PHYSICS
SECTION-A

1. **The density ρ of a uniform cylinder is determined by measuring its mass m , length l and diameter d . The measured values of m, l and d are 97.42 ± 0.02 g, 8.35 ± 0.05 mm and 20.20 ± 0.02 mm, respectively. Calculated percentage fractional error in ρ is ___。**

(a) 0.63% (b) 0.82%
(c) 0.72% (d) 0.25%

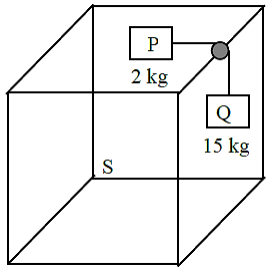
2. **The potential energy of a particle changes with distance x from a fixed origin as $V = \frac{A\sqrt{x}}{x+B}$, where A and B are constant with appropriate dimensions. The dimension of AB are ___ .**

(a) $\left[M^1\ L^{5/2}\ T^{-2}\right]$
(b) $\left[M^{3/2}\ L^{5/2}\ T^{-2}\right]$
(c) $\left[M^1\ L^2\ T^{-2}\right]$
(d) $\left[M^1\ L^{7/2}\ T^{-2}\right]$

3. **The rain drop of mass 1 g , starts with zero velocity from a height of 1 km . It hits the ground with a speed of 5 m/s. The work done by the unknown resistive force is ___ J . (take $g = 10\ m/s^2$)**

(a) -8.75 (b) -8.35
(c) -9.55 (d) -9.98

4. **Two blocks (P and Q) with respectively masses 2 kg and 1.5 kg are joined by a massless thread. These blocks are mounted on a frictionless pully which is fixed on the edge of a cube (S), as shown in the figure below. Block P is positioned on the top surface which has no friction and block Q is in contact with side-surface, having coefficient friction μ. The cube (S) moves towards the right with acceleration of $\frac{g}{2}$, where g is gravitational acceleration. During this movement the block P and Q remain stationary. The value of μ is ___。 (take $g = 10\ m/s^2$)**



(a) 0.33 (b) 0.67
(c) 1 (d) 0.5

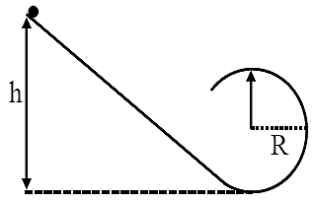
5. **A lift of mass 1600 kg is supported by thick iron wire. If the maximum stress which the wire can withstand is $4 \times 10^8\ N/m^2$ and its radius is 4 mm , then maximum acceleration the lift can take is ___ m/s^2. (take $g = 10\ m/s^2$ and $\pi = 3.14$)**

(a) 2.56 (b) 3.89
(c) 4.32 (d) 5.16

6. **A solid sphere of radius 4 cm and mass 5 kg is rotating (rotation axis is passing through the center of the sphere) with an angular velocity of 1200 rpm . It is brought to rest in 10 s by applying a constant torque. The torque applied and the number of rotations it made before it comes to rest are ___ and ___ respectively.**

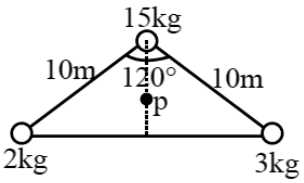
(a) 0.128πNm, 100 (b) 0.0128πNm, 50
(c) 0.128πNm, 50 (d) 0.0128πNm, 100

7. **A smooth inclined plane ends in a vertical circular loop, as shown in the figure. A small body is released from height h as shown. If the body exerts a force of three times its weight on the plane at the highest point of circle, then the height $h = \alpha$R. The value of α is**



(a) 2 (b) 4
(c) 3 (d) 6

8. **The position of center of mass of three masses 2 kg, 3 kg and 15 kg placed with respect to mid point (p) of normal bisector, as shown in the figure is ___。**



