Physics

Section-A

1. (d)

The given equation can be written as

$$x=\frac{a}{2}\cos[1.5+50.5]t+\frac{a}{2}\cos[50.5-1.5]$$

$$x=\frac{a}{2}\cos[52t]+\frac{a}{2}\cos[49t]$$

Here, $2\pi f_1 \& 2\pi f_2=49$

$$f_1=\frac{52}{2\pi}, f_2=\frac{49}{2\pi}$$

$$\therefore f_{\text{Beat}}=f_1-f_2=\frac{3}{2\pi}\text{ Hz}$$

$$\therefore T_{\text{Beat}}=\frac{1}{f_{\text{Beat}}}=\frac{2\pi}{3}\sec$$

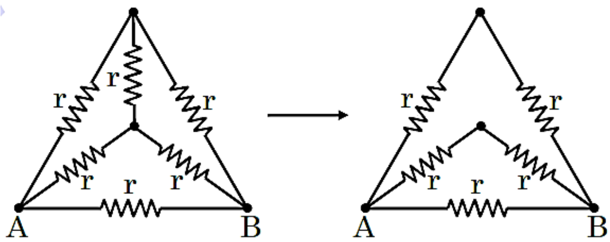
$$=2.09\sec\approx 2\sec$$

2. (c)

$$E=\sqrt{(E_0)^2+(E_0)^2+2(E_0)(E_0)\cos\frac{\pi}{3}}$$

$$E=\sqrt{2E_0^2+E_0^2}=\sqrt{3}E_0=1.73E_0$$

3. (d)



(As balanced wheat stone bridge is formed)

Now, Equivalent resistance between A and B can be written as

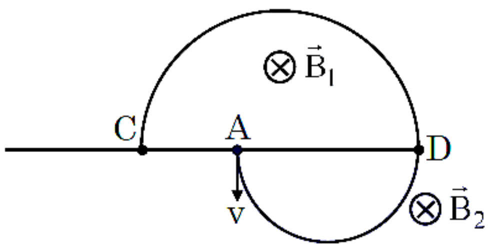
$$\frac{1}{R_{AB}}=\frac{1}{2r}+\frac{1}{2r}+\frac{1}{r}=\frac{2}{r}$$

$$R_{AB}=\frac{R}{12}$$

4. (d)

As $\vec{v}$ is $\perp$ to $\vec{B}$, so charge particle will move in circular path, whose radius is given by

$$R=\frac{mv}{qB}$$



Starting point → A

Ending point → C

$\therefore$ Net displacement = AC

AC = CD − AD

$$AC=\frac{2mv}{qB_1}-\frac{2mv}{qB_2}$$

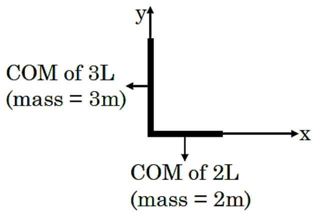
$$AC=\frac{2mv}{qB_1}\left[1-\frac{B_1}{B_2}\right]$$

5. (d)

We know that formula for displacement current is given by

$$\mathrm{id} = \varepsilon_0 \frac{d\phi_\varepsilon}{dt}$$

6. (d)



$$x_{com} = \frac{2\text{ m}(10) + 3\text{ m}(0)}{5\text{ m}} = 4\text{ cm}$$

$$y_{com} = \frac{2\text{ m}(0) + 3\text{ m}(15)}{5\text{ m}} = 9\text{ cm}$$

$$\vec{r}_{com} = 4\hat{i} + 9\hat{j}$$

7. (a)

$$\%\text{ change in } B = \frac{B_{\text{new}} - B_{\text{old}}}{B_{\text{old}}} \times 100\%$$

$$= \frac{\mu n i - \mu_0 n i}{\mu_0 n i} \times 100\% = \frac{(\mu - \mu_0)}{\mu_0} \times 100\%$$

$$= \frac{(\mu_0 \mu_r - \mu_0)}{\mu_0} \times 100\%$$

$$= (\mu_r - 1) \times 100\%$$

$$= \chi_n \times 100\%$$

$$= 1.2 \times 10^{-3}\%$$

8. (b)

As we know,

$$\frac{1}{f} = \left[\frac{\mu_L}{\mu_m} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

For air $\mu_m = 1$

$$\frac{1}{12} = [1.6 - 1]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{12} = \frac{6}{10}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{10}{72}$$

For water

$$\frac{1}{f} = \left[\frac{1.6}{1.28} - 1\right]\left[\frac{10}{72}\right] = \frac{32}{128} \times \frac{10}{72}$$

$$\frac{1}{f} = \frac{1}{4} \times \frac{10}{72}$$

$$f = 28.8\text{ cm}$$

$$f = 288\text{ mm}$$

9. (c)

$i = 5\sqrt{2} + 10\cos\left(650\pi t + \frac{\pi}{6}\right)$

$i^2 = 50 + 100\cos^2\left(650\pi t + \frac{\pi}{6}\right)$

$+(2)(5\sqrt{2})(10)\cos\left(650\pi t + \frac{\pi}{6}\right)$

$<i^2> = 50 + \frac{100}{2} + 0$

$<i^2> = 100$

$<i> = 10\text{Amp.}$

10. (d)

$f_1 = 30\text{ cm}, f_2 = 10\text{ cm}$

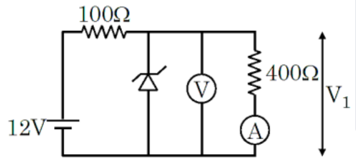
$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$, d = distance between lens

$\frac{1}{f_{eq}} = \frac{1}{0.3} + \frac{1}{0.1} - \frac{0.1}{(0.3)(0.1)}$

$\frac{1}{f_{eq}} = \frac{1}{0.1}$

power $= \frac{1}{f_{eq}} = 10D$

11. (c)



$V_1 = \frac{400}{100+400} \times 12V = \frac{4}{5} \times 12 = \frac{48}{5}V$

here, $V_1 > V_z$, (V_z = Zener Voltage)

So, Zener breakdown will be take place So, voltage across 400Ω will be 4 V

$I = \frac{4}{400}A = \frac{1}{100A} = 10\text{ mA}$

12. (d)

