1. (c) Least count = 0.2 cm

$u = (100 \pm 0.2) - (80 \pm 0.2) = (20 \pm 0.4)\text{cm}$

$v = (180 \pm 0.2) - (100 \pm 0.2) = (80 \pm 0.4)\text{cm}$

From lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{80} - \frac{1}{-20}$$

$f = 16$ cm

Also $\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{\Delta f}{f^2}$

$$\Rightarrow \frac{\Delta f}{f} \times 100 = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) \times f \times 100$$

$$\Rightarrow \%f = \left(\frac{0.4}{400} + \frac{0.4}{6400}\right) \times 16 \times 100$$

$= 1.70$

2. (a)

$$I_0 = \frac{E_0}{x_c} = \frac{E_0}{\frac{1}{\omega_c}} = E_0 \omega_c$$

$$\Rightarrow I_0 = 36 \times 120\pi \times 150 \times 10^{-6}$$
$$\Rightarrow I_0 = 2.03$$
$$\simeq 2 \text{ A}$$

3. (c)

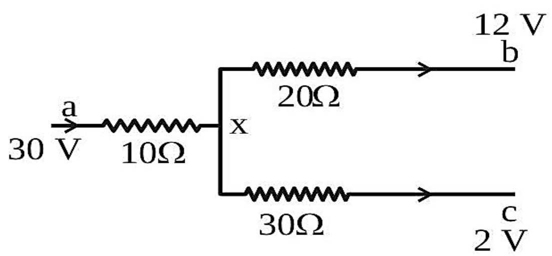
(R) is the statement of Pascal's principle & which explains the assertion (S)

4. (c) As medium changes, optical path changes.

Also, $\frac{\Delta x}{\lambda} = \frac{\Delta \phi}{2\pi}$

Hence phase difference changes.

5. (a) Sum of current at junction point will be zero:



$$\frac{x - 30}{10} + \frac{x - 12}{20} + \frac{x - 2}{30} = 0$$

$$\Rightarrow x\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{30}\right)=\frac{30}{10}+\frac{12}{20}+\frac{2}{30}$$

$$\Rightarrow x\left(\frac{6+3+2}{60}\right)=\frac{180+36+4}{60}$$

$$\Rightarrow x=\frac{220}{11}=20\ \text{V}$$

$\therefore$ Current through $20\Omega=\frac{x-12}{20}$

$$=\frac{20-12}{20}=\frac{2}{5}=0.4\ \text{A}$$

6. (d) $|\langle\vec{v}\rangle|=\frac{|\vec{r}_f-\vec{r}_i|}{\Delta t}$

$$=\frac{2R\cos\left[\frac{\pi-\theta}{2}\right]}{\frac{2\pi R}{3v}}=3\cos 30^\circ$$

$1.5\sqrt{3}\ \text{m/s}$

7. (c) $eV_s=k_{max}$

$$V_s=\left\{\frac{h}{e}\right\}f+\left\{\frac{-\phi}{e}\right\}$$

Slope is independent of nature of metal

slope $(V_s)^{\text{Gold}}=\text{slope}(V_s)^{\text{Aluimiun}}$

8. (a) $C=\sqrt{\frac{\gamma RT}{M}}$

$$C\propto\frac{1}{\sqrt{M}}$$

$$\frac{C_{H_2}}{C_{O_2}}=\sqrt{\frac{32}{2}}=4:1$$

9. (d) $\omega=\frac{2\pi}{3.14}=2\text{rad/s}$

$$|\vec{f}_{\text{centrifugal}}|=|-m\vec{a}_{\text{Re f.}}|$$

$$=M\omega^2R$$

$$=40\ \text{N}$$

10. (d) $v=u+at$

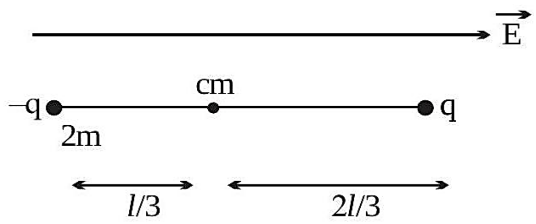
$$60=10+2t$$

$$t=25\text{sec.}$$

11. (d) Planets revolve in elliptical paths around sun. Thus their linear speed is not constant

12. (c) In forward biased condition, diffusion of majority charge carriers takes place from p-side to n-side which constitute the diffusion current.

13. (c)



If released, it will oscillate about centre of mass.

For small ' θ '

$\tau = -PE.\theta$

$$\Rightarrow \left[2\,m\frac{1^2}{9} + m\frac{41^2}{9}\right]\alpha = -qlE\cdot\theta$$

$$\Rightarrow \frac{2ml^2}{3}\alpha = -qlE\cdot\theta \;\Rightarrow \alpha = -\frac{3qE}{2ml}\theta$$

$$\omega = \sqrt{\frac{3qE}{2ml}}$$

14. (d) $U_E = \frac{1}{2}\epsilon_0 E^2, U_B = \frac{B^2}{2\mu_0}$

15. (d) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow V_{rms}\alpha\sqrt{T}$

Increasing temperature 4 times, rms speed gets doubled.

16. (c) To convert galvanometer into ammeter low resistances should be added into parallel & for voltmeter conversion, a very high resistance should be added in series.

