Physics
Section-A

1. (a)

Electric field of outside charge is zero inside conductor

2. (b)

$\rho = \frac{M}{V} = \frac{m_n \times A}{\frac{4}{3}\pi R^3} = \frac{m_n \times A}{\frac{4}{3}\pi A R_0^3}$

So ρ is almost is constant

$R = R_0 A^{1/3}$

$R \propto A^{1/3}$

3. (d)

$I = \frac{1}{2} \epsilon_0 E_0^2 \times C$

$$E_0 = \sqrt{\frac{2I}{\epsilon_0 C}}$$

E_0 : electric field

N/C

4. (d)

$$L = \frac{\mu_0 N A}{\ell}$$

$$C = \frac{A\epsilon_0}{d}$$

$$\frac{L}{C} \propto \frac{\mu_0}{\epsilon_0}$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} \propto \sqrt{\frac{L}{C}}$$

$$\frac{L}{C} = \frac{\tau R}{(\tau/R)} = R^2$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = R$$

5. (a)

$K.E = E - W$

$$\lambda_e = \frac{h}{\sqrt{2mK.E}}, E = \frac{hc}{\lambda_i}, W = \frac{hc}{\lambda_0}$$

$$\frac{h^2}{2\,m\lambda_e^2} = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_0}$$

$$\lambda_e = \sqrt{\frac{h}{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}}$$

6. (a)

$\frac{dA}{dt} = \frac{L}{2\,m}$

Due to central force torque is zero & angular momentum is constant.

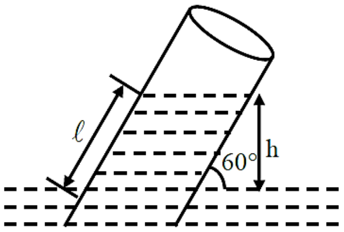
7. (d)

$K.E = \frac{f}{2} KT$

For He and Ar $f = 3$

$$\frac{K \cdot E_{He}}{K \cdot E_{Ar}} = \frac{1}{1}$$

8. (a)



$$h = \frac{2\,T\cos\theta}{\rho g r} = \frac{2 \times 70 \times 1}{1 \times 980 \times 10^{-2}}$$

$$h = \frac{100}{7}\ \text{cm}$$

$$\sin 60^\circ = \frac{h}{\ell}$$

$$\ell = \frac{h \times 2}{\sqrt{3}}$$

$$\ell = \frac{100}{7} \times \frac{2}{\sqrt{3}}$$

$$= \frac{200}{7 \times \sqrt{3}}$$

$$= 16.49\ \text{cm}$$

9. (c)

$$m = -\frac{v}{u} = -\left(\frac{v}{-u}\right) = \frac{v}{u}$$

$$\frac{1}{4} = \frac{v}{u} \Rightarrow u = 4v$$

$$v + u = 40$$

$$5v = 40$$

$$v = 8\ \text{cm}$$

$$u = 32\ \text{cm}$$

$$\frac{1}{v} + \frac{v}{u} = \frac{1}{f}$$

$$\frac{1}{8} - \frac{1}{32} = \frac{1}{f}$$

$$\frac{4-1}{32} = \frac{1}{f}$$

10. (a)

$$E = \frac{V}{d} = \frac{5}{5 \times 10^{-4}} = 10^4\ \text{V/m}$$

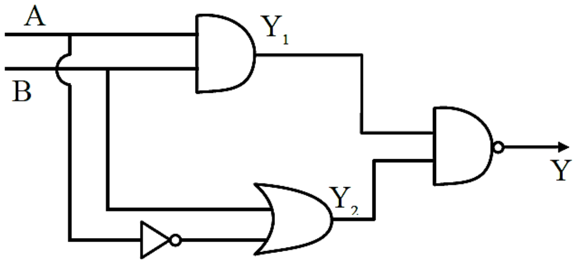
$$\tau = PE\sin\theta$$

Where $P = qa = 2 \times 10^{-6} \times 5 \times 10^{-7}$

$$= 1 \times 10^{-12}\ \text{C} - \text{m}$$

$$\tau = 1 \times 10^{-12} \times 10^4 \times \frac{1}{2} = 5 \times 10^{-9}\ \text{N} - \text{m}$$

11. (a)



$Y_1 = A \cdot B, Y_2 = \overline{A} + B$

$Y = \overline{Y_1 \cdot Y_2} = \overline{Y_1} + \overline{Y_2}$

$Y = \overline{A \cdot B} + \overline{\overline{A} + B}$

$Y = \overline{A} + \overline{B} + A \cdot \overline{B}$

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

12. (c)

(A) Mass density $= \frac{M}{V} = M^1L^{-3}$

(B) Impulse $= M \times u = M^1L^1T^{-1}$

(C) Power $= F.V = M^1L^2T^{-3}$

(D) Moment of inertia $= \mathrm{Mr}^2 = \mathrm{M}^1\ \mathrm{L}^2$

13. (a)

Minimum distance between 2 points having same speed is $\frac{\lambda}{2}$

$\lambda = \frac{2\pi}{\mathrm{k}} = \frac{1}{10}\ \mathrm{m} = 10\ \mathrm{cm}$

Distance $= \frac{\lambda}{2} = 5$ cm

14. (c)

Refractive index has no relation with mass density because both have different meaning. Hence reason is incorrect. So (A) is correct but (R) is not correct

15. (a)

Electric field of outside charge is zero inside conductor

16. (c)

Potential energy is defined for conservative force only. It is not defined for non-conservative force i.e. frictional force.

17. (c)

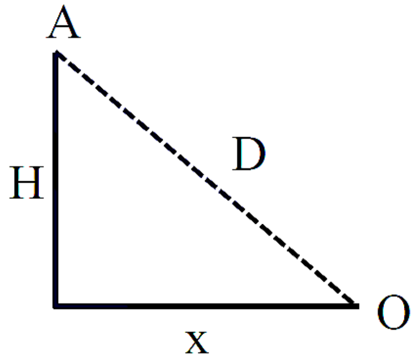
(A) Isothermal $\rightarrow \Delta T = 0 \rightarrow \Delta U = 0$ (IV)

(B) Adiabatic $\rightarrow \Delta Q = 0$ (II)

(C) Isobaric $\rightarrow \Delta P = 0 \rightarrow \Delta U \neq 0$ (III)

(D) Isochoric $\rightarrow \Delta V = 0 \rightarrow \Delta W = 0$ (I)

18. (d)

