

1. (d) For longest wavelength in Paschen's series:

$$\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

For longest $n_1 = 3$

$$n_2 = 4$$

$$\frac{1}{\lambda} = R \left[\frac{1}{(3)^2} - \frac{1}{(4)^2} \right]$$

$$\frac{1}{\lambda} = R \left[\frac{1}{9} - \frac{1}{16} \right]$$

$$\frac{1}{\lambda} = R \left[\frac{16 - 9}{16 \times 9} \right]$$

$$\Rightarrow \lambda = \frac{16 \times 9}{7R} = \frac{16 \times 9}{7 \times 1.097 \times 10^7}$$

$$\lambda = 1.876 \times 10^{-6} \text{ m}$$

2. (d)

1st law of thermodynamics

$$\Delta Q = \Delta U + W$$

$$\Rightarrow +48 = nC_v \Delta T + W$$

$$\Rightarrow 48 = (1) \left(\frac{3R}{2} \right) (2) + W$$

$$\Rightarrow W = 48 - 3 \times R$$

$$\Rightarrow W = 48 - 3 \times (8.3)$$

$$\Rightarrow W = 23.1 \text{ Joule}$$

3. (a)

$$1\text{MSD} = \frac{1 \text{ cm}}{20} = 0.05 \text{ cm}$$

$$1\text{VSD} = \frac{49}{50} \text{MSD} = \frac{49}{50} \times 0.05 \text{ cm} = 0.049 \text{ cm}$$

$$\text{LC} = 1\text{MSD} - 1\text{VSD} = 0.001 \text{ cm}$$

$$\text{For mark on paper, } L_1 = 8.45 \text{ cm} + 26 \times 0.001 \text{ cm} = 84.76 \text{ mm}$$

$$\text{For mark on paper through slab, } L_2 = 7.12 \text{ cm} + 41 \times 0.001 \text{ cm} = 71.61 \text{ mm}$$

$$\text{For powder particle on top surface, } ZE = 4.05 \text{ cm}$$

$$+1 \times 0.001 \text{ cm} = 40.51 \text{ mm}$$

$$\therefore \text{actual } L_1 = 84.76 - 40.51 = 44.25 \text{ mm}$$

$$\text{actual } L_2 = 71.61 - 40.51 = 31.10 \text{ mm}$$

$$L_2 = \frac{L_1}{\mu}$$

$$\Rightarrow \mu = \frac{L_1}{L_2} = \frac{44.25}{31.10} = 1.42$$

4. (a)

$$\text{Intensity} = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$= \frac{1}{2} \times 9 \times 10^{-12} \times (600)^2 \times 3 \times 10^8$$

$$= \frac{9}{2} \times 36 \times 3 = 486 \text{ W/m}^2$$

5. (d)

$$\text{At surface: } mg = 300 \text{ N}$$

$$m = \frac{300}{g_s}$$

$$\text{At Depth } \frac{R}{4}: g_d = g_s \left[1 - \frac{d}{R} \right]$$

$$g_d = g_s \left[1 - \frac{R}{4R} \right]$$

$$g_d = \frac{3g_s}{4}$$

weight at depth

$$= m \times g_d$$

$$= m \times \frac{3g_s}{4}$$

$$= \frac{3}{4} \times 300$$

$$= 225 \text{ N}$$

6. (c) Energy = $\frac{hc}{\lambda}$;

$$E = \frac{1240}{\lambda(\text{nm})} \text{ eV}$$

$$6 = \frac{1240}{\lambda(\text{nm})}$$

$$\lambda = \frac{1240}{6} = 207 \text{ nm}$$

7. (b) $m = 800 \text{ kg}$

$$r = 300 \text{ m}$$

$$\theta = 30^\circ$$

$$\mu_s = 0.2$$

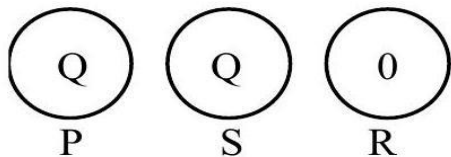
$$V_{\max} = \sqrt{Rg \left[\frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right]}$$

$$= \sqrt{300 \times g \times \left[\frac{\tan 30^\circ + 0.2}{1 - 0.2 \times \tan 30^\circ} \right]}$$

$$= \sqrt{300 \times 10 \times \left[\frac{0.57 + 0.2}{1 - 0.2 \times 0.57} \right]}$$

$$V_{\max} = 51.4 \text{ m/s}$$

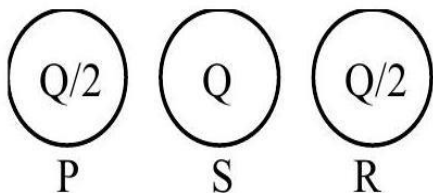
8. (b)



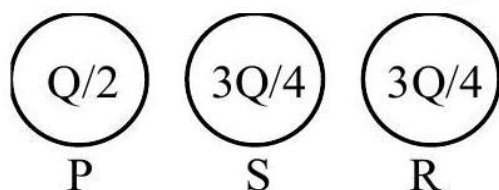
$$F_{PS} \propto Q^2$$

$$F_{PS} = 16 \text{ N}$$

Now If P & R are brought in contact then



Now If S & R are brought in contact then



New force between P & S is :

$$F_{PS} \propto \frac{Q}{2} \times \frac{3Q}{4}$$

$$F_{PS} \propto \frac{3Q^2}{8} = \frac{3}{8} \times 16 = 6 \text{ N}$$

9. (a)

$$(Emf)_{\text{induced}} = -L \frac{di}{dt}$$

In magnitude form,

$$|Emf_{\text{ind}}| = \left| (-)L \frac{di}{dt} \right|$$

$$\Rightarrow 0.1 = \frac{(L)[+2 - (-2)]}{0.2}$$

$$\Rightarrow L = \frac{0.1 \times 0.2}{4} = 5\text{mH}$$

$$10. (c) \frac{1}{f} = \left(\frac{\mu_{\text{lens}}}{\mu_{\text{dir}}} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{1}{+20} = \left(\frac{\mu}{1} - 1 \right) \left(\frac{1}{+15} - \frac{1}{(-30)} \right)$$

$$\Rightarrow \frac{1}{20} = (\mu - 1) \left(\frac{3}{30} \right)$$

$$\Rightarrow \mu - 1 = \frac{1}{2}$$

$$\Rightarrow \mu = 1 + \frac{1}{2} = \frac{3}{2} = 1.5$$

$$11. (d) \text{Degree of freedom (f)} = 5 + 2(3N - 5)$$

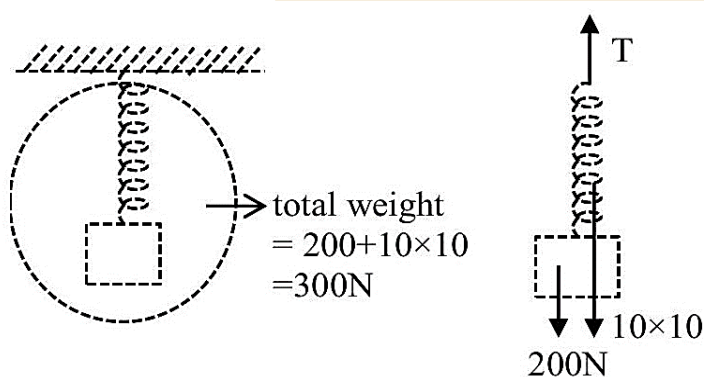
$$f = 5 + 2(3 \times 2 - 1) = 7$$

$$\text{energy of one molecule} = \frac{f}{2} K_B T$$

energy of 10 molecules

$$= 10 \left(\frac{f}{2} K_B T \right) = 10 \left(\frac{7}{2} K_B T \right) = 35 K_B T$$