(a) $\left(\frac{\sqrt{3}}{4}, 1.25\right)$ (b) $\left(\frac{\sqrt{3}}{4}, 1.0\right)$
(c) $(0,0)$ (d) $(1.25,0)$

9. **The two wires A and B of equal cross-section but of different materials are joined together. The ratio of Young's modulus of wire A and wire B is 20/11. When the joined wire is kept under certain tension the elongations in the wires A and B are equal. If the length of wire A is 2.2 m , then the length of wire *B* is ___ m.**

(a) 1.1 (b) 2.22
(c) 1.21 (d) 4.44

10. **Two closed vessels of same volume are joined through a narrow tube and both vessels are filled with air of pressure 90 kPa and temperature 400 K . Keeping the temperature of one vessel constant at 400 K the second vessel temperature is raised to 500 K . The final pressure in the vessels is ___ kPa .**

(a) 100 (b) 120
(c) 90 (d) 105

11. **In interference experiment the path difference between two interfering waves at a point A on the screen is $\lambda/3$, where λ is the wavelength of these waves, and at another point B the path difference is $\lambda/6$. The ratio of intensities at points A and B is ___。**

(a) 3 (b) 4
(c) $1/3$ (d) $1/4$

12. **A particle is executing simple harmonic motion. Its amplitude is A and time period is 5 sec . the time required by it to move from $x = A$ to $x = \frac{A}{\sqrt{2}}$ is ___ sec.**

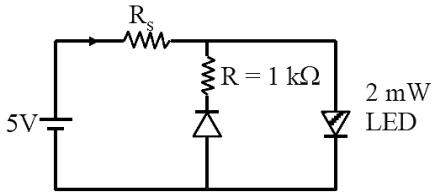
(a) ¼ (b) $5/4$
(c) $5/8$ (d) $3/8$

13. **A thin half ring of radius 35 cm is uniformly charged with a total charge of Q coulomb. If the magnitude of the electric field at center of the half ring is $100\ \mathrm{V/m}$, then the value of Q is ___ nC. ($\epsilon_0 = 8.85 \times 10^{-12}\mathrm{C^2/Nm^2}$ and $\pi = 3.14$)**

(a) 2.14 (b) 2.44
(c) 3.25 (d) 0.7

14. **The maximum rated power of the LED is 2 mW and it is used in the circuit with input voltage of 5 V as shown in the figure below. The current through resistance R_S is 0.5 mA .**

The minimum value of the resistance of R_s, to ensure that the LED is not damaged is ___ kΩ.

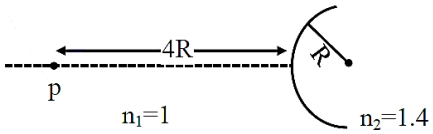


(a) 6 (b) 2
(c) 4 (d) 5

15. A point light source emits E.M. waves in free space. A detector, placed at a distance of L m , measures the intensity as I_0. The detector is now shifted to another location on the same spherical surface ensuring the angle between original location and new location as $45°$. The measured intensity at new location will be ___ .

(a) $\frac{I_0}{4}$ (b) I_0
(c) $\frac{I_0}{\sqrt{2}}$ (d) $\frac{I_0}{2}$

16. A spherical interface lens of radius R separates two media of refractive indices 1 and 1.4 respectively as shown in the figure below. A point source is placed at a distance of 4 R in front of spherical interface. The magnitude of the magnification of point source image is ___ .

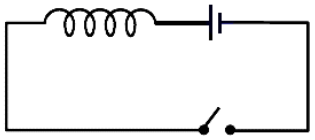


(a) 1.66 (b) 2.33
(c) 2.66 (d) 1.33

17. A small cube of side 1 mm is placed at the center of a circular loop of radius 10 cm carrying a current of 2 A . The magnetic energy stored inside the cube is $\alpha \times 10^{-14}$ J. The value of α is ___ . ($\mu_0 = 4\pi \times 10^{-7}\text{Tm/A}, \pi = 3.14$)

(a) 6.28 (b) 6.28×10^{-6}
(c) 628 (d) 6.28×10^{-4}

18. An inductor of inductance 10mH having resistance of 100Ω is connected to battery of E.M.F. 1.0 V through a switch as shown in the figure below. After switch is closed, the ratio of instantaneous voltages across the inductor when the current passing through it is 2 mA and 4 mA is ___ .