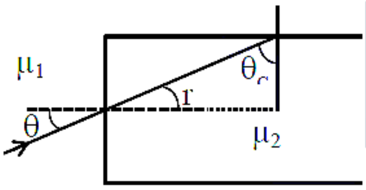


$$u = 360 \times \frac{5}{18} = 100\ m/s$$
$$x = u \times t = 2 \times 10^3\ m$$
$$t = \sqrt{\frac{2H}{g}} \Rightarrow H = \frac{t^2 g}{2}$$
$$H = \frac{400 \times 10}{2}$$
$$H = 2000\ m$$
$$D = \sqrt{x^2 + H^2}$$
$$D = 2\sqrt{2}\ km$$

19. (d)

$$F = M \times a$$
$$v = 4\sqrt{x}$$
$$v^2 = 16x$$
$$2v\frac{dv}{dx} = 16$$
$$\frac{vdv}{dx} = \frac{16}{2} = 8$$
$$F = 0.5 \times 8 = 4\ N$$

20. (c)



$$r + \theta_C = 90^\circ$$
$$\mu_1 \sin\theta = \mu_2 \sin r$$
$$\sin\theta = \frac{\mu_2}{\mu_1}\sin(90 - \theta_C)$$
$$\sin\theta = \frac{\mu_2}{\mu_1}\cos\theta_C$$
$$\sin\theta_C = \frac{\mu_1}{\mu_2}$$
$$\sin\theta = \frac{\mu_2}{\mu_1}\sqrt{1 - \frac{\mu_1^2}{\mu_2^2}}$$
$$\sin\theta = \sqrt{\frac{\mu_2^2 - \mu_1^2}{\mu_1^2}} = \sqrt{\frac{\frac{25}{16} - 1}{1}}$$
$$\sin\theta = \frac{3}{4}$$
$$\theta = \sin^{-1}\left(\frac{3}{4}\right)$$

Section-B

21. (5)

$$Q_{in} = Q\left(1 - \frac{1}{K}\right)$$
$$4 \times 10^{-6} = 5 \times 10^{-6}\left(1 - \frac{1}{K}\right)$$
$$1 - \frac{1}{K} = \frac{4}{5}$$
$$K = 5$$

22. (a)

$$P = V_{rms} I_{rms} \cos\phi$$

$$P = V_{rms} \times \frac{V_{rms}}{z} \times \frac{R}{Z}$$

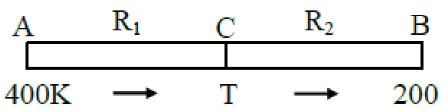
$$P = V_{ms}^2 \times \frac{R}{z^2}$$

$$Z = \sqrt{R^2 + (x_L - x_C)^2}$$

$$Z = 50\sqrt{2}\Omega$$

$$P = 100 \times \frac{50}{2500 \times 2} = 1\ \text{W}$$

23. (360)



$$R_1 = \frac{\ell_1}{K_1\, A_1}, R_2 = \frac{\ell_2}{K_2\, A_2}$$

$$\frac{dQ}{dt} = \frac{\Delta T}{R}$$

$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{\mu}\right)_2$$

$$\frac{400 - T}{R_1} = \frac{T - 200}{R_2}$$

$$\frac{400 - T}{T - 200} = \frac{R_1}{R_2} = \left(\frac{\ell_1}{\ell_2}\right)\left(\frac{r_2}{r_1}\right)^2 \times \frac{K_2}{K_1}$$

$$= \frac{1}{2} \times \left(\frac{1}{2}\right)^2 \times 2$$

$$= \left(\frac{1}{4}\right)$$

$$\frac{400 - T}{T - 200} = \frac{1}{4}$$

$$1600 - 4\,T = T - 200$$

$$5\,T = 1800$$

$$T = 360\ \text{K}$$

24. (15)

$$\phi = \vec{E} \cdot \overrightarrow{A} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3 \cdot A\hat{j}$$

$$6 = 4 \times 10^3\ A$$

$$A = 1.5 \times 10^{-3}\ m^2$$

$$= 15\ cm^2$$

25. (9)

Without cavity $I_1 = \frac{MR^2}{2}$

Mass of removed disc $= \frac{M}{\pi R^2} \times \left(\frac{R}{3}\right)^2 \pi$

$$= \left(\frac{M}{9}\right)$$

M.I. of removed disc $I_2 = \frac{\frac{M}{9}\left(\frac{R}{3}\right)^2}{2} + \frac{M}{9} \times \left(\frac{2R}{3}\right)^2$

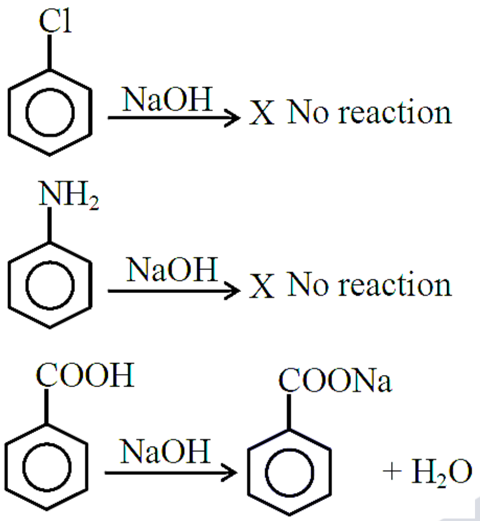
$$= \frac{MR^2}{18}$$

$$I = I_1 - I_2 = \frac{MR^2}{2} - \frac{MR^2}{18} = \frac{4MR^2}{9}$$

$(n = 9)$

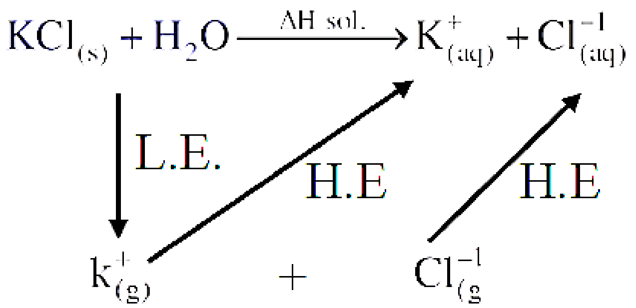
26. (c)
(i) Protein does not gives β-amino acid on hydrolysis
(ii) Fibrous protein are not water soluble
So both statement's are wrong

27. (d)



Organic layer in funnel are mixture of chloro benzene and aniline

28.(c)



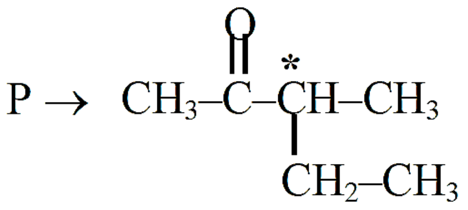
$$\Delta H_{Sol^n.} = L.E. + (H.E)_{K^+_{(g)}} + (HE)_{Cl^{-1}_{(g)}}$$
$$= Z - x - y$$
$$= z - (x + y)$$