12. (b)



Chain block system is in equilibrium so $T = 200 + 100 = 300 \text{ N}$.

$$13. (c) \frac{hc}{\lambda} - \phi = e \cdot V_s$$

$$\Rightarrow \frac{1240}{300} \text{eV} - 2.13 \text{eV} = \text{eVs}$$

$$\Rightarrow 4.13 \text{eV} - 2.13 \text{eV} = \text{eVs.}$$

$$\Rightarrow \text{So, } V_s = 2 \text{volt}$$

14. (a)

$$\text{Power (P)} = V.I$$

$$\Rightarrow 110 = (220)(I)$$

$$\Rightarrow I = 0.5 \text{ A}$$

$$\text{Now, } I = \frac{n \cdot e}{t}$$

$$\Rightarrow 0.5 = \left(\frac{n}{t}\right) (1.6 \times 10^{-19})$$

$$\Rightarrow \frac{n}{t} = \frac{0.5}{1.6 \times 10^{-19}}$$

$$\Rightarrow \frac{n}{t} = 31.25 \times 10^{17}$$

15. (a) Kinetic energy (K) = $\frac{p^2}{2m}$

$$\Rightarrow P = \sqrt{2mK}$$

$$\text{If } K_f = 36 K_i$$

$$\text{So, } P_f = 6P_i$$

$$\% \text{ increase in momentum} = \frac{P_f - P_i}{P_i} \times 100\%$$

$$= \frac{6P_i - P_i}{P_i} \times 100\%$$

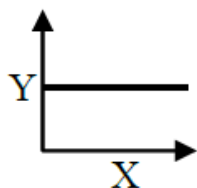
$$= 500\%$$

16. (a) There are two liquid-air surfaces in bubble so

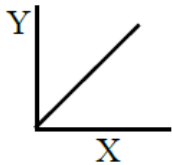
$$\Delta P = 2 \left(\frac{2S}{R} \right) = \frac{4S}{R}$$

17. (a)

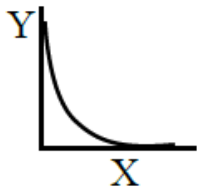
(A) Graph between Magnetic susceptibility and magnetising field is :



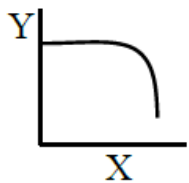
(B) magnetic field due to a current carrying wire for $x < a$: $B = \frac{\mu_0 i r}{2\pi a^2}$



(C) magnetic field due to a current carrying wire for $x > a$: $B = \frac{\mu_0 i}{2\pi a}$



(D) magnetic field inside solenoid varies as:



18. (c)

19. (c) $\Delta Q = m\Delta T$

$$s = \frac{\Delta Q}{m\Delta T}$$

$$[s] = \left[\frac{ML^2 T^{-2}}{MK} \right]$$

$$[s] = [L^2 T^{-2} K^{-1}]$$

Statement-(I) is correct

$$PV = nRT \Rightarrow R = \frac{PV}{nT}$$

$$[R] = \frac{[ML^{-1} T^{-2}][L^3]}{[mol][K]}$$

$$[R] = [ML^2 T^{-2} mol^{-1} K^{-1}]$$

Statement-II is incorrect

20. (a)

$$t_1 = \frac{u + \sqrt{u^2 + 2gh}}{g}$$

$$t_2 = \frac{-u + \sqrt{u^2 + 2gh}}{g}$$

$$t = \frac{\sqrt{2gh}}{g}$$

$$t_1 t_2 = \frac{(u^2 + 2gh) - u^2}{g^2} = \frac{2gh}{g^2} = t^2$$

$$\Rightarrow t = \sqrt{t_1 t_2}$$

21. (122)

$$10.2 \text{ eV} = \frac{hc}{\lambda}$$

$$\lambda = \frac{1245 \text{ eV} \cdot \text{nm}}{10.2 \text{ eV}} = 122.06 \text{ nm}$$

22. (10)

for maximum current, circuit must be in resonance.

$$f_0 = \frac{1}{2\pi\sqrt{L \times C}}$$

$$f_0 = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 2.5 \times 10^{-9}}}$$

$$= \frac{1}{2\pi\sqrt{25 \times 10^{-11}}}$$

$$= \frac{1}{2\pi \times 5} \times 10^5 \times \sqrt{10} \text{ Hz}$$

$$= \frac{100}{10} \times 10^3 \text{ Hz}$$

$$f_0 = 10 \times 10^3 \text{ Hz}$$

23. (3) $x^2 = 1 + t^2$

$$2x \frac{dx}{dt} = 2t$$

$$xV = t$$

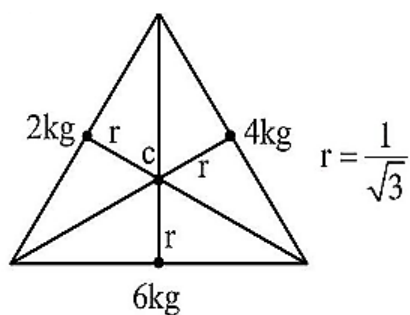
$$x \frac{dv}{dt} + v \frac{dx}{dt} = 1$$

$$x \cdot a + v^2 = 1$$

$$a = \frac{1 - v^2}{x} = \frac{1 - t^2/x^2}{x}$$

$$a = \frac{1}{x^3} = x^{-3}$$

24. (4)



