1. (a)

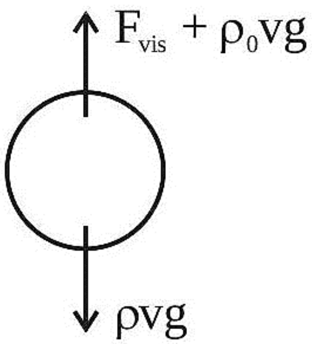
$$\frac{\text{Electric energy density}}{\text{Magnetic energy density}}=\frac{\frac{1}{2}\in_0 E_{rms}^2}{\left(\frac{B_{rms}^2}{2\mu_0}\right)}$$

$$=\left(\frac{E_{rms}}{B_{rms}}\right)^2, \mu_0 \in_0 \quad \left[C=\frac{1}{\mu_0 \in_0}\right]$$

$$=\frac{C^2}{C^2}=1$$

2. (b) Circuit is closed when neither A nor B is closed $\Rightarrow$ current flows for $A=0$ $B=0$ when either or both of A& B is closed we get current bypass from switch

3. (a)



For constant velocity $F_{net}=0$

$F_{vis}+\rho_0 vg=\rho vg$

$F_{vis}=(\rho-\rho_0)vg$

$=\rho vg\left(1-\frac{\rho_0}{\rho}\right)$

$=Mg\left(1-\frac{\rho_0}{\rho}\right)$

4. (b)

Collision frequency,

$$f=\frac{V}{\lambda}=\frac{V}{\left(\frac{1}{\sqrt{2}\pi d^2 n_v}\right)}=\sqrt{2}\pi d^2 v n_v$$

$\therefore f \propto n_v, n_v$ is number density

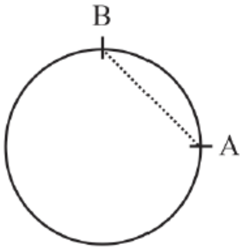
$$\frac{f_1}{f_2}=\frac{n_{v_1}}{n_{v_2}}=\frac{3\times10^{19}}{12\times10^{-19}}=0.25$$

5. (d) $dQ=dU+dw$

$$\frac{dU}{dt} = \frac{dQ}{dt} - \frac{dw}{dt}$$

$$\frac{dU}{dt} = 1000 - 200 = 800\text{ W}$$

6. (a)



$AB = R\sqrt{2}$

Let instantaneous velocity be v. time,

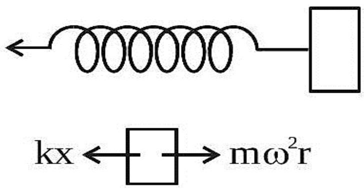
$$t = \frac{\text{Arc length}}{v} = \frac{2\pi\frac{R}{4}}{v} = \frac{\pi R}{2v}$$

average velocity,

$$\langle V \rangle = \frac{AB}{t} = \frac{R\sqrt{2}(2v)}{\pi R} = \frac{2\sqrt{2}\,V}{\pi}$$

$$\Rightarrow \frac{V}{\langle V \rangle} = \frac{\pi}{2\sqrt{2}}.$$

7. (b) $\longleftarrow 0.2 + x \longrightarrow$



Let extension in length of spring be x.

Radius of circle $r = 0.2 + x$

$Kx = m\omega^2 r$

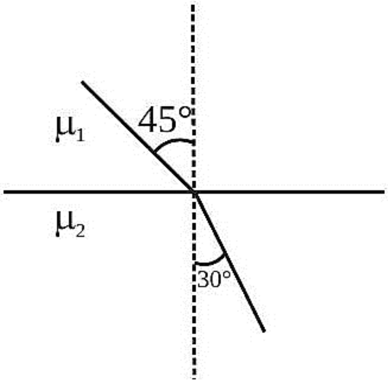
$$7.5x = \left(\frac{1}{10}\right)(5^2)(0.2 + x)$$

$$\Rightarrow \frac{15}{2}x = \frac{5}{2}\left(x + \frac{1}{5}\right)$$

$$\Rightarrow x = \frac{1}{10}$$

$\therefore$ Tension in spring $= kx = 7.5 \times \frac{1}{10} = 0.75\text{ N}$

8. (b)



Snell's law $\mu_1 \sin 45^\circ = \mu_2 \sin 30^\circ$

$$\frac{\mu_1}{\mu_2} = \frac{1}{\sqrt{2}}$$
$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1} = \frac{1}{\sqrt{2}}$$
$$\Rightarrow \lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

Frequency doesn't change on change in medium.

9. (b) $R = \frac{u^2}{g} \sin 2\theta$

R is maximum for $2\theta = 90^\circ$.

10. (c) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Differentiating both sides, we get

$$\frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2} \left[R = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 15}{10 + 15} = 6\right]$$
$$\Rightarrow \frac{\Delta R}{R} = \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right) R$$
$$= \left(\frac{0.5}{100} + \frac{0.5}{225}\right) 6$$
$$= \left(\frac{6 \times 0.5}{25}\right)\left(\frac{1}{4} + \frac{1}{9}\right) = \frac{13}{300}$$
$$\frac{\Delta R}{R} \times 100 = \frac{13}{3} = 4.33\%$$

11. (c) $m = \rho \times \frac{4}{3} \pi R^3$

$R \propto m^{\frac{1}{3}} (\rho = \text{constan t})$

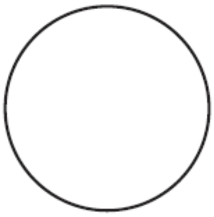
weight $= W \propto g \propto \frac{Gm}{R^2}$

$W \propto \frac{m}{m^{2/3}} \propto m^{1/3}$

So, $W^1 = (2)^{1/3}\ W$

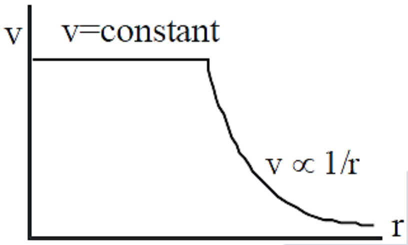
12. (d) At Moon, due to low escape velocity, the rms velocity of molecules is greater than escape velocity. Hence molecules escape and there is no atmosphere at Moon.

