1. **(b)** $m = 97.42 \pm 0.02$ g

$1 = 8.35 \pm 0.05$ mm

$d = 20.20 \pm 0.02$ mm

$$\rho = \frac{m}{\frac{\pi}{4} d^2 \ell}$$

$$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta d}{d} + \frac{\Delta\ell}{\ell}$$

$$= \frac{0.02}{97.42} + 2 \times \frac{0.02}{20.20} + \frac{0.05}{8.35} = 0.0082$$

% error $= 0.82\%$

2. **(d)** $[B] = [x] = L$

$$[U] = \left[\frac{A\sqrt{x}}{B+x}\right] \Rightarrow ML^2\,T^{-2} = \frac{[A]L^{1/2}}{L}$$

$$[A] = ML^{5/2}\,T^{-2}$$

3. **(d)** $w_g + w_{res} = \Delta k$

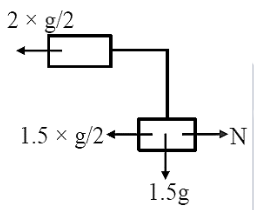
$$mgh + w_{res} = \frac{1}{2}mv^2$$

$$\therefore w_{res} = \frac{1}{2}mv^2 - mgh$$

$$= 10^{-3}\left(\frac{25}{2} - 10^4\right)$$

$$= -9.98\text{ J}$$

4. **(b)** w.r.t bigger block

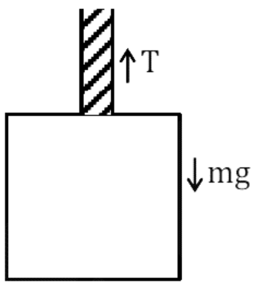


$$f = \mu N = \mu \times \frac{1.5\text{ g}}{2}$$

$$1.5\text{ g} = \text{g} + \mu + \frac{1.5\text{ g}}{2} \Rightarrow 0.5 = \mu \times \frac{1.5}{2}$$

$$\left(\mu = \frac{2}{3}\right)$$

5. **(a)**



$T - mg = ma$

$(\sigma\pi r^2) - mg = ma$

$\Rightarrow 4 \times 10^8\pi(4 \times 10^{-2})^2 - 16000 = 1600a$

$\Rightarrow a = 2.56\text{ m/s}^2$

6. **(d)** $\omega_i = 1200\text{rpm} = 1200 \times \frac{2\pi}{60} = 40\pi\text{rad/sec}$

$\rightarrow \omega_f = \omega_i + \alpha t \Rightarrow 0 = 40p + \alpha(10)$

$\alpha = -4\pi \text{rad/sec}^2$

$\rightarrow \text{I} = \frac{2}{5}\text{MR}^2 = \frac{2}{5}(5)(0.04)^2 = 2 \times 0.0016$

$\text{I} = 0.0032\text{Kg} - \text{m}^2$

$\rightarrow \tau = \text{I}\alpha = 0.0032 \times 4\pi$

$\tau = 0.0128\pi \text{ N} - \text{m}$

$\rightarrow \omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = (40\pi)^2 + 2(4\pi)\theta$

$\theta = \frac{1600\pi^2}{8\pi} = 200\pi$

No. of revolution $= \frac{\theta}{2\pi} = \frac{200\pi}{2\pi}$

No. of revolution $= 100$

7. **(b)** at highest point

$\text{mg} + \text{N} = \frac{\text{mv}^2}{\text{R}}$

$4\text{mg} = \frac{\text{mv}^2}{\text{R}}$

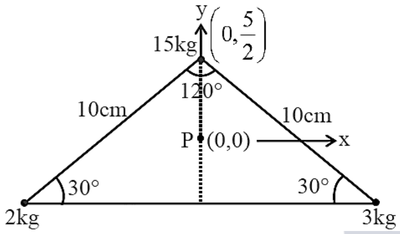
$\text{v} = \sqrt{4\text{gR}}$

Now energy conservation

$\text{mg}(\text{h} - 2\text{R}) = \frac{1}{2}\text{ m} \times 4\text{gR}$

$(\text{ h} = 4\text{R })$

8. **(a)**



$\left(-5\sqrt{3}, \frac{-5}{2}\right) \qquad \left(5\sqrt{3}, \frac{-5}{2}\right)$

$\text{X}_{\text{cm}} = \frac{2 \times (-5\sqrt{3}) + 3(5\sqrt{3}) + 15 \times 0}{20} = \frac{\sqrt{3}}{4} \text{ cm}$

$\text{Y}_{\text{cm}} = \frac{2 \times (-5/2) + 3(5/2) + 15 \times (5/2)}{20}$

$\text{Y}_{\text{cm}} = 1.25 \text{ cm}$

9. **(c)** $\Delta\ell = \frac{\text{T}\ell}{\text{YA}}$

$\ell \propto \text{Y}$

$\frac{\ell_1}{\ell_2} = \frac{\text{Y}_1}{\text{Y}_2} = \frac{20}{11}$

$\because \ell_1 = 2.2 \text{ m}$

$\therefore \ell_1 = 1.21 \text{ m}$

10. **(a)** Using mole conservation

$\frac{2 \times 9 \times 10^4 \times V}{R \times 400} = \frac{P \times V}{R \times 400} + \frac{P \times V}{R \times 500}$

$\text{P} = 100\text{kPa}$

11. **(c)** $\Delta\phi_{\text{A}} = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$

