1. **(c)** $v = \frac{4}{3}\pi R^3$

$$\frac{\Delta v}{v} = \frac{3\Delta R}{R}$$

$= 6\ \%$

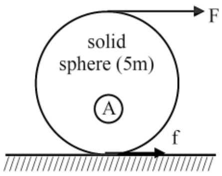
2. **(c)** $(A) \rightarrow ML^2\ T^{-2}\ K^{-1}$

$(B) \rightarrow MT^{-3}\ K^{-4}$

$(C) \rightarrow ML^2\ T^{-1}$

$(D) \rightarrow M^{-1}\ L^3\ T^{-2}$

3. **(a)**



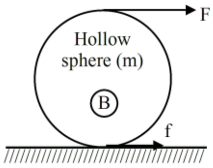
$F + f = 5\ ma_A$

$$F(R) - f(R) = \frac{2}{5}(5\ m)R^2 \cdot \alpha$$

$\alpha = R\alpha$

$2\ F = 7\ ma_A$

$$a_A = \frac{2\ F}{7\ m}$$



$F + f = ma_B$

$$F(R) - f(R) = \frac{2}{3}mR^2 \times \alpha$$

$a = R\alpha$

$$2\ F = \frac{5ma_B}{3}$$

$$\frac{a_A}{a_B} = \frac{2\ F}{7\ m} \times \frac{5\ m}{6\ F} = \frac{5}{21}$$

4. **(c)** $w.\Delta = \Delta K.E$

$$= \frac{1}{2}\ m(v_f^2 - v_i^2)$$

$$= \frac{1}{2} \times 1[(2(5)^2)^2 - (2(0))^2]^0$$

$$= \frac{1}{2} \times 50 \times 50 = 1250\ J$$

5. **(b)** $P_{Left} = P_{Right}$ (final) for right chamber

$P_i V_i^{\gamma} = P_f V_f^{\gamma}$

$$P(9)^{3/2} = \frac{275}{8} \times V_f^{3/2}$$

$$27 = \frac{27}{8}\ V_f^{3/2}$$

$\therefore V_f = 4$ litres

6. **(b)** $\vec{B} = \frac{1}{\omega}(\vec{k} \times \vec{E})$

7. **(c)** $\vec{v}_{cm}=\frac{m_1\vec{v}_1+m_2\vec{v}_2}{m_1+m_2}=\frac{m(4\hat{\imath})+m(4\hat{\jmath})}{2m}=2\hat{\imath}+2\hat{\jmath}$

$$\vec{a}_{cm}=\frac{m_1\vec{a}_1+m_2\vec{a}_2}{m_1+m_2}=\frac{m(6\hat{\imath}+6\hat{\jmath})+m(0)}{2m}=3\hat{\imath}+3\hat{\jmath}-$$

$\vec{v}_{cm}||\vec{a}_{cm}$

so, path is straight line

8. **(c)** $\frac{\Delta\ell}{w}=\tan\theta=\frac{\Delta y}{\Delta x}=\frac{1\times10^{-4}}{10}=10^{-5}$

$$\frac{w}{\Delta\ell}=10^5$$

$$\Rightarrow y=\frac{F/A}{\Delta\ell/\ell}=\frac{F\ell}{A\Delta\ell}=\left(\frac{w}{\Delta\ell}\right)\frac{\ell}{A}$$

$$\Rightarrow y=\frac{10^5\times1}{10^{-5}}=10^{10}\ \text{N/m}^2$$

9. **(b)** $v=\pi r^2h\Rightarrow528\times10^{-3}=\pi(0.4)^2hh=\frac{528\times10^{-3}\times7}{(0.4)^2\times22}=1.05\ \text{m}=105\ \text{cm}$

$$\rightarrow t=\sqrt{\frac{2H}{g}}=\sqrt{\frac{2\times(105+35)\times10^{-2}}{10}}$$

$$=\sqrt{\frac{2(140)\times10^{-2}}{10}}\sqrt{28}\times10^{-1}\text{sec}$$

$$R=ut=\sqrt{2\times10\times70\times10^{-2}}\times\sqrt{28}\times10^{-1}$$

$$=(\sqrt{14})(\sqrt{28})\times10^{-1}=14\sqrt{2}\times10^{-1}\ \text{m}$$

$$R=140\sqrt{2}\ \text{cm}$$

10. **(d)** $T_{mix}=\frac{n_1C_1T_1+n_2C_2T_2}{n_1C_1+n_2C_2}$

$$=\frac{\left(2\times\frac{3}{2}R\times T\right)+\left(6\times\frac{3}{2}R\times2T\right)}{\left(2\times\frac{3}{2}R\right)+\left(6\times\frac{3}{2}R\right)}$$

$$T_{mix}=\frac{2T+12T}{8}=\frac{7}{4}T$$

11. **(a)** $kx=0.2g;\qquad f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$k\times2\times10^{-3}=2\quad f=\frac{1}{2\pi}\sqrt{\frac{1000}{0.2}}$$

$$k=1000\ \text{N/m}$$

$$=\frac{1}{2\pi}\sqrt{5000}=\frac{5}{\pi}\sqrt{50}$$

$$E_{max}=\frac{1}{2}kx_{max}^2$$

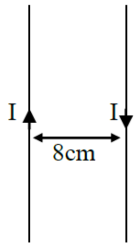
$$\frac{1}{2}\times1000\times(4\times10^{-3})^2=8\times10^{-3}\ \text{Joule}$$

12. **(a)** $V=5(x^2-y^2)V$

$$\varepsilon_x=\frac{-dv}{dx}=-10x\hat{\imath}$$

$$\varepsilon_y=\frac{-dv}{dx}=+10y\hat{\imath}\Rightarrow\varepsilon(2,3)=-20\hat{\imath}+30\hat{\jmath}$$