13. (a)

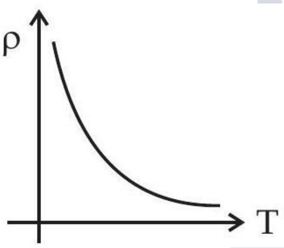


$V_{inside} = \frac{kQ}{R}$

$V_{outside} = \frac{kQ}{r}$



14. (b)



$\rho = \frac{m}{ne^2\tau}$

With rise in temperature, number density (n) of electrons and holes increases for semiconductors.

As m, e, τ are constant

$\rho \propto \frac{1}{n} \Rightarrow \rho \propto \frac{1}{T}$ [Rectangular hyperbola]

15. (c)

Mass:	$\mathbf{\frac{m}{1840}}$	4 m	m
Charge:	e	2e	e
Kinetic:			
energy	4 K	2 K	K

$\lambda = \frac{\mathbf{h}}{\sqrt{\mathbf{2mK}}}$	$\frac{\mathbf{h}}{\sqrt{2\cdot\frac{\mathbf{m}}{1840}\cdot 4\text{ K}}}$	$\frac{\mathbf{h}}{\sqrt{2\cdot 4\text{ m}2\text{ K}}}$	$\frac{\mathbf{h}}{\sqrt{2\text{mK}}}$

$\lambda_\alpha < \lambda_p < \lambda_e$

16. (c)

Range, $R = \sqrt{2Rh}$

$R_1 = \sqrt{2Rh_1}$

$h_2 = h_1 + \left(h_1 \times \frac{21}{100}\right) = 1.21\ h_1$

$\therefore\ R_2 = \sqrt{2Rh_2} = \sqrt{2R(1.21)h_1} = 1.1\sqrt{2Rh_1}$

$\therefore R_2 = 1.1R_1$

% increase in range

$= \frac{R_2 - R_1}{R_1} \times 100 = \left(\frac{R_2}{R_1} - 1\right) \times 100$

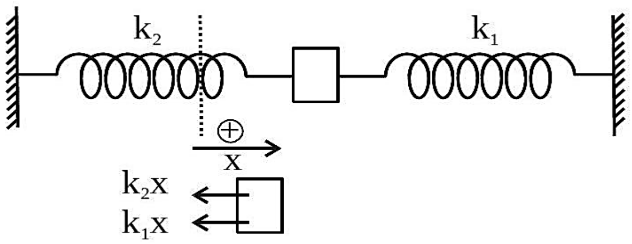
$= (1.1 - 1) \times 100 = 10\%$

17. (b) $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda\alpha\frac{1}{\Delta E}$

For shortest wavelength, energy gap should be maximum.

So, correct choice is transition from $n = 3$ to $n = 1$.

18. (c)



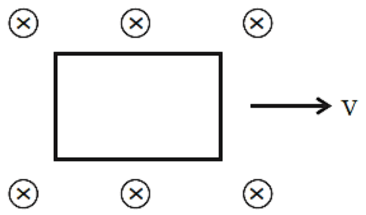
On displacing m to right by x

$F = -(k_1x + k_2x) = -(k_1 + k_2)x$

$a = \frac{F}{m} = -\left(\frac{k_1 + k_2}{m}\right)x = -\omega^2 x$

$\therefore\ \omega = \sqrt{\frac{k_1 + k_2}{m}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

19. (d)

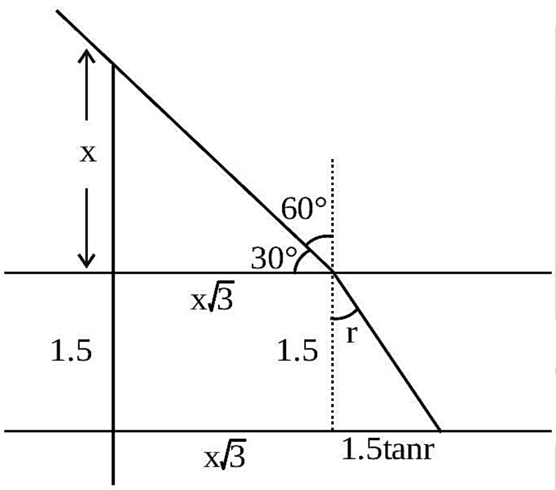


Moving a coil inside a uniform magnetic field either with uniform or non-uniform speed doesn't changes flux, so, no emf is induced.

20. (c)

$$B = \begin{cases} \frac{\mu_0 I r}{\pi a^2} & r \le a \\ \frac{\mu_0 I}{\pi r^2} & r \ge a \end{cases}$$

21. (50)



By Snell's law

$$1 \sin 60^\circ = \frac{4}{3} \sin r \rightarrow \sin r = \frac{3\sqrt{3}}{8} \rightarrow \tan r = \frac{3\sqrt{3}}{\sqrt{37}}$$

By the diagram

$$x\sqrt{3} + 1.5 \tan r = 2.15$$

$$x\sqrt{3} = 2.15 - 1.5 \times \frac{3\sqrt{3}}{\sqrt{37}}$$

$$x = \frac{2.15}{\sqrt{3}} - \frac{1.5 \times 3}{\sqrt{37}}$$

$$= 1.241 - 0.739$$

$$= 0.502$$

$$\approx 0.50 \text{ meter}$$

$$x = 50 \text{ cm}$$

22. (25) $R = \rho\frac{\ell}{A}$ be the initial resistance new resistance

$$R' = \rho\frac{1.2\ell}{0.96\ A} = 1.25\rho\frac{\ell}{A} = 1.25R$$

percentage change $= \frac{1.25R-R}{R} \times 100 = 25\%$

23. (2) Loss of K.E = work done against retarding force.

