \times 3^{\frac{1}{4}}$

$\therefore \alpha^4 = 48$