13. (b)



$$B = \frac{\mu_0 I}{2\pi(4)\times10^{-2}}\times 2$$
$$= 10^{-7}\times 30\times 10^{2}$$
$$= 30\times10^{-5}\ \mathrm{T}$$
$$= 300\mu\ \mathrm{T}$$

14. (d) $B = 0.4\sin 300t\ \theta = 60^\circ$

$$A = (0.02)^2 = 4\times10^{-4}\ \mathrm{m}^2$$
$$\phi = BA\cos\theta$$
$$\phi = 0.4\sin(300t)A\cos\theta$$
$$\text{emf } = \frac{d\phi}{dt} = 0.4\ A\cos\theta\cos(300t)\times 300$$
$$\text{max emf } = 0.4\times4\times10^{-4}\times300\times\frac{1}{2}$$
$$= 240\times10^{-4}\ \mathrm{V}$$
$$= 24\mathrm{mV}$$

15. (b) $Q_1 = CV = 100\times 100\mathrm{pC} = 10^4\mathrm{pC}$

$$V_1 = V_2 = \frac{V}{2} = 50$$
$$\varepsilon_i = \frac{1}{2}\times100\times(100)^2\mathrm{pJ} = \varepsilon$$
$$\varepsilon_f = 2\times\frac{1}{2}\times100\times(50)^2 = \frac{\varepsilon}{2}$$
$$\Delta\varepsilon = \frac{\varepsilon}{2} = 100\times(50)^2\mathrm{pJ}$$
$$= 25\times10^4\times10^{-12} = 2.5\times10^{-7}$$

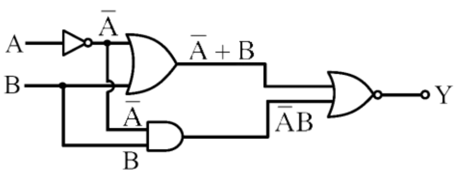
16. (b) ${}_1^2\mathrm{H} + {}_1^2\mathrm{H} \rightarrow {}_2^4\mathrm{He}$

$$Q = \mathrm{BE\ (RHS)\ \text{-}BE\ (LHS)}$$
$$= 4\times7.2 - 4\times1.1$$
$$= 4\times6.1$$
$$Q = 24.4\mathrm{MeV}$$

17. (c) $\mu_p = \mu_s \frac{\sin\left(\frac{A+\delta_{\min}}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\frac{3A}{4}}{\sin\frac{A}{2}}$

$$\mu_p = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \sqrt{2} = \sqrt{2}$$

18. (c)



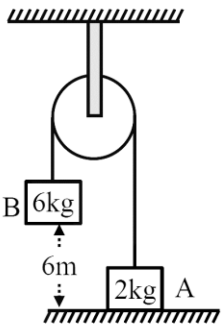
$$y = \overline{\overline{A}+B+\overline{A}B}$$
$$= \overline{\overline{A}+B} = \overline{\overline{A}}\cdot\overline{B}$$

$= A\overline{B}$

for $A = 1, B = 1, y = 0$

for $A = 0, B = 1, y = 0$

19. (a)



$W_G = \Delta K$

$m_B gh - m_A gh = \frac{1}{2}(m_A + m_B)V^2$

$(6-2) \times 10 \times 6 = \frac{1}{2}(8)v^2$

$v = \sqrt{60}\ \text{m/s}$

$v = 7.746\ \text{m/s}$

20. (c) $I_P = I_0 = 4I_0\cos^2\frac{\Delta\phi}{2}$

$\cos\frac{\Delta\phi}{2} = \frac{1}{2}$

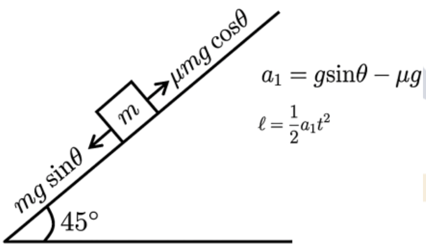
$\frac{\Delta\phi}{2} = \frac{\pi}{3}$

$\frac{2\pi}{\lambda} = \Delta x = \Delta\phi = \frac{2\pi}{3}$

$\Delta x = \frac{\lambda}{3} = 2000\ A^\circ$

21. (75) $a_1 = g\sin\theta - \mu g$

$\ell = \frac{1}{2}a \cdot t^2$



for frictionless surface $a_2 = g\sin\theta$

$\ell = \frac{1}{2}a_2\frac{t^2}{4}$

$a, t^2 = a_2\frac{t^2}{4}$

$g(\sin 45 - \mu\cos 45^\circ) = \frac{g\sin 45}{4}$

$1 - \mu = \frac{1}{4}$

$\mu = \frac{3}{4} = \frac{\alpha}{100}$

$\alpha = 75$

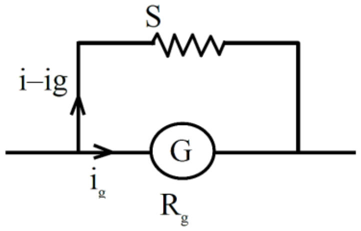
22. (4) $\lambda_1 = \frac{h}{mv} = \frac{h}{\sqrt{2meV_1}} = \lambda$

$$\lambda_2=\frac{h}{\sqrt{2meV_2}}=\frac{3\lambda}{2}=\frac{3}{2}\frac{h}{\sqrt{2meV_1}}$$

$$V_2=\frac{4}{9}\cdot\ V_1$$

$$\frac{V_1}{V_2}=\frac{9}{4}\Rightarrow\alpha=4$$

23.(50) $i_gR_g=2\left(\frac{1}{2}-ig\right)$



$$i_g=\frac{1}{R_g+2}$$

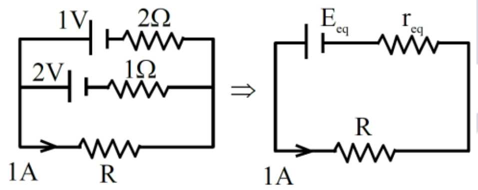
$$10=(R_g+470)i_g$$

$$10=\frac{R_g+470}{R_g+2}$$

$$10R_g+20=R_g+470$$

$$R_g=50\Omega$$

24.(3)