29.(c)

	$A_{(g)}$	$\rightarrow$	$B_{(g)}$	+	$C_{(g)}$
$t = 0$	P^o		0		0
$t = t$	$P^o - x$		x		x
$t = \infty$	0		P^o		P^o

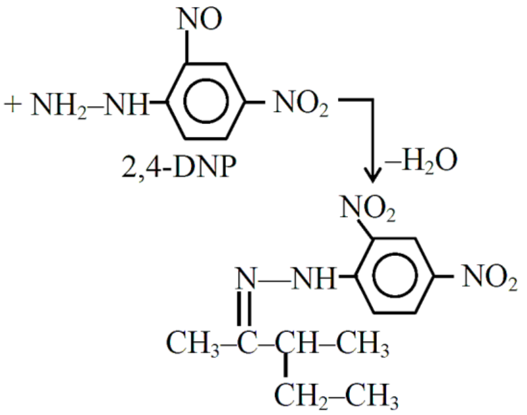
$$P_t = P^\circ + x \Rightarrow x = P_t - P^\circ = P_t - \frac{P_\infty}{2}$$
$$P_\infty = 2P^\circ \Rightarrow P^\circ = \frac{P\infty}{2}$$
$$k = \frac{1}{t}\ln\frac{P^\circ}{P^\circ - x}$$
$$k = \frac{1}{t}\ln\frac{P_\infty}{2(P_\infty - P_t)}$$

30. (b)



Chiral

Does not give
Tollen's test



Gives test with
2,4-DNP

31. (c)

Aromatic halide give nucleophilic substitution reaction at high temperature or in presence of $-I/-M$ group rate of reaction high even at low temperature.

A-IV B-III
C-II D-I

32. (d)

For standard state $\Rightarrow$ pressure $=1$ bar and temperature is specified only

$\Rightarrow (\Delta H_f^\circ)_{O_{2,(g)}} = 0$

33. (c)

$P_A^0 < P_B^0$

$\frac{P_A^\circ}{P_B^\circ} < 1$

$\frac{y_A}{y_B} = \frac{P_A^\circ}{P_B^\circ}\frac{x_A}{x_B}$

$\frac{\frac{y_A}{y_B}}{\frac{x_A}{x_B}} < 1$

$\frac{y_A}{y_B} < \frac{x_A}{x_B}$

34. (a)

$CrO_3 =$ Acidic

$Mn_2O_7 =$ Acidic

$\therefore x = 2$

$\left[\overset{III}{Co}(H_2NCH_2CH_2NH_2)_3\right]_2 (SO_4)_3 \rightleftharpoons$

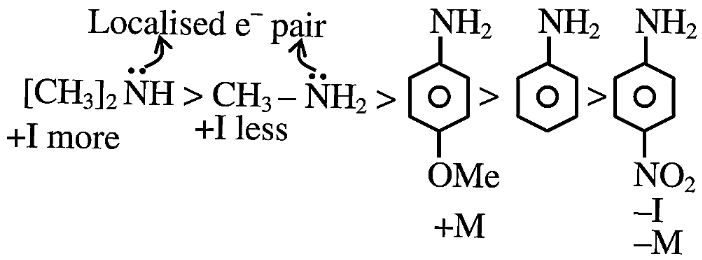
$_2\left[\overset{III}{Co}(H_2NCH_2CH_2NH_2)_3\right]^{3+} + 3SO_4^{2-}$

$\therefore$ Primary valency $= 3$

$\therefore x + y = 5$

35. (b)

E > D > B > A > C



36.(b)

Primary valency = Oxidation state

Secondary valency = Co-ordination number

	Complex	Primary valency	Secondary
(A)	$[Co(en)_2Cl_2]Cl$	3	6
(B)	$[Pt(NH_3)_2Cl(NO_2)]$	2	4
(C)	$Hg[Co(SCN)_4]$	3	4
(D)	$[Mg(EDTA)]^{2-}$	2	6

37. (a)

(A) Solution of chloroform and acetone shows -ve deviation, so maximum boiling azeotrope.

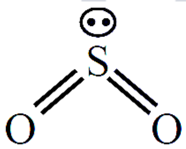
(B) Solution of ethanol & water shows +ve deviation. So minimum boiling azeotrope.

(C) Solution of benzene and toluene form ideal solution. $\Delta V_{mix} = 0$.

(D) Acetic acid in benzene form dimer.

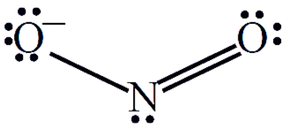
38.(a)

SO_2



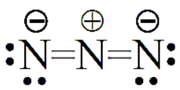
sp^2

NO_2^-



sp^2

N_3^-

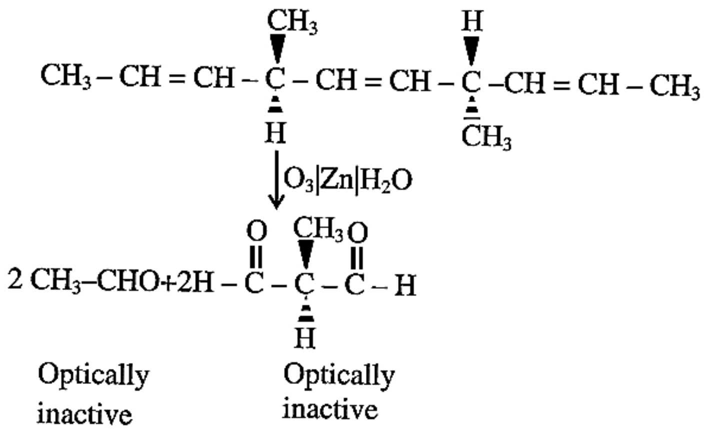


sp

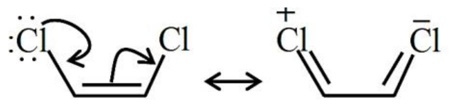
39.(a)

$[Fe(CN)_6]^{3-}$	Fe^{3+}	$3d^5$	$t_{2g}^{2,2,1}$	$e_g^{0,0}$	unpaired $e^- = 1$
$[FeF_6]^{3-}$	Fe^{3+}	$3d^5$	$t_{2g}^{1,1,1}$	$e_g^{1,1}$	unpaired $e^- = 5$
$[CoF_6]^{3-}$	Co^{3+}	$3d^6$	$t_{2g}^{2,1,1}$	$e_g^{1,}$	unpaired $e^- = 4$
$[Mn(CN)_6]^{3-}Mn^{3+}$	$3d^4$	$t_{2g}^{2,1,1}$	$e_g^{0,0}$		unpaired $e^- = 2$

