

PHYSICS

SECTION-A

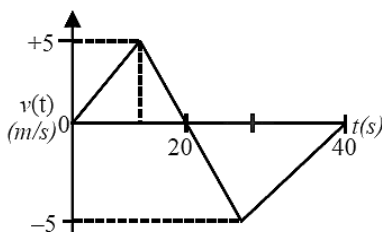
1. Match List-I with List-II.

List-I		List-II	
(A)	Meter (L)	(I)	$\sqrt{\frac{hc}{G}}$
(B)	Second (S)	(II)	$\sqrt{\frac{Gh}{c^5}}$
(C)	Kilogram (M)	(III)	$\sqrt{\frac{K^2 L^2 c^3}{Gh}}$
(D)	Kelvin (K)	(IV)	$\sqrt{\frac{Gh}{c^3}}$

where h (Planck's constant), G (gravitational constant) and c (speed of light in vacuum) as fundamental units.

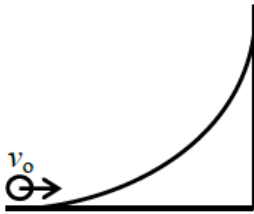
Choose the correct answer from the options given below:

- (a) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
 (b) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
 (c) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
 (d) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
2. In an experiment to determine the resistance of a given wire using Ohm's law, the voltmeter and ammeter readings are noted as 10 V and 5 A, respectively. The least counts of voltmeter and ammeter are 500 mV and 200 mA, respectively. The estimated error in the resistance measurement is ____ Ω
- (a) 0.25 (b) 2
 (c) 2.5 (d) 0.18
3. A mass of 1 kg kept in a inclined plane with 30° inclination with respect to horizontal plane and it is at rest initially. Then the whole assembly is moved up with constant velocity of 4 m/s. The work done by the frictional force in time 2 s is ____ J.
 (Take $g = 10 \text{ m/s}^2$)
- (a) 20 (b) 25
 (c) 30 (d) 10
4. The velocity (v) versus time (t) plot of a particle is shown in the figure, for a time interval of 40s. The total distance travelled by the particle and the average velocity during this period are, respectively ____



- (a) 25 m and zero (b) 50 m and zero
 (c) 100 m and zero (d) 100 m and 2.5 m/s
5. A wheel initially at rest is subjected to a uniform angular acceleration about its axis. In the first 2 s it rotates through an angle θ_1 and in the next 2 s it rotates through an angle θ_2 . The ratio $\frac{\theta_2}{\theta_1}$ is ____ .
- (a) 6 (b) 3
 (c) 4 (d) $\frac{1}{3}$

6. An object of uniform density rolls up the curved path with the initial velocity v_0 as shown in the figure. If the maximum height attained by an object is $\frac{7v_0^2}{10g}$ (g = acceleration due to gravity), the object is a ____ .



- (a) solid cylinder (b) ring
(c) disc (d) solid sphere
7. A body of mass m is taken from the surface of earth to a height equal to twice the radius of earth (R_e). The increase in potential energy will be ____ . (g is acceleration due to gravity at the surface of earth)
- (a) $\frac{1}{2}mgR_e$ (b) $\frac{3}{4}mgR_e$
(c) $\frac{1}{4}mgR_e$ (d) $\frac{2}{3}mgR_e$
8. Eight mercury drops, each of radius r , coalesce to form a bigger drop. The surface energy released in this process is ____ . (S is the surface tension of mercury).
- (a) $8\pi r^2 S$ (b) $16\pi r^2 S$
(c) $64\pi r^2 S$ (d) $4\pi r^2 S$
9. An ideal gas at pressure P and temperature T is expanding such that $PT^3 = \text{constant}$. The coefficient of volume expansion of the gas is ____ .
- (a) $\frac{2}{T}$ (b) $\frac{1}{T}$
(c) $\frac{4}{T}$ (d) $\frac{3}{T}$
10. Match List-I with List-II.