$\theta_1 = 45 + \alpha; \theta_2 = 45 - \alpha$

Time of flight, $T = \frac{2v\sin\theta}{g}$

$\frac{T_1}{T_2} = \frac{\sin(45+\alpha)}{\sin(45-\alpha)}$

$\frac{T_1}{T_2} = \frac{\frac{1}{\sqrt{2}}\cos\alpha + \frac{1}{\sqrt{2}}\sin\alpha}{\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{\sqrt{2}}\sin\alpha}$

$\frac{T_1}{T_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$

13. (a)

$\gamma = 1 + \frac{2}{f}$

f = 6, Triatomic rigid gas

f = 7, Diatomic non-rigid gas

f = 5, Diatomic rigid gas

f = 3, monoatomic rigid gas

$\gamma = 1 + \frac{2}{6} = \frac{4}{3}$ (Triatomic)

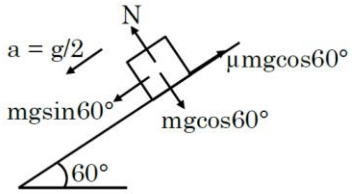
$\gamma = 1 + \frac{2}{7} = \frac{9}{7}$ (Diatomic, non-rigid)

$\gamma = 1 + \frac{2}{5} = \frac{7}{5}$ (Diatomic, rigid)

$\gamma = 1 + \frac{2}{3} = \frac{5}{3}$ (Monoatomic, rigid)

A-III, B-IV, C-I, D-II

14. (a)



mgsin60° – μmgcos60° = ma

$$mg\sin 60^\circ - \mu mg\cos 60^\circ = ma$$

$$g\sin 60 - \mu g\cos 60 = \frac{g}{2}$$

$$\frac{\sqrt{3}}{2} - \frac{\mu}{2} = \frac{1}{2}$$

$$\mu = \sqrt{3} - 1$$

15. (d)

$$\Delta E = 13.6 z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

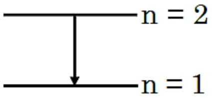
$$(13.6) z^2 \left[\frac{1}{1} - \frac{1}{9}\right] = 108.8$$

$$\frac{(13.6)(8)}{9}(z^2) = 108.8$$

$$z = 3$$

16. (b)

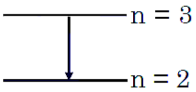
Lyman



$$\frac{1}{\lambda_1} = R\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3R}{4}$$

$$\lambda_1 = \frac{4}{3R}$$

and Balmer

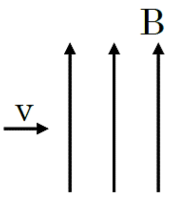


$$\frac{1}{\lambda_2} = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5R}{36}$$

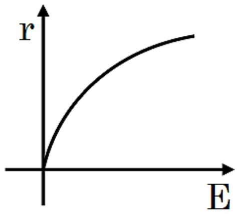
$$\lambda_2 = \frac{36}{5R}$$

Then, $\frac{\lambda_1}{\lambda_2} = \frac{5}{27}$

17. (d)



$$\frac{mv^2}{r} = qvB$$
$$mv = qBr$$
$$E = \frac{1}{2}mv^2$$
$$E = \frac{1}{2}m\left(\frac{q^2B^2r^2}{m^2}\right) = \frac{q^2B^2r^2}{2m}$$
$$E = \left(\frac{q^2B^2}{2m}\right)r^2$$
$r^2 \propto E$



18. (d)

$$\vec{F} = (2t\hat{\imath} + 3\hat{t}\hat{\jmath})\text{N}$$
$$\text{m} = 1000\text{gm} = 1\text{ kg}$$
$$\vec{F} = \text{ma}, \vec{a} = 2t\hat{\imath} + 3t^2\hat{\jmath}$$
$$\frac{d\vec{v}}{dt} = 2t\hat{\imath} + 3t^2\hat{\jmath}$$
$$\vec{v} = t^2\hat{\imath} + t^3\hat{\jmath}$$
$$\text{Power, P} = \vec{F}\cdot\vec{v}$$
$$\text{P} = (2\text{ti} + 3t^2\hat{\jmath})\cdot(t^2\hat{\imath} + t^3\hat{\jmath})$$
$$P = (2t^3 + 3t^5)W$$

19. (b)

$\frac{L_A}{L_B} = \frac{1}{3}$ and $\frac{d_A}{d_B} = 2$

$\Delta L_A = \frac{F_A L_A}{A_A Y_A}$ and $\Delta L_B = \frac{F_B L_B}{A_B Y_B}$

Given, $F_A = F_B$ and $Y_A = Y_B$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{\frac{F_A L_A}{A_A Y_A}}{\frac{F_B L_B}{A_B Y_B}} = \left(\frac{L_A}{L_B}\right)\left(\frac{A_B}{A_A}\right)$$
$$\frac{\Delta L_A}{\Delta L_B} = \left(\frac{L_A}{L_B}\right)\left(\frac{\frac{\pi}{4}d_B^2}{\frac{\pi}{4}d_A^2}\right) = \left(\frac{L_A}{L_B}\right)\left(\frac{d_B}{d_A}\right)^2$$
$$\frac{\Delta L_A}{\Delta L_B} = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{12}$$

20. (a)

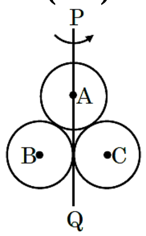
Potential at C

$$V_C = \frac{kq_1}{0.4} + \frac{kq_2}{0.5}$$

Potential at D

$$V_D = \frac{kq_1}{0.4} + \frac{kq_2}{0.1}$$
$$\Delta U = (V_D - V_C)(q_3) = \left(\frac{kq_2}{0.1} - \frac{kq_2}{0.5}\right)(q_3)$$
$$\Delta U = 8kq_2q_3 = \frac{8q_2q_3}{4\pi\varepsilon_0}$$

21. (199)



All bodies have same mass and same radius.