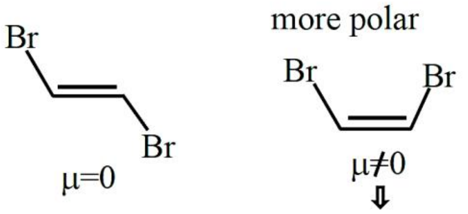
40. (b)



41. (a)

S-I


Due to vacant –d orbital
more polar

S-II


Boiling point higher

42. (c)

Extra stability of half filled is due to :

(i) Symmetrical distribution of electrons

(ii) Large exchange energy

(iii) Smaller coulombic repulsion

(iv) Smaller shielding of electrons by one another

43. (b)

(i) True, $T\ell^{+}$is more stable than $T\ell^{3+}$, due to inert pair effect. So $T\ell^{3+}$ is a powerful oxidising agent.

(ii) True, $E^{\circ}_{Al^{3+}/\alpha 1} = -1.66$ V. So it is difficult to reduce $A\ell^{3+}$. So $A\ell^{3+}$ is highly stable.

(iii) False, as $T\ell^{3+}$ is unstable

(iv) True, $T\ell^{+}$is more stable than $T\ell^{3+}$

(v) True, $A\ell^{3+}$ and $T\ell^{+}$are highly stable

44. (b)

Statement-II ⇒ At cathode, instead of Mg, $H_2O_{(\ell)}$ will reduce & evolve H_2 gas.

45.(a)

In a period from left to right atomic size decreases.

Scetion-B

46.(48)

$$pOH = pK_b + \log\frac{[NH_4^+]}{[NH_3]}$$

$\mathbf{pOH} = 4.745$

on adding 0.05 mole HCl

NH_3	+ $H^{\oplus}$	$\rightarrow$ $NH_4^{\oplus}$
0.1	0.05	0.1
0.05	0	0.15

$pOH' = 4.745 + \log 3$
$pOH' - pOH = 0.477$
$14 - pH' - 14 + pH = 0.477$
$\Delta pH = 0.477$

$47.7 \times 10^{-2} \approx 48 \times 10^{-2}$

47.(18)

Organic compound $\xrightarrow{\text{DUMA'S}}$ N_2
292mg
$V = 50ml$
$P = 715$ mmHg
$T = 300k$
Aq. tension $= 15$ mmHg
$P_{N_2} = 715 - 15 = 700mmHg$

$$P_{N_2} = \frac{700}{760} \text{ atm}$$

$$n_{N_2} = \frac{P_{N_2} \cdot V}{RT}$$

$$n_{N_2} = \frac{700}{760} \times \frac{50}{1000} \times \frac{1}{0.0821 \times 300}$$

$n_N = 2 \times n_{N_2}$

Mass of N $= 2 \times n_N \times 14$

$$\% N = \frac{\text{mass of N}}{\text{mass of organic compound}} \times 100$$

$$\% N = \frac{700}{760} \times \frac{50}{1000} \times \frac{2 \times 14}{0.0821 \times 300} \times \frac{1000}{292} \times 100$$

$\% N = 18\%$

48. (138)

$$C_4H_{10} + \frac{13}{2} O_2 \rightarrow 4CO_2 + 5H_2O$$

3×10^3 10×10^3

Moles of H_2O formed $= n_{H_2O} = 5 \times \frac{2}{13} \times 10 \times 10^3$

Then $w_{H_2O} = \frac{10^5}{13} \times 18$

$= 1.3846 \times 10^5$ g

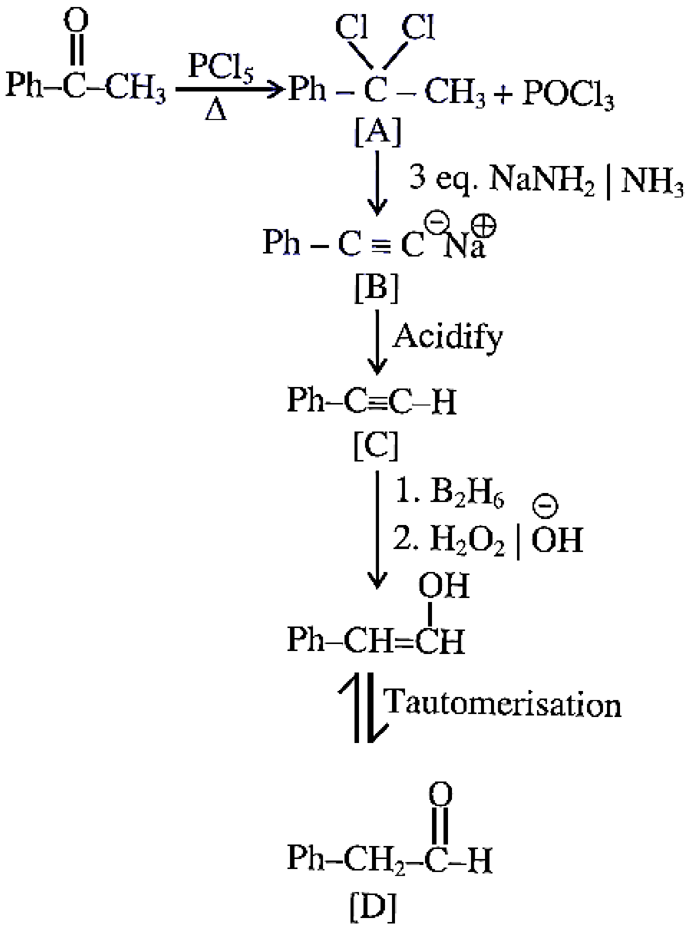
Volume of H_2O will be $= 138.46$ litre.