17. (c) using average rate of Newton's law of cooling

$$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_s\right)$$

Given $\frac{60-40}{7} = K(50 - 10)$

$$\& \frac{40 - T}{7} = K\left(\frac{40 + T}{2} - 10\right)$$

From (i) & (ii)

$T = 28°C$

18. (a) $U = \frac{1}{2}m\omega^2 r^2$

$F = -\frac{dv}{dr} = -m\omega^2 r$

Now $m\omega^2 r = \frac{mv^2}{r} \Rightarrow v = \omega r$

$\& mvr = \frac{nh}{2\pi}$

From (i) & (ii)

$m\omega r^2 = \frac{nh}{2\pi}$

$\Rightarrow r \propto \sqrt{n}$

19. (b) Modulation index $= \frac{A_m}{A_c} = 0.6$

Minimum amplitude of modulated wav

$= A_c - A_m = 3$

$\therefore A_C - 0.6A_c = 3 \Rightarrow 0.4A_c = 3$

$A_c = \frac{3}{0.4} = \frac{15}{2} = 7.5\ V$

$A_m = 0.6A_c = 4.5\ V$

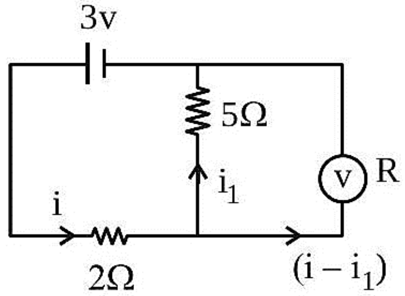
$\therefore$ Maximum amplitude $= A_c + A_m$

$= 7.5 + 4.5 = 12\ V$

20. (b) $\Rightarrow g' = \frac{gR^2}{r^2} = \frac{gR^2}{\left(R+\frac{R}{4}\right)^2} = \frac{16g}{25}$

$\therefore$ Weight $= \frac{16}{25} \times 100 = 64N$

21. (20) $i_1 = \frac{2V}{5\Omega} = \frac{2}{5}A$



$i = \frac{1V}{2\Omega} = \frac{1}{2}A$

$\therefore$ Current through voltmeter $= i - i_1$

$=\frac{1}{2}-\frac{2}{5}=\frac{5-4}{10}=\frac{1}{10}A$

$\therefore$ For voltmeter

$2=\left(\frac{1}{10}\right)R \Rightarrow R=20\Omega$

22. (11) Tensile stress, $\sigma=\frac{F}{A}=\frac{4mg}{\pi D^2}$

$\therefore m=\frac{\pi D^2\sigma}{4g}$

$=\frac{22}{7}\times\frac{(14\times10^{-3})^2\times7\times10^5}{4\times9.8}$

$=11$ kg

23. (5)

$c=\frac{\varepsilon_0 A}{(d-c)}$

$=\frac{\varepsilon_0\times200\times10^{-4}}{4\times10^{-3}}$

$\therefore x=5$

The situation is equivalent to a conducting slab placed between the plates

24. (5) For ring $I=mR_1^2=mK_1^2$

$\therefore$ Radius of gyration $K_1=R_1$

For solid sphere

$I'=\frac{2}{5}m'R_2^2=m'K_2^2$

$\therefore$ Its radius of gyration $=K_2=\sqrt{\frac{2}{5}}R_2$

$\because K_1=K_2$

$\therefore R_1=\sqrt{\frac{2}{5}}R_2$

$\therefore \frac{R_1}{R_2}=\sqrt{\frac{2}{5}}$

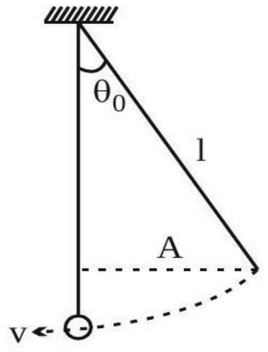
$\therefore x=5$

25. (99)

$\sin\theta_0 = \frac{A}{l} = \frac{10}{100} = \frac{1}{10}$

From conservation of energy

$\frac{1}{2}mv^2 = mgl(1-\cos\theta)$



Maximum tension occurs at mean position.

$\therefore T - mg = \frac{mv^2}{l}$

$\Rightarrow T = mg + \frac{mv^2}{l}$

$\therefore T = mg + 2mg(1-\cos\theta)$

$= mg\left[1 + 2\left(1 - \sqrt{1-\sin^2\theta}\right)\right]$

$= mg\left[3 - 2\sqrt{1 - \frac{1}{100}}\right]$

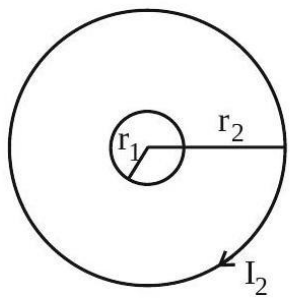
$= \frac{250}{1000} \times 9.8\left[3 - 2\left(1 - \frac{1}{200}\right)\right] = \frac{99}{40}$

$\therefore x = 99$

26. (4)

$r_1 = 1\text{ cm}, N_1 = 10$

$r_2 = 1000\text{ cm}, N_2 = 200$



$\phi_{1,2} = MI_2$

$N_2\vec{B}_2 \cdot N_1\vec{A}_1 = MI_2$

$\Rightarrow N_1N_2\frac{\mu_0 I_2}{2r_2} \cdot \pi r_1^2 = MI_2$

$\Rightarrow M = \frac{10 \times 200 \times 4\pi \times 10^{-7} \times \pi \times (0.01)^2}{2 \times 10}$

$\Rightarrow M = 4 \times 10^{-8}$

27. (462) $d = 2.5$ mm, $D = 150$ cm

Fringe width $\beta = \frac{\lambda D}{d}$

Let n^{th} bright tringle of λ_1 match with m^{th} bright triangle of λ_2

$\Rightarrow n\beta_1 = m\beta_2$

$\Rightarrow n\lambda_1 = m\lambda_2 \Rightarrow \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{5500}{7000}$

$\Rightarrow \frac{n}{m} = \frac{11}{14}$

Distance where bright fringe will match

$= n\beta_1 = \frac{11 \times 7000A^0 \times 150 \text{ cm}}{0.25 \text{ cm}}$

$= 462 \times 10^{-5}$

28. (375)

Let V_1 and V_2 are velocity just before and just after hitting the floor.