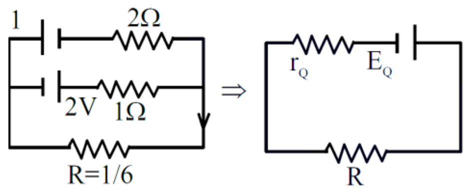
$$E_{eq}=\frac{\frac{1}{2}+\frac{2}{1}}{\frac{1}{2}+\frac{1}{1}}=\frac{\frac{5}{2}}{\frac{3}{2}}=\frac{5}{3}$$

$$r_Q=\frac{3}{2}$$

$$i=\frac{E_{eq}}{r_{eq}+R}=\frac{\frac{5}{3}}{\frac{3}{2}+R}=1$$

$$\frac{5}{3}=\frac{3}{2}+R$$

$$R=\frac{1}{6}\Omega$$



$$r_{eq}=\frac{3}{2}$$

$$E_{eq}=\frac{\frac{2}{1}-\frac{1}{2}}{\frac{1}{1}+\frac{1}{2}}=1$$

$$i=\frac{E_{eq}}{r_{eq}+R}=\frac{1}{\frac{3}{2}+\frac{1}{6}}=\frac{3}{5}\Rightarrow \alpha=3$$

25. (10) Case I v = 2u

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{-2u}+\frac{1}{-u}=\frac{1}{-f}$$

$$\frac{3}{2u}=\frac{1}{f}$$

$$u_1=\frac{3f}{2}$$

Case 2v = −2u.

$$\frac{1}{+2u}+\frac{1}{-u}=\frac{1}{-f}$$

$$\frac{-1}{2u}=\frac{1}{-f}$$

$$u_2=\frac{f}{2}$$

$\Delta u=f=10\ \text{cm}$

26. (d) N_{atoms} in $O_2=\frac{m_{O_2}}{M_{O_2}}N_A=\frac{2}{32}\times 2N_A=\frac{N_A}{8}=0.125N_A$

N_{atoms} in $SO_2=\frac{m_{SO_2}}{M_{SO_2}}=3N_A=\frac{4}{64}\times 3N_A=\frac{3N_A}{16}=0.1875N_A$

N_{atoms} in O_2 at STP $=\frac{V_{O_2}}{22.7\text{ lit}}N_A=\frac{1400\text{ml}}{22400\text{ml}}\times 2N_A=0.125N_A$

N_{atoms} in He(g) at STP $=\frac{V_{He}}{22.4\text{ lit.}}N_A=\frac{0.05}{22.4}N_A=0.002N_A$

N_{atoms} in H_2(g) = moles × atomicity × $N_A=0.0625\times 2N_A$
$=0.125N_A$

27. (d) $r_{species}=a_0\frac{n^2}{Z}$

$$70.53=52.9\left(\frac{n^2}{Z}\right)$$

$$\frac{n^2}{Z}=\frac{70.53}{52.9}=1.33=\frac{4}{3}$$

Data will satisfy if n = 2 and z = 3

$Li^{+2},2\Rightarrow\frac{4}{3}=1.33$

28.(d) SO_2,SO_3,SF_6&SF_4 do not follow octet rule NH_3,BrF_5,SF_4 have one lone pair on central atom ClF_3,XeF_4&H_2O have two lone pair on central atom.

29. (c)(A) For isothermal process, $\Delta U=0$

$W=-nRT\ell n\left(\frac{V_2}{V_1}\right)$ so $q=nRT\ell n\left(\frac{V_2}{V_1}\right)$

(B) Free expansion which is isothermal must be adiabatic, so q = 0

(C) Irreversible process $W=-P_{ext}[V_2-V_1]$

(D) Cyclic process, change in state function (S) so change in entropy must be zero.

$$dS=\frac{dq_{rev}}{T}$$

$$\oint dS=0$$

30. **(d)** On side (I) $C_1 = \frac{18/180}{0.1} = 1M$

On side (II) $C_2 = \frac{30/180}{250/1000} = \frac{2}{3}M$; $\{\pi_1 > \pi_2\}$

Hence water will flow from chamber-II to Chamber-I.

$\pi = iCRT$; $i = 3$ for K_2SO_4

And $\left(\pi = 3 \times \frac{50\times10^{-3}/174}{2}\right) \times 0.083 \times 300$

$= 0.0107$ bar.

31. **(b)** $A_xB_y \rightleftharpoons xA^{y+} + yB^{x-}$

$C(1-\alpha) \quad xC\alpha \quad yC\alpha$

$$K = \frac{(xC\alpha)^x (xC\alpha)^y}{C(1-\alpha)}$$

As $\alpha \ll 1 \Rightarrow (1-\alpha) \simeq 1$

$K = x^x y^y C^{x+y-1} \alpha^{x+y}$

$$\alpha = \left(\frac{K}{C^{x+y-1}x^x y^y}\right)^{\frac{1}{x+y}}$$

32. **(d)** Anode : $\left[Red_1 \rightarrow Ox_1^{n_1^+} + n_1e^-\right] \times n_2$

Cathode : $\left[Ox_2 + n_2e^- \rightarrow Red_2^{n_2-}\right] \times n_1$

Overall reaction :

$n_2Red_1 + n_1Ox_2 \rightarrow n_2Ox_1^{n_1^+} + n_1Red_2^{n_2^-}$

The electrons does not appear in overall reaction

$$E = E^o - 2.303\frac{RT}{nF}\log Q$$

$$\frac{(E - E^\circ)}{RT/F} = -\frac{2.303}{n}\log Q$$

Slope $= \frac{-2.303}{n}$

Straight line passing through origin.

