1. **(d)** $T = 2\pi\sqrt{\frac{m}{k}}$

$$T^2 \propto \frac{m}{k}$$

$$\frac{2\Delta T}{T}\% = \frac{\Delta m}{m}\% - \frac{\Delta k}{k}\%$$

$$\frac{\Delta k}{k}\% = \frac{\Delta m}{m}\% - \frac{2\Delta T}{T}\%$$

$$\frac{\Delta k}{k}\% = (-1)\% - 2(b)\% = |-5\%| = 5\%$$

2. **(d)**

$$K_i = \tfrac{1}{2}\, m(100)^2$$

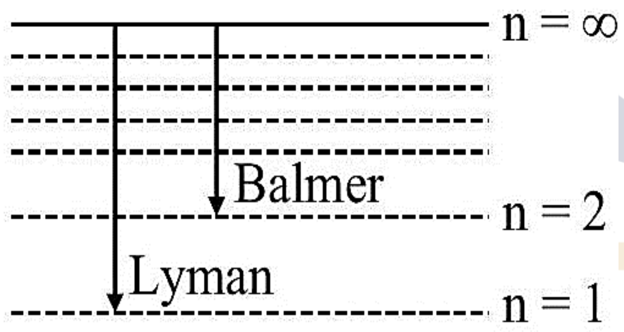
$$K_f = \frac{1}{2}\, m(40)^2$$

$$\%\text{ loss} = \tfrac{|K_f - K_i|}{K_i} \times 100$$

$$= \frac{\left|\frac{1}{2}\, m(40)^2 - \frac{1}{2}\, m(100)^2\right|}{\frac{1}{2}\, m(100)^2} \times 100$$

$$= \frac{|1600 - 100 \times 100|}{100} = 84\%$$

3. **(a)**



$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{\frac{1}{\lambda_L} = Rz^2\left(\frac{1}{1^2}\right)}{\frac{1}{\lambda_B} = Rz^2\left(\frac{1}{2^2}\right)}$$

$$\frac{\lambda_B}{\lambda_L} = 4:1$$

4. **(b)**

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R_E}$$

$$g = \frac{GM}{R_E^2}$$

$$K = mgR_E$$

5. (d)

$$\frac{\varepsilon_m \times \mu_m}{\varepsilon_0 \times \mu_0} = \frac{\frac{1}{v^2}}{\frac{1}{c^2}}$$

$$\varepsilon_r \times \mu_r = \frac{c^2}{v^2}$$

$$\varepsilon_r \times 2 = \frac{(3 \times 10^8)^2}{(1.5 \times 10^8)^2}$$

$$\varepsilon_r \times 2 = 4$$

$$\varepsilon_r = 2$$

6. (b) Photoelectric effect prove particle nature of light.

7. (b)

$$MSR = 1 \text{ mm}, CSR = 42, \text{pitch} = 1 \text{ mm}$$

$$LC = \frac{\text{pitch}}{\text{No. of CSD}} = \left(\frac{1}{100}\right) = 0.01 \text{ mm}$$

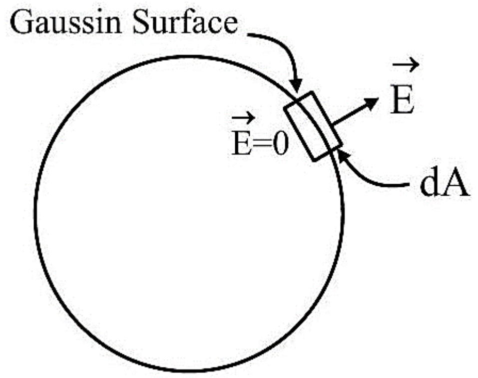
$$\text{Diameter} = MSR + LC \times CSD$$

$$\text{Diameter} = 1 + (0.01) \times 42 \text{ mm}$$

$$\text{Diameter} = 1.42 \text{ mm} = \frac{x}{50}$$

$$\therefore x = 71$$

8. (c)

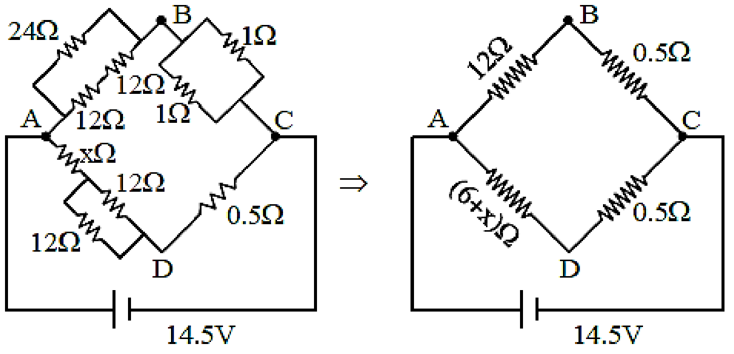


By Gauss law $\int \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

$$EdA = \frac{\sigma \times dA}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

9. (c)



In case of balanced Wheatstone Bridge

$$\frac{V_{AB}}{V_{AD}} = \frac{V_{BC}}{V_{CD}} \Rightarrow \frac{12}{6+x} = \frac{0.5}{0.5}$$

$x = 6\Omega$

10. (d)

$dQ = du + dW$

$CdT = C_V dT + PdV \ldots\ldots. (1)$

$\therefore PV^2 = RT$

$P = \text{constant}$

$P(2VdV) = RdT$

$$PdV = \frac{RdT}{2V}$$

Put in equation (1)

$$C = C_V + \frac{R}{2\,V}$$

11. (b)

$[\vec{\tau}] = [\vec{r} \times \vec{F}] = [ML^2\ T^{-2}]$

$[F] = [qVB]$

$$\Rightarrow B = \left(\frac{F}{qV}\right) = \left[\frac{MLT^{-2}}{ATLT^{-1}}\right] = [MA^{-1}\ T^{-2}]$$

$[M] = [I \times A] = [AL^2]$

$B = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$

$\Rightarrow [\mu] = \left[\frac{Br^2}{Idl}\right] = \left[\frac{MT^{-2}\ A^{-1} \times L^2}{AL}\right]$

$= [MLT^{-2}\ A^{-2}]$

12. (c)

$I_m = \frac{V_m}{\sqrt{R^2+(X_L-X_C)^2}}$ at resonance $X_L = X_C$

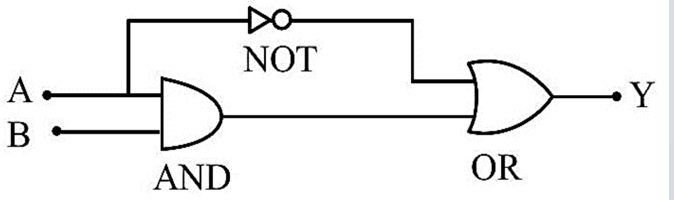
Thus, $I_m = \frac{V_m}{R}$

$\because$ Impendence is minimum therefore I is maximum at resonance.

Statement-II

$I = \left(\frac{V}{R}\right)$ in purely resistive circuit.