List-I		List-II	
(A)	$\sin^2 \omega t$	(I)	Periodic with time period $T = \frac{\pi}{\omega}$ but not simple harmonic motion (SHM)
(B)	$\sin^3(2\omega t)$	(II)	Periodic with time period $T = \frac{2\pi}{\omega}$ but Not SHM
(C)	$\sin(\omega t) + \cos(\pi\omega t)$	(III)	periodic with time period $T = \frac{\pi}{\omega}$ and SHM
(D)	$\cos \omega t + \cos 2\omega t$	(IV)	Non-periodic

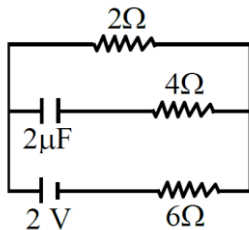
Choose the correct answer from the options given below.

- (a) A-III, B-I, C-IV, D-II
(b) A-II, B-I, C-III, D-IV
(c) A-III, B-II, C-IV, D-I
(d) A-II, B-I, C-IV, D-III

11. A metal rod of length L rotates about one end at origin with a uniform angular velocity ω . The magnetic field radially falls off as $B(r) = B_0 e^{-\lambda r}$; λ being a positive constant. The emf induced (neglecting the centripetal force on electrons in the rod) is:

- (a) $B_0 \omega \left[\frac{1}{\lambda^2} - e^{-\lambda L} \left(\frac{1}{\lambda^2} + \frac{L}{\lambda} \right) \right]$
 (b) $B_0 \omega \left[\frac{1}{\lambda^2} + e^{-\lambda L} \left(\frac{1}{\lambda^2} + \frac{L}{\lambda} \right) \right]$
 (c) $B_0 \omega \left[\frac{4}{\lambda^2} - e^{-2\lambda L} \left(\frac{1}{\lambda^2} + \frac{2L}{\lambda} \right) \right]$
 (d) $B_0 \omega \left[\frac{3}{\lambda^2} - e^{-3\lambda L} \left(\frac{3}{\lambda^2} + \frac{L}{\lambda} \right) \right]$

12. Under steady state condition the potential difference across the capacitor in the circuit is ____ V.



- (a) 0.5 (b) 1.5
 (c) 0 (d) 2

13. A particle of charge q and mass m is projected from origin with an initial velocity $\vec{v} = \left(\frac{v_0}{\sqrt{2}} \hat{x} + \frac{v_0}{\sqrt{2}} \hat{y} \right)$. There exists a uniform magnetic field $\vec{B} = B_0 \hat{z}$ and a space varying electric field $\vec{E} = E_0 e^{-\lambda x} \hat{x}$ within the region $0 \leq x \leq L$. After travelling a distance such that x -coordinate has changed from $x = 0$ to $x = L$, the change in the kinetic energy is ____.

- (a) $\frac{qE_0}{\lambda} [1 - e^{-\lambda L}]$
 (b) $\left(\frac{v_0 q B_0}{2\lambda} \right) [2 - e^{-2\lambda L}]$
 (c) $\frac{qE_0}{\lambda} [1 + e^{-\lambda L}]$
 (d) $q \left[\frac{E_0 + v_0 B_0}{\lambda} \right] [1 - e^{-\lambda L/2}]$

14. Given below are two statements : one is labelled as

Assertion (A) : and the other is labelled as Reason (R).

Assertion (A) : The electromagnetic wave exerts pressure on the surface on which they are allowed to fall.

Reason (R) : There is no mass associated with the electromagnetic waves.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A)
 (b) Both (A) and (R) are true but (R) is not the correct explanation of (A)
 (c) (A) is true but (R) is false
 (d) (A) is false but (R) is true

15. A thin convex lens and a thin concave lens are kept in contact and are co-axial. Which of the following statements is correct for this combination of two lenses?

- (a) behaves as concave lens if $|f_{\text{convex}}| > |f_{\text{concave}}|$
 (b) behaves as concave lens if $|f_{\text{convex}}| < |f_{\text{concave}}|$
 (c) behaves as convex lens if $|f_{\text{convex}}| > |f_{\text{concave}}|$
 (d) Focal length of the lens system will change if the position of two lenses are interchanged

16. An object AB is placed 15 cm on the left of a convex lens P of focal length 10 cm . Another convex lens Q is now placed 15 cm right of lens P . If the focal length of lens Q is 15 cm , the final image is ____

- (a) virtual, formed at 7.5 cm right of lens Q , with a size bigger than that of AB
 (b) real, formed at 7.5 cm right of lens Q , with a size same than that of AB