(a) 4/3 (b) 3/4
(c) 5/3 (d) 3/5

19. The ratio of momentum of the photons of the 1st and 2nd line of Balmer series of Hydrogen atoms is α/β. The possible values of α and β are :-

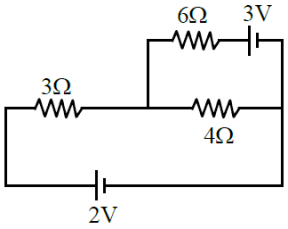
(a) 27 and 20 (b) 3 and 16
(c) 5 and 36 (d) 20 and 27

20. A LCR series circuit driven with $E_{rms} = 90$ V at frequency $f_d = 30$ Hz has resistance $R = 80\Omega$, an inductance with inductive reactance $X_L = 20.0\Omega$ and capacitance with capacitive reactance $X_C = 80.0\Omega$. The power factor of the circuit is ___ .

(a) 0.8 (b) 0.64
(c) 0.9 (d) 0.5

SECTION-B

21. Refer to the circuit diagram given below. The heat generated across the 6Ω resistance in 100 second is $\frac{\alpha}{100}$ J. The value of α is ___ . (Nearest integer)



22. An unpolarized light of intensity I_0 passes through polarizer and then through a certain optically active solution and finally it goes to analyser. If the angle between analyser and polariser is $0°$ and intensity of light emerged from analyzer is $\frac{3}{8}I_0$, the angle of rotation of the light by the solution with respect to analyser is ___ degrees.

23. The energy released when $\frac{7}{17.13}$ kg of $^{7}_{3}Li$ is converted into $^{4}_{2}He$ by proton bombardment is $\alpha \times 10^{32}$eV. The value of α is ___ . (Nearest integer)
(Mass of $^{7}_{3}Li = 7.0183u$, mass of $^{4}_{2}He = 4.004u$, mass of proton $= 1.008u$ and $1u = 931MeV/c^2$ and Avogadro number $= 6.0 \times 10^{23}$)

24. A three coulomb charge moves from the point $(0, -2, -5)$ to the point $(5,1,2)$ in an electric field expressed as $\vec{E} = 2x\hat{i} + 3y^2\hat{j} + 4\hat{k}N/C$. The work done in moving the charge is ___ J.

25. A certain gas is isothermally compressed to $\left(\frac{1}{3}\right)^{rd}$ of its initial volume ($V_0 = 3$ litre) by applying required pressure. If the bulk modulus of the gas is 3×10^5 N/m^2, the magnitude of work done on the gas is ___ J.

CHEMISTRY

SECTION-A

26. An oxide of iron contains 69.9% iron, its empirical formula, is :
(Given : Molar mass of Fe and O are 56 and 16 g mol^{-1} respectively.)
(a) FeO (b) Fe_2O_3
(c) Fe_3O_4 (d) FeO_3

27. If shortest wavelength of hydrogen atom in Lyman series is x , then longest wavelength in Balmer series of $He^+$is :
(a) $\frac{9x}{5}$ (b) $\frac{36x}{5}$
(c) $\frac{x}{4}$ (d) $\frac{5x}{9}$

28. Match the List-I wish List-II :

List-I **Orbital**	**List-II** **Radial nodes and nodal plane**		
(A)	2s	**(I)**	1 Radial node + two nodal planes
(B)	3s	**(II)**	1 Radial node + one nodal plane
(C)	3p	**(III)**	2 Radial nodes + No nodal plane
(D)	4d	**(IV)**	1 Radial node + No nodal plane.