$$= \int_0^x madx = \int_0^x m2xdx = mx^2$$
$$= (10^{-2}\ kg)x^2\ J = \left(\frac{10}{x}\right)^{-2}\ J$$

So n = 2

24. (628)

Magnetic field B_C at center $= \frac{\mu_0 i}{2r}$

$$= \frac{4\pi \times 10^{-7}}{2 \times 0.2} \times \sqrt{2}\ T$$

Net magnetic field is

$$B_C\sqrt{2} = \frac{4\pi \times 10^{-7} \times \sqrt{2}}{2 \times 0.2} \times \sqrt{2}\ T = 2\pi \times 10^{-6}\ T$$
$$= 200\pi \times 10^{-8}\ T$$
$$= 2 \times 314 \times 10^{-8}\ T$$
$$= 628 \times 10^{-8}\ T$$

25. (420) Frequency of reflected sound $= \left(\frac{v+v_c}{v-v_c}\right) f_0$

$$f = \left(\frac{330+15}{330-15}\right) \times f_0$$

$$= \frac{345}{315} f_0$$

$$\frac{345}{315} f_0 - f_0 = 40$$

$$\frac{30}{315} f_0 = 40$$

$$f_0 = \frac{4 \times 315}{3} = 420\ Hz$$

26. (425)

$$r_n = r_0\frac{n^2}{z} \rightarrow r_n = 0.51 \times \frac{25}{3} Å = 4.25 \times 10^{-10}\ m$$

$$= 425 \times 10^{-12}\ m$$

27. (25)

$$\text{Strain} = \frac{\text{stress}}{Y} = \frac{\frac{62.8\times10^3}{\pi\times(0.02)^2}}{2\times10^{11}}$$

$$= \frac{62.8\times10^3}{3.14\times4\times10^{-4}\times2\times10^{11}}$$

$$= 2.5\times10^{-4}$$

$$= 25\times10^{-5}$$

28. (240)

$$v_p = 12\times10^3 \text{ volts}$$

$$v_s = 120 \text{ volts}$$

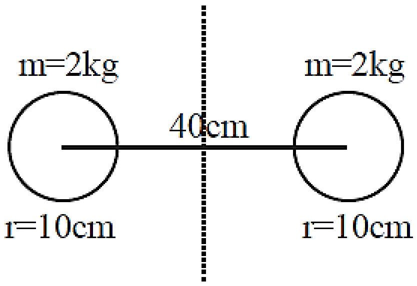
$$p_s = 60\text{KW} = v_s\times i_s$$

$$i_s = \frac{60\times10^3}{120} = 5\times10^2 \text{ A}$$

$$R_L = \frac{v_s}{i_s} = \frac{120}{5\times10^2} = 24\times10^{-2} = 240\times10^{-3}\Omega$$

$$= 240 \text{ m}\Omega$$

29. (176)



$$I = 2(I_{cm} + md^2)$$
$$= 2\left(\frac{2}{5}mr^2 + md^2\right)$$
$$= \frac{4}{5}\times2\times(0.1)^2 + 2(2)(0.20)^2$$
$$= \frac{8}{5}\times10^{-2} + 16\times10^{-2}$$
$$= (1.6+16)\times10^{-2}$$
$$= 17.6\times10^{-2}$$
$$I = 176\times10^{-3}\text{ kg m}^2$$

30. (3) For $x = \frac{d}{3}$

$$C_1=\frac{\epsilon_0\ A}{\left(\frac{d/3}{k}+\frac{2\ d}{3}\right)}=\frac{\epsilon_0\ A}{\frac{d}{12}+\frac{2\ d}{3}}$$

$$=\frac{\epsilon_0\ A}{d}\times\left(\frac{12}{9}\right)$$

$$C_1=\frac{4}{3}\frac{\epsilon_0\ A}{d}=2\mu F$$

for $x=\frac{2\ d}{3}$

$$C_2=\frac{\epsilon_0\ A}{\left(\frac{2\ d/3}{k}+\frac{d}{3}\right)}=\frac{\epsilon_0\ A}{d}\times 2$$

$$\Rightarrow\frac{6}{4}\times 2=3\mu F$$

31. (a)

$Y: CCP\Rightarrow 4Y$

$X=1/3THV=1/3\times 8\Rightarrow 8/3x$

$\therefore$ Formula: $X_{8/3}Y_4$ or X_2Y_3

32. (d)

Nitrogen detection by lassaigne's method

$$Na+C+N\rightarrow NaCN$$
$$6NaCN+FeSO_4\rightarrow Na_4[Fe(CN)_6]+Na_2SO_4$$
$$Na_4[Fe(CN)_6]+Fe^{3+}\rightarrow Fe_4[Fe(CN)_6]_3$$

(Prussian blue)

Sulphur detection by Sodium nitroprusside

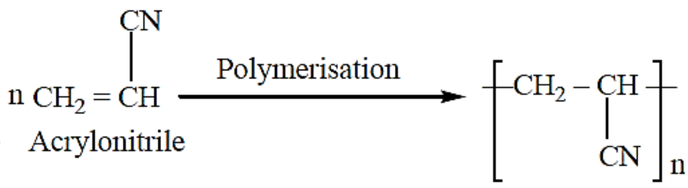
$Na_2[Fe(CN)_5NO]+Na_2\ S\rightarrow\ _{[Purple\ ]}Na_4[Fe(CN)_5NOS]$