- (c) formed at infinity.
 (d) real, formed at 7 cm right of lens Q, with a size smaller than that of AB
17. The maximum intensity in a Young's double slit experiment is I_0 . Distance between the slits (d) is 5λ , where λ is the wavelength of light used. The intensity of the fringe, exactly opposite to one of the slits on the screen, placed at $D = 10d$ is ____
- (a) $\frac{I_0}{4}$ (b) $\frac{I_0}{2}$
 (c) I_0 (d) $\frac{3I_0}{4}$
18. An electron is travelling with a velocity v in free space and when it enters a medium, its velocity is reduced by 20%. The de Broglie wavelength of electron in the medium is $\alpha\lambda_0$, where λ_0 is its de Broglie wavelength in free space. The value of α is ____
- (a) 1.20 (b) 1.0
 (c) 1.25 (d) 0.75
19. Assuming the experimental mass of $^{12}_6\text{C}$ as 12 u, the mass defect of $^{12}_6\text{C}$ atom is ____ MeV/ c^2 . (Mass of proton = 1.00727u, mass of neutron = 1.00866u, 1u = 931.5 MeV/ c^2 and c is the speed of the light in vacuum.)
- (a) 127.5 (b) 89.03
 (c) 272.0 (d) 92.0
20. In a semiconductor p – n diode, the doping concentrations on p -side and n -side are 10^{15} atoms/ cm^3 and 10^{18} atoms / cm^3 , respectively. Which one of the following statement is true?
- (a) Widths of depletion region on either side of the interface are equal
 (b) The depletion region width is more on p-side compared to that in n -side
 (c) The depletion region width is more on n-side compared to that in p -side
 (d) No depletion region forms because of unequal doping concentrations on p and n-sides

SECTION-B

21. A copper wire of length 3 m is stretched by 3 mm by applying an external force. The volume of the wire is $600 \times 10^{-6} \text{ m}^3$. The elastic potential energy stored in the wire in stretched condition would be ____ J. (Given Young modulus of copper = $1.1 \times 10^{11} \text{ N/m}^2$)
22. The heat extracted out of x gram of water initially at 50°C to cool it down to 0°C is sufficient to evaporate $(1000 - x)$ gram of water also initially at 50°C . The value of x (closest integer) is ____ .
 (Take latent heat of water 2256 kJ/kg, K, specific heat capacity of water 4200 J/kg. K)
23. A series LCR circuit with $R = 20\Omega$, $L = 1.6\text{H}$ and $C = 40\mu\text{F}$ is connected to a variable frequency a.c. source. The inductive reactance at resonant frequency is ____ Ω .
24. When an external resistance of 5Ω is connected across terminals of a cell, a current of 0.25 A flows through it. When the 5Ω resistor is replaced by a 2Ω resistor, a current of 0.5 A flows through it. The internal resistance of the cell is ____ Ω .
25. A circular loop of radius 20 cm and resistance 2Ω is placed in a time varying magnetic field $\vec{B} = (2t^2 + 2t + 3)\text{T}$. At $t = 0$, for the plane of the loop being perpendicular to the magnetic field and, the induced current in the loop at $t = 3 \text{ s}$ is $\frac{\alpha}{50} \text{ A}$. The value of α is ____ (Take $\pi = 22/7$)

CHEMISTRY

SECTION-A

26. What volume of hydrogen gas at STP would be liberated by action of 50 mL of H_2SO_4 of 50% purity (density = 1.3 g mL^{-1}) on 20 g of zinc?
 Given : Molar mass of H, O, S, Zn are 1, 16, 32, 65 gmol^{-1} respectively.
- (a) 5.824 L (b) 7.428 L
 (c) 6.892 L (d) 8.375 L
27. Which of the following statement(s) is/are true ?
 (A) If two orbitals have the same value of $(n + l)$, the orbital with lower value of n will have lower energy.

- (B) Energies of the orbitals in the same subshell increase with increase in atomic number.
 (C) The size of $2p_x$ orbital is less than the size of $3p_x$ orbital.
 (D) Among $5f, 6s, 4d, 5p$ and $5d$ orbitals, none of the orbitals have 2 radial nodes.