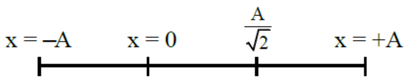
$\therefore I_A = I + I + 2\sqrt{I}\cdot\sqrt{I}\cdot\cos\left(\frac{2\pi}{3}\right) = I$

$\Delta\phi_B = \frac{2\pi}{\lambda}\times\frac{\lambda}{6} = \frac{\pi}{3}$

$\therefore I_B = I + I + 2\sqrt{I}\cdot\sqrt{I}\cdot\cos\left(\frac{\pi}{3}\right) = 3I$

$\frac{I_A}{I_B} = \frac{I}{3I} = \frac{1}{3}$

12. (c)



$x = 0 \rightarrow \theta = 0°$

$x = \frac{A}{\sqrt{2}} \rightarrow \theta = \frac{\pi}{4}$

$\therefore$ phase covered $= \frac{\pi}{4}$

time $= \frac{\text{phase}}{w} = \frac{\pi}{4w} = \frac{\pi}{4}\times\frac{T}{2\pi} = \frac{T}{8}$

$\therefore t = \frac{5}{8}$ secs.

13. (a) $E = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right)$

$100 = 2\times\frac{9\times10^9\times Q}{\pi R^2}$

$Q = 2.14\times10^{-9}C$

14. (b) Diode in reverse bias so no current in it.

$P_{bulb} = 2\times10^{-3} = V_{bulb}$

$v_{bulb} = 4\ V$

So $5\ V = V_{RS} + V_{bulb}$

$V_{rs} = 5 - 4 = 1$

$1 = 0.5\times10^{-3}R_s \Rightarrow R_s = 2k\Omega$

15. (b) Distance is same

So intensity is also same

$I = \frac{P}{4\pi R^2}$

16. (a) $m = \frac{n_1}{n_2}\frac{v}{u}$

$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

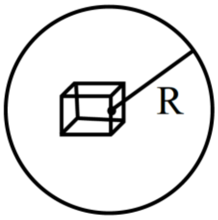
$\frac{1.4}{v} + \frac{1}{4R} = \frac{1.4-1}{R}$

$\frac{1.4}{v} = \frac{4}{10R} - \frac{1}{4R}$

$v = \frac{56R}{6} = +\frac{28}{3}R$

$m = \frac{1}{1.4}\cdot\frac{28R/3}{(-4R)} = -1.67$

17. (a) Magnetic field at center $B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi i}{R}$



$= \frac{10^{-7} \times 2 \times \pi \times 2}{10 \times 10^{-2}}$

So energy stored

$U = \frac{B^2}{2\mu_0} \times V$

$= \frac{10^{-14} \times 4 \times \pi^2 \times 4 \times 100}{2 \times 4\pi \times 10^{-7}} \times (1 \times 10^{-9})$

$= 2\pi \times 10^{-14}$ J

$= 6.28 \times 10^{-14}$ J

18. (a) Apply KVL

$E - iR - V_L = 0$

$V_L = E - iR = 1 - 100i$

at $i_1 = 2 \times 10^{-3}$ A

$V_{L_1} = 0.8$ V

at $i_2 = 4 \times 10^{-3}$ A

$V_{L_2} = 0.6$ V

$\therefore \frac{V_{L_1}}{V_{L_2}} = \frac{4}{3}$

19. (d) $p = \frac{h}{\lambda}$

$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right)$

$\frac{1}{\lambda_1} = R\frac{5}{36}$

$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{4} - \frac{1}{16}\right)$

$\frac{1}{\lambda_2} = \frac{3R}{16}$

$\frac{p_1}{p_2} = \frac{\lambda_2}{\lambda_1} = \frac{\frac{16}{3R}}{\frac{36}{5R}} = \frac{20}{27}$

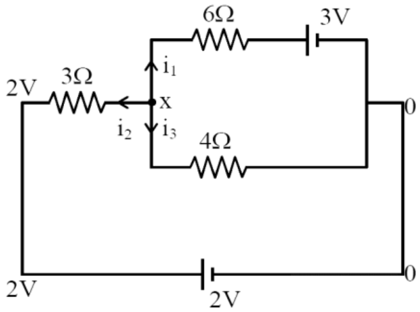
$\alpha = 20 \& \beta = 27$

20. (a) Power factor is $\cos\phi = \frac{R}{Z}$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 100\Omega$

$\cos\phi = \frac{80}{100} = 0.8$

21. (3477)



$$i_1 + i_2 + i_3 = 0$$
$$\frac{x-0-3}{6} + \frac{x-2}{3} + \frac{x-0}{4} = 0$$
$$x = \frac{14}{9} V$$
Current across 6Ω $= \frac{3-(14/9)}{6} = \frac{13}{54}$ A
$$H = \left(\frac{13}{54}\right)^2 \times 6 \times 100 = 34.77 = \frac{\alpha}{100}$$
$$\alpha = 3477$$

22. (30) $\underset{\text{(Unpolarised)}}{I_0} \rightarrow \underset{\text{(Polarised)}}{\frac{I_0}{2}} \xrightarrow{\text{Analyser}}$
$$I = I'\cos^2\theta$$
$$\frac{3I_0}{8} = \frac{I_0}{2}\cos^2\theta$$
$$\cos\theta = \frac{\sqrt{3}}{2}$$
$$\theta = 30^\circ$$

23. (6) ${}_3^7\text{Li} + {}_1^1\text{P} \rightarrow 2\,{}_2^4\text{He}$
$$\Delta m = (7.018 + 1.008) - 2(4.004) = 0.0183u$$
Energy released per reaction $E = \Delta mc^2$