Choose the correct answer from the options given below:
(a) A-IV, B-I, C-III, D-II
(b) A-IV, B-II, C-III, D-I

(c) A-III, B-I, C-IV, D-II
(d) A-IV, B-III, C-II, D-I

29. The pairs among $A = [SO_3^{2-}, CO_3^{2-}], B = [O_2^{2-}, F_2], C = [CN^-, CO]$, $D = [NH_3, H_3O^+]$ and $E = [MnO_4^{2-}, CrO_4^{2-}]$ that do not have similar Lewis dot structure are :

(a) A, B and E (b) A and E
(c) B, C and D (d) C and D

30. Arrange the following isothermal processes in order of the magnitude of the work ($p - V$) involved between states 1 and 2 .

(A) Expansion in single stage w_A
(B) Expansion in multi stages w_B
(C) Compression in single stage w_C
(D) Compression in multi stages w_D

(a) $|w_B| > |w_A| > |w_C| > |w_D|$
(b) $|w_C| > |w_D| > |w_A| > |w_B|$
(c) $|w_C| > |w_D| > |w_B| > |w_A|$
(d) $|w_B| > |w_A| > |w_D| > |w_C|$

31. When 0.25 moles of a non-volatile, non-ionizable solute was dissolved in 1 mole of a solvent the vapor pressure of solution was x% of vapor pressure of pure solvent. What is x% ?

(a) 50% (b) 60%
(c) 70% (d) 80%

32. One mole each of He and $A(g)$ are taken in a 10 L closed flask and heated to 400 K to establish the following equilibrium.

$A(g) \rightleftharpoons B(g)$

K_C for this reaction at 400 K is 4.0. The partial pressures (in atm) of He and $B(g)$ are respectively (at equilibrium).

(Assume He, $A(g)$ and $B(g)$ behave as ideal gases)

(Given : $R = 0.082$ L atm K^{-1} mol^{-1})

(a) 3.28,2.624 (b) 2.624,3.28
(c) 3.28,0.656 (d) 0.656,6.56

33. Consider the following data :

Electrolyte	$\Lambda_m^o (S\ cm^2\ mol^{-1})$
$BaCl_2$	x_1
H_2SO_4	x_2
HCl	x_3

$BaSO_4$ is sparingly soluble in water. If the conductivity of the saturated $BaSO_4$ solution is $\times$ Scm^{-1} then the solubility product of $BaSO_4$ can be given as.

(Here $\wedge_m = \wedge_m^\circ$)

(a) $\frac{10^6 x^2}{\alpha^2 (x_1 + x_2 - 2x_3)^2}$ (b) $\frac{x^2}{(x_1 + x_2 - 2x_3)^2}$
(c) $\frac{\alpha^2 (x_1 + x_2 - 2x_3)^2}{10^6 x^2}$ (d) $\frac{x^2}{(x_1 + x_2 + 2x_3)^2}$

34. Given below are two statements :

Statement-I : Aluminum is more electropositive than thallium as the standard electrode potential value of $E^o_{Al^{3+}/Al}$ is negative and $E^o_{Tl^{3+}/Tl}$ is positive.

Statement-II : The sum of first three ionization enthalpies of boron is very high when compared to that of aluminum. Due to this reason boron forms covalent compounds only and aluminium forms Al^{3+} ion.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both Statement-I and Statement-II are true.

(b) Both Statement-I and Statement-II are false.
(c) Statement-I is true but Statement-II is false.
(d) Statement-I is false but Statement-II is true.

35. The correct statements among the following are :

(A) Basic vanadium oxide is used in the manufacture of H_2SO_4.

(B) The spin-only magnetic moment value of the transition metal halide employed in ZieglerNatta polymerization is 2.84 BM .

(C) The p-block metal compound employed in Ziegler-Natta polymerization has the metal in +3 oxidation state.

(D) The number of electrons present in the outer most 'd' orbital of metal halide employed in Wacker process is 8.