Choose the correct answer from the options given below :

- (a) A, B and C only (b) A and C only
 (c) C and D only (d) A only

28. The covalent radii of atoms A and B are r_A and r_B , respectively. The covalent bond length and total length of AB molecule are respectively :

- (a) $(r_A + r_B), 2(r_A + r_B)$
 (b) $\frac{1}{2}(r_A + r_B), (r_A + r_B)$
 (c) $(r_A + r_B), (r_A + r_B)$
 (d) $2(r_A + r_B), \frac{1}{2}(r_A + r_B)$

29. Consider the following data for the reaction

$X_2(g) + Y_2(g) \rightleftharpoons 2XY(g)$ at 600 K. The $\Delta_r G^\ominus$ (in kJ mol^{-1}) for the reaction is:

Compound	$\Delta_f H_{600\text{ K}}^\ominus$ (kJ mol^{-1})	$S_{600\text{ K}}^\ominus$ ($\text{J mol}^{-1} \text{ K}^{-1}$)
XY(g)	42	200
$X_2(g)$	8	140
$Y_2(g)$	80	250

- (a) -21000 (b) -10
 (c) -1000 (d) -9.012

30. The correct order of molar heat capacities measured at 298 K and 1 bar is :

- (a) Copper (s) > Bromine (l) > Helium (g)
 (b) Bromine (l) > Copper (s) > Helium (g)
 (c) Helium (g) > Bromine (l) > Copper (s)
 (d) Helium (g) > Bromine (l) = Copper (s)

31. The reaction $A(g) \rightleftharpoons B(g) + C(g)$ was initiated with the amount 'a' of A(g). At equilibrium it is found that the amount of A(g) remaining is (a - x) at a total pressure of p.

The equilibrium constant K_p of the reaction can be calculated from the expression :

- (a) $\frac{x^2}{a^2+x^2} \times p$ (b) $\frac{x^2}{a^2-x^2} \times p$
 (c) $\frac{a+x^2}{x^2} \times p$ (d) $\frac{a-x^2}{x^2} \times p$

32. One half cell in a voltaic cell is constructed by dipping silver rod in AgNO_3 solution of unknown concentration, other half cell is Zn rod dipped in 1 molar solution of ZnSO_4 .

A voltage of 1.60 V is measured at 298 K for this cell. What is the concentration of Ag^+ ions used in terms of $\log x$ ($x = [\text{Ag}^+]$)?

$$E_{\text{Zn}^{2+}/\text{Zn}}^\ominus = -0.76 \text{ V}, E_{\text{Ag}^+/\text{Ag}}^\ominus = +0.80 \text{ V}, \frac{2.303RT}{F} = 0.059 \text{ V}$$

- (a) $\frac{2}{3.9}$ (b) $\frac{4}{5.9}$
 (c) $\frac{2.9}{2}$ (d) $\frac{5.9}{4}$

33. Given below are two statements :

Statement-I : The number of pairs among $[\text{Al}_2\text{O}_3, \text{Cr}_2\text{O}_3], [\text{Cl}_2\text{O}_7, \text{Mn}_2\text{O}_7], [\text{Na}_2\text{O}, \text{V}_2\text{O}_3]$ and $[\text{CO}, \text{N}_2\text{O}]$ that contain oxides of same nature (acidic, basic, neutral or amphoteric) is 4.

Statement-II : Among $\text{Na}_2\text{O}, \text{Al}_2\text{O}_3, \text{CO}$ and Cl_2O_7 , the most basic and acidic oxides are Na_2O and Cl_2O_7 , respectively.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement-I and Statement-II are true.
 (b) Both Statement-I and Statement-II are false.
 (c) Statement-I is true but Statement-II is false.
 (d) Statement-I is false but Statement-II is true.

34. Given below are two statements :

Statement-I : Aluminum upon reaction with NaOH forms $[\text{Al}(\text{OH})_6]^{3-}$ ion.

Statement-II : The geometry of ICl_4^- , ClO_3^- and IBr_2^- is square planar, pyramidal and linear respectively.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement-I and Statement-II are true.
 (b) Both Statement-I and Statement-II are false.
 (c) Statement-I is true but Statement-II is false.
 (d) Statement-I is false but Statement-II is true.

35. Given below are two statements :

Statement-I : Presence of large number of unpaired electrons in transition metal atoms results in higher enthalpies of their atomization.