Moment of inertia about C and perpendicular to the plane is:

$$I = r^2[2 + 4 + 6]$$

$$= \frac{1}{3} \times 12$$

$$I = 4 \text{ kg} - \text{m}^2$$

25. (100)

$$W = \Delta U = U_f - U_i$$

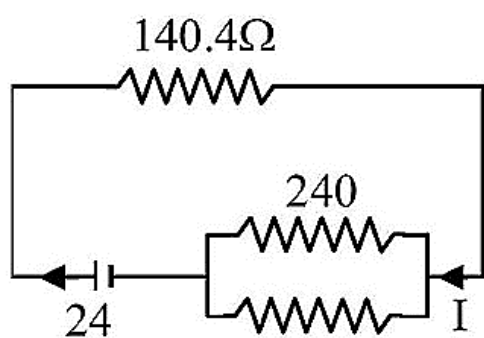
$$W = (-\vec{\mu} \cdot \vec{B})_f - (-\vec{\mu} \cdot \vec{B})_i$$

$$= 0 + (\vec{\mu} \cdot \vec{B})_i$$

$$= (100 \times 5 \times 10^{-3} \times 1 \times 10^{-3}) \times 0.2 \text{ J}$$

$$= 1 \times 10^{-4} \text{ J} = 100 \mu\text{J}$$

26. (160)



Req

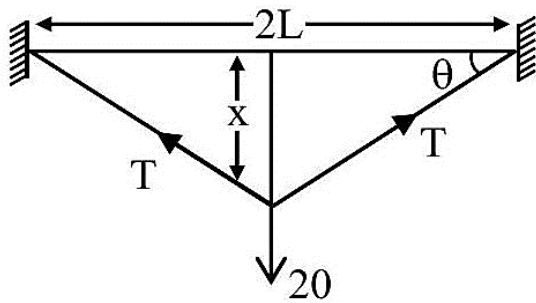
$$= 140.4 + \frac{240 \times 10}{240 + 10}$$

$$\text{Req.} = 140.4 + \frac{2400}{250}$$

$$\text{Req.} = 150 \Omega$$

$$\begin{aligned} \therefore \text{Current in ammeter} &= \frac{24}{150} \\ &= 160 \text{ mA} \end{aligned}$$

27. (1)



In vertical direction

$$2 T \sin \theta = 20 \text{ using small angle approximation } \sin \theta = \theta$$

$$\theta = \frac{1}{100}$$

$$\therefore T = \frac{10}{\theta}$$

$$T = 1000 \text{ N}$$

$$\text{Change in length } \Delta L = 2\sqrt{x^2 + L^2} - 2L$$

$$= 2L \left[1 + \frac{x^2}{2L^2} - 1 \right]$$

$$\Delta L = \frac{x^2}{L}$$

$$\therefore \text{Modulus of elasticity} = \frac{\text{stress}}{\text{strain}}$$

$$2 \times 10^{11} = \frac{10^3}{A \times \frac{x^2}{L}} \times 2L$$

$$\therefore A = 1 \times 10^{-4} \text{ m}^2$$

28. (8)

$$I_{\max} = (\sqrt{I} + \sqrt{4I})^2 = 9I$$

$$I_{\min} = (\sqrt{4I} - \sqrt{I})^2 = I$$

$$\therefore I_{\max} - I_{\min} = 8I$$

29. (740)

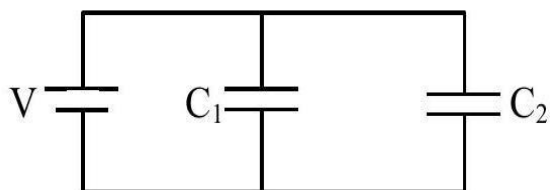
$$\text{The difference in frequency in open organ pipe} = f = \frac{nv}{2L}$$

$$\Delta f = \frac{6v}{2 \times 0.6} - \frac{5v}{2 \times 0.9}$$

$$v = 333 \text{ m/s}$$

$$\Delta f = 740 \text{ Hz}$$

30. (80)



$$C_{eq} = C_1 + C_2$$

$$C_1 = \frac{2\epsilon_0 A}{2 \times d} = 10\mu F$$

$$C_2 = \frac{3\epsilon_0 A}{2d} = 15\mu F$$

$$C_{eq} = 25\mu F$$

Now the charge on

$$C_1 = 10 V\mu C$$

$$C_2 = 1.5 V\mu C$$

Now force between the plates $\left[F = \frac{Q^2}{2A\epsilon_0}\right]$

$$\frac{100 V^2 \times 10^{-12}}{2 \times 2 \times 10^{-4} \epsilon_0} + \frac{225 V^2 \times 10^{-12}}{2 \times 2 \times 10^{-4} \times \epsilon_0} = 8$$

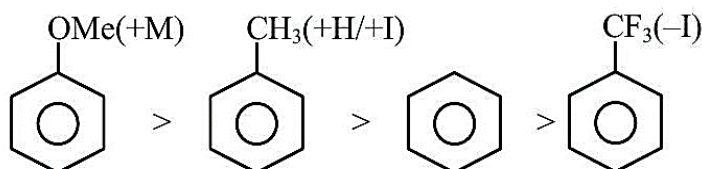
$$325 V^2 = 8 \times 4 \times 10^{-4} \times 8.85$$

$$V^2 = \frac{32 \times 8.85 \times 10^{-4}}{325}$$

$$\therefore V = \sqrt{\frac{283.2 \times 10^{-4}}{325}}$$

$$V = 0.93 \times 10^{-2}$$

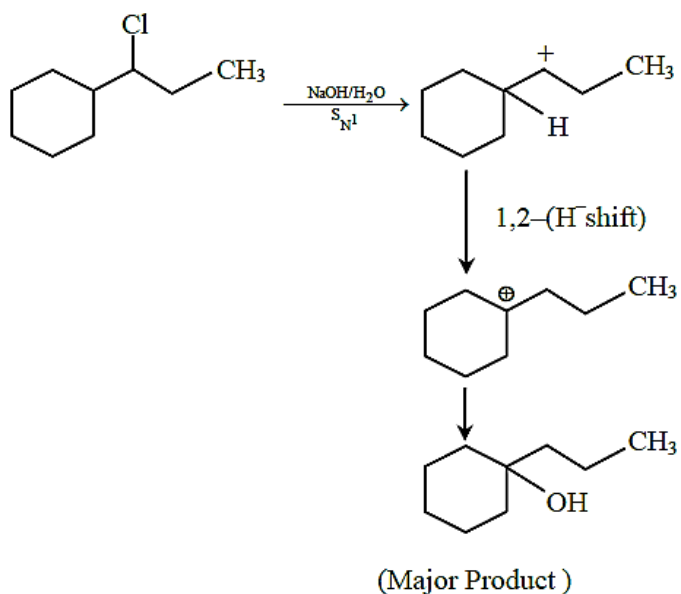
31. (b)



$$32. (b) \text{ 3 moles are present in 1 litre solution molality} = \frac{3 \times 1000}{1.25 \times 1000 - [3 \times 58.5]} = 2.79 \text{ m}$$

33. (c)

34. (b)



35. (a)

$\text{Ag}^+ + \text{I}^- \rightarrow \text{AgI}$ Yellow ppt.