Phosphorus detection by ammonium molybdate

$Na_3PO_4+3HNO_3\rightarrow H_3PO_4+3NaNO_3$

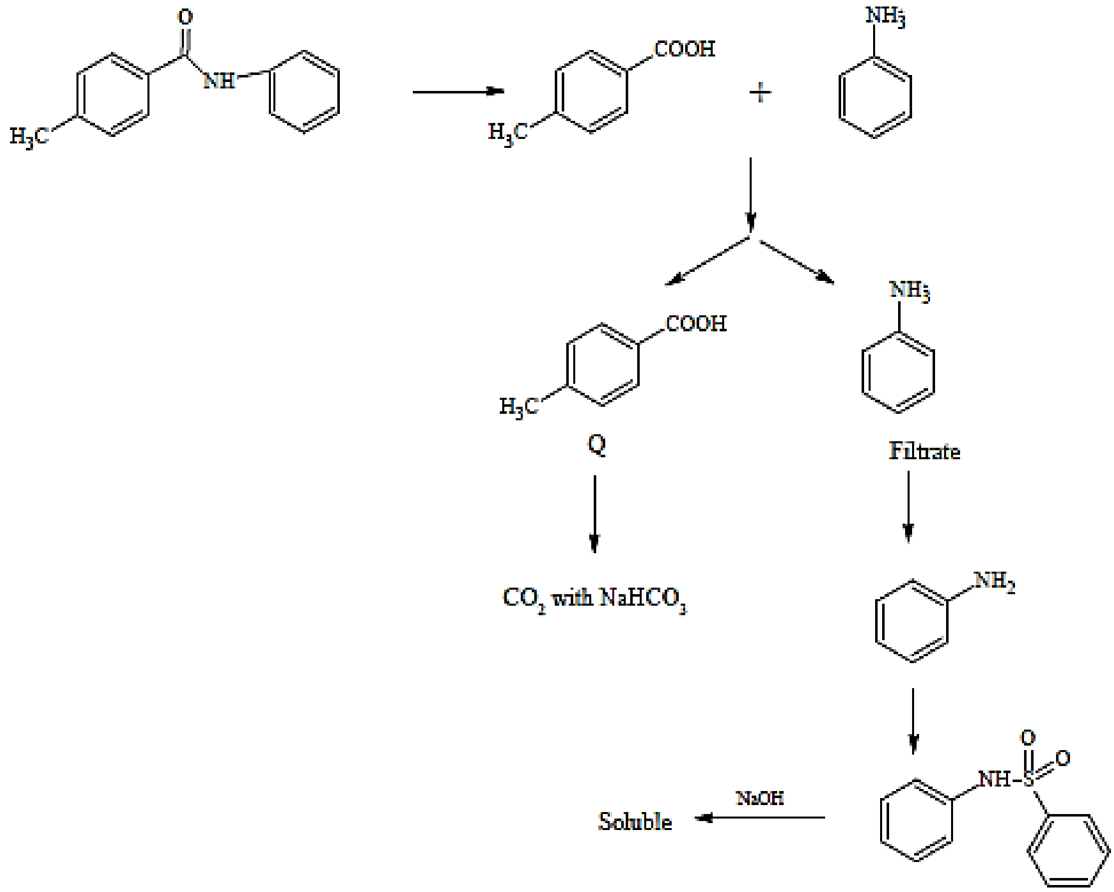
$H_3PO_4 + 12(NH_4)_2MoO_4 + 21HNO_3 \rightarrow$
$(NH_4)_3PO_4.12MoO_3 + 21NH_4NO_3 + 12H_2O$
(canary yellow)
Halogen give specific coloured ppt with
$AgNO_3(aq)$
$NaCl + AgNO_3(aq) \rightarrow AgCl + NaNO_3$
(White)
$NaBr + AgNO_3(aq) \rightarrow AgBr + NaNO_3$
(Pale yellow)
$NaI + AgNO_3(aq) \rightarrow AgI + NaNO)_3$
(Yellow)

33. **(a)**
34. **(a)**



Polyacrylonitrile
(Orlon)

35. (**b**)Cl has the most negative ΔH_{eg} among all the elements and Ne has the most positive ΔH_{eg}.
36. **(c)**
37. **(d)** Photochemical smog occurs in warm, dry and sunny climate. The main components come from the action of sunlight on unsaturated hydrocarbon and nitrogen oxides produced by automobiles and factories.
38. **(d)**
39. **(d)**
40. **(b)**



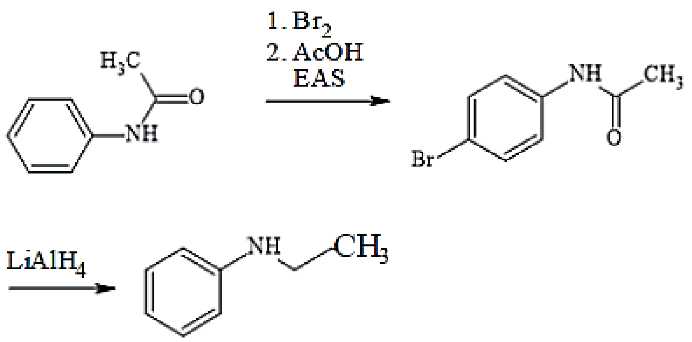
41. (d) HVZ reactions $= Br_2/$ red P

Iodoform reaction $= NaOH + I_2$

Etard reaction $=$ (i) CrO_2Cl_2, CS_2 (ii) H_2O

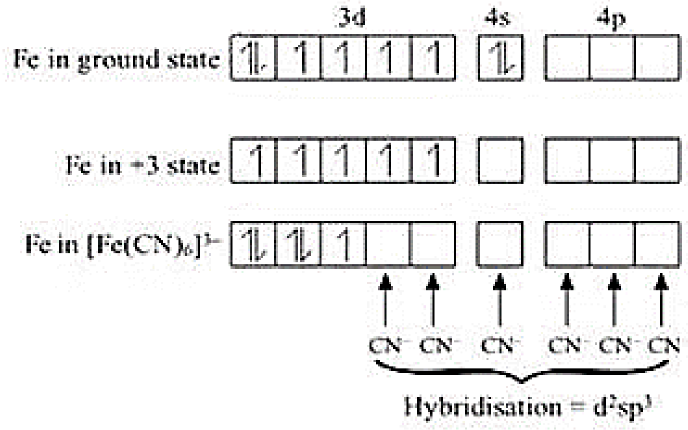
Gatterman-Koch Reaction $= CO, HCl,$ Anhydrous, $AlCl_3$

42. (d)



43. (a)

$[Fe(CN)_6]^{3-}$



Unpaired electron = 1

$\mu = \sqrt{n(n+2)} = \sqrt{1 \times 3} = 1.74$ B.M.