Statement-II : $d_{xy} = d_{xz} = d_{yz} < d_{x^2-y^2} = d_{z^2}$ and $d_{x^2-y^2} = d_{z^2} < d_{xy} = d_{xz} = d_{yz}$ are the d-orbital splitting in $[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$ and $[\text{Ni}(\text{Cl})_4]^{2-}$ complex ions respectively.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement-I and Statement-II are correct.
 (b) Both Statement-I and Statement-II are incorrect.
 (c) Statement-I is correct but Statement-II is incorrect.
 (d) Statement-I is incorrect but Statement-II is correct.

36. Identify the correct statements from the following:

- (A) $[\text{Fe}(\text{C}_2\text{O}_4)_3]^{3-}$ is the most stable complex among $[\text{Fe}(\text{OH})_6]^{3-}$, $[\text{Fe}(\text{C}_2\text{O}_4)_3]^{3-}$ and $[\text{Fe}(\text{SCN})_6]^{3-}$
 (B) The stability of $[\text{Cu}(\text{NH}_3)_4]^{2+}$ is greater than that of $[\text{Cu}(\text{en})_2]^{2+}$
 (C) The hybridization of Fe in $\text{K}_4[\text{Fe}(\text{CN})_6]$ is d^2sp^3
 (D) $[\text{Fe}(\text{NO}_2)_3\text{Cl}_3]^{3-}$ exhibits linkage isomerism
 (E) NO_2^- and SCN^- ligands are NOT ambidentate ligands

Choose the correct answer from the options given below :

- (a) A, B, C, D and E
 (b) B, C and D only
 (c) A, C and D only
 (d) A, C and E only

37. Match List-I with List-II.

List-I (Purification technique)		List-II (Used to separate)	
(A)	Simple distillation	(I)	Steam volatile compound
(B)	Fractional distillation	(II)	Two liquids with large difference in boiling points
(C)	Steam distillation	(III)	Liquid decomposing at its boiling point

(D)	Distillation under reduced pressure	(IV)	Two liquids with close boiling points
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Choose the correct answer from the options given below :

- (a) A-II, B-III, C-I, D-IV
(b) A-II, B-IV, C-I, D-III
(c) A-II, B-IV, C-III, D-I
(d) A-IV, B-III, C-II, D-I

38. IUPAC name of some alkenes are given below :

Find out the correct stability order.

- (A) 2-Methylbut-2-ene
(B) cis-But-2-ene
(C) 2,3-Dimethylbut-2-ene
(D) Prop-1-ene

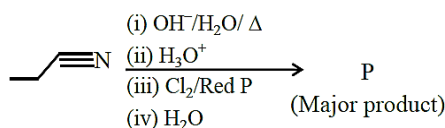
Choose the correct answer from the options given below:

- (a) C > A > B > D
(b) C > A > D > B
(c) B > D > A > C
(d) A > B > C > D

39. Identify the correct IUPAC name of hydrocarbon (x) containing three primary carbon atoms and with molar mass 72 g mol^{-1} .

- (a) 1,1-Dimethylcyclopropane
(b) 2,2-Dimethylpropane
(c) 2-Methylbutane
(d) n-pentane

40. Complete the following reaction sequence and give the name of major product 'P'.



- (a) 2-Chloropropanoic acid
(b) 3-Chloropropanoic acid
(c) 1-Chloropropane
(d) 2-Chloropropane

41. Given below are two statements :

Statement-I : The condensation reaction between $\text{CH}_3 - \text{CH} = \text{O}$ and $\text{H}_2\text{N}-\text{N}(\text{H})-\text{C}(=\text{O})-\text{NH}_2$ under optimum pH will produce $\text{CH}_3-\text{CH}=\text{N}-\text{C}(=\text{O})-\text{NH}_2$

Statement-II : The molecule, $\text{Ph}-\text{CH}(\text{OH})-\text{OCH}_3$ will generate $\text{Ph}-\text{CH}=\text{O}$ in the presence of dilute acid.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement-I and Statement-II are true.
(b) Both Statement-I and Statement-II are false.
(c) Statement-I is true but Statement-II is false.
(d) Statement-I is false but Statement-II is true.

42. Given below are two statements :

Statement-I : Heating benzamide with bromine in an ethanolic solution of sodium hydroxide will give benzylamine.

Statement-II : Nitration of aniline with $\text{HNO}_3/\text{H}_2\text{SO}_4$ at 288 K produces *m*-nitroaniline in higher amount than *o*-nitroaniline (pH adjusted).