$\frac{V_1}{V_2} = 4 \Rightarrow V_1 = 4V_2$

$KE_{before} = \frac{1}{2}mV_1^2$

$KE_{after} = \frac{1}{2}mV_2^2 = \frac{1}{2}\frac{m \cdot V_1^2}{16}$

$\Delta KE = \frac{1}{2}mV_1^2\left(\frac{1}{16} - 1\right) = \frac{-15}{32}mV_1^2$

% change $= \frac{\Delta KE}{KE_{before}} \times 100\%$

$= \frac{-15}{16} \times 100 = \frac{-375}{4}\%$

29. (16)

Binding energy of system $= \frac{ke^2}{2r}$ joule and $\frac{ke^2}{2r} = 12.8$ev

$$\frac{9\times10^9\times(1.6\times10^{-19})^2}{2r} = 12.8\times1.6\times10^{-19}$$

$$\Rightarrow r = \frac{9\times10^9\times1.6\times10^{-19}}{12.8\times2}$$

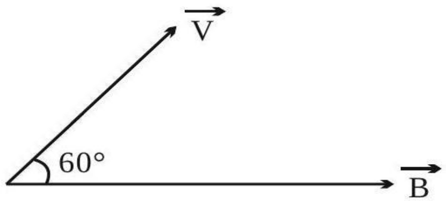
$$\Rightarrow r = \frac{9\times10^{-10}}{16}$$

30. (40)

$$B = \frac{\pi}{2}\times10^{-3}$$

$$K.E. = \frac{1}{2}mV^2$$

$$\Rightarrow V = \sqrt{\frac{2KE}{m}}$$



Pitch $= v\cos60^\circ \times$ time period of one rotation

$$= v\cos60^\circ \times \frac{2\pi m}{eB}$$

$$= \sqrt{\frac{2\times2\times1.6\times10^{-19}}{1.6\times10^{-27}}}\times\cos60^0\times\frac{2\pi\times1.6\times10^{-27}}{1.6\times10^{-19}\times\frac{\pi}{2}\times10^{-3}}$$

$$= 2\times10^4\times\frac{1}{2}\times4\times10^{-5}$$

$$= 4\times10^{-1}\ \text{m} = 40\ \text{cm}$$

31. (a) Hydration enthalpy $\propto \frac{1}{\text{size}}$

Down the group as size increases hydration enthalpy decreases

Order: $Be^{2+} > Mg^{+2} > Ca^{+2} > Sr^{+2} > Ba^{+2}$

32. (a) IUPAC name of $K_3[Co(C_2O_4)_3]$ is

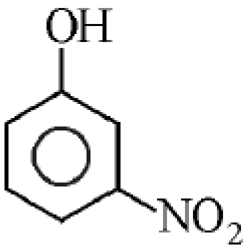
Potassium trioxalatocobaltate (III)

33. (d)

34. (d) Nessler reagent is $-K_2[HgI_4]$

35. (b) In solid state $BeCl_2$ as polymer, in vapour state it form chloro-bridged dimer while above 1200 K it is monomer.

36. (a) Strongest acid from the following is



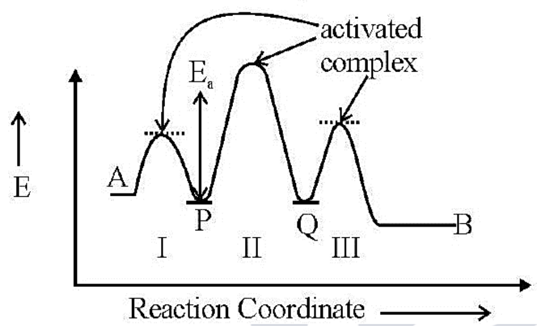
$-NO_2$ group has more EWG nature so more acidic,

37. (c) As electropositive character increases basic character of oxide increases.

$$\underbrace{B_2O_3}_{\text{acidic}} < \underbrace{Al_2O_3 < Ga_2O_3}_{\text{amphoteric}} < \underbrace{In_2O_3 < Tl_2O_3}_{\text{basic}}$$

38. (b) Step with highest activation energy is RDS, so step II is RDS

No. of activated complex = 3



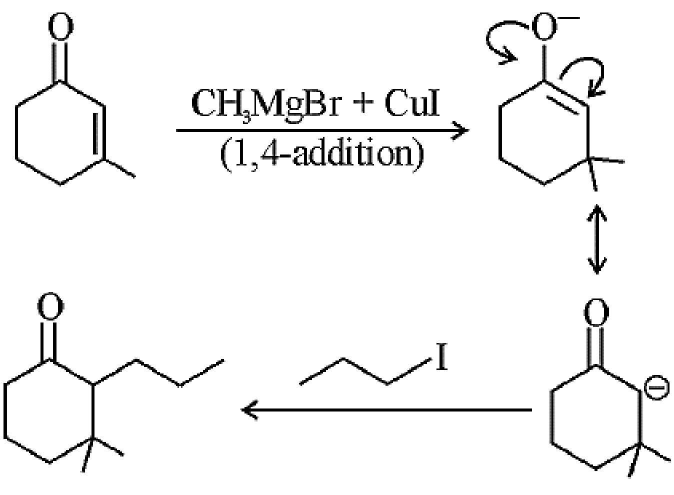
P and Q are intermediates

(Number of intermediates = 2)

39. (a) Low oxidation state of metals can stabilized by synergic bonding so ligand has to be π-acceptor.
40. (b) In neutral or weakly alkaline solution oxidation state of Mn changes by 3 unit

$$\overset{+7}{M}O_4^{-1} \rightarrow \overset{+4}{Mn}O_2$$

41. (a)



42. (c) $Pb(NO_3)_2$ is a soluble colourless compound so it cannot be used in confirmatory test of Pb^{+2} ion.