$\text{Ag}^+ + \text{Cl}^- \rightarrow \text{AgCl}$ White ppt

$\text{Ag}^+ + \text{Br}^- \rightarrow \text{AgBr}$ Pale yellow ppt

36. (a) Applied external potential should be greater than E_{cell}^0 in opposite direction.

37. (d)

Unpaired electron

Sc[Ar]4 s²3 d¹ 1

Cr[Ar]4 s¹3 d⁵ 6

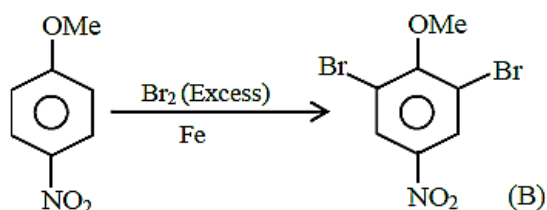
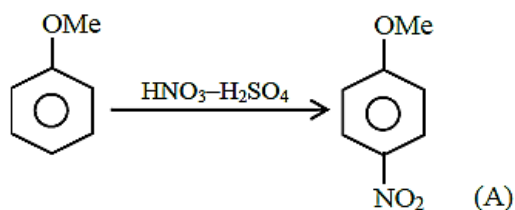
V[Ar]4 s²3 d³ 3

Ti: [Ar]4 s²3 d² 2

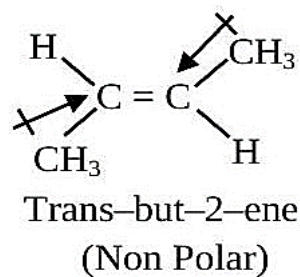
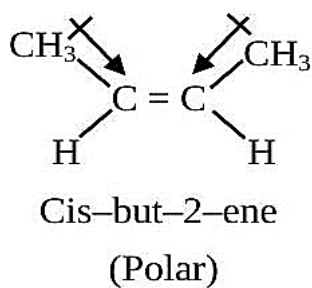
Mn: [Ar]4 s²3 d⁵ 5

38. (b)

39. (b)



40. (b)



Cis-but-2-ene has higher Dipole moment than trans-but-2-ene.

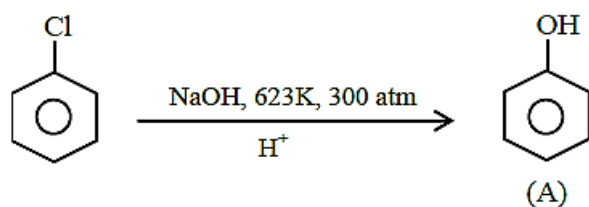
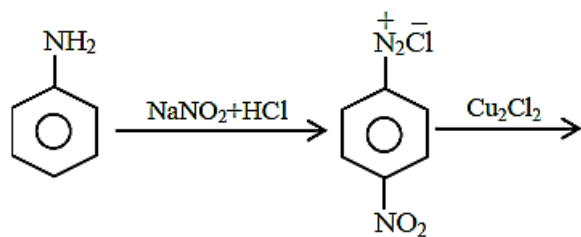
41. (c)

Hybridisation		Hybridisation	
PF ₅	sp ³ d	SF ₆	sp ³ d ²
BrF ₅	sp ³ d ²	[Co(NH ₃) ₆] ³⁺	d ² sp ³

Both Statement (1) and (2) are false.

42. (d) Due to inert pair effect; Tl⁺ and Pb²⁺ can behave as oxidising agents.

43. (b)



44. (c) Non polar compounds are having higher value of R_f than polar compound.

45. (a)

(A) Down the group; radius increases

(B) EN does not decrease gradually from C to Pb.

(C) Correct.

(D) Correct.

(E) Range of oxidation state of carbon; -4 to +4

46. (d)

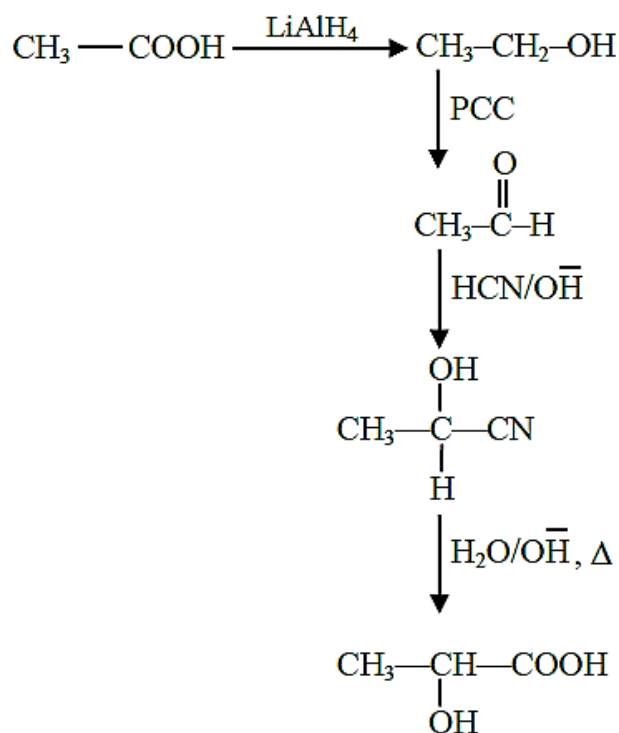
A $\rightarrow$ (IV)

B $\rightarrow$ (I)

C $\rightarrow$ (II)

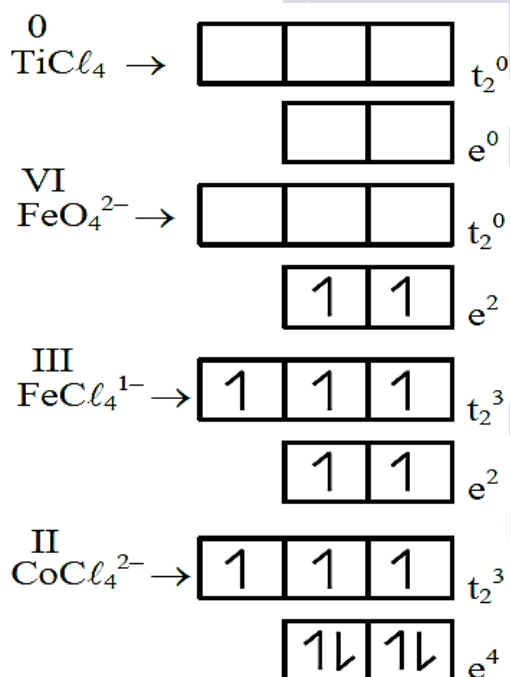
D $\rightarrow$ (III)

47. (d)

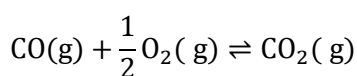


48. (c) Dibromo bis (trimethyl phosphine) platinum (II)

49. (d)