In the light of the above statements, choose the correct answer from the options given below:

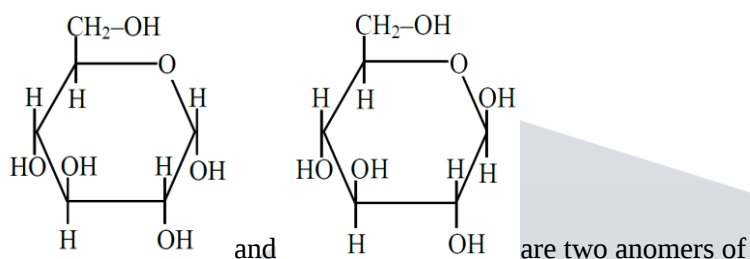
- (a) Both Statement-I and Statement-II are true.
- (b) Both Statement-I and Statement-II are false.
- (c) Statement-I is true but Statement-II is false.
- (d) Statement-I is false but Statement-II is true.

43. Identify the incorrect statement about tertiary structure of proteins.

- (a) They can be fibrous or globular in structure.
- (b) The main forces that stabilize the structure are hydrogen bonding, disulphide links, van der Waals and electrostatic forces of attraction.
- (c) The structure remains intact when exposed to pH changes.
- (d) A linear polypeptide chain will convert to a secondary structure and then further folding of the secondary structure will convert to tertiary structure.

44. Given below are two statements :

Statement-I :



D-(+)-glucose.

Statement-II : The open chain forms of D-glucose and D-fructose contain three similar chiral carbons at C_3 , C_4 and C_5 .

In the light of the above statements, choose the correct answer from the options given below :

- (a) Both Statement-I and Statement-II are true.
- (b) Both Statement-I and Statement-II are false.
- (c) Statement-I is true but Statement-II is false.
- (d) Statement-I is false but Statement-II is true.

45. A paper dipped in a dil. H_2SO_4 solution of 'X' upon treatment with SO_2 gas turns into green. The compound 'X' is :

- (a) KI-starch
- (b) $KMnO_4$
- (c) $Pb(CH_3COO)_2$
- (d) $K_2Cr_2O_7$

SECTION-B

46. The total number of unpaired electrons present in the d^3 , d^4 (low spin) d^5 (high spin), d^6 (high spin) and d^7 (low spin) octahedral complex systems is ____.

47. $RMgI$ when treated with ice cold water liberated a gas which occupied $1.4 dm^3/g$ at STP. The gas produced is further reacted with iodine in presence of HIO_3 to give compound (X). Compound (X) in presence of Na and dry ether produced compound (Y). Molar mass of compound (Y) is ____ $gmol^{-1}$. (Nearest integer)

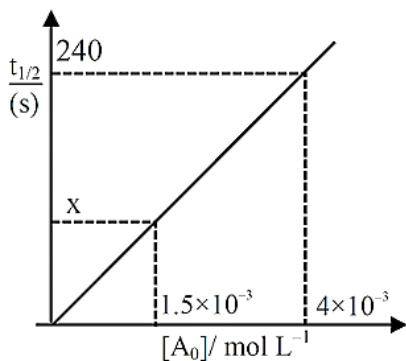
48. 20 g hemoglobin in a 1 L aqueous solution (A) at 300 K is separated from pure water by semi permeable membrane. At equilibrium the height of solution in a tube dipped in a solution (A) is found to be 80.0 mm higher than the tube dipped in water.

The molar mass of hemoglobin is ____ $kgmol^{-1}$. (Nearest integer)

(Given : $g = 10 ms^{-2}$, $R = 8.3 kPa dm^3 K^{-1} mol^{-1}$, density of solution = $1000 kg m^{-3}$)

49. At 298 K, the molar conductivity of x%(w/w) MX solution (aqueous) is $123.5 S cm^2 mol^{-1}$. The conductance of same solution is $1.9 \times 10^{-3} S$. The value of x is ____ $\times 10^{-2}$. (Given : cell constant = $1.3 cm^{-1}$; molar mass of MX is $75 g mol^{-1}$, density of aqueous solution of MX at 298 K is $1.0 g mL^{-1}$)

50. For a reaction $A \rightarrow P$ at TK, the half life ($t_{1/2}$) is plotted as a function of initial concentration $[A]_0$ of A as given below :