Number of moles of Li $= \frac{7}{17.13} \times \frac{10^3}{7} = \frac{10^3}{17.13}$

Number of molecules $= \frac{10^3}{17.13} \times 6 \times 10^{23}$

Total energy released
$$E_{\text{Total}} = 0.0183 \times 931 \times \frac{6}{17.13} \times 10^{32}$$
$$\simeq 6 \times 10^{32}$$
$$\alpha = 6$$

24. (186) $\Delta U = -w_{\text{electrostatic}}$
$$W_{\text{electrostatic}} = -q\Delta V$$
$$\Delta V = -\int 2xdx - \int 3y^2dy - \int 4dz$$
$$= (-x^2 - y^3 - 4z)_{(0,-2,-5)}^{(5,1,2)}$$
$$= -62$$
$$W_{\text{electrostatic}} = 3(62) = 186 \text{ J}$$

25. (989) For isothermal process initially $B = P_0$.
$$W = nRTn\left(\frac{V_2}{V_1}\right)$$
$$= P_0 V_0 \ln\left(\frac{1}{3}\right)$$
$$W = BV_0 \ln\left(\frac{1}{3}\right)$$
$$W = -988.75 \text{ J} \approx 989 \text{ J}$$

26. (b)

	Fe		Oxygen
Moles :	$\frac{69.9}{56}$		$\frac{30.1}{16}$
Molar Ratio :	1	:	1.5
Molar Ratio :	2	:	3

27. (a) $\frac{1}{x} = R \times 1^2 \times \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$ .....(1)

$\frac{1}{\lambda} = R \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$ ......(2)

$\frac{x}{\lambda} = R \times 4 \times \frac{5}{4 \times 9}$

$\frac{1}{\lambda} = \frac{1}{x} \times \frac{5}{9}$

$\lambda = \frac{9x}{5}$

28.(d) No. of Radial nodes $= n - \ell - 1$

No. of Angular nodes or nodal planes $= \ell$

$2\ s \Rightarrow n = 2, \ell = 0$

1 radial node + zero nodal plane

$3\ s \Rightarrow n = 3, \ell = 0$

2 radial node + zero nodal plane

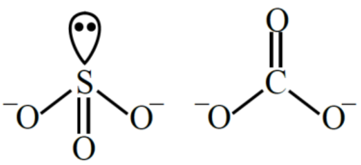
$3p \Rightarrow n = 3, \ell = 1$

1 radial node +1 nodal plane

$4\ d \Rightarrow n = 4, \ell = 2$

1 radial node +2 nodal plane

29. (b) A and E do not have similar Lewis structure

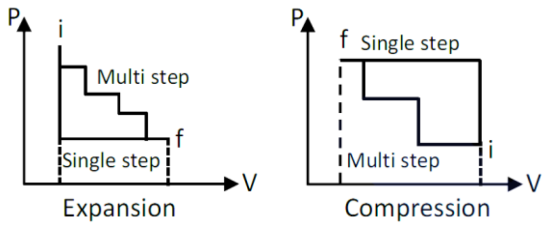


MnO_4^{2-} have are unpaired e but CrO_4^{2-} does not have unpaired e.

In B, C, D pair of species haved same no of e .

They have similar Lewis dot structure.

30. (c)



31. (d) $P_s = P^0 \cdot X_A$

$\frac{P_S}{P^0} = X_A = \frac{1}{1 + 0.25} = 0.80$

$\% = 0.8 \times 100 = 80$

32. (a) $A \rightleftharpoons B$

	A	B
$t = 0$	1 mol	–
t_{eq^m}	$1 - x$	x

$K_C = \frac{x/10}{\frac{1-x}{10}} = 4$

$x = 0.8$

$$P_{He} = \frac{1 \times 0.082 \times 400}{10} = 3.28 \text{ atm}$$

$$P_B = \frac{0.8 \times 0.082 \times 400}{10} = 2.624 \text{ atm}$$

33. (a) $BaSO_4(s) \rightleftharpoons \underset{S}{Ba^{+2}_{(aq)}} + \underset{S}{SO^{2-}_{4(aq)}}$

$K_{sp} = S^2$

k of $BaSO_4 = x\text{Scm}^{-1}$

Λ^o_m of $BaSO_4 = (x_1 + x_2 - 2x_3)$

$$\Lambda_m = \alpha\Lambda^o_M = \frac{k \times 1000}{S}$$

$$S = \frac{x \times 1000}{\alpha(x_1 + x_2 - 2x_3)}$$

$$k_{sp} = S^2 = \frac{1}{\alpha^2}\left[\frac{x \times 1000}{x_1 + x_2 - 2x_3}\right]^2$$

34. (a) $E^0_{Al^{3+}/Al} = -1.66$ V

$E^o_{Tl^{3+}/Tl} = +1.26$ V

For B: IE $+IE_2 + IE_3 = 801 + 2427 + 3659 = 6887$ kJ

Both statements are true

35. (c) A. V_2O_5 is amphoteric. It is used in manufacture of H_2SO_4.