A → Disc

B → Solid sphere

C → Spherical shell

and, $I = \frac{MR^2}{4}$

$$I_{PQ} = \frac{MR^2}{4} + \left(\frac{2}{5}MR^2 + MR^2\right) + \left(\frac{2}{3}MR^2 + MR^2\right)$$

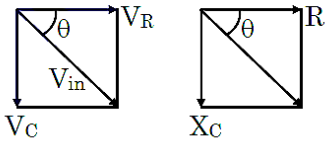
$$I_{PQ} = \frac{15MR^2 + 24MR^2 + 60MR^2 + 40MR^2 + 60MR^2}{60}$$

$$I_{PQ} = \frac{199}{60}MR^2 = \frac{199}{15}\left(\frac{MR^2}{4}\right)$$

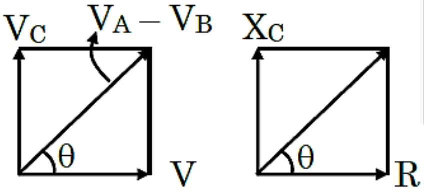
$$= \frac{199}{15}I$$

22. (d)

Input voltage



$V_A - V_B$:



$$\theta + \theta = 90°; \theta = 45°$$

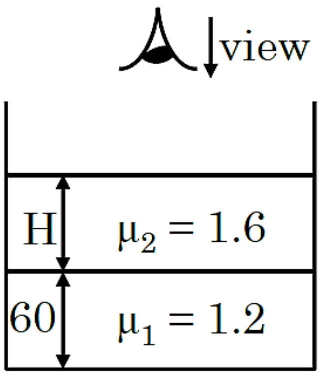
$$\tan\theta = \frac{X_C}{R}$$

$$X_C = R \Rightarrow \frac{1}{W_C} = R$$

$$W = \frac{1}{R_C} = \frac{1}{100 \times 10^3 \times 100 \times 10^{-12}}$$

$$= \frac{10^{12}}{10^7} = 10^5$$

23. (80)



y = apparent depth of bottom

$\frac{y}{1} = \frac{H}{1.6} + \frac{60}{1.2}$

Shift = 40

$H + 60 - y = 40$

$H + 60 - \frac{H}{1.6} - \frac{60}{1.2} = 40$

$\frac{6}{16} H = 30$

H = 80 cm

24.(5)

L = 10 cm, d = 0.5 mm, T = 1727°C = 2000 K Power, P = 94.2 W

$P = \varepsilon\sigma A T^4$

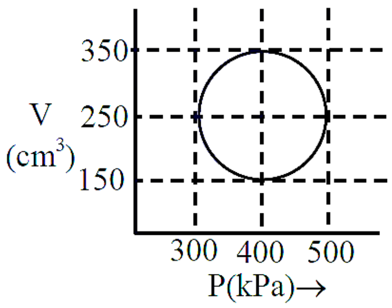
$94.2 = \varepsilon \times (6 \times 10^{-8})(\pi d L)(2000)^4$

$94.2 = \varepsilon \times (6 \times 10^{-8})(3.14)(0.5)(10^{-3})$

$(10 \times 10^{-2})(2000)^4$

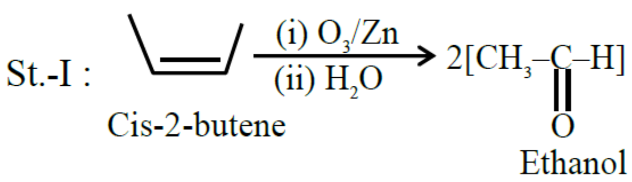
$\varepsilon = \frac{94.2}{(94.2)(16)} = \frac{5}{8}$

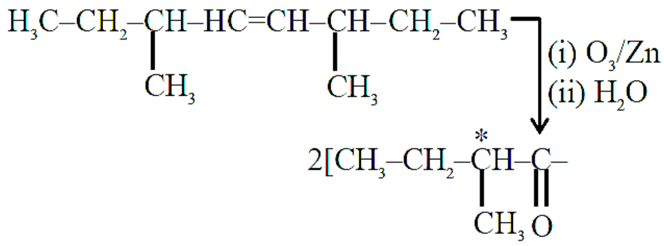
25. An ideal gas has undergone through the cyclic process as shown in the figure. Work done by the gas in the entire cycle is $\times 10^{-1}$ J.
(Take $\pi = 3.14$)



Chemistry
Section-A

26. (c)



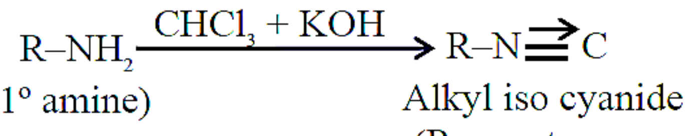


St-I : Correct statement

St-II : In correct statement because product has chiral centre.

27. (c)

Only 1° or primary amines gives positive carbylamines test.



Option (A) and (C) are primary amine and given ⊕ ve carbyl amine test

28.(c)

$A(g) \longrightarrow 2B(g) + C(g)$

$t = 0 \; P_0$

$t \rightarrow \infty \; 0 \; 2P_0 \; P_0$

$P_\infty = 3P_0 = 240$

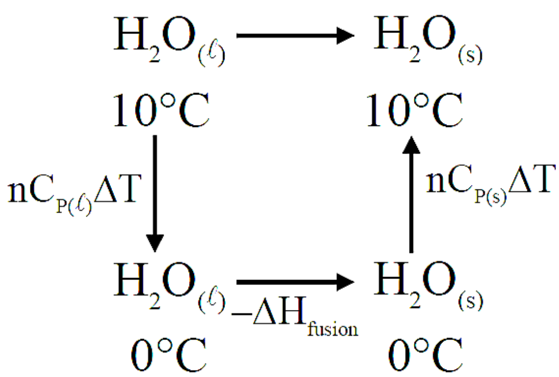
$P_0 = 80$ mm of Hg

$$Kt = \ell n\left(\frac{P_\infty - P_0}{P_\infty - Pt}\right)$$

$$K \times 10 = \ell n\left(\frac{240 - 80}{240 - 160}\right)$$

$$K = \frac{\ell n2}{10} = 0.0693 \text{ min}^{-1}$$

29. (b)



$\Delta H = 1 \times y(0 - 10) - x \times 1000 + 1 \times z(-10° - 0°)$

$\Delta H = -10(100x + y + z)$ Joule.

