

Physics
Section-A

1. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The outer body of an air craft is made of metal which protects persons sitting inside from lightning-strikes.

Reason (R) : The electric field inside the cavity enclosed by a conductor is zero.

In the light of the above statements, chose the most appropriate answer from the options given below :

- (a) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (b) (A) is correct but (R) is not correct
- (c) Both (A) and (R) are correct but (R) is not correct explanation of (A)
- (d) (A) is not correct but (R) is correct

2. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The density of the copper (${}^{64}_{29}\text{Cu}$) nucleus is greater than that of the carbon (${}^{12}_6\text{C}$) nucleus.

Reason (R) : The nucleus of mass number A has a radius proportional to $A^{1/3}$.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) (A) is correct but (R) is not correct
- (b) (A) is not correct but (R) is correct
- (c) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (d) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

3. The unit of $\sqrt{\frac{2I}{\epsilon_0 c}}$ is :

(I = intensity of an electromagnetic wave, c : speed of light)

- (a) Vm
- (b) NC
- (c) Nm
- (d) NC^{-1}

4. The dimension of $\sqrt{\frac{\mu_0}{\epsilon_0}}$ is equal to that of :

(μ_0 = Vacuum permeability and ϵ_0 = Vacuum permittivity)

- (a) Voltage
- (b) Capacitance
- (c) Inductance
- (d) Resistance

5. A photo-emissive substance is illuminated with a radiation of wavelength λ_i so that it releases electrons with de-Broglie wavelength λ_e . The longest wavelength of radiation that can emit photoelectron is λ_0 . Expression for de-Broglie wavelength is given by :

(m : mass of the electron, h : Planck's constant and c : speed of light)

$$(a) \lambda_e = \frac{h}{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)} \quad (b) \lambda_e = \sqrt{\frac{h\lambda_0}{2mc}}$$

$$(c) \lambda_e = \frac{h}{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)} \quad (d) \lambda_e = \sqrt{\frac{h\lambda_i}{2mc}}$$

6. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The radius vector from the Sun to a planet sweeps out equal areas in equal intervals of time and thus areal velocity of planet is constant.

Reason (R) : For a central force field the angular momentum is a constant.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (a) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (b) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
- (c) (A) is correct but (R) is not correct
- (d) (A) is not correct but (R) is correct

7. The helium and argon are put in the flask at the same room

temperature (300 K). The ratio of average kinetic energies (per molecule) of helium and argon is :

(Give : Molar mass of helium = 4 g/mol, Molar mass of argon = 40 g/mol)

- (a) 1: 10
- (b) 10: 1
- (c) $1:\sqrt{10}$
- (d) 1: 1

8. A capillary tube of radius 0.1 mm is partly dipped in water (surface tension 70 dyn/cm and glass water contact angle $\approx 0^\circ$) with 30° inclined with vertical. The length of water risen in the capillary is cm .

(Take $g = 9.8 \text{ m/s}^2$)

- (a) $\frac{82}{5}$ (b) $\frac{57}{2}$
(c) $\frac{71}{5}$ (d) $\frac{68}{5}$

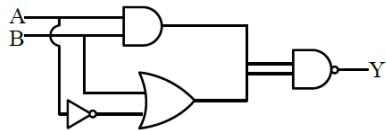
9. A mirror is used to produce an image with magnification of $\frac{1}{4}$. If the distance between object and its image is 40 cm , then the focal length of the mirror is .

- (a) 10 cm (b) 12.7 cm
(c) 10.7 cm (d) 15 cm

10. A dipole with two electric charges of $2\mu\text{C}$ magnitude each, with separation distance $0.5\mu\text{m}$, is placed between the plates of a capacitor such that its axis is parallel to an electric field established between the plates when a potential difference of 5 V is applied. Separation between the plates is 0.5 mm . If the dipole is rotated by 30° from the axis, it tends to realign in the direction due to a torque. The value of torque is:

- (a) $5 \times 10^{-9} \text{ Nm}$
(b) $5 \times 10^{-3} \text{ Nm}$
(c) $2.5 \times 10^{-12} \text{ Nm}$
(d) $2.5 \times 10^{-9} \text{ Nm}$

11. Consider the following logic circuit.



The output is $Y = 0$ when:

- (a) $A = 1$ and $B = 1$
(b) $A = 0$ and $B = 1$
(c) $A = 1$ and $B = 0$
(d) $A = 0$ and $B = 0$