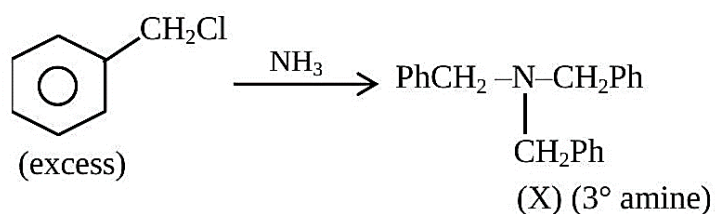
50. (d)



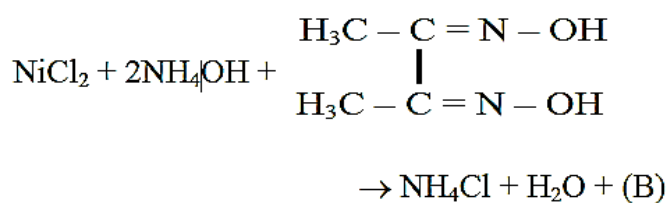
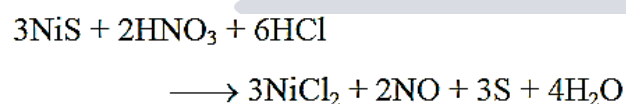
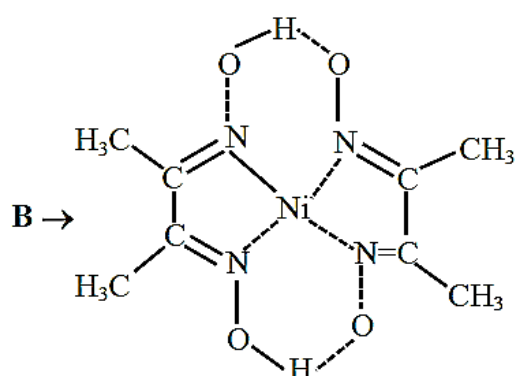
$$\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

$$\frac{K_P}{K_C} = (RT)^{\Delta n_g} = \frac{1}{\sqrt{RT}}$$

51. (287)


Molar Mass of (X) is 287 g mol^{-1}

52. (12)



53. (543)

$$\Delta T_f = 273.15 - 270.65 = 2.5 \text{ K}$$

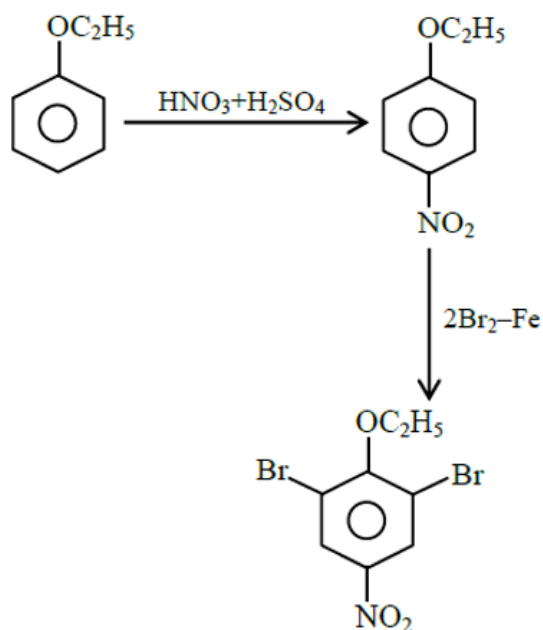
$$\Delta T_f = K_f m \Rightarrow 2.5 = 1.86 \times \frac{n}{0.1}$$

$$\Rightarrow n = 0.1344 \text{ moles}$$

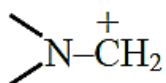
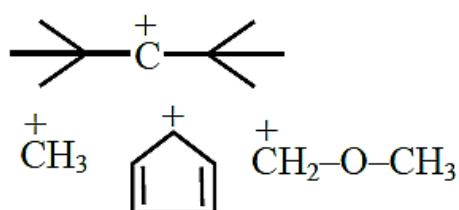
$$\Rightarrow w = 0.1344 \times 32 = 4.3 \text{ g}$$

$$\text{Volume} = \frac{4.3}{0.792} = 5.43 \text{ ml} = 543 \times 10^{-2} \text{ ml}$$

54. (15)



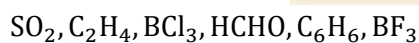
55. (5)



56. (2000)

$$\Delta G = 0$$

$$T = \frac{\Delta H}{\Delta S} = \frac{400}{0.2} = 2000 \text{ K}$$

57. (6) Central atom utilising sp^2 hybrid orbitals


58. (17)

$$\frac{(t_{1/2})_I}{(t_{1/2})_{II}} = \frac{K_2}{K_1} = \frac{5}{2}$$

$$\therefore K_1 t_1 = \ln \frac{1}{1 - \frac{2}{3}} = \ln 3$$

$$K_2 t_2 = \ln \frac{1}{1 - \frac{4}{5}} = \ln 5$$

$$\Rightarrow \frac{K_1}{K_2} \times \frac{t_1}{t_2} = \frac{0.477}{0.699}$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{0.477}{0.699} \times \frac{5}{2} = 1.7 = 17 \times 10^{-1}$$

59. (34)

60. (0)

For 3d transition series;

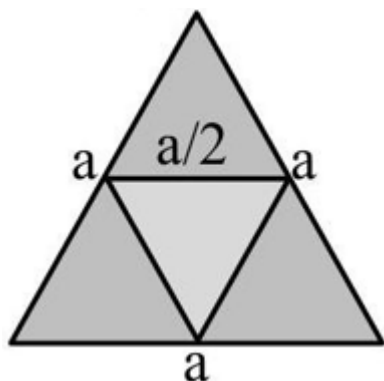
Oxidising power: $V^{+5} < Cr^{+6} < Mn^{+7}$

V^{+5} : $[Ar]4s^0 3d^0$

Number of unpaired electron = 0

$$\mu = 0$$

61. (a)



$$\text{Area of first } \Delta = \frac{\sqrt{3}a^2}{4}$$

$$\text{Area of second } \Delta = \frac{\sqrt{3}a^2}{4} \cdot \frac{a^2}{4} = \frac{\sqrt{3}a^2}{16}$$

$$\text{Area of third } \Delta = \frac{\sqrt{3}a^2}{64}$$

$$\text{sum of area} = \frac{\sqrt{3}a^2}{4} \left(1 + \frac{1}{4} + \frac{1}{16} \dots \right)$$

$$Q = \frac{\sqrt{3}a^2}{4} \cdot \frac{1}{\frac{3}{4}} = \frac{a^2}{\sqrt{3}}$$

$$\text{perimeter of 1}^{\text{st}} \Delta = 3a$$

$$\text{perimeter of 2}^{\text{nd}} \Delta = \frac{3a}{2}$$

$$\text{perimeter of 3}^{\text{rd}} \Delta = \frac{3a}{4}$$

$$P = 3a \left(1 + \frac{1}{2} + \frac{1}{4} + \dots \right)$$

$$P = 3a. 2 = 6a$$

$$a = \frac{P}{6}$$

$$Q = \frac{1}{\sqrt{3}} \cdot \frac{P^2}{36}$$

$$P^2 = 36\sqrt{3}Q$$

62. (c) Given: $4x \leq 5y$

then

$$R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4)$$

$$(2,5), (3,3), (3,4), (3,5), (4,4), (4,5), (5,4), (5,5)\}$$

i.e. 16 elements.