30. (b)

$HCl_{(aq)} pH = 1; [H^+] = 10^{-1}$

If equal volume of water is added concentration will become half

$$[H^+]_{sol} = \frac{10^{-1}}{2}$$

$pH = 1.3$

31. (d)

St-I - St-I is correct because both given ether are soluble in water → Di ethyl ether and butan-1-ol are miscible to almost same extent i.e., 7.5 and 9 gm per 100 ml water due to H -bonding

St-II : - St. II is also correct because sodium metal is not used with ethyl alcohol as H_2 gas release with ethyl a below

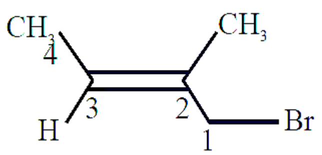
32.(d)

$$\lambda = \frac{C}{v} = \frac{3 \times 10}{5.16 \times 10^{14}}$$

$$\lambda = 581.39 \text{ nm}$$

λ_{Photon} is near & below yellow light it can show photoelectric effect.
If intensity of light decreases photocurrent decreases.
Red light will not produce photoelectric effect.
$v_{\text{Blue}} > v_{\text{yellow}}$ so photoelectric current will be produced.
White light contain all frequencies so it will show photo electric current.
Correct statements are ABD & E .

33.(c)



1-Bromo-2-methyl but-2-ene

34. (d)

$\frac{P_{N_2}}{P_{O_2}} = \frac{x_{N_2}P_T}{x_{O_2}P_T} = \frac{n_{N_2}}{n_{O_2}}$ {using Dalton's law of partial pressure } $= \frac{70/28}{27/32} = 2.96$

$\frac{P_{O_2}}{P_{Ar}} = \frac{n_{O_2}}{n_{Ar}} = \frac{27/32}{3/40} = 11.25$

35. (c)

Mohr's salt : $FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$ Using Kohlrousch law λ_m^∞ (Mohr's salt) $= x_1 + 2x_2 + 2x_3$

36. (c)

Out of Cr, Co, Fe and Ni Chromium has lowest heat of atomisation.
$Cr = [Ar]3d^5 4s^1$
$\because$ Total six valence e^- in Cr.

37. (b)

$$NH_4^+ + NaOH \longrightarrow H_2O + NH_3\uparrow$$

$$2[HgI_4]^{2-} + NH_3 + 3OH^- \longrightarrow \underbrace{HgO.Hg\begin{matrix} NH_2 \\ I \end{matrix}}_{\text{Brown ppt}} + 7I^- + 2H_2O$$

38. (c)

As per given information of ionisation energy
$A = Sn$ & $B = Pb$
$A^{+2} = Sn^{2+} =$ Reducing agent
$B^{+4} = Pb^{+4} =$ Oxidising agent

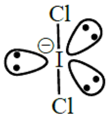
39.(c)

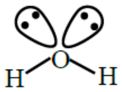
In 3d series Vanadium has highest enthalpy of atomization and colour of V^{+3} is green.
Oxide form by V^{+3} is V_2O_3 (Basic oxide)

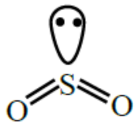
40. (d)

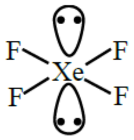
Concept - Those compounds which are more acidic than H_2CO_3 can gives effervescence of CO_2 with aq. $NaHCO_3$.
Release $CO_2 \uparrow$ gas with aq. NaHCO 3
$\Rightarrow [\text{A.S.}]_{\text{Comp.}} > [\text{A.S.}]_{H_2CO_3}$
$\rightarrow$ Option 1,2 and 3 gives effervescence of CO_2 gas with $NaHCO_3$
$\rightarrow$ Option (4) Not gives CO_2 gas with $NaHCO_3$.

41. (b)

(a)  B.P : L.P = 2: 3

(b)  B.P : L.P = 2: 2

(c)  B.P : L.P = 4: 1

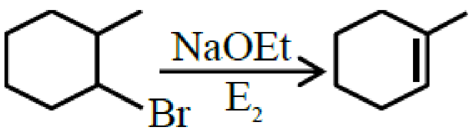
(d)  B.P : L.P = 4: 2

42. (b)
*Before applying medicine
$\frac{dA}{dt} = K[A]$ (First order growth) (Rate law)
$\frac{A}{A_0} = \frac{N}{N_0} = e^{Kt}$
*After applying medicine
Active Bacteria → Inactive Bacteria
(A) (I)
$r = -\frac{dA}{dt} = K[A]^2$(Rate law)
$y = Kx^2$ Parabola

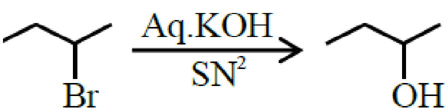
43. (b)
On hydrolysis of sucrose gives D-(+)-glucose and D-(-)-fructose while in St. (1) D-(+)-fructose is given evince St -(1) is incorrect.
St. II - It is correct because sucrose on hydrolysis gives invert sugar

44. (c)
(A) complex is $[Co(NH_3)_5(SO_4)]Cl$
(B) complex is $[Co(NH_3)_5Cl]SO_4$
Both (A) and (B) are Ionisation isomers.

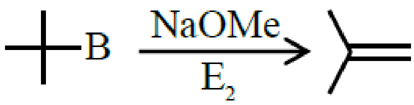
45. (d)



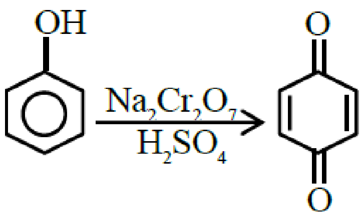
(A)



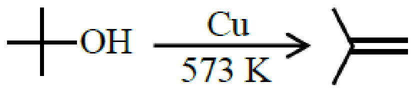
(B)



(C)



(D)



(E) (3^{o} Alcohol)

Option (B) and (D) reaction are not able to form alkene as a product.