49.(2)

$[Co(NH_3)_6]^{3+}$ Co^{3+} $3d^6$ $t_{2g}^{2,2,2} e_g^{0,0}$
Diamagnetic (unpaired electron = 0)
$[Co(C_2O_4)_3]^{3-}$ Co^{3+} $3d^6$ $t_{2g}^{2,2,2} e_g^{0,0}$
Diamagnetic (unpaired electron = 0)
$[MnCl_6]^{3-}$ Mn^{3+} $3d^4$ $t_{2g}^{1,1,1}$ $e_g^{1,0}$
Paramagnetic (unpaired electron = 4)
$[Mn(CN)_6]^{3-}$ Mn^{3+} $3d^4$ $t_{2g}^{2,11} e_g^{0,0}$
Paramagnetic (unpaired electron = 2)
$[CoF_6]^{3-}$ Co^{3+} $3d^6$ $t_{2g}^{2,1,1} e_g^{1,1}$
Paramagnetic (unpaired electron = 4)
$[Fe(CN)_6]^{3-}$ Fe^{3+} $3d^5$ $t_{2g}^{2,2,1} e_g^{0,0}$

Paramagnetic (unpaired electron = 1)
$[FeF_6]^{3-}$ Fe^{3+} $3d^5$ $t_{2g}^{1,1,1}e_g^{1,1}$
Paramagnetic (unpaired electron = 5)

50. (7)

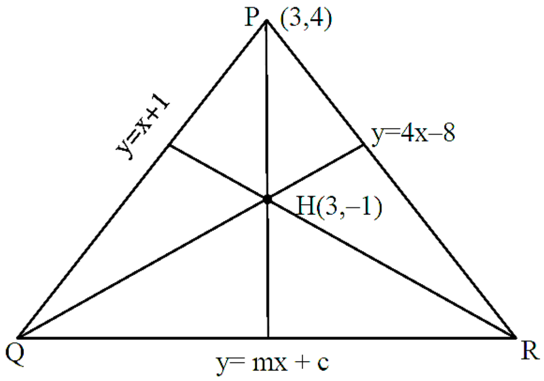


$\Rightarrow$ Number of sp^2 C-atoms in product D = 7
NTA Ans. = 7 ALLEN Ans. = 7

Mathematics

Section-A

51. (a)



Solve line PQ & QR

Point Q $\left(\frac{1-c}{m-1}, \frac{1-c}{m-1}+1\right)$

$$m_{2H}=\frac{\frac{1-c}{m-1}+2}{\frac{1-c}{m-1}-3}=\frac{1-c+2m-2}{1-c-3m+3}=-\frac{1}{4}$$

$\because m_{PH}=\frac{5}{0}\to\infty$

$\Rightarrow$ Slope of line QR(m) $=0$

Put value of m in equation (1)

$\frac{1-c-2}{1-c+3}=-\frac{1}{4}\Rightarrow c=0$

so $m-c=0$ Ans.

52. (c)

$\frac{|\bar{a}+\bar{b}|+|\bar{a}-\bar{b}|}{|\bar{a}+\bar{b}|-|\bar{a}-\bar{b}|}=\sqrt{2}+1$

Apply componendo and dividendo

$$\Rightarrow\frac{2|\bar{a}+\bar{b}|}{2|\bar{a}-\bar{b}|}=\frac{\sqrt{2}+2}{\sqrt{2}}$$

$$\Rightarrow|\bar{a}+\bar{b}|=(1+\sqrt{2})|\bar{a}-\bar{b}|$$

$$\Rightarrow|\bar{a}+\bar{b}|^2=(3+2\sqrt{2})|\bar{a}-\bar{b}|^2$$

$$\Rightarrow 2|\bar{a}|^2+2\bar{a}\cdot\bar{b}=(3+2\sqrt{2})(2|\bar{a}|^2-2\bar{a}\cdot\bar{b})$$

$$\Rightarrow 2|\bar{a}|^2(2+2\sqrt{2})=2\bar{a}\cdot\bar{b}(4+2\sqrt{2})$$

$$\Rightarrow\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|^2}=\frac{2+2\sqrt{2}}{4+2\sqrt{2}}=\frac{1}{\sqrt{2}}$$

Now

$$\frac{|\bar{a}+\bar{b}|^2}{|\bar{a}|^2}=1+\frac{|\bar{b}|^2}{|\bar{a}|^2}+\frac{2\bar{a}\cdot\bar{b}}{|\bar{a}|^2}$$

$$=1+1+2\left(\frac{1}{\sqrt{2}}\right)=2+\sqrt{2}$$

53. (a)

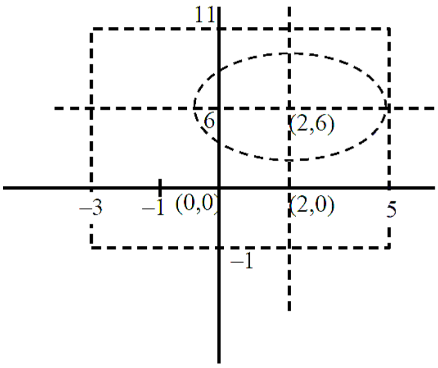
A : $|x-1|\le 4$ and $|y-5|\le 6$

$\Rightarrow -4\le x-1\le 4\Rightarrow -6\le y-5\le 6$

$\Rightarrow -3\le x\le 5\ \Rightarrow -1\le y\le 11$

B : $16(x-2)^2+9(y-6)^2\le 144$

B : $\frac{(x-2)^2}{9}+\frac{(y-6)^2}{16}\le 1$



From Diagram $B\subset A$

54.(d)

$$y=\frac{5-x}{x^2-3x+2}$$

$$yx^2-3xy+2y+x-5=0$$

$$yz^2+(-3y+1)x+(2y-5)=0$$

Case I : If $y=0$ (Accepted)

$\Rightarrow x = 5$

Case II : If $y \neq 0$

$D \geq 0$

$(-3y + 1)^2 - 4(y)(2y - 5) \geq 0$

$9y^2 + 1 - 6y - 8y^2 + 20y \geq 0$

$y^2 + 14y + 1 \geq 0$

$(y + 7)^2 - 48 \geq 0$

$|y + 7| \geq 4\sqrt{3}$

$\Rightarrow y + 7 \geq 4\sqrt{3}$ or $y + 7 \leq -4\sqrt{3}$

$\Rightarrow y \geq 4\sqrt{3} - 7$ or $y \leq -4\sqrt{3} - 7$

From Case I and Case II

$y \in (-\infty, -4\sqrt{3} - 7] \cup [4\sqrt{3} - 7, \infty)$

So $\alpha = -4\sqrt{3} - 7$

$\beta = 4\sqrt{3} - 7$

$\Rightarrow a^2 + b^2 = (-4\sqrt{3} - 7)^2 + (4\sqrt{3} - 7)^2$

$= 2(48 + 49)$

$= 194$

55. (a)

$$P(H) = \underbrace{\frac{19}{20}}_{\text{Selection of unbiased}} \times \underbrace{\frac{1}{2}}_{\text{Head occurs}} + \underbrace{\frac{1}{20}}_{\text{Selection of biased coin}} \times \underbrace{1}_{\text{Head occurs}}$$

$$\text{Required probability } = \frac{\frac{19}{20} \times \frac{1}{2}}{\frac{19}{20} \times \frac{1}{2} + \frac{1}{20} \times 1} = \frac{19}{21}$$