Choose the correct answer from the options given below :

(a) A and B only (b) A, C and D only
(c) C and D only (d) B, C and D only

36. Match the List-I wish List-II

List-I Electronic configuration of tetrahedral metal ion		**List-II Crystal Field Stabilization Energy(Δ_t)**	
(A)	d^2	**(I)**	-0.6
(B)	d^4	**(II)**	-0.8
(C)	d^6	**(III)**	-1.2
(D)	d^8	**(IV)**	-0.4

Choose the correct answer from the options given below :

(a) A-III, B-IV, C-II, D-I
(b) A-III, B-I, C-IV, D-II
(c) A-III, B-IV, C-I, D-II
(d) A-II, B-I, C-IV, D-III

37. Which of the following are true about the energy of the given d-orbitals of a tetrahedral complex ?

(A) $d_{xy} = d_{xz} > d_{x^2-y^2}$

(B) $d_{xy} = d_{yz} > d_{z^2}$

(C) $d_{x^2-y^2} > d_{z^2} > d_{xz}$

(D) $d_{x^2-y^2} = d_{z^2} < d_{xz}$

Choose the correct answer from the options given below :

(a) A, B and D only (b) A and B only
(c) B and D only (d) B, C and D only

38. R_f value for 2-methylpropene in a solvent system (Ethyl acetate + ether) is 0.42 . 2-methylpropene is treated with dilute H_2SO_4 to give major organic product (X). R_f value for (X) in the same solvent system under identical condition will be :

(a) 0.42 (b) 0.82
(c) 0.62 (d) 0.12

39. Given below are two statements :

Statement-I : 2, 6-diethylcyclohexanone and 6-methyl-2-n-propylcyclohexanone are metamers.
Statement-II : 2, 2, 6, 6-tetramethylcyclohexanone exhibits keto-enol tautomerism.
In the light of the above statements, choose the correct answer from the options given below.

(a) Both Statement-I and Statement-II are true.
(b) Both Statement-I and Statement-II are false.
(c) Statement-I is true but Statement-II is false.
(d) Statement-I is false but Statement-II is true.

40. Given below are two statements :

Statement-I : Methane can be prepared by decarboxylation of sodium ethanoate, Kolbe's electrolysis of sodium acetate and reaction of CH_3MgBr with water.

Statement-II : Methane cannot be prepared from unsaturated hydrocarbons and by Wurtz reaction.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both Statement-I and Statement-II are true.
(b) Both Statement-I and Statement-II are false.
(c) Statement-I is true but Statement-II is false.
(d) Statement-I is false but Statement-II is true.

41. Given below are two statements :

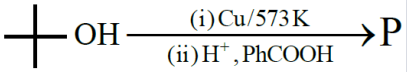
Statement-I : 3-phenylpropene reacts with HBr and gives secondary alkyl bromide having a chiral carbon atom as the major product.

Statement-II : Aryl chlorides and aryl cyanides can be prepared by Sandmeyer reaction as well as Gattermann reaction.

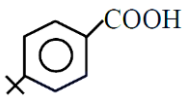
In the light of the above statements, choose the correct answer from the options given below.

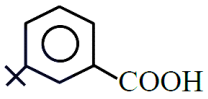
(a) Both Statement-I and Statement-II are true.
(b) Both Statement-I and Statement-II are false.
(c) Statement-I is true but Statement-II is false.
(d) Statement-I is false but Statement-II is true.

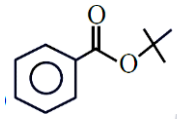
42. Consider the following sequence of reactions

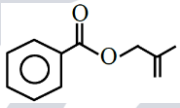


The major product P is :

(a) 

(b) 

(c) 

(d) 

43. Arrange the following compounds according to increasing order of boiling points.

$n-C_4H_9OH$(A), $n-C_4H_9NH_2$(B), $n-C_4H_{10}$(C) and $C_2H_5NHC_2H_5$(D).