Section-B

46. (12)

Organic compound $\underset{\Delta}{\overset{CuO}{\rightarrow}}$ $CO_2 + H_2O$ $\underset{\Delta}{\overset{CuO}{\rightarrow}}$ $CO_2 + H_2O$

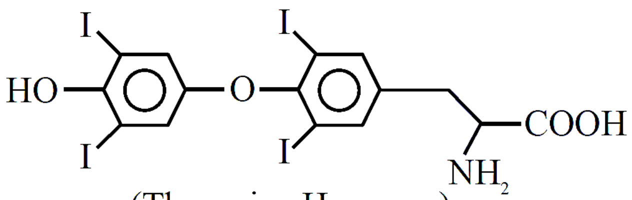
$n_{CO_2} = \frac{220\times10^{-3}}{44} = 5\times10^{-3}$ moles

$m_C = 5\times10^{-3}\times12$

$\%$ m carbon $= \frac{5\times10^{-3}\times12}{500\times10^{-3}}\times100 = 12\%$

Correct answer is 12

47. (65)



(Thyroxine Hormone)

$\rightarrow$ Molecular formula of Thyroxine $\Rightarrow C_{15}H_{11}O_4NI_4$

$\rightarrow$ Molecular mass of Thyroxine -

$C \rightarrow 15\times12 = 180$

$H \rightarrow 11\times1 = 11$

$O \rightarrow 16\times4 = 64$

$N \rightarrow 14\times1 = 14$

$I \rightarrow 127\times4 = 508$

$\rightarrow$ Molecular mass of Thyroxine $\Rightarrow 777$

$\rightarrow$ % of Iodine $= \frac{508}{777}\times100$

$= 65.38\%$

Nearest integer $= 65$

48. (400)

$*\ Cu^{+2} + 2e^{-} \longrightarrow Cu$

(1 faraday = charge on 1 mole electron)

$t = 0$ 1.5 1 mole

$t = t$ 1 $-$ 0.5 mole

$[Cu^{+2}] = 1M$ after electrolysis

$*$	$Ag\oplus + e^{-}$		$\longrightarrow Ag$	
$t = 0$	0.2	0.1 mole		
$t = t$	0.1	$-$	$-$	

$[Ag^{+}] = 0.1M$ after electrolysis

Cell $Cu_{(s)} + 2Ag^{+}_{(aq)} \rightarrow Cu^{+2}_{(aq)} + 2Ag_{(s)}$

reaction

$$E = E^{\circ} - \frac{0.06}{n}\log\frac{[Cu^{+2}]}{[Ag^{+}]^2}$$

$$E = (0.8 - 0.34) - \frac{0.06}{2}\log\frac{1}{(0.1)^2} = 0.4\ V$$

Correct answer $= 400mV$

49. (33)

$MX_3 \rightarrow M^{+3} + 3X^{\ominus}$

$i = 1 + (n-1)\alpha$

$i = 1 + (4-1)\alpha = 2$

$\alpha = \frac{1}{3} = 33.33\% \approx 33\%$

50. (2)

$[FeF_6]^{3-}$	Paramagnetic	$sp^3\,d^2$
$[Fe(CN)_6]^{3-}$	Paramagnetic	d^2sp^3
$[Mn(CN)_6]^{3-}$	Paramagnetic	d^2sp^3
$[Co(C_2O_4)_3]^{3-}$	Diamagnetic	d^2sp^3
$[MnCl_6]^{3-}$	Paramagnetic	sp^3d^2
$[CoF_6]^{3-}$	Paramagnetic	sp^3d^2

Only $[Fe(CN)_6]^{3-}$ and $[Mn(CN)_6]^{3-}$ are paramagnetic and d^2sp^3 hybridisation of metal.

Mathematics

Section-A

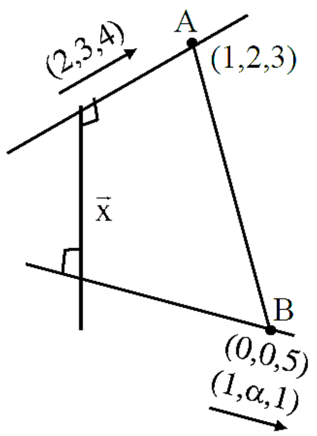
51. (c)

$$\lim_{x\to 0^+}\left(\frac{\tan(5x^{1/3})}{5x^{1/3}}\right)\cdot\left(\frac{(3\sqrt{x})^2}{(\tan^{-1}3\sqrt{x})^2}\right)$$

$$\left(\frac{\ell(1+3x^2)}{3x^2}\right)\left(\frac{5x^{4/3}}{e^{5x^{\frac{4}{3}}}-1}\right)\times\frac{5x^{1/3}\cdot 3x^2}{5x^{4/3}\cdot 9x}$$

$$=\frac{1}{3}$$

52. (d)



$L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

$L_1: \frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$

$\vec{x} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 1 & \alpha & 1 \end{vmatrix} = \hat{\imath}(3-4\alpha) - \hat{\jmath}(-2) + \hat{k}(2\alpha - 3)$

S.D. $= \left|\frac{\overrightarrow{BA} \cdot \vec{n}}{|\vec{n}|}\right| = \left|\frac{(\hat{\imath} + 2\hat{\jmath} - 2\hat{k}) \cdot \vec{n}}{|\vec{n}|}\right|$

$\Rightarrow 6(13-8\alpha)^2 = 25((4\alpha-3)^2 + (2\alpha-3)^2 + 16)$

$6(64a^2 - 280\alpha + 169) = 25(20\alpha^2 - 36\alpha + 34)$

$\Rightarrow 116\alpha^2 + 348\alpha - 164 = 0$

$\alpha_1 + \alpha_2 = \frac{-348}{116} = -3$

53. (a)

$f(x) = x^2 + ax^2 + b\ell n|x| + 1, x \neq 0$

$f'(x) = 3x^2 + 2ax + \frac{b}{x}$

$f'(-1) = 3 - 2a - b = 0$

$f'(-2) = 12 + 4a - \frac{b}{2} = 0$

$a = \frac{-9}{2}, b = 12$

$f'(x) = 3x^2 - 9x + \frac{12}{x} = \frac{3(x+1)(x+2)^2}{x}$

Max. at $\mathrm{n} = -1$

$f(x) = x^2 - \frac{9}{2}x^2 + 12\ln|x| + 1$

$\mathrm{f}(-1) = -1 - \frac{9}{2} + 1 = -\frac{9}{2}$

$\mathrm{M} = -4.5$

Min. value at $\mathrm{x} = -2$

$\mathrm{f}(-2) = -8 - 18 + 12\ell\mathrm{n}2 + 1$

$\mathrm{m} = -25 + 12\ell\mathrm{n}2 = -16.6$

$|\mathrm{M} + \mathrm{m}| = 21.1$

54. (b)