$\therefore \frac{m}{n} = \frac{19}{21}$

$\Rightarrow n^2 - m^2 = 441 - 361 = 80$

56. (b)

$2p + 2q = \frac{1}{2}$

$p + q$

$$E(x^2) = \sum_{i=0}^{3} x_i^2 p(x_i) = 0 \cdot p + 1 \cdot p + 4 \cdot q + 9q$$

$= p + 13q$

$$E(x) = \sum_{i=0}^{3} x_i^2 p(x_i) = 0 \cdot p + 1 \cdot p + 2q + 3q = p + 5q$$

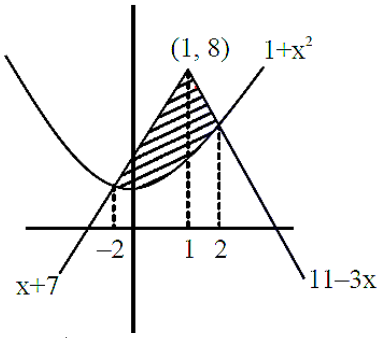
$p + 13q = 2(p + 5q)$

$p = 3q$

So, $q = \frac{1}{8}$ & $p = \frac{3}{8}$

So, $8p - 1 = 2$

57. (a)



$$A = \int_{-2}^{1} (x + 7 - x^2 - 1)dx +$$
$$\int_{1}^{2} (11 + 3x - x^2 - 1)dx$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_{-2}^{1} + \left[10x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_{1}^{2}$$

$= \frac{50}{3} \Rightarrow 3A = 50$ Option (1)

58.(b)

$$\lim_{x \to 0} \frac{f(x)}{x^2} = 5$$

$$\lim_{x \to 0} \frac{ax^4 + bx^3 + cx^2 + dx + e)}{x^2} = 5$$

$c = 5$ and $d = e = 0$

$f(x) = ax^4 + bx^3 + 5x^2$

$f'(x) = 4ax^3 + 3bx^2 + 10x$

$= x(4ax^2 + 3bx + 10)$

has extremes at 4 and so $f'(4) = 0 \& f'(5) = 0$

so $a = \frac{1}{8} \& b = \frac{-3}{2}$

so $f(2) = \frac{1}{8} \times 2^4 - \frac{3}{2} \times 2^3 + 5 \times 2^2$

$= 2 - 12 + 20 = 10$

59.(a)

$$\cos 2\theta \cos\frac{\theta}{2} + \cos\frac{5\theta}{2} = 2\cos^3\frac{5\theta}{2}$$

$$\frac{1}{2}\left(2\cos 2\theta \cos\frac{\theta}{2}\right) + \cos\frac{50}{2}$$

$$= \frac{1}{2}\left(\cos\frac{15\theta}{2} + 3\cos\frac{5\theta}{2}\right)$$

or solving

$\cos\frac{3\theta}{2}=\cos\frac{15\theta}{2}$

$\cos\frac{15\theta}{2}-\cos\frac{3\theta}{2}=0$

$2\sin 30\sin\frac{9\theta}{2}=0$

$3\theta=\mathrm{n}\pi$ or $\frac{9\theta}{2}=\mathrm{m}\pi$

$\theta=\frac{\mathrm{n}\pi}{3}\ \theta=\frac{2\ \mathrm{m}\pi}{9}$

$\theta=\left\{-\frac{\pi}{2},\frac{\pi}{3},0\right\}$

$\theta=\left\{-\frac{4\pi}{9},\frac{-2\pi}{9},\frac{4\pi}{9},\frac{2\pi}{9}\right\}$

60. (c)

$\mathrm{S_n}=700=\frac{\mathrm{n}}{2}[2\mathrm{a}+(\mathrm{n}-1)\mathrm{d}]$

$\mathrm{a}_6=7\Rightarrow \mathrm{a}+5\ \mathrm{d}=7$

$\mathrm{S}_7=7\Rightarrow\frac{7}{2}(2\mathrm{a}+6\ \mathrm{d})=7$

$a+3d=1$

Solve (ii) and (iii)

$\frac{\mathrm{n}}{2}(-16+3\mathrm{n}-3)=700\Rightarrow 3\mathrm{n}^2-19\mathrm{n}-1400=0$

$(3\mathrm{n}+56)(\mathrm{n}-25)=0$

$\therefore \mathrm{a}_{25}=\mathrm{a}+24\ \mathrm{d}=-8+24\times 3$

$=-8+72$

$=64$ Ans. $\rightarrow 3$

61. (c)

$\operatorname{Re}\left(\frac{z-1}{2z+i}\right)+\operatorname{Re}\left(\frac{\bar{z}-1}{2\bar{z}-i}\right)=2$

Here, $\frac{z-1}{2z+i}=\left(\overline{\frac{\bar{z}-1}{2\bar{z}-i}}\right)=2$

$=\operatorname{Re}\left(\frac{z-1}{2z+i}\right)+\operatorname{Re}\left(\overline{\frac{z-1}{2z+i}}\right)=2$

$=2\operatorname{Re}\left(\frac{z-1}{2z+1}\right)=2\Rightarrow\operatorname{Re}\left(\frac{z-1}{2z+i}\right)=1$

Let $z=x+iy$

$\operatorname{Re}\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right)=1$

$\Rightarrow\operatorname{Re}\left[\frac{((x-1)+iy)(2x-i(y+1)}{(2x+i(2y+1)(2x-i(2y+1))}\right]=1$

$\Rightarrow\frac{2x(x-1)+y(2y+1)}{4x^2+(2y+1)^2}=1$

$\Rightarrow 2x^2-2x+2y^2+y=4x^2+4y^2+1+4y$

$\Rightarrow 2x^2+2y^2+3y+2x+1=0$

$\Rightarrow x^2+y^2+x+\frac{3}{2}y+\frac{1}{2}=0$

centre $=\left(\frac{-1}{2},\frac{-3}{4}\right),r=\sqrt{\frac{1}{4}+\frac{9}{16}-\frac{1}{2}}=\frac{\sqrt{5}}{4}$

$a=\frac{-1}{2}, b=\frac{-3}{4}, r^2=\frac{5}{16}$

$15\frac{ab}{r^2}=15\times\left(\frac{-1}{2}\right)\times\left(\frac{-3}{4}\right)\times\frac{16}{5}=18$

62.(b)