Electrical work = Charge × Potential difference

33. **(c)** Left most element in a period will be of group 1 which will have lowest IE, while right most element will be of group 17 which will have most negative ΔH_{eg}.

34. **(c)**

Specie	**Bond angle**
OCl_2	111°
H_2O	104.5°
OF_2	103°

SiF_4 is covalent.

35. **(b)** $Cr - [Ar]3\,d^5 4\,s^1$

$Mn - [Ar]3\,d^5 4\,s^2$

$IE_1 - Mn > Cr$

$IE_2 - Mn < Cr$

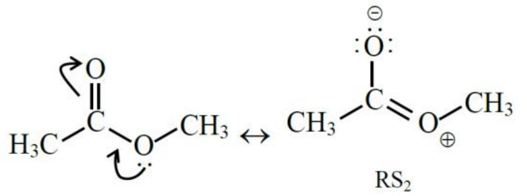
36. **(d)** $[Ni(CO)_4]$ is sp^3, tetrahedral and diamagnetic

37. **(a)** Sugar is soluble in alcohol not NaCl

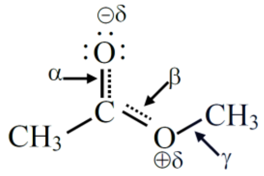
So they are separated by solvent extraction (or differential extraction)

Rose essence oil is steam volatile, so it is separated by steam distillation.

38. **(b)**



RS$_1$ RS$_2$



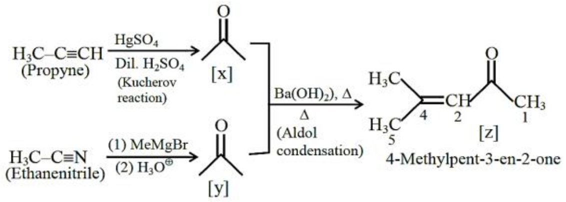
R.Hybrid

RS 1 is more stable than RS 2

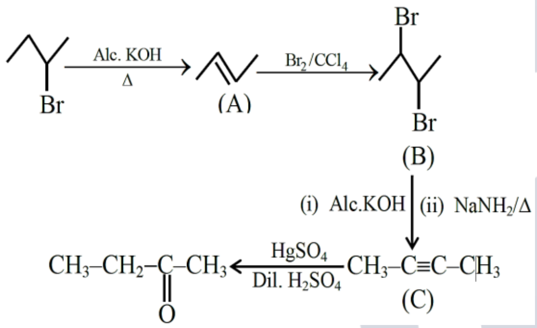
$\therefore$ contribution of RS1 is more than RS2 in its R.H.

$\therefore$ Order of bond length $\alpha < \beta < \gamma$

39.(b)



40. (a)



Final product gives +ve haloform test

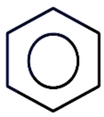
41. (d) $Ph - O - Ch_2 - Ph \xrightarrow{HI} \underset{Phenol}{Ph - OH} + \underset{Benzyl\ Iodide}{I - Ch_2 - Ph}$

42. (a)

$$CH_3\text{–}CH\text{–}COOH \xrightarrow[\Delta]{NH_3} CH_3 - CH_2 - \overset{\overset{O}{||}}{C} - NH_2$$

$$\xrightarrow{NaOH + Br_2} CH_3\text{–}CH_2\text{–}NH_2 \xrightarrow[H_2O/0\,^{\circ}C]{HNO_2}$$

$$CH_3\text{–}CH_2\text{–}OH \xrightarrow[0\,^{\circ}C]{Ph\text{–}N_2^+Cl^-}$$



$$+$$

$$CH_3 - \overset{\overset{O}{||}}{C} - H + N_2 + HCl$$

43. (c) (ii), (iii), (vi)

$$Ph-\overset{O}{\overset{\|}{C}}-NH_2 \xrightarrow[\Delta]{Br_2 + KOH} Ph-NH_2$$

(i)

Aniline is not formed by Gabriel phtalimide synthesis.

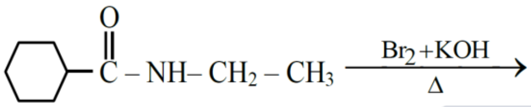
(ii)

$$Ph-CH_2-\overset{O}{\overset{\|}{C}}-NH_2 \xrightarrow[\Delta]{Br_2 + KOH} Ph-CH_2-NH_2$$

(iii)

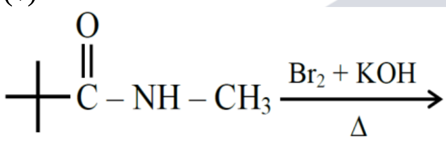
$$CH_3-\overset{O}{\overset{\|}{C}}-NH_2 \xrightarrow[\Delta]{Br_2 + KOH} CH_3-NH_2$$

(iv)



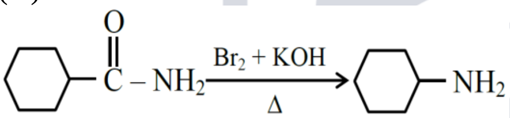
No 1° amine is form

(v)