B. Ziegler natta catalyst is $\underset{+4}{TiCl_4} + \underset{+3}{AlMe_3}$

$Ti^{4+}: 3\ d^0 4\ s^0 n = 0 \mu = 0$

C. O.S. of Al is +3

D. $\underset{+2}{PdCl_2}$ is used in wacker process

$Pd^{2+}: 4\ d^8 5\ s^0$

36. (b) In tetrahedral complex

$d^2: \underbrace{e^{1,1}}_{n_1=2}\ \underbrace{t^{0,0,0}}_{n_2=0}$ CFSE $= \left[-\frac{3}{5}n_1 + \frac{2}{5}n_2\right]\Delta_t$

$d^4: \underbrace{e^{1,1}}_{n_1=2}\ \underbrace{t^{1,1,0}}_{n_2=2}$ C.F.S.E $= \left[\frac{-3}{5}(2)\right]\Delta t = -1.2\Delta t$

Δt

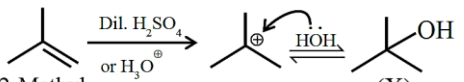
$d^6: \underbrace{e^{2,1}}_{n_1=3}\ \underbrace{t_2^{1,1,1}}_{n_2=3}$; C.F.S.E $= \left[\frac{-3}{5}(3) + \frac{2}{5}(3)\right]\Delta t = -0.6\Delta t$

$d^8: \underbrace{e^{2,2}}_{n_1=4}\ \underbrace{t_2^{2,1,1}}_{n_2=4}$; C.F.S.E $= \left[\frac{-3}{5}(4) + \frac{2}{5}(4)\right]\Delta t = -0.8\Delta t$

37. (a) Order of energy in tetrahedral splitting

$(dxy = dyz = dzx) < d_{x^2-y^2} = d_{z^2}$

38. (d)

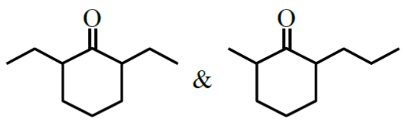


2-Methyl propene (X) Ter.butanol

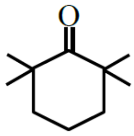
Ter butanol (X) is more polar than 2-Methyl propene and $R_f \propto \frac{1}{\text{polarity}}$ so R_f of ter.butanol is less than 2-Methylpropen

So. is 0.12

39. (c) Statement-I is correct, and Statement-II is incorrect



are metamers



No $\alpha-H$
No Keto enol tautomerism

40. (d) Methane cannot be prepared by Kolbe's electrolysis process. So, statement-1 is false

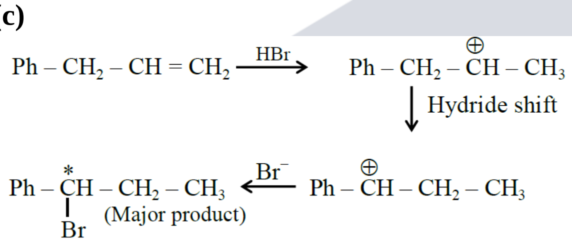
$2CH_3-\overset{\overset{O}{\|}}{C}-ONa \xrightarrow{\text{Kolbe electrolysis}}$

$CH_3-CH_3+2CO_2+2NaOH+H_2$
Ethane
Methane also cannot be prepared by Wurtz reaction, so statement-2 is true

$2R-X \xrightarrow[\text{Wurtz reaction}]{\text{Na.oxyether}} R-R+2NaX$

If $R=-CH_3$ then ethane is formed not methane.

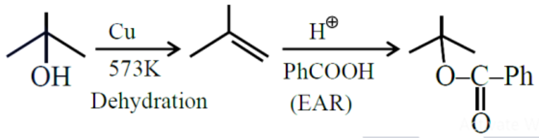
41. (c)



1 Chiral carbon

Aryl cyanide is not formed by Gattermann reaction.

42. (c)



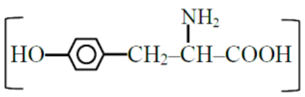
43. (c) Boiling point ∝ Intermolecular attraction
œ Number of H-bonding
n-butanol has strong H-bonding.

44. (c) Vitamin-A → Xerophthalmia
Vitamin-C (Ascorbic acid) → Scurvy
Vitamin-B $_2$ (Riboflavin) → Cheilosis
Vitamin-B $_6$ (Pyridoxine) → Convulsions

45. (b) Glutamine

$\left[H_2N-\overset{\overset{O}{\|}}{C}-CH_2-CH_2-\overset{\overset{NH_2}{|}}{CH}-COOH\right]$ has amide functional group inside chain. So, it gives Hoffmann bromamide reaction.

Lysine $H_2N-(CH_2)_4-\overset{\overset{NH_2}{|}}{CH}-COOH$ has 1° amine group. So, it gives + ve Hinsberg test.

Tyrosine  has phenolic OH . So it gives +ve test with neutral $FeCl_3$.

Serine $HO-CH_2-\overset{\overset{NH_2}{|}}{CH}-COOH$ has alcoholic -OH group. So, it gives +ve test with ceric ammonium nitrate.

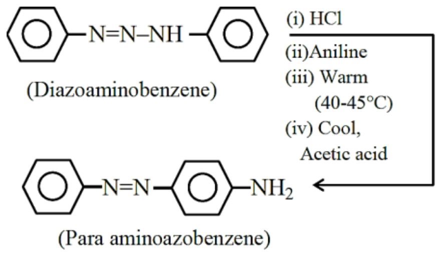
46.(4) Number of moles of Li $= \frac{3.5\times10^{-3}}{7} = 5\times10^{-4}$

Total energy needed to convert

$Li(g) \rightarrow Li^{2+}(g)$ is

$5\times10^{-4}(520+7297) = 3.9085\ kJ = 4\ kJ$

47. (21)

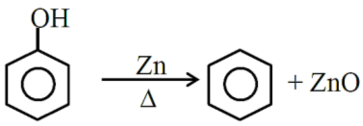


• HCl catalyzed hydrolysis.
• Aniline acts as a solvent and reactant.
• The mixture is gently heated to facilitate rearrangement.
• After the reaction is complete, the mixture is cooled and treated with acetic acid to neutralise excess acid and precipitate the final product.