43. (a)
44. (d) Statement-I- Morphine relieves in pain and produce sleep (incorrect)

Statement-II - Correct

45. (c) $N_1 v_1 = N_2 v_2$

$\Rightarrow 0.02 v_1 = 0.02 \times 10$

$\Rightarrow v_1 = 10 ml$

46. (d) Brine is aq. Solution of NaCl

$NaCl_{(aq)} \rightarrow Na^+ + Cl^-$

Cathode reaction

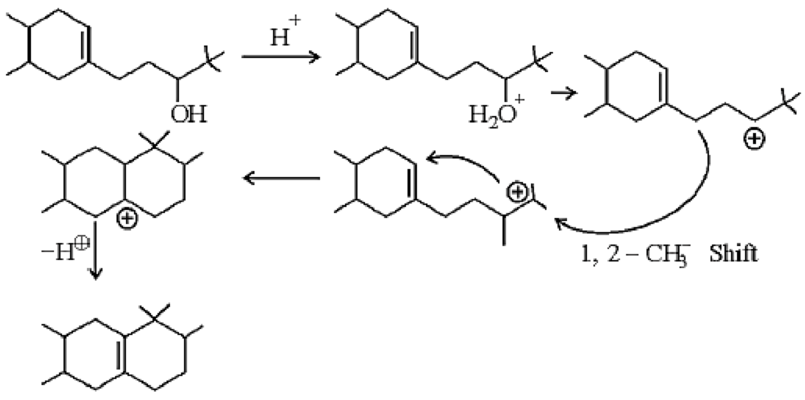
$2H_2O + 2e^- \xrightarrow{\text{reduction}} H_{2(g)} + 2OH$

Anode reaction

$2Cl^- \xrightarrow{\text{oxidation}} Cl_{2(g)} + 2e$

So HCl will not form during electrolysis.

47. (c) Pesticides → D.D.T and Aldrin
48. (b)



49. (b) Li, Cs reacts vigorously with water.

Br_2 changes in vapour state in boiling water ($BP = 58°C$)

Ga reacts with water above $100°C (MP = 29°C, BP = 2400°C)$

50. (c) $(r_3)_H = \frac{a_0 n^2}{Z} = a_0 \times 3^2 = 9a_0$

$2\pi r = n\lambda$

$\Rightarrow 2\pi \times 9a_0 = 3\lambda$

$\Rightarrow \lambda = 6\pi a_0$

51. (4) In ice each water molecule is hydrogen bonded with four other water molecules.

52. (48)

$$\begin{array}{ccc} PCl_3 + Cl_2 \rightleftharpoons & & PCl_5 \\ 0.2M & (0.1+0.2)M & 0.4M \\ \mathbf{Eq}^m \cdot 0.2 - x & 0.3 - x & 0.4 + x \end{array}$$

$$\frac{(0.4+x)}{(0.2-x)(0.3-x)} = 20$$

$$\Rightarrow x \simeq 0.086$$

$$[PCl_5]_{eq} = 0.486M = 48.6 \times 10^{-2}M$$

53. (4) $\pi = icRT$

A, B, C **and** D **are isotonic pairs.**

54. (3) Metal having lower SRP than 0.97 V will be oxidised by NO_2^-.

55. (5) Paints, milk, hair cream, froth, soap lather.

56. (4) $XeF_4, BrF_4^{-1}, [Cu(NH_3)_4]^{+2}, [PtCl_4]^{-2}$ has square planar shape.

57. (278)

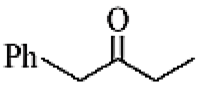
$$2C_{(s)} + 3H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow C_2H_5OH_{(l)}$$

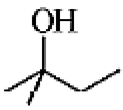
$$(\Delta H_f)_{C_2H_5OH} = \Sigma(\Delta H_{comb})_{reac\ tant} - \Sigma(\Delta H_{comb})_{product}$$

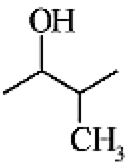
$$= 2 \times (-393.5) + 3(-241.8) - (-1234.7)$$

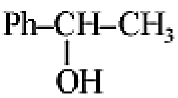
$$= -277.7\ kJ/mol$$

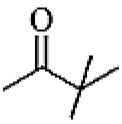
58. (4)

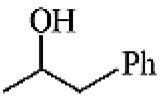
(a) 

(b) 

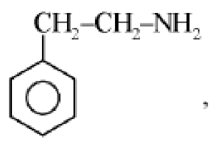
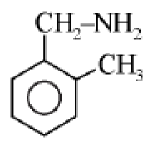
(c)  (Positive for iodoform)

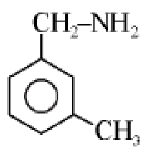
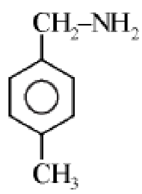
(d)  (Positive for iodoform)

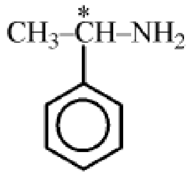
(e)  (Positive for iodoform)

(f)  (Positive for iodoform)

59. (5)

 , 

 , 



(d + l)

60. (3) Cubic, tetragonal and orthorhombic have body centered unit céll.

61. (a) Total number of ways $= 6^3 = 216$

Favourable outcomes $^6p_3 = 120$

$\Rightarrow$ Probability $= \frac{120}{216} = \frac{5}{9}$

$\Rightarrow p = 5, q = 9$

$\Rightarrow q - p = 4$

62. (c)

$S_1 = (1999 + 24)^{2022} - (1999)^{2022}$

$\Rightarrow\ ^{2022}C_1(1999)^{2021}(24) +\ ^{2022}C_2(1999)^{2020}(24)^2 + \cdots$. so on S_1 is divisible by 8

$S_2: 13(13^n) - 11n - 13$

$13^n = (1+12)^n = 1 + 12n +\ ^nC_2 12^2 +\ ^nC_3 12^3 \ldots\ldots$

$13(13^n) - 11n - 13 = 145n +\ ^nC_2 12^2 +\ ^nC_3 12^3 \ldots\ldots$

If ($n = 144m, m \in N$), then it is divisible by 144 For infinite value of n.