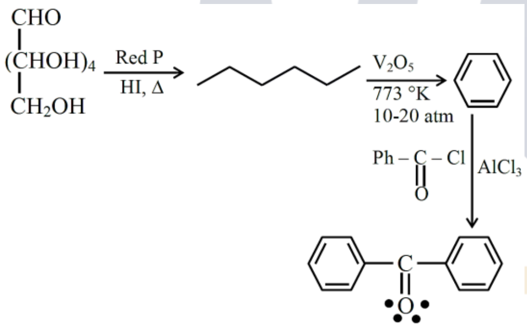


No 1° amine is form

(vi)



44. (c)



Total number of π and lone pair electrons in final product = 18

45. (a) $NO + FeSO_4 \xrightarrow{\text{dil. } H_2SO_4} [\overset{I}{Fe}(NO)]SO_4$

46.(5) $[Cr(H_2O)_4Cl_2]Cl + AgNO_3 \rightarrow AgCl\downarrow +$ $[Cr(H_2O)_4Cl_2]NO_3$

$\Rightarrow$ 1 mole of complex will precipitate 1 mole of AgCl .

$\Rightarrow$ Given moles of complex =

$M \times V = (0.05 \times 100 \times 10^{-3})$ moles

$\Rightarrow$ Moles of AgCl $= 5 \times 10^{-3}$ moles

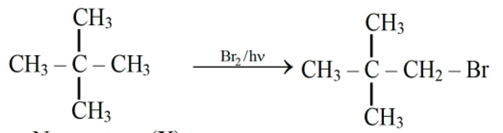
$\Rightarrow x = 5$

47. (4) $C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \rightarrow nCO_2 + (n+1)H_2O$

$\frac{3n+1}{2} = 8$

$3n + 1 = 8 \times 2$

$n = 5(\therefore C_5H_{12})$



Neopentane (X)

Only one mono-bromo product

Number of primary caroms = 4

48. (1) $3e^- + MnO_4^- + 2H_2O \rightarrow MnO_2 + 4OH^{\ominus}$

$2I^{\ominus} \rightarrow I_2 + 2e^-$

$I_2 + 2\,S_2O_3^{-2} \rightarrow 2I^{\ominus} + S_4O_6^{-2}$

gm equivalent of MnO_4^-

$= 0.2 \times \frac{500}{1000} \times 3 = 0.3$ (Limiting reagent)

gm equivalent of KI

$= 1.5 \times \frac{500}{1000} \times 1 = 0.75$

gm equivalent of MnO_4^-

= gm equivalent of hypo

$0.3 = x \times \frac{300}{1000} \times 1$

$x = 1$

49. (8) $X_2Y_4(g) \rightleftharpoons 2XY_2(g)$

$t = 0 \quad P^\circ$

$t = t \quad P^\circ(1-\alpha)\ 2P^\circ\alpha$

$P^\circ(1+\alpha) = 1$

$\alpha = \frac{3}{4}; P^\circ = \frac{4}{7}$

$k_P = \frac{4P_o\alpha^2}{1-\alpha^2} = \frac{36}{7}$

$\Delta G^\circ = -RT\ell nk_p$

$|\Delta G^\circ| = 8.314 \times 600 \ln\left(\frac{36}{7}\right)$

$= 8169.5\ J\ mol^{-1}$

$= 8.169\ kJ/mol$

50.(232) $K = (5.5 \times 10^{11}\ S^{-1})e^{-\frac{28000\ K}{T}}$

$K = Ae^{-E_a/RT}$

$\frac{E_a}{R} = 28000$

$E_a = \frac{28000 \times 8.3}{1000}\ kJ/mol = 232.4\ kJ/mol$

51. (d) $\frac{2x^2-3x+2}{3x^2+x+3} = y$

$(3y-2)x^2 + (y+3)x + (3y-2) = 0$

$D \geq 0$

$(7y-1)(5y-7) \leq 0$

$y \in \left[\frac{1}{7}, \frac{7}{5}\right]$

Hence $f(x)$ is into and $f(x)$ is many-one because f(x) is non-monotonic

52. (c) $\alpha = n^2 - 2n + 2$

$\alpha = (n-1)^2 + 1$

$\therefore$ minimum value of α is 1

Similarly $\beta = \frac{3}{(n-1)^2+1}$

$\therefore$ maximum value of β is 3

required G.P. is $1, \frac{1}{3}, \frac{1}{3^2}, \ldots$

$$S_6 = \frac{1\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}} = \frac{3}{2}\left[1-\left(\frac{1}{3}\right)^6\right] = \frac{364}{243}$$

53. (a) $z = \frac{-\sqrt{6}i \pm \sqrt{-6+12}}{2}$

$= \dfrac{-\sqrt{6}i \pm \sqrt{6}}{2}$

$= \dfrac{\sqrt{6}}{2}(\pm 1 - i)$

$\alpha = \dfrac{-\sqrt{6}}{2}(1+i), \beta = \dfrac{\sqrt{6}}{2}(1-i)$

$\alpha = \dfrac{-\sqrt{6}}{2} \times \sqrt{2}e^{i\pi/4}, \beta = +\dfrac{\sqrt{6}}{2} \times \sqrt{2} - i\pi/4$

$\alpha = -\sqrt{3}e^{i\pi/4}, \beta = +\sqrt{3}e^{-i\pi/4}$

$\alpha^8 + \beta^8 = 81 \cdot e^{i2\pi} + 81 \cdot e^{i(-2\pi)}$

$= 81(1+1) = 162$

54. (d) $\begin{vmatrix} \cos 3\theta & -8 & -12 \\ \cos 2\theta & 3 & 3 \\ 1 & 1 & 3 \end{vmatrix} = 0$

$\begin{vmatrix} \cos 3\theta & -8 & -4 \\ \cos 2\theta & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$(C_1 \to C_1 - C_2) \& C_2 \to C_2 - C_3$