Mass percentage of nitrogen in final product

$= \frac{42}{197}\times 100 = 21.30\% \approx 21$

48. (3)



Moles of phenol $= \frac{4.7}{94.0} = 0.05$ mol

Reaction is 1: 1 so 0.05 mole of phenol will give 0.05 mole of benzene on 100% yield

For 60% yield $= 0.05\times0.60$

$= 0.03$ mol.

mol of ' X ' $= 3\times10^{-2}$ mol

49. (25) Sucrose $+H_2O \rightarrow$ glucose + fructose

$t_{1/2} = 3hr$

For 1st order $\quad t_{75\%} = 2\times t_{1/2}$

So t = 6hr, $\quad$ 75% of sucrose decomposed

So 25% of sucrose remains after 6 hr .

50. (34) $X \rightleftharpoons Y K_{eq} = 1.8\times10^{-7}$

$$\ln(K_{eq}) = \frac{\Delta_r S^\circ}{R} - \frac{\Delta_r H^\circ}{R}\left(\frac{1}{T}\right)$$

$$2.3\log(1.8\times10^{-7}) = \frac{\Delta_r S^\circ}{R} - \frac{28400}{R}\times\frac{1}{300}$$

$$\Delta_r S^o = 2.3\times R\log(1.8\times10^{-7}) + \frac{284}{3}$$

$$= 19.09[-8+2\times0.48+0.3] + \frac{284}{3}$$

$$= -128.66 + \frac{284}{3}$$

$\Delta_r S^\circ = -33.99 \simeq -34\ J/K.mol$

51. (b) $-1 \le \frac{x+[x]}{3} \le 1$

$\Rightarrow -3 \le x - 1[x] \le 3$

$\Rightarrow -3-[x] \le x \le 3-[x]$

if $[x] = -1 \Rightarrow x \in [-1,0)$

if $[x] = 0 \Rightarrow x \in [0,1)$
if $[x] = 1 \Rightarrow x \in [1,2)$
Hence $x \in [-1,2)$
$\Rightarrow \alpha = -1$ and $\beta = 2$
$\Rightarrow \alpha^2 + \beta^2 = 5$

52. (d) $\alpha + 2\alpha = \frac{-9(k-1)}{k^2-15k+27} = 3\alpha \quad \ldots(1)$
$\alpha \times 2\alpha = \frac{18}{k^2-15k+27} = 2\alpha^2 \ldots(2)$
solving (1) & (2)
$k = 2$
$\Rightarrow$ parabola will be $y^2 = 6kx = 12x$
length of latus rectum $= 12$

53. (c) $e_1 + e_2 = a$
$e_1 e_2 = 2 \Rightarrow e_2 = \frac{2}{e_1}$
Case 1 : Both are eccentricities of hyperbola
Now, $e_1 > 1$ and $e_2 > 1$
$\Rightarrow \frac{2}{e_1} > 1 \Rightarrow e_1 < 2 \Rightarrow e_1 \in (1,2)$
Now, $a = e_1 + \frac{1}{e_1}$
Minimum occurs at $e_1 = \sqrt{2}$
$\Rightarrow a_{min} = 2\sqrt{2}$
if $e = 1,2$ (end points)
$a \rightarrow 2,3 \Rightarrow a \in (2\sqrt{2}, 3)$
$\Rightarrow \alpha = 2\sqrt{2}$ and $\beta = 3$
Case 2 : One ellipse, one hyperbola
(e_1)
$0 < e_1 < 1$ and $e_2 < 1$
$e_2 = \frac{2}{e_1} > 2$
Now $a = e_1 + \frac{2}{e_1}$
as $e_1 \rightarrow 1 \Rightarrow a \rightarrow 3$
$e_1 \rightarrow 0 \Rightarrow a \rightarrow \infty$
$\therefore (3, \infty) \Rightarrow r = 3$
Now $\alpha^2 + \beta^2 + \gamma^2 = 8 + 9 + 9 = 26$

54. (a) Put $z = x + iy, \bar{z} = x - iy$
$\Rightarrow \bar{z} + 2 + i = x - iy + 2 + i$
$\Rightarrow z(\bar{z} + 2 + i) = (x + iy)(x + 2 + i(1 - y))$
$\Rightarrow$ Real part $= x(x + 2) - y(1 - y)$
Imaginary part $= x(1 - y) + y(x + 2)$
$\Rightarrow x^2 + y^2 + 2x - y + 2k = 0 \quad \ldots(1)$
and $x + 2y + 3k = 0 \quad \ldots(2)$
eliminating x from (1) and (2)
$5y^2 + (12k - 5)y + 9k^2 - 4k = 0$
since $y \in R$
use $D \geq 0$
$(12k - 5)^2 - 4.5(9k^2 - 4k) \geq 0$
$\Rightarrow 36k^2 + 40k - 25 \leq 0$
$\Rightarrow \alpha + \beta = \frac{-10}{9}$