63. (d) $\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n < \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{7}}\right)$

$---\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right) < \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$

$\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n < L < \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$

$\lim_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n = 0$ and $\lim_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n = 0$

$\Rightarrow \lim_{n\to\infty} L = 0$

64. (c) $\mathrm{Re}(az^2+bz)=a$

$az^2+bz+a\bar{z}^2+b\bar{z}=2a$

$a(z^2+\bar{z}^2)+b(z+\bar{z})=2a$

$\mathrm{Re}(bz^2+az)=b$

$bz^2+az+b\bar{z}^2+a\bar{z}=2\,b$

$b(z^2+\bar{z}^2)+a(z+\bar{z})=2\,b$

$(a)\times b-(b)\times(a)$

$\Rightarrow (b^2-a^2)(z+\bar{z})=0$
$\Rightarrow\ (z+\bar{z})=0\ (a^2\neq b^2)$

$(a)\times a-(b)\times(b)$

$\Rightarrow (a^2-b^2)(z+\bar{z})=2(a^2-b^2)\ (a^2\neq b^2)$

$z^2+\bar{z}^2=2$

$\Rightarrow (z+\bar{z})^2-2z\bar{z}=2$

$z\bar{z}=-1$

$\Rightarrow 1+1^2=-1$

$\Rightarrow$ No solution

But when $a=-b$,

$\mathrm{Re}(az^2-az)=a$

$\Rightarrow \mathrm{Re}(a(x^2-y^2+i2xy)-a(x+iy))=a$

$\Rightarrow a(x^2-y^2)-ax=a$

$\Rightarrow x^2-y^2-x=1$

$\Rightarrow x^2-x-1=y^2$

For any real values of y there two values of x, hence infinite complex numbers are possible.

65. (a) $f(x)=\frac{1}{\sqrt{[x]-x}}$

If $x\in I[x]=[x]$ (greatest integer function)

If $x\notin I[x]=[x]+1$

$$\Rightarrow f(x)=\begin{cases}\dfrac{1}{\sqrt{[x]-x}}, x\in I\\ \dfrac{1}{\sqrt{[x]+1-x}}, x\notin I\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}\dfrac{1}{\sqrt{-\{x\}}}, x\in I, \text{(does not exist)}\\ \dfrac{1}{\sqrt{1-\{x\}}}, x\notin I\end{cases}$$

$\Rightarrow$ domain of $f(x)=R-I$

Now, $f(x)=\frac{1}{\sqrt{1-\{x\}}}, x\notin I$

$\Rightarrow 0<\{x\}<1$

$\Rightarrow 0<\sqrt{1-\{x\}}<1$

$\Rightarrow \dfrac{1}{\sqrt{1-\{x\}}}>1$

$\Rightarrow$ Range $(1,\infty)$

$\Rightarrow A=R-I$

$B=(1,\infty)$

So, $A\cap B=(1,\infty)-N$

$A\cup B\neq(1,\infty)$

$\Rightarrow$ S1 is only correct

66. (d) $(1+\ln x)\frac{dx}{dy}-x\ln x=e^y$

Let $x\ln x=t$

$(1+\ln x)\dfrac{dx}{dy}=\dfrac{dt}{dy}$

$\dfrac{dt}{dy}-t=e^y$

If $=e^{\int -dy}=e^{-y}$

t.e $^{-y}=\int e^ye^{-y}dy+c$

$te^{-y}=y+c$

$x\ln xe^{-y}=y+c$

$x\ln x=ye^y+ce^y$

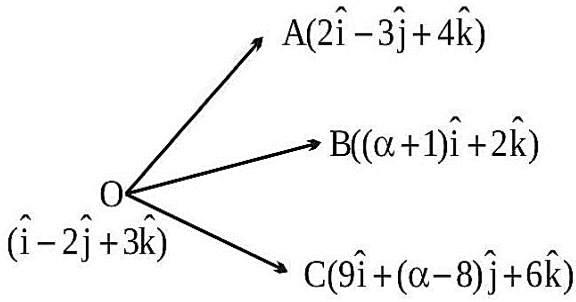
$(1,0)\ 0=C$

$\Rightarrow x\ln x=ye^y$

$\Rightarrow \alpha\ln\alpha=2e^2$

$\alpha^\alpha=e^{2e^2}$

67. (d)



$[OA\ OB\ OC] = 0$

$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha-6 & 3 \end{vmatrix} = 0$$

$\Rightarrow \alpha^2 - 2\alpha - 8 = 0$

$\Rightarrow (\alpha - 4)(\alpha + 2) = 0$

$\therefore \alpha = 4, -2$

68. (a)

$x + y + z$

$x + 2y + \alpha z = 10$

$x + 3y + 5z = \beta$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix} = 1(10 - 3\alpha) - 1(5 - \alpha) + 1(3 - z)$$

$= 10 - 3\alpha - 5 + \alpha + 1$

$= 6 - 2\alpha$

For unique solution $6 - 2\alpha \neq 0 \Rightarrow \alpha \neq 3$

69. (b) $y = |x - 1| + |x - 2|$ and $y = 3$

$\therefore$ Required area $= \frac{1}{2}(1 + 3) \times 2 = 4$

70. (c) $P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 29P, P^\alpha - P^\beta = \delta I - 13P$

$P^4 = (I - P)^2 = I - 2P + P^2 = 2I - 3P$

$P^6 = (2I - 3P)(I - P) = 5I - 8P$

$P^8 = (2I - 3P)^2 = 4I - 12P + 9(I - P) = 13I - 21P$

$P^8 + P^6 = 18I - 29P$

$P^8 - P^6 = 8I - 13P$

$\alpha = 8; \beta = 6; \gamma = 18, \delta = 8$

$\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$

71. (b) B $\rightarrow 5! = 120$

C .... $\rightarrow 5! = 120$

I .... $\rightarrow 5! = 120$

L .... $\rightarrow 5! = 120$

PB ... ... $4\,! = 24$

PC ... ... $+\ 4\,! = 24$

PL .... $\rightarrow 4\,! = 24$

PI ... ... $4\,! = 24$

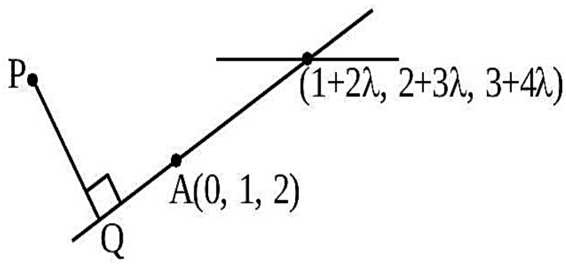
P ∪ BC ... ...$2!\,! = 2$
P ∪ BI ... ... $2! = 2$