$[Fe(H_2O)_6]^{3+}$ No pairing because H_2O is WFL

Number of unpaired electrons $= 5, \mu = 5.92$BM

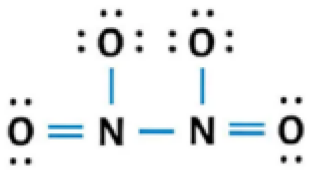
Assertion is true, Reason is true but not correct explanation.

44. (d)
45. (a)

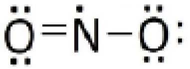
$$2\left[\overset{+1}{Au}(CN)_2\right]^- + \overset{0}{Z}n(s) \longrightarrow 2\overset{0}{Au} + \left[\overset{+2}{Zn}(CN)_4\right]^{-2}$$

Zn displaced Au^+
Reduction and Oxidation both are taking place.

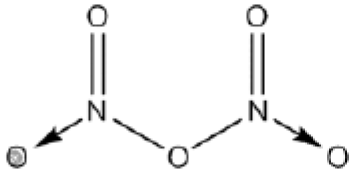
46. (d) N_2O_4



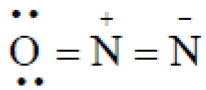
NO_2



N_2O_5

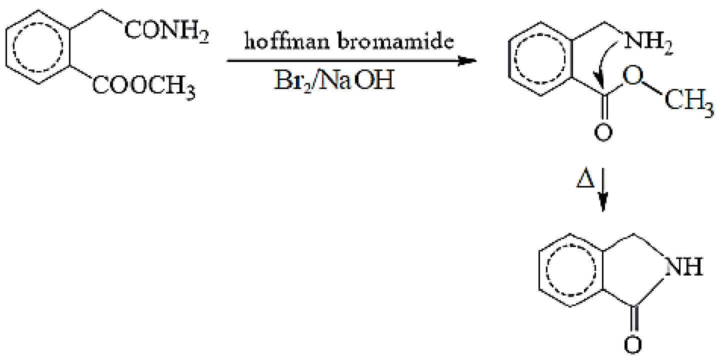


N_2O

 and 

47. (d)

48. (c)



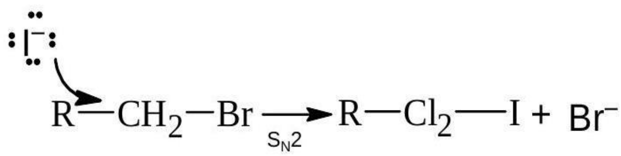
49. (a) $A_2\,B_3(aq.) \rightleftharpoons 2\,A^{3+}_{(aq.)} + 3\,B^{2-}_{(aq)}$

$c(1-\alpha)$ $2c\alpha$ $3c\alpha$

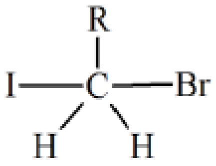
$$K_{eq} = \frac{[A^{3+}]^2[B^{2-}]^3}{[A_2\,B_3]} = \frac{4c^2\alpha^2 \times 27c^3\alpha^3}{c(1-\alpha)}$$

$$K_{eq} = \frac{108c^5\alpha^5}{c}\ \alpha = \left(\frac{K_{eq}}{108c^4}\right)^{\frac{1}{5}}$$

50. (a) This is finkelstein reaction



Transition state



Clearly, the transition state is less polar than free anions. Br and I^-

Acetic acid is protic which does not support S_N2

Acetone does not solvate anion

Br^-gets precipitated and hence can not compete with I

51. (7)

$$\lambda_d = \frac{h}{mv} = \frac{h}{\sqrt{2mKE}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9 \times 10^{-31} \times 4.5 \times 10^{-29}}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{9^2 \times 10^{-60}}}$$

$$= \frac{6.6 \times 10^{-34}}{9 \times 10^{-30}} = \frac{6.6}{9} \times 10^{-4}$$

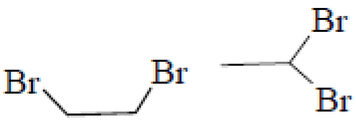
$$= 7.3 \times 10^{-5}\ \text{m}$$

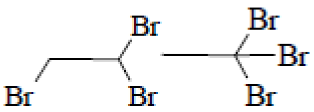
Therefore Ans = 7

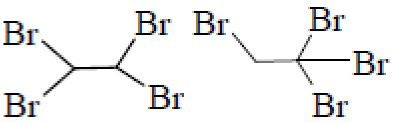
52. (9)

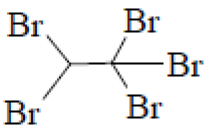
$CH_3 - CH_3 + Br_2$ (Excess) $\xrightarrow{h\nu}$

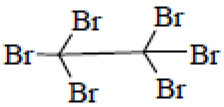
Monobromo 

Dibromo 

Tribromo 

Tretrabromo 

Pentabromo 

Hexabromo 

53. (2)

For, Spontaneous process $dG < 0$
For, Equilibrium $dG = 0$
For, Nonspontaneous process $dG > 0$
$\therefore$ A Wrong
B Correct
C Correct
D Wrong