(a) C < B < A < D (b) D < C < B < A
(c) C < D < B < A (d) D < B < A < C

44. Match the List-I wish List-II :

List-I Deficiency Disease		**List-II Vitamin**	
(A)	Scurvy	**(I)**	Pyridoxine
(B)	Convulsions	**(II)**	Vitamin A
(C)	Cheilosis	**(III)**	Ascorbic Acid
(D)	Xerophthalmia	**(IV)**	Riboflavin

Choose the correct answer from the options given below :

(a) A-I, B-III, C-II, D-IV
(b) A-I, B-III, C-IV, D-II
(c) A-III, B-I, C-IV, D-II
(d) A-III, B-I, C-II, D-IV

45. Match the List-I wish List-II

List-I Amino acid		List-II Positive reaction/Test for functional group present in side chain of amino acid	
(A)	Glutamine	**(I)**	Heinsberg's test
(B)	Lysine	**(II)**	Neutral $FeCl_3$ test
(C)	Tyrosine	**(III)**	Ceric ammonium nitrate test
(D)	Serine	**(IV)**	Hoffman bromamide degradation

Choose the correct answer from the options given below :

(a) A-IV, B-II, C-I, D-III

(b) A-IV, B-I, C-II, D-III

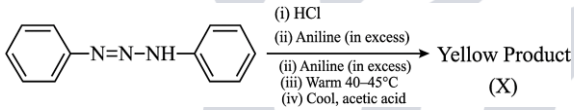
(c) A-III, B-II, C-I, D-IV

(d) A-IV, B-I, C-III, D-II

SECTION-B

46. First and second ionization enthalpies of lithium are 520 kJ mol^{-1} and 7297 kJ mol^{-1} respectively. Energy required to convert 3.5 mg lithium (g) into $Li^{2+}(g)[Li(g) \rightarrow Li^{2+}(g)]$ is ___ $kJmol^{-1}$. (nearest integer) [Molar mass of Li = 7 g mol^{-1}]

47. Consider the following sequence of reactions.



The percentage of nitrogen in the yellow product (X) formed is ___ %.

(Nearest Integer) (Given Molar mass in $gmol^{-1}$ H: 1, C: 12, N: 14)

48. 4.7 g of phenol is heated with Zn to give product X. If this reaction goes to 60% completion then the number of moles of compound X formed will be

___ $\times 10^{-2}$ (Nearest Integer) (Given molar mass in $gmol^{-1}$: H: 1, C: 12, O: 16)

49. Sucrose hydrolyses in acidic medium into glucose and fructose by first order rate law with $t_{1/2} = 3$ hour. The percentage of sucrose remaining after 6 hours is ___ . (Nearest integer)

(Given : $\log 2 = 0.3010$ and $\log 3 = 0.4771$)

50. Consider the reaction X ⇌ Y at 300 K . If ΔH^{θ} and K are 28.40 kJ mol^{-1} and 1.8×10^{-7} at the same temperature, then the magnitude of ΔS^{θ} for the reaction in JK^{-1} mol^{-1} is ___ .(Nearest Integer). (Given : R = 8.3JK^{-1} mol^{-1}, $\ln 10 = 2.3$, $\log 3 = 0.48$, $\log 2 = 0.30$

MATHEMATICS

SECTION-A

51. Let [.] denote the greatest integer function. If the domain of the function $f(x) = \sin^{-1}\left(\frac{x+[x]}{3}\right)$ is $[\alpha, \beta)$, then $\alpha^2 + \beta^2$ is equal to:

(a) 2 (b) 5

(c) 10 (d) 13

52. Let one root of the quadratic equation in x :

$(k^2 - 15k + 27)x^2 + 9(k-1)x + 18 = 0$ be twice the other. Then the length of the latus rectum of the

parabola $y^2 = 6kx$ is equal to:

(a) 4 (b) 6

(c) 8 (d) 12

53. Let e_1 and e_2 be two distinct roots of the equation $x^2 - ax + 2 = 0$. Let the sets $\{a \in R: e_1$ and e_2 are the eccentricities of hyperbolas $\} = (\alpha, \beta)$, and $\{a \in R: e_1$ and e_2 are the eccentricities of an ellipse and a hyperbola, respectively $\} = (\gamma, \infty)$.