13. (b)



14. (b) $V_{rms} = \sqrt{\frac{3RT}{M_w}}$

$\Rightarrow \frac{V_{O_2}}{V_{He}} = \sqrt{\frac{M_{w,He}}{M_{w,O_2}}}$

$= \sqrt{\frac{4}{32}} = \frac{1}{2\sqrt{2}}$

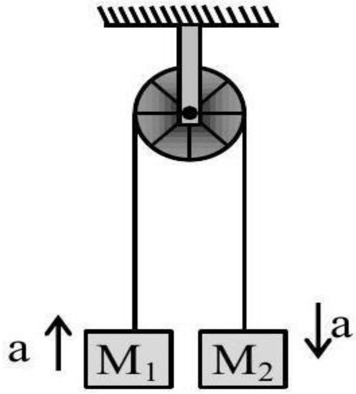
$\frac{V_{He}}{V_{O_2}} = \frac{2\sqrt{2}}{1}$

15. (a)

$a = \left(\frac{M_2 - M_1}{M_1 + M_2}\right)g$

$\frac{g}{\sqrt{2}} = \left(\frac{M_2 - M_1}{M_1 + M_2}\right)g$

$(M_1 + M_2) = \sqrt{2}M_2 - \sqrt{2}M_1$



$\frac{M_1}{M_2} = \left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$

16. (c)

$KE = \frac{p^2}{2\ m}$

Same momentum, so less mass means more KE.

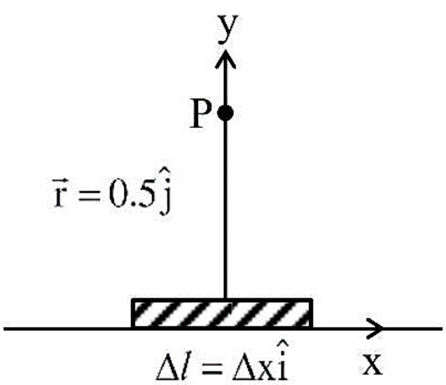
So $\frac{m}{2}$ will have max. KE.

17. (b)

Average speed $= \frac{\text{total distance}}{\text{time taken}}$

$= \frac{\frac{80 \times t}{2} + 80 \times 3t}{4t} = 70\ \text{km/hr}$

18. (a)



$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi} \frac{(\overrightarrow{dl} \times \vec{r})}{r^3}$ (Tesla)

$= \frac{10^{-7} \times 10 \times \left(\frac{1}{2} \times \frac{1}{100}\right)(+\hat{k})}{\left(\frac{1}{2}\right)^3} = 4 \times 10^{-8}\ T(+\hat{k})$

19. (d) $mg - F_B - F_v = ma$

$a = 0$ for constant velocity

$mg - F_B = F_V$

$F_v = mg - v\rho_0 g = mg - \frac{m}{\rho}\rho_0 g = mg\left(1 - \frac{\rho_0}{\rho}\right)$

20. (d) $eV_s = h\nu - \phi$

$0.5\ V = 2.48 - \phi$

work function $(\phi) = 2.48\ V - 0.5\ V = 1.98\ V$

21. (13) By conservation of angular momentum

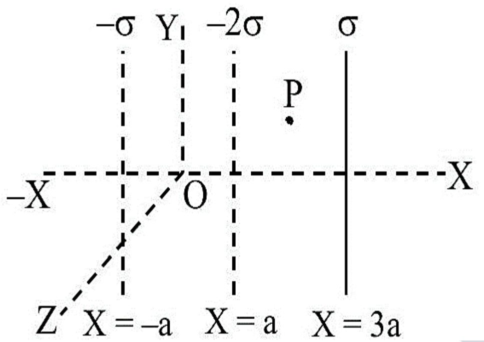
$I_1\omega_1 = I_2\omega_2$

$\left(\frac{2}{5}MR^2\right)\frac{2\pi}{T_1} = \frac{2}{5}M\left(\frac{3}{4}R\right)^2\frac{2\pi}{T_2}$

$\frac{1}{T_1} = \frac{9}{16\ T_2}$

$\frac{1}{T_2} = \frac{9}{16} \times T_1 = \frac{9}{16} \times 24hr = \frac{27}{2}hr = 13hr30$ mins.

22. (2)



$\vec{E}_p = \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right)(-\hat{i})$
$= -\frac{2\sigma}{\varepsilon_0}\hat{i}$

23. (1)

1000 drops Big drop

$$1000\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$
$$10r = R$$
$$R = 10r$$

$$\frac{\text{S.E. of 1000 drops}}{\text{S.E. of Big drop}} = \frac{1000(4\pi r^2)T}{4\pi R^2\ T}$$
$$= \frac{1000 \times r^2}{(10r)^2} = 10 = \frac{10}{x}$$
$$\therefore\ x = 1$$

24. (250)

For DC voltage

$$R = \frac{V}{I} = \frac{100}{5} = 20\Omega$$

for AC voltage

$$X_L = 20\sqrt{3}\Omega$$

$$R = 20\Omega$$

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{3 \times 400 + 400} = 40\Omega$$

$$\text{Power} = i_{rms}^2\ R$$

$$= \left(\frac{V_{rms}}{Z}\right)^2 \times R = \left(\frac{\frac{200}{\sqrt{2}}}{40}\right)^2 \times 20 = 250\ \text{W}$$

25. (60)

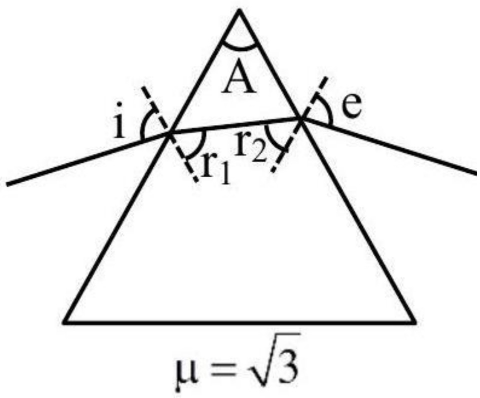
For δ_{min}

$$i = e$$

$$r_1 = r_2 = \frac{A}{2}$$

$$\frac{\delta_{min}}{A} = 1$$

$$\frac{2i - A}{A} = 1$$



$2i = 2A$

$i = A$

Snell's law

$1 \times \sin i = \mu \sin r$

$\sin i = \mu \sin\left(\frac{A}{2}\right)$

$\sin A = \mu \sin\left(\frac{A}{2}\right)$

$2\sin\frac{A}{2}\cos\frac{A}{2} = \sqrt{3}\sin\left(\frac{A}{2}\right)$

$\cos\left(\frac{A}{2}\right) = \frac{\sqrt{3}}{2}$

$\therefore \frac{A}{2} = 30^\circ$

$\therefore A = 60^\circ$

26. (16)

We know $R = \frac{\rho l}{A}, R \propto \frac{l}{r^2}$

As we starch the wire, its length will increase but its radius will decrease keeping the volume constant

$$V_i = V_f$$
$$\pi r^2 l = \pi \frac{r^2}{4} l_f$$
$$l_f = 4l$$
$$\frac{R_{new}}{R_{old}} = \left(\frac{4l}{\frac{r^2}{4}}\right)\frac{r^2}{l} = 16$$
$$R_{new} = 16R$$
$$\therefore x = 16$$

27. (16)