54. (1111)

$$\frac{P^0 - P_s}{P_s} = \frac{n_{solute}}{n_{solvent}} = \frac{\frac{x}{60}}{\frac{1000}{18}} = \frac{P^0 - 0.75P^0}{0.75P^0}$$

$$\Rightarrow x = \frac{10000}{9} = 1111\text{gm}$$

55. (10)

$$\Delta G^0 = \Delta H^0 - T\Delta S$$

$$\Rightarrow \Delta G^0 = (-54070 - 10 \times 298)$$

Also, $\Delta G^0 = (-2.303RT\log K)$

$$\Rightarrow (-54070 - 10 \times 298)$$

$$= (-2.303 \times 8.134 \times 298\log K)$$

$\Rightarrow \log K = 10$ Ans: 10

56. (3)

. PF_5

$sp^3\ d$ (0 lone pair)

Trigonal bipyramidal

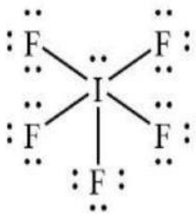
BrF_4^-,

$sp^3\ d^2$ (2 lone pair)

square planar

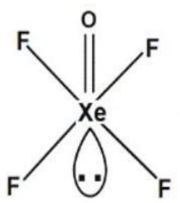
IF_5

$sp^3\ d^2$ (1 lone pair)



square pyramidal

BrF_5
$sp^3\ d^2$ (1 lone pair)
$XeOF_4$
$sp^3\ d^2$ (1 lone pair)



square pyramidal

ICl_4^-

$sp^3\ d^2$ (2 lone pair) $\begin{bmatrix} Cl_1 & Cl \\ & Cl \end{bmatrix}^-$ square planar

57. (4)
58. (2)

$K = Ae^{-\frac{E_a}{RT}}$

$K_1 = Ae^{-\frac{(E_a)_1}{RT}}$

$K_2 = Ae^{-\frac{(E_a)_2}{RT}}$

$\frac{K_2}{K_1} = e^{\frac{(E_a)_1-(E_a)_2}{RT}}$

$\log\frac{K_2}{K_1} = \frac{(E_a)_1 - (E_a)_2}{2.3RT}$

$= \frac{(41.4 - 30) \times 1000}{2.3 \times 8.3 \times 300} = 1.99$

59. (1)

$\underset{5}{3BaCl_2} + \underset{2}{2Na_3PO_4} \rightarrow Ba_3(PO_4)_2 + 6NaCl$

Na_3PO_4 is limiting reagent.
2 mole Na_3PO_4 gives 1 mole of $Ba_3(PO_4)_2$

60. (6)

$(NH_4)_3PO_4 \cdot 12MoO_3$

Let X = oxidation state of Mo in MoO_3

$X + (-2) \times 3 = 0$
$X = +6$

61. (a) $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$

replace $x \rightarrow \frac{1}{x}$

$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3$

Eq. (a) $\times 5 -$ eq. (b) $\times 4$

$f(x) = \frac{1}{9}\left(\frac{5}{x} - 4x + 3\right)$

$I = 18\int_1^2 \frac{1}{9}\left(\frac{5}{x} - 4x + 3\right) dx = 10\log_e 2 - 6$

62. (b) Probability of success $= \frac{1}{9} = p$

Probability of failure $q = \frac{8}{9}$

P(at least 4 success) = P(4 success) + P(5 success)

$= {}^5C_4p^4q + {}^5C_5p^5 = \frac{41}{3^{10}} = \frac{123}{3^{11}}$

$k = 123$

63. (d) $\frac{^{2n}C_3}{^{n}C_3} = 10 \Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 10$

$n = 8$

So $(n^2 + 3n) : (n^2 - 3n + 4) = 2$

64. (b)

$$\frac{^{n}C_4 2^{\frac{n-4}{4}} \cdot \left(3^{\frac{-1}{4}}\right)^4}{^{n}C_4 3^{-\left(\frac{n-4}{4}\right)} \cdot \left(2^{\frac{1}{4}}\right)^4} = \frac{\sqrt{6}}{1}$$

$\Rightarrow n = 10$

So $T_3 = {}^{10}C_2 2^{\frac{1}{4}\cdot 8} \cdot 3^{-\frac{1}{4}\cdot 2} = \frac{45\cdot 4}{\sqrt{3}} = 60\sqrt{3}$

65. (d) $a = \lambda(b \times c)$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$\vec{d} = \lambda(2\hat{i} - \hat{j} + 2\hat{k})$

$\vec{a}.\vec{d} = 18$

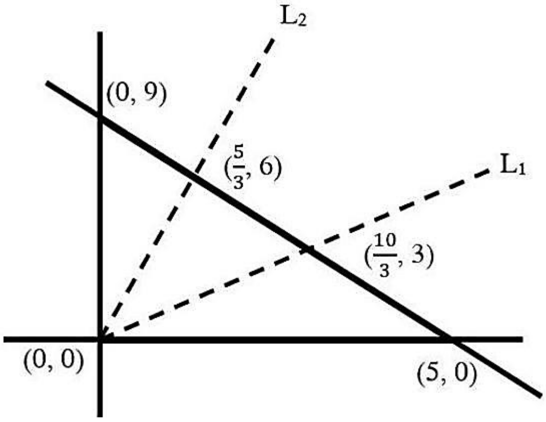
$\lambda = 2$

So $\vec{d} = 2(2\hat{i} - \hat{j} + 2\hat{k})$

$$\vec{d} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 4 \\ 2 & 3 & 4 \end{vmatrix} = -20\hat{i} - 8\hat{j} + 16\hat{k}$$

$|\vec{d} \times \vec{a}|^2 = 720$

66. (c)



$m_{L_1} = \frac{3.3}{10} = \frac{9}{10}$

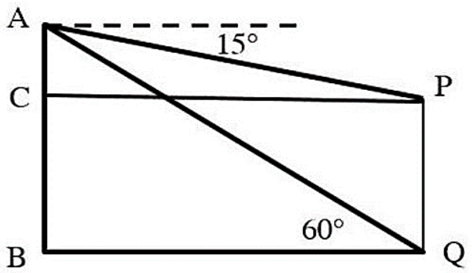
$m_{L_2} = \frac{6.3}{5} = \frac{18}{5}$