Let $\mathrm{N} = ((64)^{64})^{64}$

$\mathbf{N} = (64)^{64^2}$

$\mathbf{N} = (1+63)^{64^2}$, let $64^2 = \mathrm{n}$

Expanding by binomial

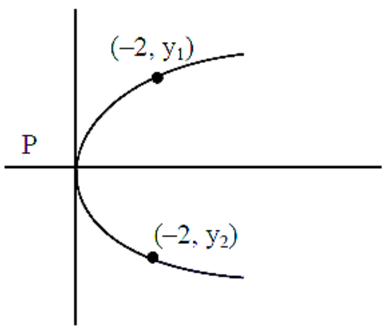
$N = (1+63)^n = 1 + {}^nC_1 63 + {}^nC_2(63)^2 + \cdots \ldots$

$= 1 + 63\lambda = 1 + 7(9\lambda)$

Remainder when divided by 7 is 1

55. (a)

Sol. Equation of parabola



$$(x+2)^2+(y-1)^2=\left(\frac{2x+y+2}{\sqrt{5}}\right)^2$$

$5[(x+2)^2+(y-1)^2]=(2x+y+2)^2$

Put $x=-2, 5(y-1)^2=(y-2)^2$

$5(y^2-2y+1)=y^2-4y+4$

$$\Rightarrow 4y^2-6y+1=0 \Rightarrow y_1+y_2=\frac{3}{2}$$

56.(d)

$x(x^2+e^x)dy+(e^x(x-2)y-x^3)dx=0$

$$x(x^2+e^x)\frac{dy}{dx}+e^x(x-2)y=x^3$$

$$\frac{dy}{dx}+\frac{e^x(x-2)}{x(x^2+e^x)}y=\frac{x^2}{x^2+e^x}$$

$$\text{I.F.}=e^{\int\frac{e^x(x-2)}{x(x^2+e^x)}dx}=e^{\int\frac{e^x\left(\frac{1}{x^2}\frac{2}{x^2}\right)dx}{\left(1+\frac{e^x}{x^2}\right)}dx}$$

$$\text{Let } 1+\frac{e^x}{x^2}=t\Rightarrow\frac{x^2e^x-e^x2x}{x^4}dx=dt$$

$$\Rightarrow \text{ I.F. } e^{\ln\left(1+\frac{e^x}{x^2}\right)}=1+\frac{e^x}{x^2}$$

$$\text{Now } y\left(1+\frac{e^x}{x^2}\right)=\int\frac{x^2}{x^2+e^x}\cdot\frac{x^2+e^x}{x^2}dx+C$$

$$y\left(1+\frac{e^x}{x^2}\right)=x+C$$

Passing through (1,0)

$\Rightarrow C=-1$

$$y=\frac{x-1}{1+\frac{e^x}{x^2}}$$

$$y(2)=\frac{1}{1+\frac{e^2}{4}}=\frac{4}{4+e^2}$$

57. (b)

7 Batsmen & 6 Bowlers

To select 10 players including atleast

4 Batsmen & 4 Bowlers

Captain & vice-captain already selected

No. of ways $= {}^6C_5\times{}^5C_3+{}^6C_4\times{}^5C_4+{}^6C_3\times{}^5C_5$

$=6\times10+15\times5+20\times1$

$=60+75+20=155$

58.(d)

$x=3\left(\dfrac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}\right)$

$x-\sqrt{3}\tan\theta=3\tan\theta+3\sqrt{3}$

$\tan\theta=\dfrac{x-3\sqrt{3}}{3+\sqrt{3}x}\ldots(1)$

$2\left(\dfrac{\tan\theta+\frac{1}{\sqrt{3}}}{1-\frac{\tan\theta}{\sqrt{3}}}=y\right)$

$2(\sqrt{3}\tan\theta+1)=y(\sqrt{3}-\tan\theta)\ldots.$

using (1) and (2)

$2\left(\dfrac{x-3\sqrt{3}}{\sqrt{3}+x}+1\right)=y\left(\sqrt{3}-\dfrac{(x-3\sqrt{3})}{\sqrt{3}(\sqrt{3}+x)}\right)$

$2\sqrt{3}(x-3\sqrt{3}+x+\sqrt{3})=y(3(\sqrt{3}+x)-x+3\sqrt{3})$

$4\sqrt{3}x-12=y(2x+6\sqrt{3})$

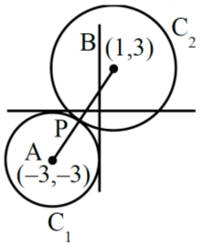
$xy-2\sqrt{3}x+3\sqrt{3}y-6=0$

$\Rightarrow\alpha=-2\sqrt{3},\beta=3\sqrt{3},\gamma=-6$

$\alpha^2+\beta^2+\gamma^2=12+27+36=75$

59. (c)

$C_1:(x+3)^2+(y+3)^2=3^2$



Let C_1 and C_2 has centres $A(-3_1-3)$ and $B(1,3)$

$AB=\sqrt{16+36}=2\sqrt{13}$

$r_1=3$ and $r_2=2\sqrt{13}-3$

$P(\alpha,\beta),\alpha=\dfrac{r_1(1)+r_2(-3)}{r_1+r_2},\beta=\dfrac{r_1(3)+r_2(-3)}{r_1+r_2}$

$\alpha=\dfrac{3-3(2\sqrt{13}-3)}{2\sqrt{13}},\beta=\dfrac{18-6\sqrt{13}}{2\sqrt{13}}$

$(\beta-\alpha)^2=\left(\dfrac{6}{2\sqrt{13}}\right)^2$

$(\beta-\alpha)^2=\left(\dfrac{6}{2\sqrt{13}}\right)^2$, $m+n=22$

60. (c)

$$I=\int_0^{\pi}\frac{(x+3)\sin x}{1+3\cos^2 x}dx$$

$$I=\int_0^{\pi}\frac{(\pi-x+3)\sin x}{(1+3\cos^2 x)}dx$$

$$2I=\int_0^{\pi}\frac{(\pi+6)\sin x\cdot dx}{(1+3\cos^2 x)}=2\int_0^{\pi/2}\frac{(\pi+6)\sin x}{(1+3\cos^2 x)}$$

$$I=\int_0^{\pi/2}\frac{(\pi+6)\sin x\cdot dx}{(1+3\cos^2 x)}=\frac{\pi}{3\sqrt{3}}(\pi+6)$$