We know

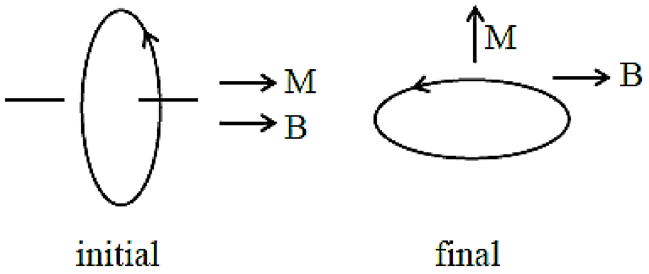
$r = 0.529\frac{n^2}{Z} \Rightarrow 8.48 = 0.529\frac{n^2}{1}$

$n^2 = 16 \Rightarrow n = 4$

We know

$$E \propto \frac{1}{n^2}$$
$$E_{n^{th}} = \frac{E}{16}$$
$$x = 16$$

28. (5)



We know

$$\begin{aligned} W_{ext} &= \Delta U + \Delta KE\ (\text{P.E.} = -\vec{M}\cdot\vec{B}) \\ &= -\vec{M}\cdot\vec{B}_f + \vec{M}\cdot\vec{B}_i + 0 \\ &= -MB\cos 90 + MB\cos 0 \\ &= MB \\ &= NIAB \\ &= 200 \times 100 \times 10^{-6} \times \frac{5}{2} \times 10^{-4} \times 1 = 5\mu J \end{aligned}$$

29. (12)

We know

$v_{max} = \omega A$ at mean position

$= \frac{2\pi}{T}A = \frac{2\pi}{\pi} \times 0.06 = 0.12\ m/sec$

$v_{max} = 12\ cm/sec$

30. (4)

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 15\hat{i} - 21\hat{j} + 33\hat{k}$$

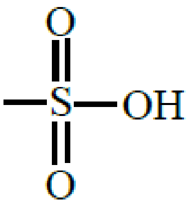
$\vec{A}\cdot(\vec{B}\times\vec{C}) = (-x\hat{i} - 6\hat{j} - 2\hat{k})\cdot(15\hat{i} - 21\hat{j} + 33\hat{k})$

$0 = -15x + 126 - 66$

$15x = 60$

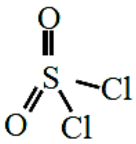
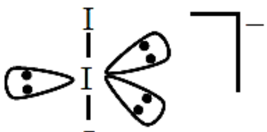
$x = 4$

31. (b)

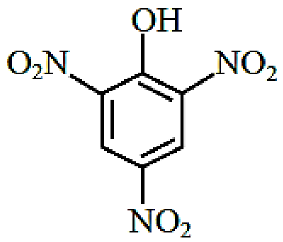


Group present in sulphonic acids

32. (b)

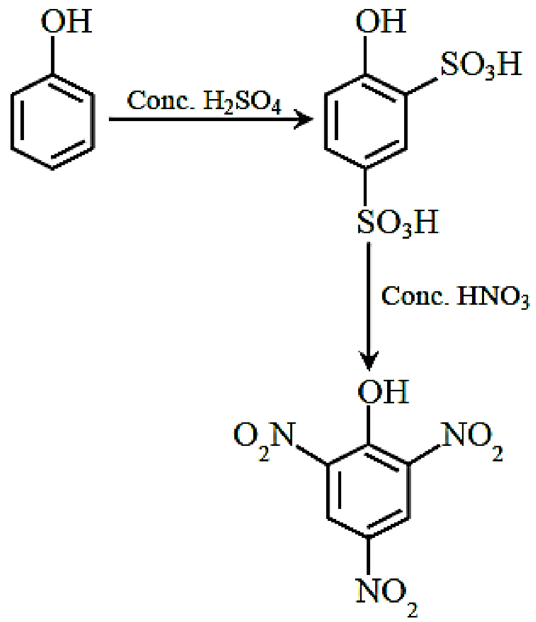
(A)	SO_2Cl_2	sp^3	 Tetrahedral
(B)	NO		Paramagnetic
(C)	NO_2^-		Diamagnetic
(D)	I_3^-	sp^3d	 Linear

33. (a)



picric acid

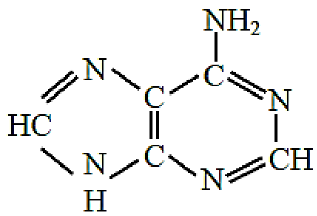
(2, 4, 6 – trinitrophenol)



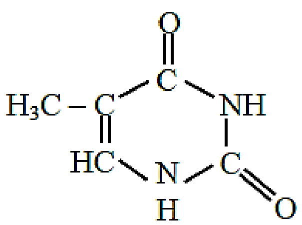
Picric acid

34. (d) Metamer ⇒ Isomer having same molecular formula, same functional group but different alkyl/aryl groups on either side of functional group.

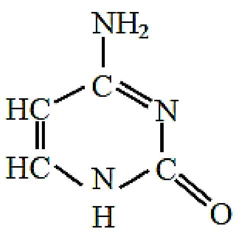
35. (c)



⇒ Adenine



⇒ Thymine



⇒ Cytosine

Are bases of DNA molecule. As DNA contain four bases, which are adenine, guanine, cytosine and thymine.

36. (d)

$sp^3 \rightarrow$ Tetrahedral

$dsp^2 \rightarrow$ Square planar

$sp^3d \rightarrow$ Trigonal Bipyramidal

$sp^3d^2 \rightarrow$ Octahedral

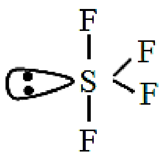
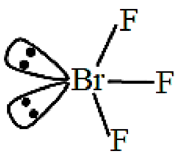
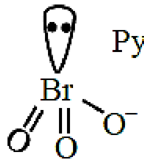
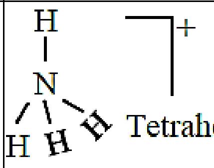
37. (d) Statement - I $\Rightarrow$ Correct

Statement - II $\Rightarrow$ False

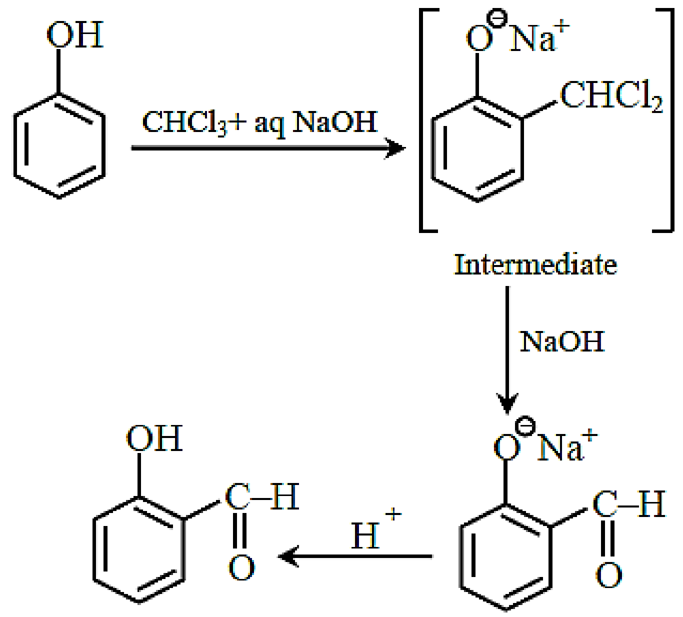
Ga is used to measure high temperature

38. (b) Option (B) is incorrect because aniline is immisible in water.