$\Rightarrow 9(\alpha+\beta)=-10$

55. **(b)** $(1^3-2^3)+(3^3-4^3)+\cdots.13^3-14^3+15^3$

use $n^3-(n+1)^3=-3n^2-3n-1$

put n $=1,3,5,\ldots.13$ (7 terms)

sum of pairs $=-1519$

add last term $=15^3$

$\Rightarrow 3375-1519=1856$

56. **(a)** Sum of 10 terms of A.P, $S_{10}=160$

$5[2a+9d]=160$

$2a+9d=32$

G.P. $\Rightarrow d, da, da^2 \ldots$

$\Rightarrow d+da=8$

$\Rightarrow \dfrac{d+d(32-9d)}{2}=8$

$\Rightarrow 2d+32d-9d^2=8$

$\Rightarrow 9d^2-34d+8=0$

$d_1+d_2=\dfrac{34}{9}$

57. **(d)** INCONSEQUENTIAL contains (I, E,O, A,U) and (N,C,S,Q,T,L)

Total number of 4 letter words using two vowels and two consonants is

$= {}^5C_2\times{}^6C_2\times 4!$

$=3600$

58. **(d)** $(1+\alpha x)^{26}$

Middle term, $T_{14}={}^{26}C_{13}\alpha^{13}$

For $(1-\alpha x)^{28}$

Middle term $={}^{28}C_{14}\alpha^{14}$

$\Rightarrow {}^{26}C_{13}\alpha^{13}={}^{28}C_{14}\alpha^{14}$

$\Rightarrow {}^{26}C_{13}={}^{28}C_{14}\alpha$

$\Rightarrow {}^{26}C_{13}=\dfrac{28}{14}\times\dfrac{27}{14}.\,{}^{26}C_{13}\alpha$

$\Rightarrow \alpha=\dfrac{7}{27}$

59. **(b)** $\sum_{i=1}^{20}(x_i+5)^2=2500 \quad \ldots(1)$

$\sum_{i=1}^{20}(x_i-5)^2=100 \quad \ldots(2)$

on (1) - (2) $\Rightarrow 20\sum x_i=2400\Rightarrow\sum x_i=120$

mean, $\bar{x}=6$

(1) + (2) $\Rightarrow 2\sum x_i^2+40(5^2)=2600$

$\sum x_i^2=800$

$\sigma^2=\dfrac{\sum x_i^2}{n}-(\bar{x})^2=\dfrac{800}{20}-36=4$

$\bar{x}:\sigma=6:2=3:1$

60. **(b)** Total $(N+1)$ coins

$N\Rightarrow$ fair coins

$1\Rightarrow$ Head on both side

$P(1^+)=\left(\dfrac{N}{N+1}\right)\left(\dfrac{1}{2}\right)+\left(\dfrac{1}{N+1}\right)1$

$=\dfrac{N+2}{2(N+1)}=\dfrac{9}{16}$

$8N+16=9N+9$

$\Rightarrow N=7$

61. (a) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

It passs through $(6,4\sqrt{3})$

$\Rightarrow \frac{36}{a^2}-\frac{48}{b^2}=1 \quad \ldots(1)$

also $15e^2-34e+15=0$

$\Rightarrow 15e^2-25e-9e+15=0$

$e=\frac{5}{3}$ or $\frac{3}{5} \Rightarrow e=\frac{5}{3}$

$1+\frac{b^2}{a^2}=\frac{25}{9} \Rightarrow \frac{b^2}{a^2}=\frac{16}{9} \quad \ldots(2)$

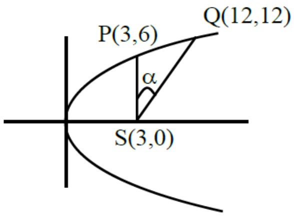
using (1) and (2)

$$\Rightarrow \frac{36}{a^2}-\frac{48}{16a^2}\times 9=1 \Rightarrow a=3, b=4$$

Length of L.R.

$$\frac{4(a^2+1)}{b}=\frac{4(10)}{4}=10$$

62. (a)



$P(3t^2,6t), Q(12t^2,12t)$

$PQ=\sqrt{81t^4+36t^2}$

$81t^4+36t^2=117$

$\Rightarrow 9t^4+4t^2-13=0$

$(t^2-1)(9t^2+13)=0 \Rightarrow t=1$

$P(3,6), Q(12,12)$

slope of QS $=\frac{4}{3}$

$$\tan\alpha=\frac{3}{4}, \sin\alpha=\frac{3}{5}$$

63. (b) $\beta=\frac{1}{3\alpha}, \tan^{-1}(1-\alpha)+\tan^{-1}(1-\beta)=\frac{\pi}{4}$

$\Rightarrow \tan(\tan^{-1}(1-\alpha)+\tan^{-1}(1-\beta))=1$

$$\Rightarrow \frac{1-\alpha+1-\beta}{1-(1-\alpha)(1-\beta)}=1$$

$\Rightarrow 2-\alpha-\beta=1-(1-\alpha-\beta+\alpha\beta)$

$\Rightarrow 2-\alpha-\beta=\alpha+\beta-\alpha\beta$

$\Rightarrow 2(\alpha+\beta)=2+\alpha\beta$

$$\Rightarrow 2(\alpha+\beta)=2+\frac{1}{3}=\frac{7}{3}$$

$$\alpha+\beta=\frac{7}{6}$$

$6(\alpha+\beta)=7$

64.(b) $\cos\theta+1=\sqrt{3}\sin\theta$

$$\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}+1=\sqrt{3}\left(\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)$$

$$2=2\sqrt{3}\tan\frac{\theta}{2}$$

$\Rightarrow \tan\frac{\theta}{2}=\frac{1}{\sqrt{3}}\ \theta\in(-2\pi,2\pi),\frac{\theta}{2}\in(-\pi,\pi)$