$\sqrt{3}\cos x=t$

$\sqrt{3}\sin x=dt$

61. (c)

$S_1:|z|=1,\frac{z-i}{z+i}=\frac{\bar{z}+i}{\bar{z}-i}$

$\Rightarrow(z-i)(\bar{z}-i)=(z+i)(\bar{z}+i)$

$|z|^2-i(z+\bar{z})-1=|z|^2+i(z+\bar{z})-1$

$i(z+\bar{z})=0$

$z+\bar{z}=2\cos\theta=0\Rightarrow\cos\theta=0$

$z=0+0i,|z|\neq1$

$S_1:\frac{z-1}{z+1}+\frac{\bar{z}-1}{\bar{z}+1}=0$

$(z-1)(\bar{z}+1)+(z+1)(\bar{z}-1)=0$

$\Rightarrow|z|^2+(z-\bar{z})-1+|z|^2+(z-\bar{z})-1=0$

$|z|^2=1$

62.(d)

Actual means $=\mu=\frac{100(40)-50+40}{100}$ $\mu=40-\frac{1}{10}=39.9$

Incorrect variance

$(5.1)^2=\frac{\sum x_i^2}{100}-(\bar{x})^2$

$\sum x_i^2=100\times(40^2)+100(5.1)^2$

$\sum x_i^2=16\times10^4+(5.1)^2\times100=162601$

$\sigma^2=\frac{\sum x_i^2-50^2+40^2}{100}-(\mu)^2$

$\sigma^2=1617.01-(39.9)^2=25$

$\sigma=5$

$10(\mu+\sigma)=10(39.9+5)$

$=10\times44.9=449$

63.(d)

$x_1,x_2,x_3,x_4\to$ G.P.

Let $a,ar,ar^2,ar^3\to$ G.P.

Now a -2, ar - 7, ar $^2-9,ar^3-5\to$ A.P.

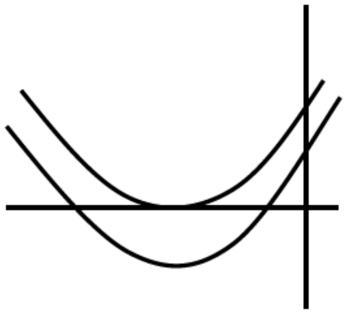
$2(ar-7)=a-2+ar^2-9$ .... (i)

$2(ar^2-9)=ar-7+ar^3-5$ ... (ii)

Solving $r=2,a=-3$

$\therefore$ Product $=x_1,x_2,x_3,x_4=a^4r^6=81\times64$

64. (c)



(i) D ≥ 0

(ii) $\frac{-b}{2a} < 0$

(iii) a. $f(0) > 0$

$(p+2)^2 - 4(2p+9) \geq 0$

$(p+4)(p-8) \geq 0 \; p+2 < 0 \; 2p+9 > 0$

Intersection p $\in \left(-\frac{9}{2}, -4\right]$

$\therefore \beta - 2\alpha = -4 + 9 = 5$

65.(d)

$|\text{adj}(\text{adj})(\text{adj}A)| = 81$

$\Rightarrow |\text{adj}A|^4 = 81$

$\Rightarrow |\text{adj}A| = 3$

$\Rightarrow |A|^2 = 3$

$\Rightarrow |A| = \sqrt{3}$

$(|A|^4)^{\frac{(n-1)^2}{2}} = |A|^{3n^2-5n-4}$

$\Rightarrow 2(n-1)^2 = 3n^2 - 5n - 4$

$\Rightarrow 2n^2 - 4n + 2 = 3n^2 - 5n - 4$

$\Rightarrow n^2 - n - 6 = 0$

$\Rightarrow (n-3)(n+2) = 0$

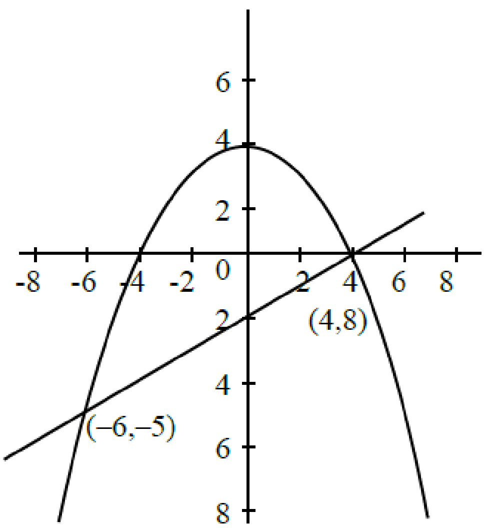
$\Rightarrow n = 3, -2$

$\sum |A^{n^2+n}|$

$= |A^2| + |A^{12}|$

$= 3 + 36 = 3 + 729 = 732$

66. (a)



Area $= \int_{-6}^{4}\left\{\left(4-\frac{x^2}{4}\right)-\left(\frac{x-4}{2}\right)\right\}dx$

$= \int_{-6}^{4}\left\{-\frac{x^2}{4}-\frac{x-6}{2}\right\}dx$

$\alpha = -\frac{x^3}{12}-\frac{x^2}{4}+6x|_{-6}^{4} = \frac{125}{3}$

$\therefore 6\alpha = 250$

67. (b)

$$\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(\lambda+4) \end{vmatrix} = 0$$

$\Rightarrow 12(-21)-3(-39)-(\lambda+4)(-15)=0$

$\Rightarrow -252+117+15(1+4)=0$

$\Rightarrow 15\lambda+177-252=0$

$\Rightarrow 15\lambda-75=0 \Rightarrow \lambda=5$

$$\begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 7 \\ \mu-8 & 0 & 7 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0$$

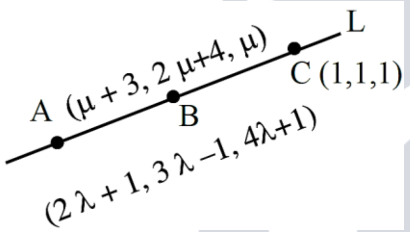
$\Rightarrow 7-7(\mu-8)=0 \Rightarrow 1-(\mu-8)=0 \Rightarrow \mu=9$