39. (b)

(A)	SF_4	sp^3d hybridisation	
(B)	BrF_3	sp^3d hybridisation	 Bent T-Shape
(C)	BrO_3^-	sp^3 hybridisation	 Pyramidal
(D)	NH_4^+	sp^3 hybridisation	 Tetrahedral

40. (d)



41. (b) Graphite is conductor

42. (b) As ligand field increases, light of more energy is absorbed

Energy ∝ wave number

$(\bar{U})$

43. (c)

44. (d)

(A) $Be + e^- \rightarrow Be^-$, $E.A = -$ ive

(B) $N + e^- \rightarrow N^-$ $E.A = -$ ive

(C) $O + e^- \rightarrow O^-$ $O^- + e^- \rightarrow O^{-2}$

$E.\ A = -$ ive

(D) $Na + e^- \rightarrow Na^-$ $E.\ A = +$ive

(E) $A\ell + e^- \rightarrow A\ell^-$ $E.\ A = +$ive

Ans. A,B and C only

45. (c) Cm is Actinide

46. (b)

$$\text{Molality} = \frac{1000 \times M}{1000 \times d - M \times (Mw)_{solute}}$$

$$3 = \frac{1000 \times x}{1000 \times 1.12 - (x \times 40)}$$

$x = 3$

47. (a) $H-\overset{\overset{\displaystyle O}{\|}}{C}-H$ has low steric hindrance at carbonyl carbon and high partial positive charge at carbonyl carbon.

48. (c)

$\Delta n_g = 0$ $\quad\quad K_p = \dfrac{(n_{HI})^2}{n_{H_2} n_{I_2}}\left(\dfrac{P_T}{n_T}\right)^{\Delta n_g}$

$n_{HI} = n_{H_2} = n_{I_2}$ $\quad$ so $K_P = 1$

$1 = x \times 10^{-1}$ $\quad\quad x = 10$

49. (c)

Iodoform - Antiseptic

CCl_4 $-$ Fire extinguisher

CFC-Refrigerants

DDT - Insecticide

50. (b) Solution is already infinitely dilute, hence no change in molar conductivity upon addition of water

51. (76)

$HX \rightleftharpoons H^+ + X^-$ $K_a = 1.2 \times 10^{-5}$

0.03M

$0.03 - x$ x x

$K_a = 1.2 \times 10^{-5} = \dfrac{x^2}{0.03 - x}$

$0.03 - x \approx 0.03$(K_a is very small)

$\dfrac{x^2}{0.03} = 1.2 \times 10^{-5}$

$x = 6 \times 10^{-4}$

Final solution: $0.03 - x + x + x$

$= 0.03 + x = 0.03 + 6 \times 10^{-4}$

$\Pi = \left(0.03 + (6 \times 10^{-4})\right) \times 0.083 \times 300$

$= 76.19 \times 10^{-2} \approx 76 \times 10^{-2}$

52. (6) Spin only magnetic moment of Mn in $KMnO_4 = 0$ Spin only value of manganese product fromed during titration of $KMnO_4$ aganist oxalic acid in acidic medium is $= 6$

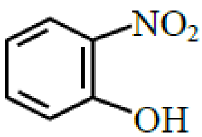
53. (3)

$K = \dfrac{1}{t_{99.9\%}} \ln\left(\dfrac{100}{0.1}\right) = \dfrac{1}{t_{90\%}} \ln\left(\dfrac{100}{10}\right)$

$$t_{99.9\%} = t_{90\%} \frac{\ell n(10^3)}{\ell n 10}$$

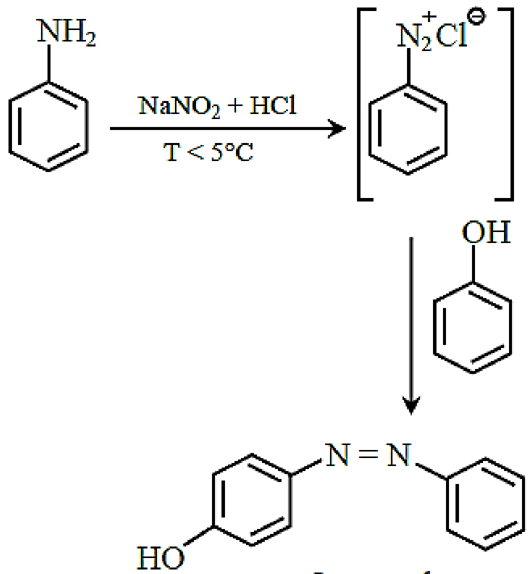
$$t_{99.9\%} = t_{90\%} \times 3$$

54. (5)

CH_3OH, H_2O,  HF, NH_3

Can show H-bonding.

55. (20)



Orange dye

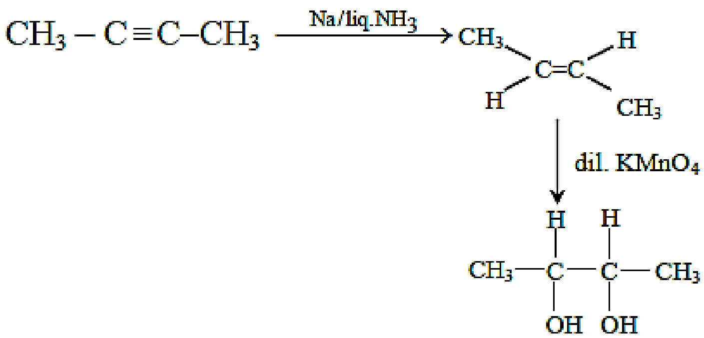
Reaction suggests that 1 mole of aniline give 1 mole of orange dye.

so $(\text{mol})_{\text{aniline}} = (\text{ mole })_{\text{orange dye}}$

$$\frac{9.3 \text{ g}}{93 \text{ g mol}^{-1}} = \frac{\text{mass of orange dye}}{199 \text{ g mol}^{-1}}$$

mass of orange dye $= 19.9 \text{ g} \simeq 20 \text{ g}$

56. (2)



(Product P)

57. (661)

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{hv}{mv^2}$$

$\frac{mv^2}{h} = \frac{v}{\lambda} = v$ (frequency)

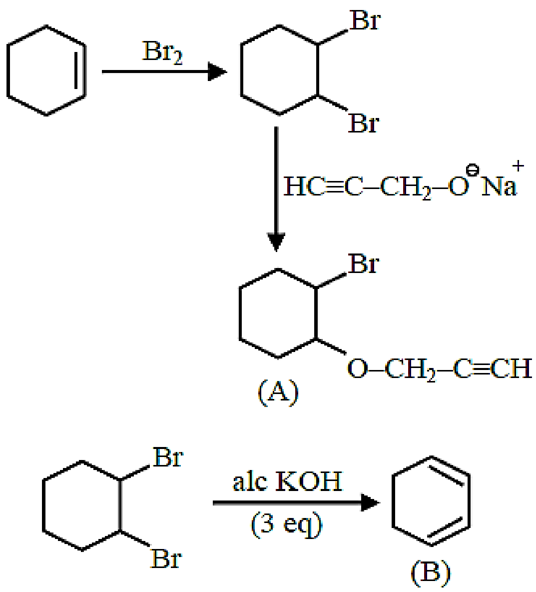
Given $\frac{1}{2}mv^2 = 2.18 \times 10^{-18}$ J