$\begin{vmatrix} \cos 3\theta + 8 & -4 & -3 \\ \cos 2\theta - 3 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$

$2\cos 3\theta + 16 + 4\cos 2\theta - 12 = 0$

$(4\cos^3\theta - 3\cos\theta) + 2(2\cos^2\theta - 1) + 2 = 0$

$4\cos^3\theta + 4\cos^2\theta - 3\cos\theta = 0$

$\cos\theta(4\cos^2\theta + 4\cos\theta - 3) = 0$

$\cos\theta(2\cos\theta + 3)(2\cos\theta - 1) = 0$

$\cos\theta = 0, \frac{1}{2}, -\frac{3}{2}$ (rejected)

$\theta = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

Sum $= 4\pi$

55. (c) $A = P + I$

$A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A = P + I$ where $P^3 = 0$

$A^{99} = I + 99P + {}^{99}C_2 P^2$

$A^{99} - I = 99\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + {}^{99}C_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 99 \times 3 & 0 & 0 \\ 44550 & 99 \times 3 & 0 \end{bmatrix}$$

$b_{31} = 44550,\ b_{21} = 99 \times 3,\ b_{32} = 99 \times 3$

$$\frac{44550 - 297}{297} = 149$$

56.(c) $T_n = \frac{1}{n}(1^2 + 2^2 + \cdots + n^2)$

$$T_n = \frac{n(n+1)(2n+1)}{6n}$$

$$T_n = \frac{1}{6}[2n^2 + 3n + 1]$$

$$S_n = \Sigma T_n = \frac{1}{6}[2\Sigma n^2 + 3\Sigma n + \Sigma 1]$$

$$\Sigma T_n = \frac{1}{6}\left[2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n\right]$$

$$S_{10} = \frac{1}{6}\left[\frac{2 \times 10 \times 11 \times 21}{6} + \frac{3}{2} \times 10 \times 11 + 10\right]$$

$$= \frac{10}{6}\left[77 + \frac{33}{2} + 1\right] = \frac{315}{2}$$

57. (c) Form group of 4 and 5 person $\frac{9!}{4!5!} = 126$

choose two floor out of 8 floor ${}^8C_2 = 28$

Total ways in which 4 & 5 persons can leave at different floor $126 \times 28 \times 2 = 7056$

58. (c) $\frac{2+4+\alpha+\beta+8+12+14}{7} = 8$

$\alpha + \beta = 56 - 40 = 16 \Rightarrow \alpha + \beta = 16$ …(i)

Variance $= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$16 = \frac{2^2+4^2+\alpha^2+\beta^2+8^2+12^2+14^2}{7} - 64$

$560 = \alpha^2 + \beta^2 + 424$

$\Rightarrow \alpha^2 + \beta^2 = 136$ …(ii)

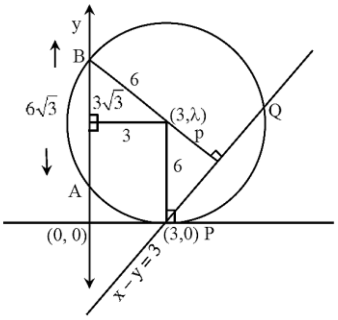
From (i) & (ii) $\alpha = 6 \& \beta = 10$

$x_1 = 3\alpha + 2 = 20; x_2 = 2\beta + 1 = 21$

Q.E. : $x^2 - 41x + 420 = 0$

59.(c) Probability $= \frac{\underline{6} \times \underline{6}}{\left(\frac{\underline{12}}{(\underline{2})^6 \underline{6}}\right) \times \underline{6}} = \frac{16}{231}$

60. (c) Centre of circle is (3,6) and radius = 6 units.



$$PQ = 2\sqrt{6^2 - p^2} \ \therefore p = \frac{6}{\sqrt{2}}$$

$$PQ = 2\sqrt{36 - \frac{36}{2}}$$

$$= 2 \times \frac{6}{\sqrt{2}} = 6\sqrt{2}$$

61. (d) $e = \frac{\sqrt{3}}{2}$

$\frac{a}{e} = \frac{4\sqrt{6}}{3}$

$a = 2\sqrt{2}$

$(ae)^2 = a^2 - b^2$

$b^2 = 2$

so $E: \frac{x^2}{8} + \frac{y^2}{2} = 1$

Now $e_H = 2\sqrt{2} \Rightarrow\ b^2 = 7a^2$

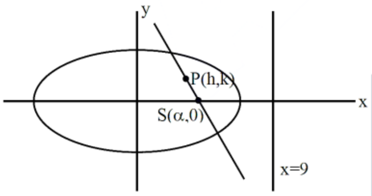
$\frac{2\,b^2}{a} = 2\sqrt{2} \Rightarrow a = \frac{\sqrt{2}}{7}$

$(ae_H)^2 = a^2 + b^2 = 8a^2 = \frac{16}{49}$

$ae_H = \frac{4}{7}$

Distance between foci $= 2ae_H = \frac{8}{7}$

62. (a)



$\frac{a}{e} = 9 \Rightarrow a = 3$

$b^2 = a^2(1 - e^2) = 9\left(1 - \frac{1}{9}\right) = 8$

$\frac{x^2}{9} + \frac{y^2}{8} = 1$

$S(\alpha, 0) \equiv S(1,0)$

$T = S_1 \Rightarrow \frac{hx}{9} + \frac{ky}{8} = \frac{h^2}{9} + \frac{k^2}{8}$