$\frac{\theta}{2}=-\frac{5\pi}{6},\frac{\pi}{6}$

$\theta=\frac{-5\pi}{3},\frac{\pi}{3}$

$\text{Sum}=-\frac{-5\pi}{3}+\frac{\pi}{3}=\frac{-4\pi}{3}$

65. **(c)** Let Q be the point $\perp^{r}$ from $P(1,6,a)$ to the line

$L:\frac{x}{1}=\frac{y-1}{2}=\frac{z-a+1}{b}$

The image of point P in line L is $P'\left(\frac{a}{3},0,a+c\right)$

$\because$ Q is the mid-point PP' :

$$Q\equiv\left(\frac{1+\frac{a}{3}}{2},\frac{6+0}{2},\frac{a+a+c}{2}\right)\equiv\left(\frac{3+a}{6},3,\frac{2a+c}{2}\right)$$

comparing x and $y,\frac{3+a}{6}=1\Rightarrow a=3$

$\therefore Q\equiv\left(1,3,\frac{6+c}{2}\right)$

Direction vector of L is $\vec{v}=(1,2,b)$

vector PQ is $\perp^{r}$ to $\vec{V}$

$\therefore \overrightarrow{PQ}=\left(1-1,3-6,\frac{6+c}{2}-3\right)$

$=\left(0,-3,\frac{c}{2}\right)$

$\overrightarrow{PQ}\cdot\vec{v}=0$

$(0).(1)+(-3)(2)+\left(\frac{c}{2}\right)(b)=0$

$bc=12$

Now comparing z with $Q\left(1,3,\frac{6+c}{2}\right)$

$$\frac{3-1}{2}=\frac{\frac{6+c}{2}-3+1}{b}$$

$2b-c=2$

solving $bc=12$ and $c=2b-2$

$b=3,-2$

$b=3\ (b>0)$

$c=4$

$\therefore Q(1,3,5)$

Now, any point S on the line L can be written as $(k,2k+1,3k+2)$

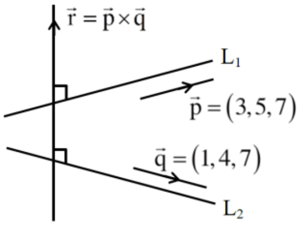
$SQ=\sqrt{(k-1)^2+(2k-2)^2+(3k-3)^2}=2\sqrt{14}$

$k=3,-1$

for $k=3, S\equiv(3,7,11)$

$\therefore \alpha+\beta+\gamma=21$

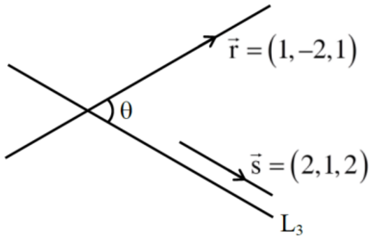
66. (b)



$$\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 7\hat{i} - 14\hat{j} + 7\hat{k}$$

$\equiv 7(\hat{i} - 2\hat{j} + \hat{k})$

Now,



$$\because \cos\theta = \frac{2 - 2 + 2}{\sqrt{1 + 4 + 1}\sqrt{4 + 1 + 4}}$$

$$\Rightarrow \cos\theta = \frac{2}{3\sqrt{6}} \Rightarrow \tan\theta = \frac{5\sqrt{2}}{2}$$

67. (b) $\lim_{x\to 0} \frac{x^2 \sin^2 x}{x^2 - \sin^2 x}$

Applying expansion

$$\lim_{x\to 0} \frac{x^2 \sin^2 x}{x^2 - \left(x - \frac{x^3}{3!} + \cdots\right)^2}$$

$$\lim_{x\to 0} \frac{x^2 \sin^2 x}{x^2 - x^2 + \frac{2x^4}{3!} \ldots} = \frac{3!}{2} = 3$$

68. **(b)** Apply (P-5)

$$I = \int_{-\pi/4}^{\pi/4} \frac{32\cos^4\theta}{1 + e^{-\sin\theta}} d\theta$$

Add $2I = \int_{-\pi/4}^{\pi/4} 32\cos^4\theta \, d\theta = 2\int_0^{\pi/4} 32\cos^4\theta \, d\theta$

$$I = 32\int_0^{\pi/4} \cos^4\theta d\theta$$

$$= 8\int_0^{\pi/4} (2\cos^2\theta)^2 \, d\theta = 8\int_0^{\pi/4} (1 + \cos 2\theta)^2 d\theta$$

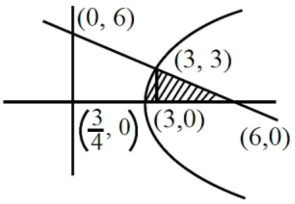
$$= 8\int_0^{\pi/4} 1 + 2\text{co}2\theta + \cos^2 2\theta d\theta$$

$$= 8\int_0^{\pi/4} 1 + 2\cos 2\theta + \frac{1 + \cos 4\theta}{2} d\theta$$

$$= 8\left[\frac{3\theta}{2} + \frac{2\sin 2\theta}{2} + \frac{\sin 4\theta}{8}\right]_0^{\pi/4}$$

$$= 8\left[\left(\frac{3\pi}{8} + 1 + 0\right)\right] = 3\pi + 8$$

69.(b) $A=\int_0^3\left(6-y-\frac{y^2+3}{4}\right)dy=9$



70.(a) $f(x)=e^{x-1}$

$g(x)=\frac{1}{\log_e x+1}$

$f^{-1}(g^{-1}(1/2))=f^{-1}(e)=2$

$\phi(x)=2^x$

$\therefore \int_0^1(2^x-x^2)dx=\left(\frac{2^x}{\ln 2}-\frac{x^3}{3}\right)_0^1=\frac{1}{\ln 2}-\frac{1}{3}$