The $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

(a) 18 (b) 22

(c) 26 (d) 34

54. Let the set of all values of $k \in R$ such that the equation $z(\bar{z} + 2 + i) + k(2 + 3i) = 0, z \in C$, has at least one solution, be the interval $[\alpha, \beta]$. Then $9(\alpha + \beta)$ is equal to:

(a) -10 (b) -8

(c) $10\sqrt{13}$ (d) $8\sqrt{13}$

55. The value of $1^3 - 2^3 + 3^3 - \cdots . + 15^3$ is:

(a) 1706 (b) 1856

(c) 1982 (d) 2403

56. The sum of the first ten terms of an A.P. is 160 and the sum of the first two terms of a G.P. is 8 . If the first term of the A.P. is equal to the common ratio of the G.P. and the first term of the G.P. is equal to common difference of the A.P., then the sum of all possible values of the first term of the G.P. is:

(a) $\frac{34}{9}$ (b) $\frac{34}{13}$

(c) $\frac{32}{9}$ (d) $\frac{32}{13}$

57. The number of 4-letter words, with or without meaning, each consisting of two vowels and two consonants that can be formed from the letters of the word INCONSEQUENTIAL, without repeating any letter, is:

(a) 2670 (b) 2840

(c) 2920 (d) 3600

58. If the coefficients of the middle terms in the binomial expansions of $(1 + \alpha x)^{26}$ and $(1 - \alpha x)^{28}$, $\alpha \neq 0$, are equal, then the value of α is:

(a) 1 (b) $\frac{14}{13}$

(c) $\frac{27}{7}$ (d) $\frac{7}{27}$

59. A data consists of 20 observations $x_1, x_2, \ldots\ldots, x_{20}$. If $\sum_{i=1}^{20}(x_i + 5)^2 = 2500$ and $\sum_{i=1}^{20}(x_i - 5)^2 = 100$, then the ratio of mean to standard deviation of this data is :

(a) 2: 1 (b) 3: 1

(c) 3: 2 (d) 4: 1

60. A bag contains $(N + 1)$ coins -N fair coins, and one coin with 'Head' on both sides. A coin is selected at random and tossed. If the probability of getting 'Head' is $\frac{9}{16}$, then N is equal to:

(a) 5 (b) 7

(c) 8 (d) 9

61. If the eccentricity e of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, passing through $(6, 4\sqrt{3})$, satisfies $15(e^2 + 1) = 34e$, then the length of the latus rectum of the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{2(a^2+1)} = 1$ is:

(a) 10 (b) 20

(c) 25 (d) 30

62. Let chord PQ of length $3\sqrt{13}$ of the parabola $y^2 = 12x$ be such that the ordinates of points P and Q are in the ratio 1: 2. If the chord PQ subtends an angle α at the focus of the parabola, then $\sin\alpha$ is equal to :

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$

(c) $\frac{5}{13}$ (d) $\frac{12}{13}$

63. Let $0 < \alpha < 1, \beta = \frac{1}{3\alpha}$ and $\tan^{-1}(1-\alpha) + \tan^{-1}(1-\beta) = \frac{\pi}{4}$. Then $6(\alpha+\beta)$ is equal to:

(a) 6 (b) 7

(c) 8 (d) 9

64. Let $S = \{\theta \in (-2\pi, 2\pi): \cos\theta + 1 = \sqrt{3}\sin\theta\}$.

Then $\sum_{\theta \in S} \theta$ is equal to:

(a) $-\frac{2\pi}{3}$ (b) $-\frac{4\pi}{3}$

(c) $\frac{2\pi}{3}$ (d) $\frac{4\pi}{3}$

65. Let the image of the point $P(1, 6, a)$ in the line $L: \frac{x}{1} = \frac{y-1}{2} = \frac{z-a+1}{b}$, $b > 0$, be $\left(\frac{a}{3}, 0, a+c\right)$. If $S(\alpha, \beta, \gamma), \alpha > 0$, is the point on L such that the distance of S from the foot of perpendicular from the point P on L is $2\sqrt{14}$, then $\alpha + \beta + \gamma$ is equal to :

(a) 19 (b) 20

(c) 21 (d) 22

66. Let a line L be perpendicular to both the lines

$L_1: \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $L_2: \frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$.