i.e. $m = 16$

Now to make R a symmetric relation add

$$\{(2,1)(3,2)(4,3)(3,1)(4,2)(5,3)(4,1)(5,2)(5,1)\}$$

i.e. $n = 9$

$$\text{So } m + n = 25$$

63. (a) Total method = 5^3

$$\text{favorable} = {}^5C_2(2^3 - 2) = 60$$

$$\text{probability} = \frac{60}{125} = \frac{12}{25}$$

64. (c)

65. (c)

let $P(x, y)$

$$\frac{(x-2)^2 + (y-1)^2}{(x-1)^2 + (y-3)^2} = \frac{25}{16}$$

$$9x^2 + 9y^2 + 14x - 118y + 170 = 0$$

$$\begin{aligned} a^2 + 2b + 3c + 4d + e \\ = 81 + 18 + 0 + 56 - 118 \\ = 155 - 118 \\ = 37 \end{aligned}$$

66. (b)

$$\lim_{n \rightarrow \infty} \frac{\sum_{r=1}^{n-1} (r^2 - r)(n - r)}{\sum_{r=1}^n r^3 - \sum_{r=1}^n r^2}$$

$$\lim_{n \rightarrow \infty} \frac{\sum_{r=1}^{n-1} (-r^3 + r^2(n+1) - nr)}{\left(\frac{n(n+1)}{2}\right)^2 - \frac{n(n+1)(2n+1)}{6}}$$

$$\lim_{n \rightarrow \infty} \frac{\left(\frac{((n-1)n)}{2}\right)^2 + \frac{(n+1)(n-1)n(2n-1)}{6} - \frac{n^2(n-1)}{2}}{\frac{n(n+1)}{2} \left(\frac{n(n+1)}{2} - \frac{2n+1}{3}\right)}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{n(n-1)}{2} \left(\frac{-n(n-1)}{2} + \frac{(n+1)(2n-1)}{3} - n\right)}{\frac{n(n+1)}{2} \frac{3n^2 + 3n - 4n - 2}{6}}$$

$$\lim_{n \rightarrow \infty} \frac{(n-1)(-3n^2 + 3n + 2(2n^2 + n - 1) - 6)}{(n+1)(3n^2 - n - 2)}$$

$$\lim_{n \rightarrow \infty} \frac{(n-1)(n^2 + 5n - 8)}{(n+1)(3n^2 - n - 2)} = \frac{1}{3}$$

67. (c)

$$\frac{{}^{n+1}C_r}{{}^nC_r} = \frac{55}{35}$$

$$\frac{(n+1)!}{(r+1)!(n-r)!} \cdot \frac{r!(n-r)!}{n!} = \frac{11}{7}$$

$$\frac{(n+1)}{r+1} = \frac{11}{7}$$

$$7n = 4 + 11r$$

$$\frac{{}^nC_r}{{}^{n-1}C_{r-1}} = \frac{35}{21}$$

$$\frac{n!}{r!(n-r)!} = \frac{(r-1)!(n-r)!}{(n-1)!} = \frac{5}{3}$$

$$\frac{n}{r} = \frac{5}{3}$$

$$3n = 5r$$

By solving $r = 6$

$$n = 10$$

$$2n + 5r = 50$$

68. (b)

$$17m = m + (m-4) + (m-4 \times 2) + \dots + (m-4 \times 24)$$

$$17m = 25m - 4(1 + 2 \dots 24)$$

$$8m = \frac{4 \cdot 24 \cdot 25}{2} = 150$$

69. (a)

$$\frac{z_1 - 2z_2}{\frac{1}{2} - z_1 \bar{z}_2} \times \frac{\bar{z}_1 - 2\bar{z}_2}{\frac{1}{2} - \bar{z}_1 z_2} = 4$$

$$|z_1|^2 2z_1 \bar{z}_2 - 2\bar{z}_1 z_2 + 4|z_2|^2$$

$$= 4 \left(\frac{1}{4} - \frac{\bar{z}_1 z_2}{2} - \frac{z_1 \bar{z}_2}{2} + |z_1|^2 |z_2|^2 \right)$$

$$z_1 \bar{z}_1 + 2z_2 \cdot 2\bar{z}_2 - z_1 \bar{z}_1 2z_2 \cdot 2\bar{z}_2 - 1 = 0$$

$$(z, \bar{z}_1 - 1)(1 - 2z_2 \cdot 2\bar{z}_2) = 0$$

$$(|z_1|^2 - 1)(|2z_2|^2 - 1) = 0$$

70. (c)

$$\text{Let } y = \left(\frac{1}{x}\right)^{2x}$$

$$\ell ny = 2x \ell n \left(\frac{1}{x}\right)$$

$$\ell ny = -2x \ell nx$$

$$\frac{1}{y} \frac{dy}{dx} = -2(1 + \ell nx)$$

$$\text{for } x > \frac{1}{e} \text{ } f^n \text{ is decreasing}$$

$$\text{so, } e < \pi$$

$$\left(\frac{1}{e}\right)^{2e} > \left(\frac{1}{\pi}\right)^{2\pi}$$

$$e^\pi > \pi^e$$

71. (d)

$$|(\vec{a} \times \vec{b} \times \vec{c})| = |\vec{a} \times \vec{b}| |\vec{c}| \frac{\sqrt{3}}{2}$$

$$|\vec{c} - \vec{a}| = 2\sqrt{2}$$

$$|c|^2 + |a|^2 - 2\vec{c} \cdot \vec{a} = 8$$

$$|z|^2 + 38 - 12|z| = 8$$

$$|z|^2 - 12|z| + 30 = 0$$

$$|z| = \frac{12 \pm \sqrt{144 - 120}}{2}$$

$$= \frac{12 \pm 2\sqrt{6}}{2}$$

$$|z| = 6 + \sqrt{6}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\hat{i} - \hat{j} + 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{27}$$

$$|(\vec{a} \times \vec{b}) \times \vec{z}| = \sqrt{27}(6 + \sqrt{6}) \frac{\sqrt{3}}{2}$$

$$\frac{9}{2}(6 + \sqrt{6})$$

72. (c)