$\Rightarrow$ centre of circle (5,9)

radius $=$ length of $\perp$ from centre (5,9) $=$

$\left|\frac{20-27}{5}\right| = \frac{7}{5}$

68. (d)



Dr's of AC $\Rightarrow 2\lambda, 3\lambda-2, 4\lambda$

Dr's of BC $\Rightarrow \mu+2, 2\mu+3, \mu-1$

$\Rightarrow \frac{\mu+2}{2\lambda} = \frac{2\mu+3}{3\lambda-2} = \frac{\mu-1}{4\lambda}$

$\Rightarrow 2(\mu+2) = \mu-1 \Rightarrow \mu=-5$

$\Rightarrow$ Dr's of BC $\Rightarrow 3,7,6$

$\Rightarrow$ equation of L $\Rightarrow \frac{x-1}{3} = \frac{y-1}{7} = \frac{z-1}{6}$

(7,15,13) satisfies.

69. (c)

$\vec{c} = 3\vec{a}+6\vec{b}+9(\vec{a}\times\vec{b})$

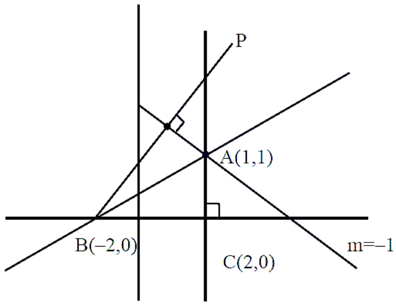
$\sin^{-1}\left(\frac{\sqrt{65}}{9}\right) \Rightarrow \sin\theta = \frac{\sqrt{65}}{9} \Rightarrow \cos\theta = \frac{4}{9}$

$\vec{c}\cdot\vec{a} = 3|\vec{a}|^2+6\vec{a}\cdot\vec{b} = 3+\frac{6\cdot 4}{9} = \frac{51}{9}$

$\vec{c}\cdot\vec{a} = 3\vec{a}\cdot\vec{b}+6|\vec{b}|^2 = \frac{3\cdot 4}{9}+6 = \frac{22}{3}$

$\therefore 9(\vec{c}\cdot\vec{a})-3(\vec{c}\cdot\vec{b}) = 51-22 = 29$

70. (d)



Equation of Altitude AP: $x = 1$
Equation of Altitude BP: $y - 0 = 1(x + 2)$
$\Rightarrow x = 1 \&$
$x - y + 2 = 0$
$P(1,3)$

Area of $\triangle PBC = \frac{1}{2} \times 4 \times 3 = 6$

Section-B

71. (8)

Check for $\left[\frac{x^2}{2}\right]$ and $[\sqrt{x}]$ becomes integers. $\{0,1,\sqrt{2},2,\sqrt{6},\sqrt{8},\sqrt{10},\sqrt{12},\sqrt{14},4\}$ Continuous at 0^+, continuous at 4^-

$\left[\frac{x^2}{2}\right] = [\sqrt{x}]$, occurs at $x = \sqrt{2}$

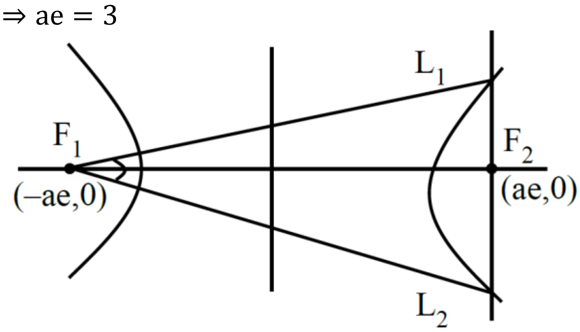
$\Rightarrow$ Not continuous

72. (6)

$A = \{1,2,3\}$
$(1,1),(2,2),(3,3),(1,2) \in R$
Remaining elements are
$(2,1),(2,3),(1,3),(3,1),(3,2)$
(1) If relation contains exactly 4 elements $= 1$ way
(2) if relation contains exactly 5 elements
It can be $(1,3),(3,2) \Rightarrow 2$ ways
(3) If relation contain exactly 6 elements
It can be
$((2,3),(1,3)),((1,3),(3,2)),((3,1),(3,2))$
$\Rightarrow 3$ ways.
Total $= 6$ wavs

73. (1944)

Sol. $f_1 \equiv (-ae, 0) \equiv P(-3,0)$
$\Rightarrow ae = 3$



$\tan 45^\circ = \dfrac{b^2/a}{2ae}$

$2ae = \dfrac{b^2}{a}$

$b^2 = 6a$

Also $a^2e^2 = a^2 + b^2$

$9 = a^2 + 6a$

$a^2 + 6a - 9 = 0$

$a = -3 \pm 3\sqrt{2} = -3(1 \pm \sqrt{2})$

$\therefore a^2b^2 = a^2 \cdot 6a = 6a^3$

$= 6(135\sqrt{2} - 189)$

$\alpha = 810$ and $\beta = 1134$

$\therefore \alpha + \beta = 1944$

74. (36)

$\begin{vmatrix} a & d \\ b & c \end{vmatrix} = ad - bc \Rightarrow ad = bc$

Case-I Exactly 1 no. is used

$\Rightarrow$ All singular $= {}^4C_1$

Case-II Exactly 2 no. is used

$\Rightarrow {}^4C_2 \times 2 \times 2$

Case-III Exactly 3 no. is used

None will be singular

Case-IV Exactly 4 No. is used ad = bc

$\Rightarrow 2 \times 9 = 3 \times 6$

$\begin{vmatrix} 9 & - \\ - & 2 \end{vmatrix} \Rightarrow {}^4C_1 \times 21$

Total = 36

75. (13)

A = {1,2,3,4,5 ... ... n}

No. of subsets having r elements such that no two are consecutive is $= {}^{n-r+1}C_r$

for n = 5, no. of ways $= {}^{6-r}C_r$

Subsets having no element = 1

Subsets having exactly 1 element $= {}^5C_1 = 5$

Subsets having exactly 2 element $= {}^4C_2 = 6$

Subsets having exactly 3 element $= {}^3C_3 = 1$

$\Rightarrow 5 + 6 + 1 + 1 = 13$