P ∪ BLC ... ... $\rightarrow 1! = 1$

P ∪ BLIC ... ... $\rightarrow = 1$

Serial number $= 4(120) + 4(24) + 6 = 582$

72. (d)



$\overrightarrow{AB} \cdot \vec{n}$

$\Rightarrow [(1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 + 4\lambda)\hat{k}] \cdot (2\hat{i} + \hat{j} - 3\hat{k})$

$2 + 4\lambda + 1 + 3\lambda - 3 - 12\lambda = 0$

$5\lambda = 0 \Rightarrow \lambda = 0$

Line $\overrightarrow{AB}, \vec{r} = \hat{j} + 2\hat{k} + \mu(\hat{i} + \hat{j} + \hat{k})$

General form: $Q(\mu, 1 + \mu, 2 + \mu)$

$\therefore \overrightarrow{PQ} \cdot \overrightarrow{AB} = 0$

$(\mu - 1) + (10 + \mu) + \mu = 0$

$3\mu = -9 \Rightarrow \mu = -3$

$\therefore$ distance $= \sqrt{16+49+9} = \sqrt{74}$

73. (b) Equation of plane P is

$(x+y+z-6)+\lambda(2x+3y+4z+5)=0$

Plane passes through the point $(0,2,-2)$

$\therefore (2-2-6)+\lambda(6-8+5)=0$

$-6+\lambda(c)=0$

$\lambda = 2$

Equation of plane p is

$(x+y+z-6)+2(2x+3y+4z+5)=0$

$5x+7y+9z+4=0$

$$d = \left|\frac{5\times 12+7\times 12+9\times 18+4}{\sqrt{5^2+7^2+9^2}}\right|$$

$$d = \left|\frac{60+84+162+4}{\sqrt{25+49+81}}\right|$$

$$d = \frac{310}{\sqrt{155}}$$

$$d^2 = \frac{310\times 310}{155} = 620$$

74. (d) $f(x)+f(\pi-x)=\pi^2$

$I = \int_0^{\pi} f(x)\sin x\,dx$

Applying King's Rule

$I = \int_0^{\pi} f(\pi-x)\cdot\sin(\pi-x)dx$

$2I = \int_0^{\pi}[f(x)+f(\pi-x)]\sin x\,dx$

$2I = \int_0^{\pi}\pi^2\sin x\,dx$

$2I = \pi^2\cdot\int_0^{\pi}\sin x\,dx$

$2I = \pi^2\times 2$

$I = \pi^2$

75. (b) $\left(ax^2+\frac{1}{2bx}\right)^{11}$

$$T_{r+1} = {}^{11}C_r(ax^2)^{11-r} \cdot \left(\frac{1}{2bx}\right)^r$$

$$= {}^{11}C_r a^{11-r} \cdot \left(\frac{1}{2\,b}\right)^r \cdot x^{22-2r-r} = {}^{11}C_r a^{11-r} \cdot \left(\frac{1}{2\,b}\right)^r \cdot x^{22-3r}$$

$\therefore 22 - 3r = 7$

$3r = 15$

$r = 5$

Again $\left(ax - \frac{1}{3bx^2}\right)^{11}$

$$T_{r+1} = {}^{11}C_r(ax)^{11-r}\left(-\frac{1}{3bx}\right)^r$$

$$= {}^{11}C_r a^{11-r} \cdot \left(\frac{-1}{3\,b}\right)^r \cdot x^{11-r-2r}$$

$\therefore 11 - 3r = -7$

$3r = 18$

$r = 6$

Now, $\frac{{}^{11}C_5 a^6}{32\,b^5} = \frac{{}^{11}C_6 \cdot a^5}{3^6 \cdot b^6}$

$729ab = 32$

76. (a) $(p \to q) \vee ((\sim p) \wedge q)$

p	q	$p \to q$	$\sim p \wedge q$	$(p \to q) \vee (\sim p) \wedge q)$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	F	T

Not a tautology

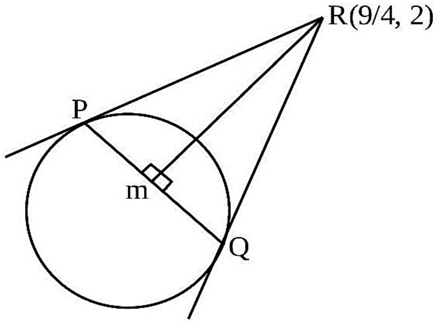
p	q	$q \to p$	$(\sim p) \wedge q$	$(q \to p) \vee (\sim p) \wedge q)$
T	T	T	F	F
T	F	T	F	F

F	T	F	T	T
F	F	T	F	F

Not a contradiction

77. (d) Equation of circle is $x^2 + y^2 - 2x + y - 5 = 0$

$$R = \frac{5}{2}$$



Length of $PR = QR = \sqrt{S_1}$

$$= \sqrt{\frac{81}{16} + 4 - \frac{2 \times 9}{4} + 2 - 5} = \frac{5}{4}$$

Area of triangle $PQR = \frac{RL^3}{R^2+L^2} = \frac{\frac{5}{2} \cdot \frac{125}{64}}{\frac{25}{4} + \frac{25}{16}} = \frac{5}{8}$