NAGPUR

$$A \rightarrow 5! = 120$$

$$GB5! = 120 \quad 240$$

$$NA@4! = 24 \quad 264$$

$$NG@4! = 24 \quad 288$$

$$NPB4! = 24 \quad 312$$

$$NRAGPU = 1 \quad 313$$

$$NRAGUP \quad 314$$

$$NRAPGU \quad 315$$

73. (d)

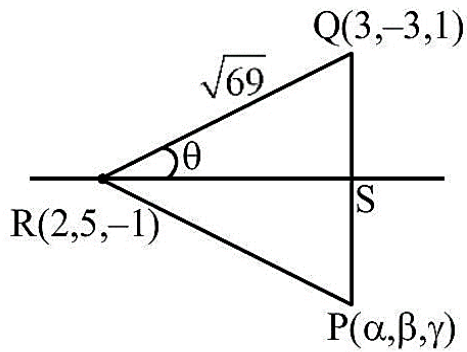
$$g(x) = h(e^x) \cdot e^{h(x)}$$

$$g'(x) = h(e^x) \cdot e^{h(x)} \cdot h'(x) + e^{h(x)} h'(e^x) \cdot e^x$$

$$g'(0) = h(a)e^{h(0)}h'(0) + e^{h^x(0)}h'(a)$$

$$= 2 + 2 = 4$$

74. (d)



$$RQ = \sqrt{1 + 64 + 4} = \sqrt{69}$$

$$\overrightarrow{RQ} = \hat{i} - 8\hat{j} + 2\hat{k}$$

$$\overrightarrow{RS} = \hat{i} + \hat{j} - \hat{k}$$

$$\cos \theta = \frac{\overrightarrow{RQ} \cdot \overrightarrow{RS}}{|\overrightarrow{RQ}| |\overrightarrow{RS}|} = \frac{|1 - 8 - 2|}{\sqrt{69} \sqrt{3}} = \frac{9}{3\sqrt{23}}$$

$$\cos \theta = \frac{3}{\sqrt{23}} = \frac{RS}{RQ} = \frac{RS}{\sqrt{69}}$$

$$RS = 3\sqrt{3}$$

$$\sin \theta = \frac{\sqrt{14}}{\sqrt{23}} = \frac{QS}{\sqrt{69}}$$

$$QS = \sqrt{42}$$

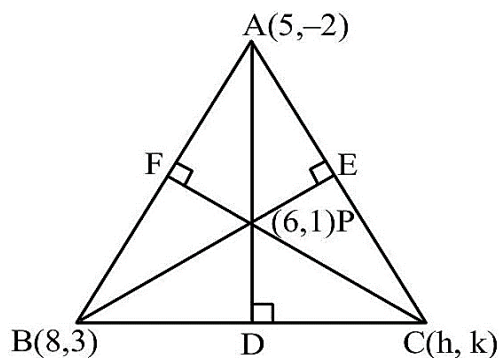
$$\text{area} = \frac{1}{2} \cdot 2QS \cdot RS = \sqrt{42} \cdot 3\sqrt{3}$$

$$\lambda = 9\sqrt{14}$$

$$\lambda^2 = 81 \cdot 14 = 14k$$

$$k = 81$$

75. (a)



$$\text{Slope of AD} = 3$$

$$\text{Slope of BC} = -\frac{1}{3}$$

$$\text{equation of BC} = 3y + x - 17 = 0$$

slope of BE = 1

Slope of AC = -1

equation of AC is $x + y - 3 = 0$

point C is $(-4, 7)$

76. (d)

$$\sin 5x \in [-1, 1]$$

$$-\sin 5x \in [-1, 1]$$

$$7 - \sin 5x \in [6, 8]$$

$$\frac{1}{7 - \sin 5x} \in \left[\frac{1}{8}, \frac{1}{6}\right]$$

77. (b)

$$\vec{a} \times (\hat{i} + \hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \hat{i} - \hat{j} + \hat{k}$$

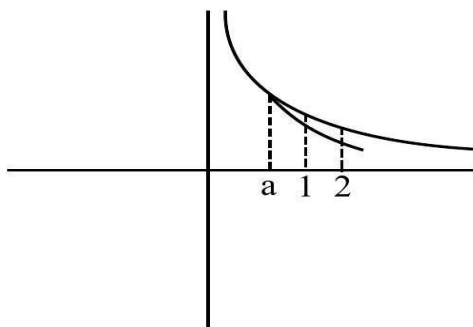
$$(\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i} = \hat{k} + \hat{j}$$

$$((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i} = \hat{j} - \hat{k}$$

$$\text{projection of } \vec{a} \text{ on } \hat{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= \frac{1 + 1}{\sqrt{2}} = \sqrt{2}$$

78. (c)



$$\text{area} \int_1^2 \left(\frac{1}{x} - \frac{a}{x^2} \right) dx$$

$$\left[\ln x + \frac{a}{x} \right]_1^2$$

$$\ln 2 + \frac{a}{2} - a = \log_e 2 - \frac{1}{2}$$

$$\frac{-a}{2} = -\frac{1}{7}$$

$$a = \frac{2}{7}$$

$$7a = 2$$

$$7a - 3 = -1$$

79. (a)

$$\int \frac{\sec^2 x dx}{a^2 \tan^2 x + b^2}$$

$$\text{let } \tan x = t$$

$$\sec^2 x dx = dt$$

$$\int \frac{dt}{a^2 t^2 + b^2}$$

$$\frac{1}{a^2} \int \frac{dt}{t^2 + \left(\frac{b}{a}\right)^2}$$

$$\frac{1}{a^2} \frac{1}{b} \tan^{-1} \left(\frac{t}{b/a} \right) + c$$

$$\frac{1}{ab} \tan^{-1} \left(\frac{a}{b} \tan x \right) + c$$

$$\text{on comparing } \frac{a}{b} = 3$$

$$ab = 12$$

$$a = 6, b = 2$$

maximum value of

$$6 \sin x + 2 \cos x \text{ is } \sqrt{40}$$

80. (c) $|A| = 3$

$$\left| \text{adj} \left(-4 \text{adj} \left(-3 \text{adj} \left(3 \text{adj} \left((2A)^{-1} \right) \right) \right) \right) \right|$$

$$|-4 \text{adj}(-3 \text{adj}(3 \text{adj}(2A)^{-1})|$$

$$4^6 |\text{adj}(-3 \text{adj}(3 \text{adj}(2A)^{-1})|^2$$

$$2^{12} \cdot 3^{12} |3 \text{adj}(2A)^{-1}|^8$$

$$2^{12} \cdot 3^{12} \cdot 3^{24} |\text{adj}(2A)^{-1}|^8$$

$$2^{12} \cdot 3^{36} |(2A)^{-1}|^{16}$$

$$2^{12} \cdot 3^{36} \frac{1}{|2A|^{16}}$$

$$2^{12} \cdot 3^{36} \frac{1}{2^{48} |A|^{16}}$$

$$2^{12} \cdot 3^{36} \frac{1}{2^{48} \cdot 3^{16}}$$

$$\frac{3^{20}}{2^{36}} = 2^{-36} \cdot 3^{20}$$

$$m = -36 \quad n = 20$$

$$m + 2n = 4$$

81. (17)