$(1,0) \Rightarrow \frac{h}{9} + 0 = \frac{h^2}{9} + \frac{k^2}{8}$

$h = h^2 + \frac{9}{8}k^2$

$9y^2 = 8x(1 - x)$

63.(a) $\frac{x\sqrt{2}}{\sqrt{2x^2+1}} = \frac{1}{\sqrt{1-x^2}}$

$2(1 - x^2) = (2x^2 + 1)$

$2 - 2x^2 = 2x^2 + 1$

$x^2 = \frac{1}{4}$

$x = \frac{1}{2}$

64. (c) $SD = \frac{\begin{vmatrix} 4-(-2) & 3-6 & 2-5 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}$

$SD = \frac{|6(-10+12)-(-3)(-5+6)-3(4-4)|}{|2\hat{i}-\hat{j}|}$

$SD = \left|\frac{12+3}{\sqrt{5}}\right|$

$= \frac{15}{\sqrt{5}}$

$SD = 3\sqrt{5}$

65.(c) $2\vec{a} + 3\vec{b} = 2(2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}) + 3(6\hat{\imath} + 3\hat{\jmath} + 3\hat{k})$

$\Rightarrow 22\hat{\imath} + 15\hat{\jmath} + 15\hat{k}$

$\vec{a} - \vec{b} = -4\hat{\imath}$

$\text{Area} = \frac{1}{2}|(2\vec{a} + 3\vec{b}) \times (\vec{a} - \vec{b})|$

$= \frac{1}{2}|-60\hat{\jmath} + 60\hat{k}| = \frac{1}{2}\sqrt{(60)^2 \times 2}$

$A = \frac{60}{\sqrt{2}}$

Square of area is $A^2 = 1800$

66. (c) $\lim_{x\to 2} \frac{\tan(x-2)}{x-2} \cdot \frac{[rx^2+(p-2)x-2p]}{x-2} = 5$

$\Rightarrow \lim_{x\to 2} \frac{rx^2 - 2x + p(x-2)}{x-2} = 5$

$\because D^r \to 0 \therefore N^r \to 0 \Rightarrow r = 1$

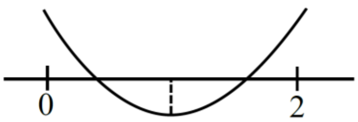
$\Rightarrow \lim_{x\to 2} \frac{x(x-2) + p(x-2)}{x-2} = 5$

$\Rightarrow 2 + p = 5$

$\Rightarrow p = 3$

Now, quadratic equation is $x^2 - 3x + q = 0$

its both roots lie in $(0,2)$



$D \geq 0, 0 < \frac{-b}{2a} < 2, f(0) > 0, f(2) > 0$

$\Rightarrow 9 - 4q \geq 0$ and $q > 2$

$\Rightarrow q \in \left(2, \frac{9}{4}\right]$

$\alpha = 2, \beta = \frac{9}{4} \Rightarrow \left(2 + \frac{9}{4}\right) = 17$

67.(b) $|A| = 0$

$\Rightarrow 1.(-1-\alpha) - 3(-2-0) - 1(2-0) = 0$

$\Rightarrow \alpha = 3$

$f(x) = \int_0^x (t^2 + 2t + 3)dt; x \in [1,3]$

$f(x) = \frac{x^3}{3} + x^2 + 3x$

$f'(x) = x^2 + 2x + 3$ where $D < 0$

$\Rightarrow f(x)$ is strictly increasing

$M = f(3) = 9 + 9 + 9 = 27$

$m = f(1) = \frac{1}{3} + 1 + 3 = \frac{13}{3}$

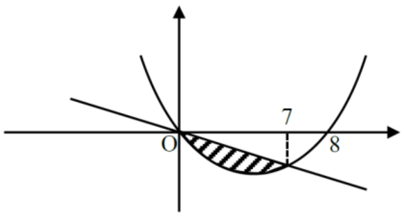
$3(M - m) = 3\left(27 - \frac{13}{3}\right) = 68$

68. (c) $\because f(xy) = f(x)f(y)$

$\therefore f(x) = x^n$ or $f(x) = k$

But $f(0) \neq 0 \Rightarrow f(x) \neq x^n$
For $f(x) = k \Rightarrow k = k \cdot k \Rightarrow k = 1 (\because f(0) \neq 0)$
$\Rightarrow f(x) = 1$
$x^2\, g(x) = \int_1^x (t^2(1) - tg(t)) \cdot dt$
diff w/r to x
$\Rightarrow x^2\, g'(x) + g(x).2x = x^2 - xg(x)$
$\Rightarrow xg'(x) + 3\, g(x) = x$
$\Rightarrow g'(x) + \frac{3}{x} g(x) = 1$
$IF = e^{\int \frac{3}{x} dx} = e^{3\ln(x)} = x^3$
Solution of DE :
$\Rightarrow g(x) \cdot x^3 = \int x^3 dx$
$\Rightarrow x^3 \cdot g(x) = \frac{x^4}{4} + C$
$\Rightarrow C = -\frac{1}{4} \{\because\ g(1) = 0\}$
$g(x) \cdot x^3 = \frac{x^4}{4} - \frac{1}{4}$
Put $x = 2$
$(8)\ g(2) = 4 - \frac{1}{4}$
$g(2) = \frac{15}{32}$.