The value of x in the given figure is ____ s
(Nearest integer)

MATHEMATICS

SECTION-A

51. Let α, β be the roots of the equation $x^2 - x + p = 0$ and γ, δ be the roots the equation $x^2 - 4x + q = 0$; $p, q \in \mathbb{Z}$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then $|p + q|$ equals:
(a) 16 (b) 32
(c) 34 (d) 38
52. Let $z_1, z_2 \in \mathbb{C}$ be the distinct solutions of the equation $z^2 + 4z - (1 + 12i) = 0$. Then $|z_1|^2 + |z_2|^2$ is equal to :
(a) 18 (b) 22
(c) 29 (d) 34
53. If $f: \mathbb{N} \rightarrow \mathbb{Z}$ is defined by $f(n) = \begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}, k \in \mathbb{N}$, and $\sum_{n=1}^k f(n) = 98$, then k is equal to:
(a) 3 (b) 4
(c) 5 (d) 6
54. Let M be a 3×3 matrix such that $M \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, M \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ and $M \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$. If $M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 11 \end{pmatrix}$, then $x + y + z$ equals :
(a) 4 (b) 5
(c) 7 (d) 11
55. If the sum of the first 10 terms of the series $\frac{1}{1+1^4 \times 4} + \frac{2}{1+2^4 \times 4} + \frac{3}{1+3^4 \times 4} + \frac{4}{1+4^4 \times 4} + \dots$ is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to :
(a) 256 (b) 264
(c) 276 (d) 284
56. Let $A_1, A_2, A_3, \dots, A_{39}$ be 39 arithmetic means between the numbers 59 and 159. Then the mean of A_{25}, A_{28}, A_{31} and A_{36} is equal to:
(a) 129 (b) 136
(c) 131.50 (d) 134
57. The coefficient of x^2 in the expansion of $\left(2x^2 + \frac{1}{x}\right)^{10}, x \neq 0$, is :
(a) 3240 (b) 3360
(c) 3480 (d) 3600

58. The probabilities that players A and B of a team are selected for the captaincy for a tournament are 0.6 and 0.4, respectively. If A is selected the captain, the probability that the team wins the tournament is 0.8 and if B is selected the captain, the probability that the team wins the tournament is 0.7. Then the probability, that the team wins the tournament, is:
- (a) 0.74 (b) 0.76
(c) 0.72 (d) 0.78
59. A box contains 5 blue, 6 yellow and 4 red balls. The number of ways, of drawing 8 balls containing at least two balls of each colour, is:
- (a) 4100 (b) 4140
(c) 4230 (d) 4290
60. A variable X takes values $0, 0, 2, 6, 12, 20, \dots, n(n-1)$ with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, {}^nC_4, {}^nC_5, \dots, {}^nC_n$, respectively. If the mean of this data is 60, then its median is:
- (a) 56 (b) 42
(c) 72 (d) 90
61. Let the point P be the vertex of the parabola $y = x^2 - 6x + 12$. If a line passing through the point P intersects the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at the points R and S, then the maximum value of $(PR + PS)^2$ is:
- (a) 10 (b) 20
(c) 25 (d) 5
62. Let the directrix of the parabola $P: y^2 = 8x$, cut x-axis at the point A. Let $B(\alpha, \beta)$, $\alpha > 1$, be a point on P such that the slope of AB is $3/5$. If BC is a focal chord of P, then six times the area of $\triangle ABC$ is:
- (a) 80 (b) 160
(c) 174 (d) 192
63. Let the eccentricity e of a hyperbola satisfy the equation $6e^2 - 11e + 3 = 0$. If the foci of the hyperbola are $(3, 5)(3, -4)$, then the length of its latus rectum is:
- (a) $\frac{11}{3}$ (b) $\frac{17}{3}$
(c) $\frac{15}{2}$ (d) $\frac{17}{2}$
64. Let a triangle PQR be such that P and Q lie on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ and are at a distance of 6 units from $R(1, 2, 3)$. If (α, β, γ) is the centroid of $\triangle PQR$, then $\alpha + \beta + \gamma$ is equal to:
- (a) 4 (b) 5
(c) 6 (d) 8
65. If the distance of the point $(a, 2, 5)$ from the image of the point $(1, 2, 7)$ in the line $\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{2}$ is 4, then the sum of all possible values of a is equal to:
- (a) 11 (b) 9
(c) 6 (d) 4
66. Let O be the origin, $\overrightarrow{OP} = \vec{a}$ and $\overrightarrow{OQ} = \vec{b}$. If R is the point on $\overrightarrow{OP}$ such that $\overrightarrow{OP} = 5\overrightarrow{OR}$, and M is the point such that $\overrightarrow{OQ} = 5\overrightarrow{RM}$, then $\overrightarrow{PM}$ is equal to:
- (a) $\frac{1}{5}(\vec{a} - 4\vec{b})$ (b) $\frac{1}{5}(\vec{b} - 4\vec{a})$
(c) $\frac{1}{5}(-\vec{a} + 4\vec{b})$ (d) $\frac{1}{5}(-\vec{b} + 4\vec{a})$
67. Let $f(x) = \lim_{y \rightarrow 0} \frac{(1 - \cos(xy)) \tan(xy)}{y^3}$. Then the number of solutions of the equation $f(x) = \sin x$, $x \in \mathbb{R}$ is:
- (a) 0 (b) 2
(c) 3 (d) 1
68. Let $(2^{1-a} + 2^{1+a}), f(a), (3^a + 3^{-a})$ be a A. P. and α be the minimum value of $f(a)$. Then the value of the integral $\int_{\log_e(\alpha-1)}^{\log_e(\alpha)} \frac{dx}{(e^{2x} - e^{-2x})}$ is:

- (a) $\frac{1}{2} \log_e \left(\frac{4}{3}\right)$ (b) $\frac{1}{4} \log_e \left(\frac{4}{3}\right)$
(c) $\frac{1}{2} \log_e \left(\frac{8}{5}\right)$ (d) $\frac{1}{4} \log_e \left(\frac{8}{5}\right)$

69. Let $f: [1, \infty) \rightarrow \mathbb{R}$ be a differentiable function defined as $f(x) = \int_1^x f(t) dt + (1-x)(\log_e x - 1) + e$. Then the value of $f(f(1))$ is :

- (a) $(1 + e^e)$ (b) $(1 + e)$
(c) $(1 + e + e^e)$ (d) $1 + 2e$

70. Let $f(x)$ and $g(x)$ be twice differentiable functions satisfying $f''(x) = g''(x)$ for all $x \in \mathbb{R}$, $f'(1) = 2$, $g'(1) = 4$ and $g(2) = 3f(2) = 9$. Then $f(25) - g(25)$ is equal to :

- (a) 20 (b) 40
(c) -20 (d) -40

SECTION-B

71. Let $A = \{1, 4, 7\}$ and $B = \{2, 3, 8\}$. Then the number of elements, in the relation $R = \{(a_1, b_1), (a_2, b_2)\} \in ((A \times B) \times (A \times B)) : a_1 + b_2 \text{ divides } a_2 + b_1\}$ is ____

72. From the point $(-1, -1)$, two rays are sent making angles of 45° with the line $x + y = 0$. These rays get reflected from the mirror $x + 2y = 1$. If the equations of the reflected rays are $ax + by = 9$ and $cx + dy = 7$, $a, b, c, d \in \mathbb{Z}$, then the value of $ad + bc$ is ____

73. If $S = \left\{ \theta \in [-\pi, \pi] : \cos \theta \cos \frac{5\theta}{2} = \cos 7\theta \cos \frac{7\theta}{2} \right\}$, then $n(S)$ is equal to

74. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) + 3f\left(\frac{\pi}{2} - x\right) = \sin x$, $x \in \mathbb{R}$. Let maximum value of f on $\mathbb{R}$ be α . If the area of the region bounded by the curves $g(x) = x^2$ and $h(x) = \beta x^3$, $\beta > 0$, is α^2 , then $30\beta^3$ is equal to ____

75. Let $y = y(x)$ be the solution of the differential equation $(\tan x)^{1/2} dy = (\sec^3 x - (\tan x)^{3/2} y) dx$, $0 < x < \frac{\pi}{2}$, $y\left(\frac{\pi}{4}\right) = \frac{6\sqrt{2}}{5}$. If $y\left(\frac{\pi}{3}\right) = \frac{4}{5}\alpha$, then α^4 equals ____