$\left[\frac{x}{2} + 3\right]$ is discontinuous at $x = 2, 4, 6, 8$

$\sqrt{x}$ is discontinuous at $x = 1, 4$

$F(x)$ is discontinuous at $x = 1, 2, 6, 8$

$$\sum a = 1 + 2 + 6 + 8 = 17$$

82. (61)

$$\frac{2b^2}{a} = 9 \text{ and } \pm \frac{a}{e} = \pm \frac{4}{\sqrt{3}}$$

Equation of tangent $y = \sqrt{3}x - \sqrt{3}$

$$m = \sqrt{3}$$

$$c = -\sqrt{3}$$

Condition of tangency $c = \pm \sqrt{a^2 m^2 - b^2}$

$$-\sqrt{3} = \sqrt{3a^2 - b^2}$$

$$3 = 3a^2 - b^2$$

$$6a^2 - 9a - 6 = 0$$

$$2a^2 - 3a - 2 = 0$$

$$2a^2 - 4a + a - 2 = 0$$

$$(a - 2)(2a + 1) = 0$$

$$a = 2 \text{ or } -\frac{1}{2}$$

when $a = 2$, $b^2 = 9$ put $a = 2$ in

$$\frac{2}{e} = \frac{4}{\sqrt{3}} \Rightarrow e = \frac{\sqrt{3}}{2}$$

Which is impossible

83. (3660)

$$S(x) = (1+x) + 2(1+x)^2 + 3(1+x)^3 + \dots + 60(1+x)^{60}$$

$$(1+x)S = (1+x)^2 + \dots + 59(1+x)^{60} + 60(1+x)^{61}$$

$$-xS = \frac{(1+x)(1+x)^{60} - 1}{x} - 60(1+x)^{61}$$

Put $x = 60$

$$-60S = \frac{61((61)^{60} - 1)}{60} - 60(61)^{61}$$

on solving 3660

84. (23)

$$\int_0^3 [x^2] dx + \int_0^3 \left[\frac{x^2}{2} \right] dx$$

$$= \int_0^1 0 dx + \int_1^{12} 1 dx + \int_{\sqrt{2}}^{\sqrt{3}} 2 dx$$

$$+ \int_{\sqrt{3}}^2 3 dx + \int_2^{\sqrt{5}} 4 dx + \int_{\sqrt{5}}^{\sqrt{6}} 5 dx$$

$$+ \int_{\sqrt{6}}^{\sqrt{7}} 6 dx + \int_{\sqrt{7}}^{\sqrt{8}} 7 dx + \int_{\sqrt{8}}^3 8 dx$$

$$+ \int_0^{\sqrt{2}} 0 dx + \int_{\sqrt{2}}^2 1 dx$$

$$+ \int_2^{\sqrt{6}} 2 dx + \int_{\sqrt{6}}^{\sqrt{8}} 3 dx + \int_{\sqrt{8}}^3 4 dx = 31 - 6\sqrt{2} - \sqrt{3} - \sqrt{5}$$

$$- 2\sqrt{6} - \sqrt{7}$$

$$a = 31 \quad b = -6 \quad c = -2$$

$$a + b + c = 31 - 6 - 2 = 23$$

85. (71)

$$a = 1 - \frac{{}^3C_5}{{}^{12}C_5}$$

$$b = 3 \cdot \frac{{}^9C_4}{{}^{12}C_5}$$

$$c = 3 \cdot \frac{{}^9C_3}{{}^{12}C_5}$$

$$d = 1 \cdot \frac{{}^9C_2}{{}^{12}C_5}$$

$$u = 0.a + 1.b + 2.c + 3.d = 1.25$$

$$\sigma^2 = 0.a + 1.b + 4.c + 9d - u^2$$

$$\sigma^2 = \frac{105}{176}$$

$$\text{Ans. } 176 - 105 = 71$$

86. (39)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\frac{2}{3} = \frac{8^2 + c^2 - 7^2}{2 \times 8 \times c}$$

$$C = 9$$

$$\cos C = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8} = \frac{2}{7}$$

$$49 \cos 3C + 42$$

$$49(4 \cos^3 C - 3 \cos C) + 42$$

$$49 \left(4 \left(\frac{2}{7} \right)^3 - 3 \left(\frac{2}{7} \right) \right) + 42$$

$$= \frac{32}{7}$$

$$m + n = 32 + 7 = 39$$

87. (43)

$$\vec{a}_1 = \lambda \hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{a}_2 = -2\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\vec{p} = 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{q} = -3\hat{i} + 2\hat{j} + 4\hat{k}$$

$$(\lambda + 2)\hat{i} + 7\hat{j} - 3\hat{k} = \vec{a}_1 - \vec{a}_2$$

$$\vec{p} \times \vec{q} = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$\frac{44}{\sqrt{30}} = \frac{|-6\lambda - 12 - 105 - 9|}{\sqrt{(-6)^2 + (-15)^2 + 3^2}}$$

$$\frac{44}{\sqrt{30}} = \frac{|6\lambda + 126|}{3\sqrt{30}}$$

$$132 = |6\lambda + 126|$$

$$\lambda = 1, \lambda = -43$$

$$|\lambda| = 43$$

88. (4)

$$\frac{\alpha^{10} + \beta^{10} + \sqrt{2}(\alpha^9 + \beta^9)}{2(\alpha^8 + \beta^8)}$$

$$\frac{\alpha^8(\alpha^2 + \sqrt{2}\alpha) + \beta^8(\beta^2 + \sqrt{2}\beta)}{2(\alpha^8 + \beta^8)}$$

$$\frac{8\alpha^8 + 8\beta^8}{2(\alpha^8 + \beta^8)} = 4$$

89. (38)

$$D = D_1 = D_2 = D_3 = 0$$

$$D_3 = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 2 & 4 \\ 1 & \mu & -1 \end{vmatrix} = 0 \Rightarrow \mu = -39$$

$$D = \begin{vmatrix} 2 & 7 & \lambda \\ 3 & 2 & 5 \\ 1 & -39 & 32 \end{vmatrix} = 0 \Rightarrow \lambda = -1$$

$$\lambda - \mu = 38$$

90. (3)

$$(e^y + 1)\cos x + e^y \sin x dy = 0$$

$$\Rightarrow d((e^y + 1)\sin x) = 0$$

$$(e^y + 1)\sin x = C$$

$$\text{It passes through } \left(\frac{\pi}{2}, 0\right)$$

$$\Rightarrow C = 2$$

$$\text{Now, } x = \frac{\pi}{6}$$

$$\Rightarrow e^y = 3$$