69. **(a)** Required area $= \int_0^7 (7x - x^2) dx$
$= \left[\frac{7x^2}{2} - \frac{x^3}{3}\right]_0^7 = \frac{343}{6}$



70. **(b)** $I = \int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} . dx$
$I = \int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1} \cdot dx + \int_{-1}^{1} \frac{|x| + 1}{x^2 + 2|x| + 1} \cdot dx$
$I = 0 + \int_{-1}^{1} \frac{|x| + 1}{|x|^2 + 2|x| + 1} \cdot dx$
$2\int_0^1 \frac{1}{x + 1} \cdot dx$
$I = 2[\ln|x + 1|]_0^1$
$I = 2\ell n(2)$

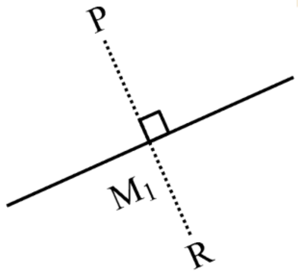
71. **(15)** $x + y \leq e^2 (e^2 \simeq 7.29)$
$R = \{(1,1), (1,2), (1,4), (1,5), (1,6)\}$
$(2,1), (2,2), (2,3), (2,4), (2,5)$
$(3,1), (3,2), (3,3), (3,4)$
$(4,1), (4,2), (4,3)$
$(5,1), (5,2)$
$(6,1)$

For R to be transitive, we have to add
(6,2), (6,3), (6,4), (6,5), (6,6)
(5,3), (5,4), (5,5), (5,6)
(4,4), (4,5), (4,6)
(3,5), (3,6)
(2,3)
= 15 elements

72. **(30)** $(1-x)^3 = 1 - x^3 - 3x(1-x)$
$(1-x)^3 = 1 - x^3 - 3x + 3x^2$
$(1-x)^3 + 3x - 3x^2 = 1 - x^3$
$(1-x^3)^{10} = [(1-x)^3 + 3x(1-x)]^{10}$
$\Rightarrow \; {}^{10}C_r(3x(1-x))^r \cdot ((1-x)^3)^{10-r}$
$(1-x^3)^{10} = \; {}^{10}C_r \cdot 3^r \cdot x^r \cdot (1-x)^{30-2r}$
$\therefore a_r = \; {}^{10}C_r \cdot 3^{\,r}$
$\frac{a_9}{a_{10}} = \frac{{}^{10}C_9 3^9}{{}^{10}C_{10}.\, 3^{10}}$
$\frac{a_9}{a_{10}} = \frac{10}{3}$
$\therefore \frac{9a_9}{a_{10}} = 30$

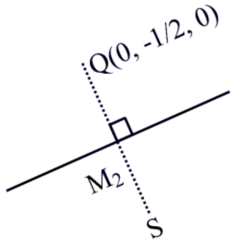
73. **(18)** $x - y = 4$ …(1)
$(x-4)^2 + (y+3)^2 = 9$ …(2)
(α, β) lies on circle
$\therefore (\alpha-4)^2 + (\beta+3)^2 = 9$ …(3)
eq. (1) & (2) $\Rightarrow y^2 + (y+3)^2 = 9 \Rightarrow 2y^2 + 6y = 0$
$\Rightarrow y = 0, -3$
$\therefore x = 4, 1$
$Q(4,0), R(1,-3)$
now
$(PQ)^2 = (PR)^2$
$\Rightarrow (\alpha-4)^2 + \beta^2 = (\alpha-1)^2 + (\beta+3)^2$
$\Rightarrow \alpha + \beta = 1$ …(4)
eq. (3) & (4)
$\Rightarrow \alpha = 4 + \frac{3}{\sqrt{2}}, \beta = -3 - \frac{3}{\sqrt{2}}$
$\therefore (6\alpha + 8\beta)^2 = 18$

74. **(162)** $P(0,-5,0)$

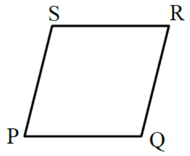


$\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{-2} = k$
Let $R(x_1, y_1, z_1)$
$2k+1, k, -2k-1$
$(2k+1)(2) + (k+5)(1) + (-2k-1)(-2) = 0$
$\Rightarrow 9k + 9 = 0 \Rightarrow k = -1$
$\therefore \; M_1 \Rightarrow (-1,-1,1)$

$R \Rightarrow (-2,3,2)$
Q(0, -1/2, 0)
$\frac{x-1}{-1}=\frac{y+9}{4}=\frac{z+1}{1}=\lambda$
$(-\lambda+1, 4\lambda-9, \lambda-1)$

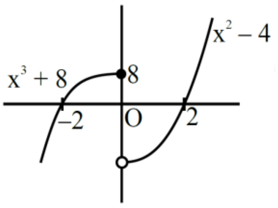


$S(x_2, y_2, z_2)$
$(-\lambda+1)(-1)+\left(4\lambda-\frac{17}{2}\right)(4)+(\lambda-1)(1)=0$
$\Rightarrow 18\lambda-36=0 \Rightarrow \lambda=2$
$M_2 \Rightarrow (-1,-1,1)$
$S \Rightarrow (-2,-3/2,2)$



$PR=\sqrt{4+64+4}=\sqrt{72}=6\sqrt{2}$
$QS=\sqrt{4+1+4}=\sqrt{9}=3$
$\text{Ar.} =\frac{1}{2}\cdot 6\sqrt{2}\cdot 3=9\sqrt{2}$
$(\text{Ar.})^2=(9\sqrt{2})^2=162$ Sq.unit

75. (c) $g(f(x))=\begin{cases}(f(x)-8)^{1/3} & f(x)<0\\ (f(x)+4)^{1/2} & f(x)\geq 0\end{cases}$



$$g(f(x))=\begin{cases}(x^3)^{1/3}, & x<-2\\ (x^2-12)^{1/3}, & 0<x<2\\ (x^3+12)^{1/2}, & -2\leq x\leq 0\\ (x^2)^{1/2}, & x\geq 2\end{cases}$$

$$g(f(x))=\begin{cases}x & , \quad x<-2\\ (x^3+12)^{1/2} & , \quad -2\leq x\leq 0\\ (x^2-12)^{1/3} & , \quad 0<x<2\\ x & , \quad x\geq 2\end{cases}$$

Number of points of discontinuity is equal to 3 .