If θ is the acute angle between the lines L and $L_3: \frac{x-\frac{8}{7}}{2} = \frac{y-\frac{4}{7}}{1} = \frac{z}{2}$, then $\tan\theta$ is equal to:

(a) $\frac{3}{2}\sqrt{2}$ (b) $\frac{5}{2}\sqrt{2}$

(c) $\frac{5}{3}\sqrt{2}$ (d) $\frac{4}{3}\sqrt{2}$

67. The value of $\lim_{x\to 0}\left(\frac{x^2\sin^2 x}{x^2 - \sin^2 x}\right)$ is:

(a) 2 (b) 3

(c) 4 (d) 6

68. The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{32\cos^4 x}{1+e^{\sin x}}\right) dx$ is :

(a) $4\pi + 2$ (b) $3\pi + 8$

(c) $3\pi + 4$ (d) $4\pi + 3$

69. The area of the region $\{(x, y): 0 \le y \le 6 - x, y^2 \ge 4x - 3, x \ge 0\}$ is :

(a) 8 (b) 9

(c) 12 (d) 15

70. Let e be the base of natural logarithm and let $f: \{1, 2, 3, 4\} \to \{1, e, e^2, e^3\}$ and $g: \{1, e, e^2, e^3\} \to \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$ be two bijective functions such that f is strictly decreasing and g is strictly increasing. If $\phi(x) = \left[f^{-1}\left\{g^{-1}\left(\frac{1}{2}\right)\right\}\right]^x$, then the area of the region $R = \{(x, y): x^2 \le y \le \phi(x), 0 \le x \le 1\}$ is :

(a) $\frac{3-\log_e(2)}{3\log_e(2)}$ (b) $\frac{1}{3\log_e(2)}$

(c) $3 + \log_e(2)$ (d) $\frac{3+\log_e(2)}{2+\log_e(3)}$

SECTION-B

71. Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ satisfy $A^2 + \alpha(\text{adj}(\text{adj}(A))) + \beta(\text{adj}(A)(\text{adj}(\text{adj}(A))))$

$= \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$ for Some $\alpha, \beta \in$ R. Then $(\alpha - \beta)^2$ is equal to ___

72. Let the centre of the circle $x^2 + y^2 + 2gx + 2fy + 25 = 0$ be in the first quadrant and lie on the line $2x - y = 4$. Let the area of an equilateral triangle inscribed in the circle be $27\sqrt{3}$. Then the square of the length of the chord of the circle on the line $x = 1$ is___ .

73. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\jmath} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$, then $\vec{c} \cdot (\vec{a} - 2\vec{b})$ is equal to ___ .

74. For the functions $f(\theta) = \alpha\tan^2\theta + \beta\cot^2\theta$, and $g(\theta) = \alpha\sin^2\theta + \beta\cos^2\theta, \alpha > \beta > 0$, let $\min_{0<\theta<\frac{\pi}{2}} f(\theta) = \max_{0<\theta<\pi} g(\theta)$. If the first term of a G.P. is $\left(\frac{\alpha}{2\beta}\right)$, its common ratio is $\left(\frac{2\beta}{\alpha}\right)$ and the sum of its first 10 terms is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to ___ .

75. Let $y = y(x)$ be the solution of the differential equation $\left(x^2 - x\sqrt{x^2 - 1}\right)dy + \left(y\left(x - \sqrt{x^2 - 1}\right) - x\right)dx = 0$, $x \geq 1$. If $y(1) = 1$, then the greatest integer less than $y(\sqrt{5})$ is ___ .