$h = 6.6 \times 10^{-34}$

$$v = \frac{4.36 \times 10^{-18}}{6.6 \times 10^{-34}} = 660.60 \times 10^{13} \text{ Hz}$$

$\approx 661 \times 10^{13}$ Hz

58. (8)



59. (877)

CrO Basic oxide

Cr_2O_3 Amphoteric oxide

In CrO, Cr exist as Cr^{+2} and have μ only = 4.90

In Cr_2O_3, Cr exist as Cr^{+3} and have μ only = 3.87

Sum of spin only magnetic moment

$= 4.90 + 3.87 = 8.77$

$\mu_{only} = 877 \times 10^{-2}$

Ans. 877

60. (274)

$\Delta U = q + w(q = 0)$

$nC_V\Delta T = -P_{ext}(V_2 - V_1)$

$V_2 = 2\ V_1$

$\frac{nRT_2}{P_2} = \frac{2nRT_1}{P_1}$

$P_1 = 5,\ T_1 = 298$

$P_2 = \frac{5\ T_2}{2 \times 298}$

$n\frac{5}{2}R(T_2 - T_1) = -1\left(\frac{nRT_2}{P_1} - \frac{nRT_1}{P_1}\right)$

Put $T_1 = 298$

and $P_2 = \frac{5\ T_2}{2\times298}$

Solve and we get $T_2 = 274.16$ K

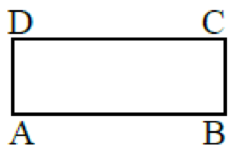
$T_2 \approx 274$ K

61. (b)

$f'(x) = 3x^2\sin\left(\frac{1}{x}\right) - x\cos\left(\frac{1}{x}\right)$

$f''(x) = 6x\sin\left(\frac{1}{x}\right) - 3\cos\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) - \frac{\sin\left(\frac{1}{x}\right)}{x} f''\left(\frac{2}{\pi}\right) = \frac{12}{\pi} - \frac{\pi}{2} = \frac{24-\pi^2}{2\pi}$

62. (a)



Area $= \frac{1}{2}|\overline{BD} \times \overline{AC}|$

$\overline{BD} = \frac{5}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{2}{3}\hat{k}$

$\overline{AC} = \hat{i} - \hat{j} - 2\hat{k}$

63. (c)

Divide Nr&Dr by cosx

$\int_0^{\pi/4} \frac{\tan^2 x \sec^2 x dx}{(1+\tan^3 x)^2} dx$

Let $1+\tan^3 x = t$

$\tan^2 x \sec^2 x dx = \frac{dt}{3}$

$\frac{1}{3}\int_1^2 \frac{dt}{t^2} = \frac{1}{6}$

64. (c) Mean $(\bar{x}) = 10$

$\Rightarrow \frac{\sum x_i}{20} = 10$

$\Sigma x_i = 10 \times 20 = 200$

If 8 is replaced by 12 , then $\sum x_i = 200 - 8 + 12 = 204$
$\therefore$ Correct mean $(\bar{x}) = \frac{\Sigma x_i}{20}$

$= \frac{204}{20} = 10.2$

$\because$ Standard deviation $= 2$

$\therefore$ Variance $= (\text{ S.D. })^2 = 2^2 = 4$

$\Rightarrow \frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2 = 4$

$\Rightarrow \frac{\Sigma x_i^2}{20} - (10)^2 = 4$

$\Rightarrow \frac{\Sigma x_i^2}{20} = 104$

$\Rightarrow \Sigma x_i^2 = 2080$

Now, replaced '8' observations by '12'

Then, $\Sigma x_i^2 = 2080 - 8^2 + 12^2 = 2160$

$\therefore$ Variance of removing observations

$\Rightarrow \frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2$

$\Rightarrow \frac{2160}{20} - (10.2)^2$

$\Rightarrow 108 - 104.04$

$\Rightarrow 3.96$

Correct standard deviation

$= \sqrt{3.96}$

65. (c)

66. (a) $n(3) \Rightarrow$ multiple of 3

$102, 105, 108, \ldots, 699$

$T_n = 699 = 102 + (n-1)(3)$

$n = 200$

$n(3) = 200$

$\because n(4) \Rightarrow$ multiple of 4

$100, 104, 108, \ldots, 700$

$T_n = 700 = 100 + (n-1)(4)$

$n = 151$

$n(4) = 151$

$n(3 \cap 4) \Rightarrow$ multiple of 3&4 both

$108, 120, 132, \ldots, 696$

$T_n = 696 = 108 + (n-1)(12)$

$n = 50$

$n(3 \cap 4) = 50$

$n(3 \cup 4) = n(c) + n(d) - n(3 \cap 4)$

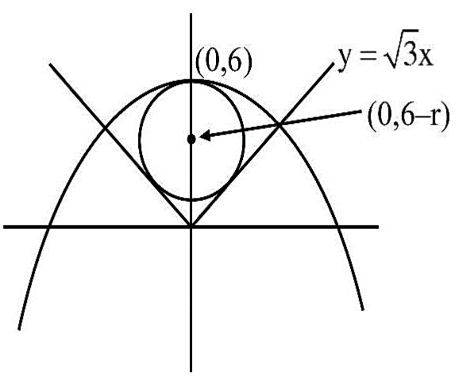
$= 200 + 151 - 50$

$= 301$

$n(\overline{3 \cup 4}) =$ Total $- n(3 \cup 4) =$ neither a multiple of 3 nor a multiple of 4

$= 601 - 301 = 300$

67. (a)



Equation of circle

$x^2 + (y - (6 - r))^2 = r^2$

touches $\sqrt{3}x - y = 0$

$p = r$

$\frac{|0 - (6 - r)|}{2} = r$

$|r - 6| = 2r$

$r = 2$

$\therefore$ Circle $x^2 + (y - 4)^2 = 4$

(2,4) Satisfies this equation

68. (a)

$$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix}$$

$$2\,A_{10} - A_8 = \begin{vmatrix} 20 & 1 & \frac{n^2}{2} + \alpha \\ 40 & 2 & n^2 - \beta \\ 56 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix} - \begin{vmatrix} 8 & 1 & \frac{n^2}{2} + \alpha \\ 16 & 2 & n^2 - \beta \\ 22 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 12 & 1 & \frac{n^2}{2} + \alpha \\ 24 & 2 & n^2 - \beta \\ 34 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 1 & \frac{n^2}{2} + \alpha \\ 0 & 2 & n^2 - \beta \\ -2 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix}$$

$\Rightarrow -2\left((n^2 - \beta) - (n^2 + 2\alpha)\right)$

$\Rightarrow -2(-\beta - 2\alpha) \Rightarrow 4\alpha + 2\beta$

69. (b)

$\frac{x - 3}{2} = \frac{y + 15}{-7} = \frac{z - 9}{5} \,\&\, \frac{x + 1}{2} = \frac{y - 1}{1} = \frac{z - 9}{-3}$

$S.D = \frac{|(\bar{a}_2 \cdot \bar{a}_1) \cdot (\bar{b}_1 \cdot \bar{b}_2)|}{|\bar{b}_1 \times \bar{b}_2|}$

$a_1 = 3, -15, 9$

$b_1 = 2, -7, 5$

$a_2 = -1,1,9$

$b_2 = 2,1,-3$

$a_2 - a_1 = -4,16,0$

$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = \hat{i}(16) - \hat{j}(-16) + \hat{k}(16)$

$16(\hat{i} + \hat{j} + \hat{k})$

$\left|\bar{b}_1 \times \bar{b}_2\right| = 16\sqrt{3}$

$\therefore (\bar{a}_2 - \bar{a}_1) \cdot \left(\bar{b}_1 - \bar{b}_2\right) = 16[-4 + 16] = (16)(12)$

$\text{S.D.} = \frac{(16)(12)}{16\sqrt{3}} = 4\sqrt{3}$

70. (a)

	A	B
Manufactured	60%	40%
Standard quality	80%	90%

P (Manufactured at B/ found standard quality) = ?