$=\frac{3-\ell n2}{3\ell n2}$

71. (d) $|A|=-1$

$\operatorname{adj}(\operatorname{adj}(A))=|A|^{n-2}A=|A|^{3-2}\cdot A=|A|\cdot A$

$\operatorname{adj}(A)(\operatorname{adj}(\operatorname{adj}(A)))=\operatorname{adj}(A)(|A|.A)$

$=|A|(\operatorname{adj}(A).A)$

since $\operatorname{adj}(A)A=|A|I$,

then $\operatorname{adj}(A)(\operatorname{adj}(\operatorname{adj}(A)))=|A|^2I$

or $\operatorname{adj}(A)(\operatorname{adj}(\operatorname{adj}(A)))=I$

solving the matrix equation

$A^2+\alpha(-A)+\beta(I)=\begin{bmatrix}2&-2&2\\-2&0&-1\\0&0&-1\end{bmatrix}$

$A^2-\alpha A+\beta I=M$

$\begin{bmatrix}2&-1&1\\-1&1&0\\0&0&1\end{bmatrix}-\alpha\begin{bmatrix}-1&1&-1\\1&0&1\\0&0&1\end{bmatrix}+\beta\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}2&-2&2\\-2&0&-1\\0&0&1\end{bmatrix}$

$\alpha=1$ and $\beta=-1$

$\therefore(\alpha-\beta)^2=4$

72. (80) $x^2+y^2+2gx+2fy+25=0$

$C\equiv(-g,-f)$ lies online

$\therefore -2g+f=4$ ...(i)

Also $g^2+f^2-25=r^2$ ...(ii)

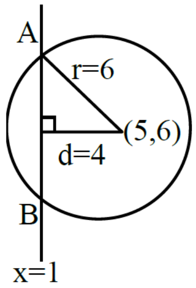
Area of equation triangle inside the circle

$=\frac{3\sqrt{3}}{4}r^2$

$27\sqrt{3}=\frac{3\sqrt{3}}{4}\times r^2$

$r^2=36$

Solving (i) and (ii) $g=-5, f=-6$



$\ell_{AB} = \sqrt{36-16} = 2\sqrt{20}$
$(\ell_{AB})^2 = 80$

73. (3) Using triple product identity with $\vec{a}\times\vec{c} = \vec{b}$

$\vec{a}\times(\vec{a}\times\vec{c}) = \vec{a}\times\vec{b}$

$(\vec{a}\cdot\vec{c})\vec{a} - (\vec{a}\cdot\vec{a})\vec{c} = \vec{a}\times\vec{b}$

As $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -2\hat{\imath} + \hat{\jmath} + \hat{k}$

Substituting given values

$3\vec{a} - 3\vec{c} = -2\hat{\imath} + j + k$

$3(\hat{\imath}+\hat{\jmath}+\hat{k}) + 2\hat{\imath} + \hat{\jmath} + \hat{k} = 3\vec{c}$

$\vec{c} = \frac{1}{3}(5\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

Now $\vec{c}\cdot(\vec{a} - 2\vec{b}) = \frac{1}{3}(5\hat{\imath} + 2\hat{\jmath} + 2\hat{k})\cdot(\hat{\imath} - \hat{\jmath} + 3\hat{k})$

$= \frac{1}{3}[5 - 2 + 6] = 3$

74. (1279) $f(\theta)|_{\min} = 2\sqrt{\alpha\beta}(AM \geq GM)$

$g(\theta) = \alpha\sin^2\theta + \beta(1-\sin^2\theta) = (\alpha-\beta)\sin^2\theta + \beta$

$\therefore g(\theta)|_{\max} = \alpha$

Equation $2\sqrt{\alpha\beta} = \alpha$

$4\alpha\beta = \alpha^2$

$\alpha = 4\beta(\alpha \neq 0)$

Now first term $\left(\frac{\alpha}{2\beta}\right) = 2$ and common ratio

$\left(\frac{2\beta}{\alpha}\right) = \frac{1}{2}$

$S_{10} = \frac{a(1-r^{10})}{1-r} = \frac{1023}{256} = \frac{m}{n}$

$m + n = 1279$

75. (3) $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x-\sqrt{x^2-1}}$

$\frac{dy}{dx} + \frac{y}{x} = x + \sqrt{x^2-1}$

I. $f = e^{\ln x} = x$

$y\cdot x = \int\left(x + \sqrt{x^2-1}\right)x dx$

$= \frac{x^2}{2} + \frac{(x^2-1)^{3/2}}{3} + C$

Given $y(1) = 1$

$1 = \frac{1}{2} + C \Rightarrow C = \frac{1}{2}$

$[y(\sqrt{5})] = \left[\frac{\sqrt{5}}{2} + \frac{8}{3} + \frac{1}{2\sqrt{5}}\right] = 3$