78. (c) $V = [\vec{a}\vec{b}\vec{c}]$

$[\vec{a}, \vec{b} + \vec{c}, \vec{a} + 2\vec{b} + 3\vec{c}]$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} [\vec{a}\vec{b}\vec{c}] = 1(3-2)V = V.$$

79. (b)

$1^2 - 2^2 + 3^2 - 4^2 + \cdots (2021)^2 - (2022)^2 + (2023)^2 = 1012\ m^2 n$

$= (1-2)(1+2) + (3-4)(3+4) + \cdots . + (2021 - 2022)\ (2021$

$+ 2022) + (2023)^2$

$= (-1)(1 + 2 + 3 + 4 + \cdots .. + 2022) + (2023)^2$

$$= (-1) \cdot \frac{(2022)(2023)}{2} + (2023)^2$$

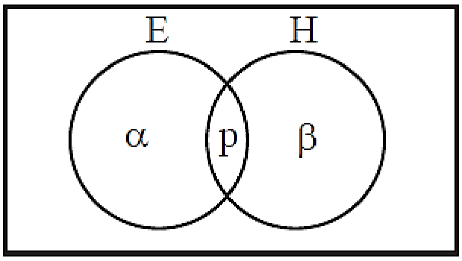
$= 2023(2023 - 1011) = 2023 \times 1012$

$m^2 n = 2023 = 17^2 . 7$

$m = 17, n = 7$

$m^2 - n^2 = 17^2 - 7^2 = 240$

80. (c)



$\alpha + p = 75$

$\beta + p = 40$

$\alpha + \beta + p = 100$

From (a), (b) and (c)

$P = 15, \alpha = 60$ and $\beta = 25$

Now equation of ellipse: $25\left(\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2}\right) = 1$

$\frac{x^2}{144} + \frac{y^2}{25} = 1$

$\Rightarrow e = \frac{\sqrt{119}}{12}$

81. (0) Let $f(x) = \frac{x}{(1+x^n)^{1/x}}, x \in R - \{-1\}, n \in N, n > 2$

$F^n(x) = (\text{ fofof... upto n times})(x),$

then $\lim_{n\to\infty} \int_0^1 x^{n-2}(f^n(x))dx$

$$f(f(x)) = \frac{x}{(1+2x^n)^{1/n}}$$
$$f(f(f(x))) = \frac{x}{(1+3x^n)^{1/n}}$$

Similarly $f^n(x) = \frac{x}{(1+n\cdot x^n)^{1/n}}$

Now $\lim_{n\to\infty} \int \frac{x^{n-2}\cdot xdx}{(1+n\cdot x^n)^{1/n}} = \lim_{n\to\infty} \int \frac{x^{n-1}\cdot dx}{(1+n\cdot x^n)^{1/n}}$

Now $1 + nx^n = t$

$n^2 \cdot x^{n-1}dx = dt$

$x^{n-1}dx = \frac{dt}{n^2}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n^2} \int_1^{1+n} \frac{dt}{t^{1/n}}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n^2}\left[\frac{t^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right]_1^{1+n}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n(n-1)}\left((1+n)^{\frac{n-1}{n}}-1\right)$ Now let $n=\frac{1}{h}$

$\Rightarrow \lim_{h\to 0} \frac{\left(1+\frac{1}{h}\right)^{1-h}-1}{\frac{1}{h}\frac{(1-h)}{h}}$

Using series expansion.

$\Rightarrow 0$

82. (4) The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$\Rightarrow \tan 9^\circ + \cot 9^\circ - \tan 27^\circ - \cot 27^\circ$

$\Rightarrow \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$

$\Rightarrow \frac{2\times 4}{\sqrt{5}-1} - \frac{2\times 4}{\sqrt{5}+1}$

$\Rightarrow 4$

83. (18) If the lines $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$

And $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ intersect

Point on first line $(1,2,3)$ and point on second line $(4,1,0)$.

Vector joining both points is $-3\hat{\imath}+\hat{\jmath}+3\hat{k}$

Now vector along first line is $2\hat{\imath}+3\hat{\jmath}+\alpha\hat{k}$

Also vector along second line is $5\hat{\imath}+2\hat{\jmath}+\beta\hat{k}$

Now these three vectors must be coplanar

$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix} = 0$

$\Rightarrow 2(6-\beta)-3(15+3\beta)+\alpha(11)=0$

$\Rightarrow \alpha-\beta=3$

Now $\alpha=3+\beta$

Given expression $8(3+\beta)\cdot\beta = 8(\beta^2+3\beta)$

$$= 8\left(\beta^2 + 3\beta + \frac{9}{4} - \frac{9}{4}\right) = 8\left(\beta + \frac{3}{2}\right)^2 - 18$$

So, magnitude of minimum value $= 18$

84. (400)

If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \cdots + 20(21)^{19} = k(20)^{19}$ then k is

$$20^{19}\left(1 + 2\cdot\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \cdots . +20\left(\frac{21}{20}\right)^{19}\right) = k(20)^{19}$$

$$\Rightarrow k = 1 + 2\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \cdots . +20\left(\frac{21}{20}\right)^{19}$$

$$\Rightarrow k\left(\frac{21}{20}\right) = \frac{21}{20} + 2\cdot\left(\frac{21}{20}\right)^2 + \cdots .$$

$$\ldots . +19\left(\frac{21}{20}\right)^{19} + 20\cdot\left(\frac{21}{20}\right)^{20}$$

Subtracting equation (b) from (a)

$$\Rightarrow k\left(\frac{-1}{20}\right) = 1 + \frac{21}{20} + \left(\frac{21}{20}\right)^2 + \cdots . + \left(\frac{21}{20}\right)^{19} - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = \frac{1\left(\left(\frac{21}{20}\right)^{20} - 1\right)}{\left(\frac{21}{20} - 1\right)} - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = 20\left(\frac{21}{20}\right)^{20} - 20 - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = -20$$