A: Found S.Q

B: Manufacture B

C: Manufacture A

$P(E_1) = \frac{40}{100}$

$P(E_2) = \frac{60}{100}$

$P(A/E_1) = \frac{90}{100}$

$P(A/E_2) = \frac{80}{100}$

$\because P(E_1/A) = \frac{P(A/E_1)P(E_1)}{P(A/E_1)P(E_1) + P(A/E_2)P(E_2)} = \frac{3}{7}$

$\therefore 126P = 54$

71. (c) by newton's theorem

$a_{n+2} - (t^2 - 5t + 6)a_{n+1} + a_n = 0$

$\therefore a_{2025} + a_{2023} = (t^2 - 5t + 6)a_{2024}$

$\therefore \dfrac{a_{2025} + a_{2023}}{a_{2024}} = t^2 - 5t + 6$

$\because t^2 - 5t + 6 = \left(t - \dfrac{5}{2}\right)^2 - \dfrac{1}{4}$

$\therefore$ minimum value $= -\frac{1}{4}$

72. (d) $x = \{1,2,3, \ldots \ldots .20\}$

$R_1 = \{(x, y): 2x - 3y = 2\}$

$R_2 = \{(x, y): -5x + 4y = 0\}$

$R_1 = \{(4,2), (7,4), (10,6), (13,8), (16,10), (19,12)\}$

$R_2 = \{(4,5), (8,10), (12,15), (16,20)\}$

in R_1 6 element needed

in R_2 4 element needed

So, total $6 + 4 = 10$ element

73. (25)

equation of line is

$y + 9 = m(x - 4)$

$\therefore A = \left(\dfrac{9 + 4m}{m}, 0\right)$

$B = (0, -9 - 4m)$

$\therefore OA + OB = \frac{9+4\,m}{m} + 9 + 4\,m \because m > 0$

$= 13 + \dfrac{9}{m} + 4m$

$\because \dfrac{4\,m + \dfrac{9}{m}}{2} \geq \sqrt{36} \Rightarrow 4\,m + \dfrac{9}{m} \geq 12$

$\therefore OA + OB \geq 25$

74. (d) $f(x) = x^x; x > 0$

$\ell\,ny = x\ell\,nx$

$\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ell\,nx$

$\dfrac{dy}{dx} = x^x(1 + \ell nx)$

for strictly increasing

$$\frac{dy}{dx} \geq 0 \Rightarrow x^x(1 + \ell nx) \geq 0$$

$$\Rightarrow \ell nx \geq -1$$

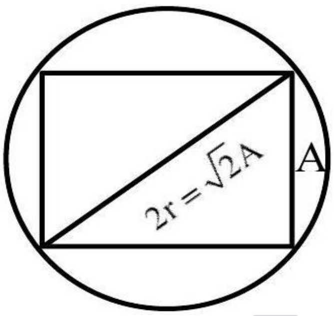
$$x \geq e^{-1}$$

$$x \geq \frac{1}{e}$$

$$x \in \left[\frac{1}{e}, \infty\right)$$

75. (b)

$$r = \frac{\Delta}{s} = \frac{\sqrt{3}a^2}{4 \cdot \frac{3a}{2}} = \frac{a}{2\sqrt{3}} = \frac{12}{2\sqrt{3}} = 2\sqrt{3}$$



$\therefore$ $\mathbf{A} = r\sqrt{2} = 2\sqrt{6}$

Area $= m = A^2 = 24$

Perimeter $= n = 4A = 8\sqrt{6}$

$\therefore m + n^2 = 24 + 384$

$= 408$

76. (c) $\because$ no. of triangles having no side common with a n sided polygon $= \frac{{}^nC_1 \cdot {}^{n-4}C_2}{3}$

$$= \frac{{}^8C_1 \cdot {}^4C_2}{3} = 16$$

77. (b)

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$$

$$\text{I.F.} = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$$

$$y.e^{\tan^{-1}x} = \int \left(\frac{e^{\tan^{-1}x}}{1+x^2}\right) e^{\tan^{-1}x}.dx$$

Let $\tan^{-1} x = z \therefore \frac{dx}{1+x^2} = dz$

$\therefore y.e^z = \int e^{2z}dz = \frac{e^{2z}}{2} + C$

$y.e^{\tan^{-1} x} = \frac{e^{2\tan^{-1} x}}{2} + C$

$\Rightarrow y = \frac{e^{\tan^{-1} x}}{2} + \frac{C}{e^{\tan^{-1} x}}$

$\because y(a) = 0 \Rightarrow 0 = \frac{e^{\pi/4}}{2} + \frac{C}{e^{\pi/4}} \Rightarrow C = \frac{-e^{\pi/2}}{2}$

$\therefore y = \frac{e^{\tan^{-1} x}}{2} - \frac{e^{\pi/2}}{2e^{\tan^{-1} x}}$

$\therefore y(0) = \frac{1 - e^{\pi/2}}{2}$

78. (c) $\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{3}{2x^2}$

$\therefore$ I.F. $= e^{\int \frac{1}{x\ln x}dx} = e^{\ln(\ln(x))} = \ell nx$

$\therefore y\ell nx = \int \frac{3\ell nx}{2x^2}dx$

$= \frac{3\ln x}{2}\int x^{-2}dx - \int \left(\frac{3}{2x} \cdot \int x^{-2}dx\right)dx$

$= \frac{3\ln x}{2}\left(-\frac{1}{x}\right) - \int \frac{3}{2x}\left(-\frac{1}{x}\right)dx$

$y.\ln x = \frac{-3\ln x}{2x} - \frac{3}{2x} + C$

$\because y(e^{-1}) = 0$

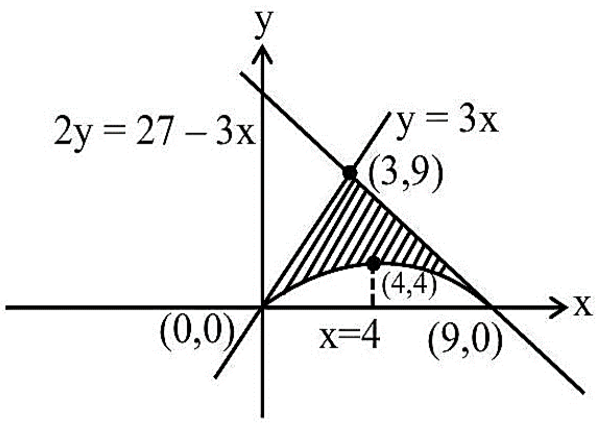
$\therefore 0(-1) = \frac{3e}{2} - \frac{3e}{2} + C \Rightarrow C = 0$