Length of $LR=\frac{2b^2}{a}=10\Rightarrow 5a=b^2$

$f(t)=t^2+t+\frac{11}{12}$

$\frac{df(t)}{dt}=2t+1=0\Rightarrow t=\frac{-1}{2}$

Min value of $f(t)=\left(\frac{-1}{2}\right)^2+\left(\frac{-1}{2}\right)+\frac{11}{12}$

$=\frac{1}{4}\frac{-1}{2}+\frac{11}{12}=\frac{3-6+11}{12}=\frac{8}{12}=\frac{2}{3}=e$

$e^2=\frac{1-b^2}{a^2}\Rightarrow\frac{4}{9}=\frac{1-b^2}{a^2}$

$\Rightarrow\frac{b^2}{a^2}=\frac{1-4}{a}=\frac{5}{a}\Rightarrow b^2=\frac{5a^2}{a}$

From (1) & (2)

$5a=\frac{5a^2}{a}\Rightarrow a=9, b=\sqrt{45}=3\sqrt{5}$

$\therefore a2+b2=81+45=126$

63.(a)

$(x^2+1)\frac{dy}{dx}-2xy=(x^4+2x^2+1)\cos x$

$\frac{dy}{dx}-\left(\frac{2x}{x^2+1}\right)y=\frac{(x^2+1)^2\cos x}{cx^2+1}=(x^2+1)\cos x$

(Linear D.E)

$P=\frac{-2x}{x^2+1}, Q=(x^2+1)\cos x$

I.F $=e^{\int Pdx}=e^{\int\frac{-2x}{x^2+1}dx}=\frac{1}{x^2+1}$

$y\cdot\frac{1}{x^2+1}=\int(x^2+1)\cos x\cdot\frac{1}{x^2+1}dx$

$\frac{y}{x^2+1}=\sin x+c\Rightarrow y\cos=1\Rightarrow c=1$

$y=(x^2+1)(\sin x+1)$

$\int_{-3}^{3}ydx=\int_{-3}^{3}(x^2+1)(\sin x+1)$

$dx=\int_{-3}^{3}x^2\sin x+x^2\sin x+1dx$

$\Rightarrow\int_{-3}^{3}x^2\sin xdx+\int_{-3}^{3}x^2dx+\int_{-3}^{3}\sin xdx+\int_{-3}^{3}1dx$

$=0+18+0+6=24$

64. (b)

Line is $\perp^r$ to 2 line $\Rightarrow$ line will be parallel to

$(i+a\hat{j}+b\hat{k})\times(-b\hat{i}+a\hat{j}+5\hat{k})$

Parallel vector along the required line is

$\hat{i}(5a-ab)-\hat{j}(b^2+5)+\hat{k}(a+ab)$

Dr's of required line $\alpha(5a-ab), -(b^2+5), (a+ab)$

Also Dr's of required line $\alpha-2, d, -4$

$\therefore \frac{5a-ab}{-2}=\frac{-(b^2+5)}{d}=\frac{a+ab}{-4}$

Also point $\left(0,\frac{-1}{2},0\right)$ will lie on $\frac{x-1}{-2}=\frac{y+4}{d}=\frac{z-c}{-4}$

$\frac{0-1}{-2}=\frac{\frac{-1}{2}+4}{d}=\frac{0-c}{-4}\Rightarrow d=7, c=2$

From (1) $\frac{5a-ab}{-2}=\frac{-b^2-5}{7}=\frac{a+ab}{-4}$

$\frac{5a-ab}{-2}=\frac{a+ab}{-4};\frac{-b^2-5}{7}=\frac{a+ab}{-4}$

$-20a+4ab=-2a-2ab$	$4b^2+20=70+7ab$
$18a=6ab$	$36+20=70+21a$
$\boxed{b=3}$	$56=28a\Rightarrow\boxed{a=2}$

$a+b+c+d=2+3+2+7=14$

65. (d)

Total trangles $==\Rightarrow=\ {}^hC_3$

Total auadrilaterals $=\ {}^hC_4=q$

${}^nC_3+{}^nC_4=126\Rightarrow {}^{n+1}C_4=126$

$\Rightarrow n+1=9\Rightarrow n=8$

$\frac{x^2}{16}+\frac{y^2}{n}=1\Rightarrow\frac{x^2}{16}+\frac{y^2}{8}=1$

$e=\sqrt{1-\frac{8}{16}}=\sqrt{\frac{8}{16}}=\frac{1}{\sqrt{2}}$

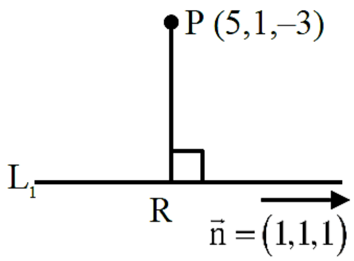
66.(b)

$L_1:\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-0}{2}$

Let $Q(\lambda+1,\lambda+2,\lambda)$

$\overrightarrow{PQ}=(\lambda-4,\lambda-1,\lambda+3)$

$\overrightarrow{PQ}\cdot\vec{m}=0$



$\Rightarrow \lambda - 4 + \lambda + 1, \lambda + 3 = 0$

$\Rightarrow 3\lambda = 0$

$\lambda = 0$

$\Rightarrow Q(1,2,0)$

$L_2: \frac{x-2}{1} = \frac{y-0}{1} = \frac{z-1}{2}$

Let $R(\mu+2, \mu, \mu+1) \overrightarrow{PR} = (\mu-3, \mu-1, \mu+4)$

$PR \cdot \vec{n} = 0$

$\mu - 3 + \mu - 1 + \mu + 4 = 0$

$\neq \mu = 0$

$R(2,0,1)$

$$\text{Area of } \Delta PQR(A) = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$$

$$A = \frac{1}{2}|(-4\hat{i} + \hat{j} + 3\hat{k}) \times (-3\hat{i} + \hat{j} + 4\hat{k})|$$

$$A = \frac{1}{2}|7(\hat{i} + \hat{j} + \hat{k})|$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 1 & 3 \\ -3 & -1 & 4 \end{vmatrix}$$

$= 7\hat{i} + 7\hat{j} + 7\hat{k}$

$4A^2 = 49 \times 3 = 147$

67. (c)

(I) $x < 2$

$-x^2 + 2x - 3x + 9 + 1 = 0$

$\Rightarrow x^2 + x - 10 = 0$

$$\Rightarrow x = \frac{-1+\sqrt{41}}{2}, \frac{-1-\sqrt{41}}{2}$$

$\times$

(II) $2 \leq x < 3$

$\Rightarrow x^2 - 2x - 3x + 9 + 1 = 0$

$\Rightarrow x^2 - 5x + 10 = 0$

$D < 0$

(III) $x \geq 3$

$x^2 - 2x + 3x - 9 + 2 = 0$

$\Rightarrow x^2 + x - 8 = 0$

$$x = \frac{-1+\sqrt{32}}{2}, \frac{-1-\sqrt{32}}{2}$$

$\times \times$

1 real roots

68. (b)