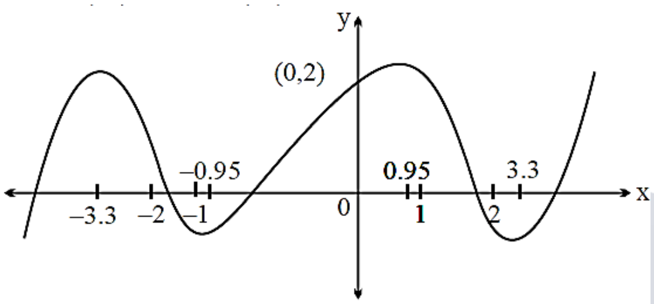
$$\Rightarrow k = 400$$

85. (432)

UNIVERSE
Vowels: E, I, U
Consonants: N, V, R, S
$\rightarrow\ {}^3C_2 \times {}^4C_2 \times 4! = 3 \times 6 \times 24 = 432$

86. (5)

$y = x^5 - 20x^3 + 50x + 2$
$\frac{dy}{dx} = 5x^4 - 60x^2 + 50 = 5(x^4 - 12x^2 + 10)$
$\frac{dy}{dx} = 0 \Rightarrow x^4 - 12x^2 + 10 = 0$
$\Rightarrow x^2 = \frac{12 \pm \sqrt{144 - 40}}{2}$
$\Rightarrow x^2 = 6 \pm \sqrt{26} \Rightarrow x^2 \approx 6 \pm 5.1$
$\Rightarrow x^2 \approx 11.1, 0.9$
$\Rightarrow x \approx \pm 3.3, \pm 0.95$
$f(0) = 2, f(1) = +ve, f(2) = -ve$
$f(-1) = -ve, f(-2) = +ve$



87. (2) For circle:

$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$

$r = \frac{|z_1 - z_2|}{2} = \frac{|\alpha - \beta|}{2} = \sqrt{\lambda - 1}$

$2\lambda = |\alpha - \beta|^2$

$|\alpha - \beta| = 2\sqrt{\lambda - 1}$

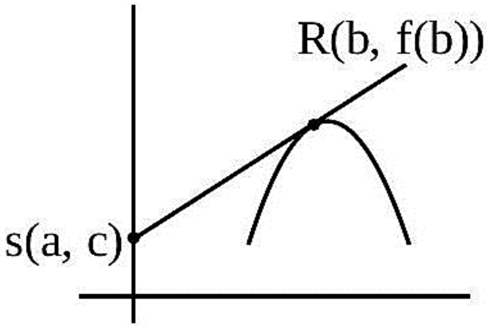
$|\alpha - \beta|^2 = 4\lambda - 4 = 2\lambda$

$\lambda = 2$

$\Rightarrow |\alpha - \beta|^2 = 4$

$|\alpha - \beta| = 2$

88. (5)



Equation of tangent at $R(b, f(b))$ is

$y - f(b) = f'(b) \cdot (x - b)$

which passes through $(0, c)$

$\Rightarrow c - f(b) = f'(b) \cdot (-b)$

$\Rightarrow \frac{3}{b} - f(b) = f'(b) \cdot (-b)$

$\Rightarrow bf'(b) - f(b) = -\frac{3}{b}$

$\Rightarrow \frac{bf'(b) - f(b)}{b^2} = -\frac{3}{b^3}$

$\Rightarrow d\left(\frac{f(b)}{b}\right) = -\frac{3}{b^3} \Rightarrow \frac{f(b)}{b} = \frac{3}{2\,b^2} + \lambda$

Which passes through $(1,3/2)$

$\Rightarrow \frac{3}{2} = \frac{3}{2} + \lambda \Rightarrow \lambda = 0$

$\Rightarrow f(b) = \frac{3}{2\,b}$

$f(a) = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{3}{2\,b} \Rightarrow b = 3$

$\Rightarrow c = 1 \Rightarrow Q(3,1/2)$

$\Rightarrow PQ^2 = 2^2 + (1)^2 = 5$

89. (2) $e_H = \sqrt{2}$

$e_E = \frac{1}{\sqrt{2}}$

Since the curves intersect each other orthogonally

The ellipse and the hyperbola are confocal

$H: \frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$

$\Rightarrow$ foci $= (1,0)$

For ellipse a. $e_E = 1$

$\Rightarrow a = \sqrt{2}$

$(e_E)^2 = \frac{1}{2} \Rightarrow 1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$

$\Rightarrow b^2 = 1$

Length of L.R. $= \frac{2b^2}{a} = \frac{2}{\sqrt{2}} = \sqrt{2}$

90. (25)

$2x_i$	f_i	f_ix_i	$f_ix_i{}^2$
2	4	8	16
4	4	16	64
6	α	6α	36α
8	15	120	960
10	8	80	800
12	β	12β	144β
14	4	56	784
16	5	80	1280

$N = \sum f_i = 40 + \alpha + \beta$

$\sum f_ix_i = 360 + 6\alpha + 12\beta \; \sum f_ix_i^2 = 3904 + 36\alpha + 144\beta$

$\text{Mean}(\bar{x}) = \frac{\sum f_ix_i}{\sum f_i} = 9$

$\Rightarrow 360 + 6\alpha + 12\beta = 9(40 + \alpha + \beta)$

$3\alpha = 3\beta \Rightarrow \alpha = \beta$

$\sigma^2 = \frac{\sum f_ix_i^2}{\sum f_i} - \left(\frac{\sum f_ix_i}{\sum f_i}\right)^2$

$\Rightarrow \frac{3904 + 36\alpha + 144\beta}{40 + \alpha + \beta} - (\bar{x})^2 = 15.08$

$\Rightarrow \frac{3904 + 180\alpha}{40 + 2\alpha} - (9)^2 = 15.08$

$\Rightarrow \alpha = 5$

Now, $\alpha^2 + \beta^2 - \alpha\beta = \alpha^2 = 25$