$\therefore y = \frac{-3\ell\ nx}{2x} - \frac{3}{2x}$

$\therefore y(e) = \frac{-3}{2e} - \frac{3}{2e} = \frac{-3}{e}$

79. (d)

$y = 3x, 2y = 27 - 3x \& y = 3x - x\sqrt{x}$



$$A = \int_0^3 3x - (3x - x\sqrt{x})dx + \int_3^9 \left(\frac{27 - 3x}{2} - (3x - x\sqrt{x})\right) dx$$

$$A = \int_0^3 x^{3/2}dx + \int_3^9 \frac{27}{2} - \frac{9x}{2} + x^{3/2}dx$$

$$A = \left[\frac{2x^{5/2}}{5}\right]_0^3 + \frac{27}{2}[x]_3^9 - \frac{9}{2}\left[\frac{x^2}{2}\right]_3^9 + \left[\frac{2x^{5/2}}{5}\right]_3^9$$

$$A = \frac{2}{5}(3^{5/2}) + \frac{27}{2}(6) - \frac{9}{4}(72) + \frac{2}{5}(9^{5/2} - 3^{5/2})$$

$$A = \frac{2}{5}(3^{5/2}) + 81 - 162 + \frac{2}{5} \times 3^5 - \frac{2}{5} \times 3^{5/2}$$

$$A = \frac{486}{5} - 81 = \frac{81}{5}$$

$10\ A = 162$

80. (c) f: $(-\infty, \infty) - \{0\} \to R$

$$f'(a) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$$

$$\lim_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\ln(a)$$

$$\lim_{a\to\infty} a^2\left(\frac{\left(1+\frac{1}{a}\right)}{2}\tan^{-1}\left(\frac{1}{a}\right) + 1 - \frac{2}{a^2}\ln(a)\right)$$

$$f(x) = \frac{1}{2}(1+x)\tan^{-1}(x) + 1 - 2x^2\ln(x)$$

$$f'(x) = \frac{1}{2}\left(\frac{1+x}{1+x^2} + \tan^{-1}(x) + 4x\ell n(x)\right) + 2x$$

$$f'(a) = \frac{1}{2}\left(1 + \frac{\pi}{4}\right) + 2$$

$$f'(a) = \frac{5}{2} + \frac{\pi}{8}$$

81. (55) $\alpha\beta\gamma = 45, \alpha\beta\gamma \in R$

$x(\alpha, 1,2) + y(1, \beta, 2) + z(2,3, \gamma) = (0,0,0)$

$x, y, z \in R, xyz \neq 0$

$\alpha x + y + 2z = 0$

$x + \beta y + 3z = 0$

$2x + 2y + \gamma z = 0$

$xyz \neq 0 \Rightarrow$ non-trivial

$$\begin{vmatrix} \alpha & 1 & 2 \\ 1 & \beta & 3 \\ 2 & 2 & \gamma \end{vmatrix} = 0 \Rightarrow \alpha(\beta\gamma - 6) - 1(\gamma - 6) + 2(2 - 2\beta) = 0$$

$\Rightarrow \alpha\beta\gamma - 6\alpha - \gamma + 6 + 4 - 4\beta = 0$

$\Rightarrow 6\alpha + 4\beta + \gamma = 55$

82. (25) $P(x, y) \& x \geq 3$

Slope of line at $P(x, y)$ will be $\frac{dy}{dx} = \frac{1}{2}\left(\frac{y+5}{x-3}\right)$

$$\Rightarrow 2\frac{dy}{(y+5)} = \frac{1}{(x-3)}dx$$

$\Rightarrow 2\ln(y+5) = \ln(x-3) + C$

Passes through $(4, -2)$

$\Rightarrow 2\ell n(c) = \ln(a) + C$

$\Rightarrow C = 2\ell n(c)$

$\Rightarrow 2\ln(y+5) = \ln(x-3) + 2\ln(c)$

$$\Rightarrow 2\left(\ln\left(\frac{y+5}{3}\right)\right) = \ln(x-3)$$

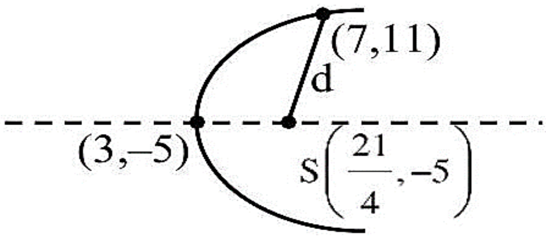
$$\Rightarrow \left(\frac{y+5}{3}\right)^2 = (x-3)$$

$\Rightarrow (y+5)^2 = 9(x-3)$

Parabola

$4a = 9$

$$a = \frac{9}{4}$$



$$d = \sqrt{\left(\frac{7}{4}\right)^2 + 6^2}$$

$$d = \frac{\sqrt{625}}{4}$$

$$d = \frac{25}{4}$$

$12\ d = 75$

83. (65)

$I_K = \int 1.(1-x^7)^K dx$

$$I_K = (1-x^7)^K x\Big|_0^1 + 7K\int_0^1 (1-x^7)^{K-1}x^6 \cdot x dx$$

$I_K = -7K\int_0^1 (1-x^7)^{K-1}\left((1-x^7)-1\right)dx$

$I_K = -7\ KI_K + 7\ KI_{K-1}$

$$\Rightarrow \frac{I_K}{I_{K+1}} = \frac{7\ K+8}{7\ K+7}$$

$$r_K = \frac{7\ K+8}{7\ K+7}$$

$$r_K - 1 = \frac{1}{7(K+1)}$$

$$\Rightarrow 7(r_K - 1) = \frac{1}{K+1}$$

$\sum_{K=1}^{10}(K+1) = 11(6) - 1 = 65$

84. (221)

$4x^4 + 8x^3 - 17x^2 - 12x + 9$

$= 4(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

Put $x = 2i\& - 2i$

$64 - 64i + 68 - 24i + 9 = (2i-x_1)(2i-x_2)(2i-x_3)\ (2i-x_4)$

$= 141 - 88i$

$64 + 64i + 68 + 24i + 9 = 4(-2i - x_1)(-2i - x_2)(-2i$

$-x_3)(-2i - x_4)$

$= 141 + 88i$

$$\frac{125}{16} m = \frac{141^2 + 88^2}{16}$$

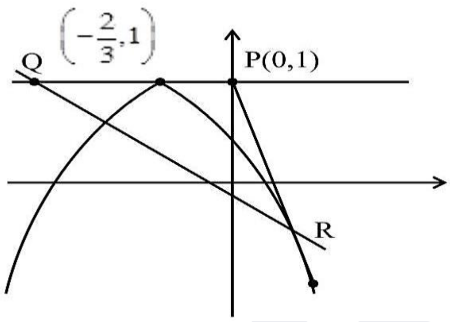
$m = 221$

85.(68)

$9x^2 + 12x + 4 = -18(y - 1)$

$(3x + 2)^2 = -18(y - 1)$

$$\left(x + \frac{2}{3}\right)^2 = -2(y - 1)$$



(0,1)