$y = (m_1 + m_2)x$

$y = \frac{9}{2}x$

Point of intersection with L is $\left(\frac{10}{7}, \frac{45}{7}\right)$

67. (a)



$\tan 60^\circ = \sqrt{3} = \frac{30}{BQ}$

$BQ = 10\sqrt{3\ m} = CP$

$\tan 15^\circ = 2 - \sqrt{3} = \frac{AC}{CP}$

$AC = 10\sqrt{3}(2 - \sqrt{3})$

$\text{Area} = 10\sqrt{3}(60 - 20\sqrt{3}) = 600(\sqrt{3} - 1)$

68. (d)

$S_{20} = 5 + 11 + 19 + 29 + \cdots ....$

Let $T_r = ar^2 + br + c$

$T_1 = a + b + c = 5$

$T_2 = 4a + 2\ b + c = 11$

$T_3 = 9a + 3\ b + c = 19$

$a = 1, b = 3, c = 1$

Hence $S_{20} = \sum_{r=1}^{20} r^2 + 3\sum_{r=1}^{20} r + \sum_{r=1}^{20} 1 = 3520$

69. (d) Combine var. $= \frac{n_1\sigma^2 + n_2\sigma^2}{n_1 + n_2} + \frac{n_1 n_2(m_1 - m_2)^2}{(n_1 + n_2)^2}$

$13 = \frac{15.14 + 15 \cdot \sigma^2}{30} + \frac{15.15(12 - 14)^2}{30 \times 30}$

$13 = \frac{14 + \sigma^2}{2} + \frac{4}{4}$

$\sigma^2 = 10$

70. (d) Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$A^2 = \begin{bmatrix} p^2 + qr & pq + qs \\ pr + rs & qs + s^2 \end{bmatrix}$

$\Rightarrow p^2 + qr = 1(a) pq + qs = 0 \Rightarrow q(p+s) = 0(c)$

$\Rightarrow s^2 + qr = 1(b) pr + rs = 0 \Rightarrow r(p+s) = 0$

Equation (a) - equation (b)

$p^2 = s^2 \Rightarrow p + s = 0$

Now $3a^2 + 4b^2$

$= 3(p+s)^2 + 4(ps - qr)^2$

$= 3.0 + 4(-p^2 - qr)^2 = 4(p^2 + qr)^2 = 4$

71. (c)

$$I(x) = \int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2} dx$$

Let $x\tan x + 1 = t$

$$I = x^2\left(\frac{-1}{x\tan x + 1}\right) + \int \frac{2x}{x\tan x + 1} dx$$

$$I = x^2\left(\frac{-1}{x\tan x + 1}\right) + 2\int \frac{x\cos x}{x\sin x + \cos x} dx$$

$$I = x^2\left(\frac{-1}{x\tan x + 1}\right) + 2\ln|x\sin x + \cos x| + C$$

As $I(0) = 0 \Rightarrow C = 0$

$$I\left(\frac{\pi}{4}\right) = \ln\left(\frac{(\pi+4)^2}{32}\right) - \frac{\pi^2}{4(\pi+4)}$$

72. (a) Equation of family of plane

$(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$

$x(2 + 4\lambda) - y(1 + 3\lambda) + z(1 + 5\lambda) - 3 + 9\lambda = 0$

Parallel to the line

$-2(2 + 4\lambda) - (1 + 3\lambda)4 + (1 + 5\lambda)5 = 0$

$5\lambda = 3$

$\lambda = \frac{3}{5}$

equation of plane

$11x - 7y + 10z + 6 = 0$

$a + b + c = 14$

73. (a) $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$

We known that $P \Rightarrow Q \equiv \sim P \vee Q$

$\Rightarrow (\sim P \vee Q) \wedge (\sim R \vee Q)$

$\Rightarrow (\sim P \wedge \sim R) \vee Q$

$\Rightarrow \sim (P \vee R) \vee Q$

$\Rightarrow (P \vee R) \Rightarrow Q$

74. (a) For $x \leq 3$ or $x \geq 5$

$x^2 - 8x + 15 - 2x + 7 = 0$

$x = 5 + \sqrt{3}$

For $3 < x < 5, x^2 - 8x + 15 + 2x - 7 = 0$

$x = 4$

Hence sum $= 9 + \sqrt{3}$

75. (a)

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \left(\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \cdots \ldots \ldots . + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \right)$$

On rationalising each term

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_n} - \sqrt{a_1}}{d} \right)$$

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \left(\frac{(n-1)d}{(\sqrt{a_n} + \sqrt{a_1})d} \right) = 1$$

76. (a) $\Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11 - 4 - a = 0$

$a = 7$

$$\Delta_1 = \begin{vmatrix} b & 1 & a \\ 6 & 5 & 2 \\ 3 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11\,b - 12 - 21 = 0$$

$b = 3$

$2a + 3\,b = 23$

77. (b)

$2x^y + 3y^x = 20$

$$2x^y\left[\frac{y}{x} + (\ln x)y'\right] + 3y^x\left[\frac{xy'}{y} + \ln y\right] = 0$$

$$y' = \frac{-(12\ln 2 + 8)}{12 + 8\ln 2} = -\left(\frac{2 + \log_e 8}{3 + \log_e 4}\right)$$

78. (d) Equation of OP is $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

$a_1 = (0,0,0)$
$a_2 = (3,0,5)$
$b_1 = (3,4,5)$
$b_2 = (0,0,1)$

Equation of edge parallel to z axis

$$\frac{x-3}{0} = \frac{y-0}{0} = \frac{z-5}{1}$$

$$\text{S.D} = \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\frac{\begin{vmatrix} 3 & 0 & 5 \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}} = \frac{3(4)}{|4\hat{i} - 3\hat{j}|} = \frac{12}{5}$$

79. (a)