$e_1^2 = 1 - \frac{b^2}{25}$ $e_2^2 = 1 - \frac{b^2}{16}$

$\therefore e_1^2 e_2^2 = 1$

$\left(1 - \frac{b^2}{25}\right)\left(1 + \frac{b^2}{16}\right) = 1$

$\Rightarrow 2 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^2}{400} = 1$

$\Rightarrow \frac{9b^2}{400} = \frac{b^4}{400}$

$b^2 = 9$

$\frac{x^2}{9} + \frac{y^2}{25} = 1$ $\frac{x^2}{16} - \frac{y^2}{9} = 0$

$e_1 \sqrt{1 - \frac{9}{25}}$ $e_2 = \frac{5}{4}$

$e_1 = \frac{4}{5}$

Focii :- $(0, \pm 4)$ $(\pm 5, 0)$

ellipse passing through all four foci

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

69. (a)

For infinitely many solution

$\Delta = 0$

$\begin{vmatrix} 1 & 5 & -1 \\ 4 & 3 & -3 \\ 24 & 1 & \lambda \end{vmatrix} = 0$

$\Rightarrow 1(3\lambda + 3) - 5(4\lambda + 72) - 1(4 - 72) = 0$

$\Rightarrow -17\lambda + 3 - 4 \times 72 - 4 = 0$

$\Rightarrow 17\lambda = -289$

$\Rightarrow \lambda = -17$

$\Delta 1 = 0$

$\Rightarrow \begin{vmatrix} 1 & 5 & -1 \\ 7 & 3 & -3 \\ \mu & 1 & -17 \end{vmatrix} = 0$

$\Rightarrow 1(-51 + 3) - 5(-119 + 3\mu) - 1(7 - 3\mu) = 0$

$\Rightarrow -48 + 595 - 15\mu - 7 + 3\mu = 0$

$\Rightarrow 12\mu = 540$

$\mu = 45$

$x+5y-z=1$
$4x+3y-3z=7$
$24x+y-17z=45$
Let $z=1$
$x+5y=1+\lambda]\times 4$
$4x+3y=7+3\lambda$
$4x+20y=4+4\lambda$
$--17y=3-\lambda$
$y=\frac{\lambda-3}{17}, x=1+\lambda-\frac{5\lambda-15}{17}$
$7\le\frac{\lambda-3}{17}+\frac{32+12\lambda}{17}+\lambda\le 77$
$7\le\frac{30\lambda+29}{17}\le 77$
$3\le\lambda\le 42$
$\lambda=3,20,37$

70. (d)

$ar+ar^3+ar^5=21, ar^7+ar^9+ar^{11}=15309 \Rightarrow \mathrm{ar}(1+\mathrm{r}^2+\mathrm{r}^4)=21, \mathrm{ar}^7(1+\mathrm{r}^2+\mathrm{r}^4)=15309$

$\mathrm{Eq}^{\mathrm{n}}(2)\div \mathrm{eq}^{\mathrm{n}}(1)$

$\Rightarrow\frac{\mathrm{a}\cdot\mathrm{r}^7}{\mathrm{ar}}=\frac{15309}{21}\Rightarrow \mathrm{r}^6=729$

$\Rightarrow\frac{\mathrm{a}\cdot(\mathrm{r}^9-1)}{\mathrm{r}-1}=\frac{\frac{7}{91}(19683-1)}{2}=\frac{7\times 19682}{91\times 2}$

$=\frac{9841}{13}=757$

Section-B

71. (b)

$$\lim_{x\to 0}\frac{\frac{\tan(\tan x)-\tan x}{\tan^3 x}\frac{\tan^3 x}{x^3}+\frac{\tan x-\sin x}{x^3}+\frac{\sin x-\sin(\sin x)}{\sin^3 x}\frac{\sin^3 x}{x^3}}{\frac{\tan x-\sin x}{x^3}}$$

$$=\frac{\frac{1}{3}+\frac{1}{2}+\frac{1}{6}}{\frac{1}{2}}=2$$

72. (19)

$$\int\left(\frac{1}{x^2}+\frac{1}{x^4}\right)\left(\frac{3}{x}+\frac{1}{x^3}\right)^{\frac{1}{23}}dx$$

using $t=\frac{3}{x}+\frac{1}{x^3}\Rightarrow dt=-3\left(\frac{1}{x^2}+\frac{1}{x^4}\right)dx$

$$\int\frac{t^{1/23}dt}{-3}=\frac{t^{24/23}}{\left(\frac{24}{23}\right)(-3)}+C$$

$\Rightarrow \alpha=23\beta=-1\gamma=-3$

$\alpha+\beta+\gamma=19$

73. (98)

$$a+b=\lim_{t\to -1^+}(\alpha+\beta)=\lim_{t\to -1^+}-\frac{(t+2)^{\frac{1}{6}}-1}{(t+2)^{\frac{1}{7}}-1}$$

let $\mathrm{t}+2=\mathrm{y}$

$$a+b=\lim_{y\to 1^+}\frac{y^{1/6}-1}{y^{1/7}-1}=\frac{7}{6}$$

$$72(a+b)^2=72\frac{49}{36}=98$$

74. (189)

$$PF_1 \cdot PF_2 = (e\alpha - a)(e\alpha + a)$$

$$P = e^2\alpha^2 - a^2 = \frac{25}{9} \cdot 9 \cdot \frac{9}{4} - 9 = \frac{189}{4}$$

$$\left.\begin{matrix} ae = 5 \\ a = 9/5 \end{matrix}\right\} \begin{matrix} a = 3 \\ e = 5/3 \\ b = 4 \end{matrix}$$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$\frac{\alpha^2}{9} - \frac{4(5)}{16} = 1 \Rightarrow \alpha^2 = 9\left(\frac{36}{16}\right)$$

75. (34)

$$(1-x)^{20} = {}^{20}C_0 - {}^{20}C_1 x + {}^{20}C_2 x^2 \ldots\ldots + {}^{20}C_{20} x^{20}$$

$$\frac{(1-x)^{20}}{x^2} = \frac{{}^{20}C_0}{x^2} - \frac{{}^{20}C_1}{x} + {}^{20}C_2 - {}^{20}C_3 x + {}^{20}C_4 x^2 \ldots$$

Diff twice and put $x = 1$

$$= 6 - {}^{20}C_1(2) + A$$

$$A = 40 - 6 = 34$$