$y = mx + 1$

$$\left(x + \frac{2}{3}\right)^2 = -2(y - 1)$$

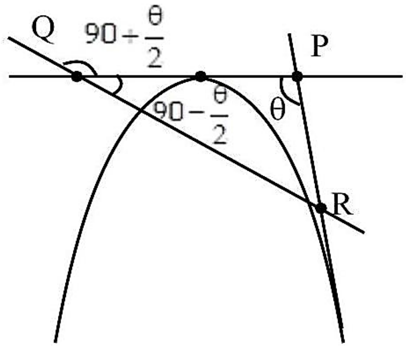
$(3x + 2)^2 = -18mx$

$9x^2 + (12 + 18\ m)x + 4 = 0$

$4(6 + 9m)^2 = 4(36)$

$6 + 9m = 6, -6$

$$m = 0, \frac{-4}{3}$$



$\tan\theta = -\frac{4}{3}$

$\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{-4}{3}$

$\left(\tan\frac{\theta}{2} - 2\right)\left(2\tan\frac{\theta}{2} + 1\right) = 0$

$\tan\frac{\theta}{2} = 2, \frac{-1}{2}$

$m_{QR} = \tan\left(90 + \frac{\theta}{2}\right) = -\cot\frac{\theta}{2}$

$m_1 = \frac{-1}{2} \quad m_2 = \frac{-1}{-1/2} = 2$

$16(m_1^2 + m_2^2) = 16\left(\frac{1}{4} + 4\right)$

$= 4 + 64 = 68$

86. (806)

${}^nC_1 x^{n-1} y = 135$………… (i)

${}^nC_2 x^{n-2} y^2 = 30$……….. (ii)

${}^nC_3 x^{n-3} y^3 = \frac{10}{3}$………. (iii)

By $\frac{(i)}{(ii)}$

$\frac{{}^nC_1}{{}^nC_2}\frac{x}{y} = \frac{9}{2}$………. (IV)

By $\frac{(ii)}{(iii)}$

$\frac{{}^nC_2}{{}^nC_3}\frac{x}{y} = 9$……………..(v)

By $\frac{(iv)}{(v)}$

$$\frac{{}^{n}C_{1}\,{}^{n}C_{3}}{{}^{n}C_{2}\,{}^{n}C_{2}}=\frac{1}{2}$$

$$\frac{2n^2(n-1)(n-2)}{6}=\frac{n(n-1)}{2}\frac{n(n-1)}{2}$$

$4n-8=3n-3$

$\Rightarrow n=5$

put in (v)
$\frac{x}{y}=9$

$x=9y$

put in (i)

$${}^{5}C_1x^4\left(\frac{x}{9}\right)=135$$

$x^5=27\times 9$

$$\Rightarrow x=3,\ y=\frac{1}{3}$$

$6(n^3+x^2+y)$

$$=6\left(125+9+\frac{1}{3}\right)$$

$=806$

87. (6)

$T_r=3\,T_{r-1}+6^r, r=2,3,4,\dots n$

$T_2=3.T_1+6^2$

$T_2=3.6+6^2\dots\dots\dots(1)$

$T_3=3\,T_2+6^3$

$T_3=3\,T_2+6^3$

$T_3=3(3.6+6^2)+6^3$

$T_3=3^2.6+3.6^2+6^3\dots\dots\dots\dots\dots..(2)$

$T_r=3^{r-1}\cdot 6+3^{r-2}\cdot 6^2+\cdots+6^r$

$$T_r=3^{r-1}\cdot 6\left[1+\frac{6}{3}+\left(\frac{6}{3}\right)^2+\cdots+\left(\frac{6}{3}\right)^{r-1}\right]$$

$T_r=3^{r-1}\cdot 6(1+2+2^2+\cdots+2^{r-1})$

$$T_r = 6 \cdot 3^{r-1} 1 \cdot \frac{(1-2^r)}{(-1)}$$

$$T_r = 6.3^{r-1} \cdot (2^r - 1)$$

$$T_r = \frac{6 \cdot 3^r}{3} \cdot (2^r - 1)\ T_r = 2.(6^r - 3^r)$$

$$S_n = 2\Sigma(6^r - 3^r)$$

$$S_n = 2 \cdot \left[\frac{6 \cdot (6^n - 1)}{5} - \frac{3 \cdot (3^n - 1)}{2}\right]$$

$$S_n = 2\left[\frac{12(6^n - 1) - 15(3^n - 1)}{10}\right]$$

$$S_n = \frac{3}{5}[4.6^4 - 5.3^n + 1]$$

$$\therefore n^2 - 12n + 39 = 3$$

$$n^2 - 12n + 36 = 0$$

$$n = 6$$

88. (47)

$$\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{1} n = \frac{\pi}{4}$$

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{n} = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{46}{48}\right) + \tan^{-1}\frac{1}{n} = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{23}{24}\right) + \tan^{-1}\frac{1}{n} = \frac{\pi}{4}$$

$$\tan^{-1}\frac{1}{n} = \tan^{-1} 1 - \tan^{-1}\frac{23}{24}$$

$$\tan^{-1}\frac{1}{n} = \tan^{-1}\left(\frac{1 - \frac{23}{24}}{1 + \frac{23}{24}}\right)$$

$$\tan^{-1}\frac{1}{n} = \tan^{-1}\left(\frac{\frac{1}{24}}{\frac{47}{24}}\right)$$

$$\tan^{-1}\frac{1}{n} = \tan^{-1}\frac{1}{47}$$

$$n = 47$$

89. (13)
90. (46)

$\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} = (1,8,13)$

$$\vec{a} \times (\vec{a} \times \vec{b}) + \vec{a} \times (\vec{a} \times \vec{c}) + \vec{a} \times (\vec{b} \times \vec{c})$$
$$= \vec{a} \times (\hat{\imath} + 8\hat{\jmath} + 13\hat{k})$$

$(\vec{a} \cdot \vec{b})\vec{a} - a^2\vec{b} + (\vec{a} \cdot \vec{c})\vec{a} - a^2\vec{c} + (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{a} \times (\hat{\imath} + 8\hat{\jmath} + 13\hat{k})$

$\Rightarrow -26\vec{a} - 29\vec{b} + 13\vec{a} - 29\vec{c} + 13\vec{b} + 26\vec{c} = \vec{a} \times (\hat{\imath} + 8\hat{\jmath} + 13\hat{k})$

$-13\vec{a} - 16\vec{b} - 3\vec{c} = \vec{a} \times (\hat{\imath} + 8\hat{\jmath} + 13\hat{k})$

$-13\vec{a} \cdot \vec{b} - 16\, b^2 - 3\vec{b} \cdot \vec{c} = \{\vec{a} \times (\hat{\imath} + 8\hat{\jmath} + 13\hat{k})\} \cdot \vec{b}$

$$(-13)(-26) - 16(50) - 3\vec{b} \cdot \vec{c} = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 8 & 13 \\ 3 & 4 & -5 \end{vmatrix}$$

$-462 - 3\vec{b} \cdot \vec{c} = -396$

$\vec{b} \cdot \vec{c} = -22$

Hence $24 - \vec{b} \cdot \vec{c} = 46$