Since A, B, C, D are coplanner

Hence $[\overrightarrow{BA}\quad \overrightarrow{CA}\quad \overrightarrow{DA}] = 0$

$$\begin{vmatrix} 4 & 3 & 2\lambda - 3 \\ 7 & 5 - \lambda & 2\lambda - 4 \\ 6 & 0 & 2\lambda - 6 \end{vmatrix} = 0$$

$\lambda = 2,3$ Hence $\sum_{\lambda \in S}(\lambda + 2)^2 = 41$

80. (b) $[x] + 3 + [x] + 4 \leq 3$

$2[x] \leq -4$

$[x] \leq -2 \Rightarrow x \in (-\infty, -1)$

$$3^x\left(\frac{3\cdot\frac{1}{10}}{1-\frac{1}{10}}\right)^{x-3} < 3^{-3x}$$

$27 < 3^{-3x}$

$-3x > +3$

$x < -1$

$A = B$

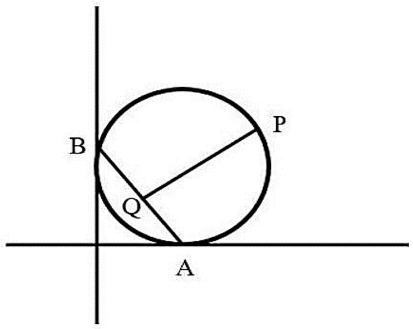
81. (25)

$f(x) = [a + 13\sin x], x \in (0, \pi)$

For $[n\sin x]$; Total number of non differentiable points are $= 2n - 1$ for $x \in (0, \pi)$

So number of non-differentiable points for $[13\sin x] \Rightarrow 25$ Points

82. (121)



Let equation of circle is $(x-a)^2 + (y-a)^2 = a^2$

which is passing through $P(\alpha, \beta)$

then $(\alpha - a)^2 + (\beta - a)^2 = a^2$

$\alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 = 0$

Here equation of AB is $x + y = a$

Let $Q(\alpha', \beta')$ be foot of perpendicular of P on AB

$$\frac{\alpha'-\alpha}{1} = \frac{\beta'-\beta}{1} = \frac{-(\alpha+\beta-a)}{2}$$

$PQ^2 = (\alpha'-\alpha)^2 + (\beta'-\beta) = \frac{1}{4}(\alpha+\beta-a)^2 + \frac{1}{4}(\alpha+\beta-)\ 121 = \frac{1}{2}(\alpha+\beta-a)^2$

$242 = \alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 + 2\alpha\beta$

$242 = 2\alpha\beta$

$\Rightarrow \alpha\beta = 121$

83. (171)

84. (5) $x^2+y^2-2y \geq 0$ & $x^2-2y \leq 0, x \geq y$

Hence required area $=\frac{1}{2} \times 2 \times 2-\int_0^2 \frac{x^2}{2} dx-\left(\frac{\pi}{4}-\frac{1}{2}\right)$

$=\frac{7}{6}-\frac{\pi}{4} \Rightarrow n=5$

85. (3) $3-x \leq y \leq \sqrt{9-x^2}$

Points $(p, p+1)$ lies on $y=x+1$

So, point of intersection between

$y=x+1$ & $y=3-x$ is $x=1, y=2$

and point of intersection between

$x+1=\sqrt{9-x^2}$ is $x=\frac{-1+\sqrt{17}}{2}$

Hence $p \in\left(1, \frac{-1+\sqrt{17}}{2}\right)$

Hence $b^2+b-a^2=3$

86. (2)

$(x\cos x)dy+(xy\sin x+y\cos x-1)dx=0, 0<x<\frac{\pi}{2}$

$$\frac{dy}{dx}+\left(\frac{x\sin x+\cos x}{x\cos x}\right)y=\frac{1}{x\cos x}$$

$\text{IF}=x\sec x$

$$y \cdot x\sec x=\int \frac{x\sec x}{x\cos x} dx=\tan x+c$$

Since $y\left(\frac{\pi}{3}\right)=\frac{3\sqrt{3}}{\pi}$

Hence $c=\sqrt{3}$

Hence $\left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right)+y'\left(\frac{\pi}{6}\right)\right|=|-2|=2$

87. (5005)

$$\left(x^4-\frac{1}{x^3}\right)^{15}$$

$$T_{r+1}={}^{15}C_r(x^4)^{15-r}\left(\frac{-1}{x^3}\right)^r$$

$60-7r=18$

$r=6$

Hence coeff. of $x^{18} = {}^{15}C_6 = 5005$

88. (18)

$A = \{1,2,3, \ldots \ldots .10\}$

$B = \{0,1,2,3,4\}$

$R = \{(a,b) \in A \times A: 2(a-b)^2 + 3(a-b) \in B\}$

Now $2(a-b)^2 + 3(a-b) = (a-b)(2(a-b)+3)$

$\Rightarrow a = b$ or $a - b = -2$

When $a = b \Rightarrow 10$ order pairs

When $a - b = -2 \Rightarrow 8$ order pairs

Total $= 18$

89. (594) Let $Q(\alpha, \beta, \gamma)$ be the image of P, about the plane

$2x - y + z = 9$

$$\frac{\alpha-1}{2} = \frac{\beta-2}{-1} = \frac{\gamma-3}{1} = 2$$

$\Rightarrow \alpha = 5, \beta = 0, \gamma = 5$

Then area of triangle PQR is $= \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$

$= |-12\hat{\imath} - 3\hat{\jmath} + 21\hat{k}| = \sqrt{144+9+441} = \sqrt{594}$

Square of area $= 594$

90. (292)

Equation of tangent at $P(1,3)$ to the curve $x^2 + 2x - 4y + 9 = 0$ is $y - x = 2$

Then the point A is $(0,2)$

Equation of line passing through P and parallel to the line $x - 3y = 6$.

The possible coordinate of B are $(4,4)$ or $(16,8)$

But $(4,4)$ does not satisfy $2x - 3y = 8$

Thus, the point B is $(16,8)$

Then $(AB)^2 = 292$