12. Match List-I with List-II.

List-I		List-II	
(A)	Mass density	(I)	$[\text{ML}^2 \text{T}^{-3}]$
(B)	Impulse	(II)	$[\text{MLT}^{-1}]$
(C)	Power	(III)	$[\text{ML}^2 \text{T}^0]$
(D)	Moment of inertia	(IV)	$[\text{ML}^{-3} \text{T}^0]$

Choose the correct answer from the options given below :

- (a) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(b) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
(c) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(d) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

13. The equation of a wave travelling on a string is $y = \sin[20\pi x + 10\pi t]$, where x and t are distance and time in SI units. The minimum distance between two points having the same oscillating speed is :

- (a) 5.0 cm (b) 20 cm
(c) 10 cm (d) 2.5 cm

14. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A) : Refractive index of glass is higher than that of air.

Reason (R) : Optical density of a medium is directly proportionate to its mass density which results in a proportionate refractive index.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) (A) is not correct but (R) is correct
- (b) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (c) (A) is correct but (R) is not correct
- (d) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

15. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason(R).

Assertion (A) : Magnetic monopoles do not exist.

Reason (R) : Magnetic field lines are continuous and form closed loops.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (a) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
- (b) (A) is correct but (R) is not correct
- (c) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (d) (A) is not correct but (R) is correct

16. Which one of the following forces cannot be expressed in terms of potential energy?

- (a) Coulomb's force
- (b) Gravitational force
- (c) Frictional force
- (d) Restoring force

17. Match List-I with List-II.

List-I		List-II	
(A)	Isothermal	(I)	ΔW (work done) = 0
(B)	Adiabatic	(II)	ΔQ (supplied heat) = 0
(C)	Isobaric	(III)	ΔU (change in internal energy) $\neq 0$
(D)	Isochoric	(IV)	$\Delta U = 0$

Choose the correct answer from the options given below:

- (a) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)
- (b) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
- (c) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
- (d) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

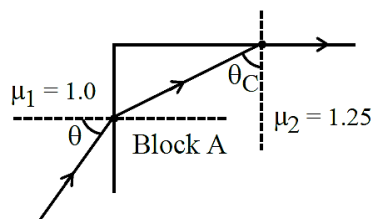
18. A helicopter flying horizontally with a speed of 360 km/h at an altitude of 2 km , drops an object at an instant. The object hits the ground at a point O , 20 s after it is dropped. Displacement of 'O' from the position of helicopter where the object was released is :(use acceleration due to gravity $g = 10 \text{ m/s}^2$ and neglect air resistance)

- (a) $2\sqrt{5}$ km
- (b) 4 km
- (c) 7.2 km
- (d) $2\sqrt{2}$ km

19. An object with mass 500 g moves along x -axis with speed $v = 4\sqrt{x}$ m/s. The force acting on the object is :

- (a) 8 N
- (b) 5 N
- (c) 6 N
- (d) 4 N

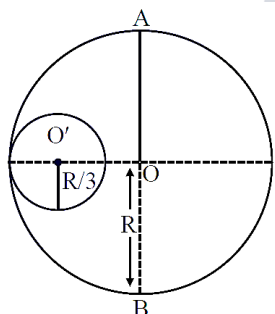
20. A transparent block A having refractive index $\mu = 1.25$ is surrounded by another medium of refractive index $\mu = 1.0$ as shown in figure. A light ray is incident on the flat face of the block with incident angle θ as shown in figure. What is the maximum value of θ for which light suffers total internal reflection at the top surface of the block ?



- (a) $\tan^{-1}(4/3)$
 (b) $\tan^{-1}(3/4)$
 (c) $\sin^{-1}(3/4)$
 (d) $\cos^{-1}(3/4)$

Section-B

21. A parallel plate capacitor has charge $5 \times 10^{-6} \text{ C}$. A dielectric slab is inserted between the plates and almost fills the space between the plates. If the induced charge on one face of the slab is $4 \times 10^{-6} \text{ C}$ then the dielectric constant of the slab is.
22. An inductor of reactance 100Ω , a capacitor of reactance 50Ω , and a resistor of resistance 50Ω are connected in series with an AC source of 10 V , 50 Hz . Average power dissipated by the circuit is _____ W.
23. Two cylindrical rods A and B made of different materials, are joined in a straight line. The ratio of lengths, radii and thermal conductivities of these rods are : $\frac{L_A}{L_B} = \frac{1}{2}$, $\frac{r_A}{r_B} = 2$ and $\frac{k_A}{k_B} = \frac{1}{2}$. The free ends of rods A and B are maintained at 400 K , 200 K , respectively. The temperature of rods interface is K , when equilibrium is established.
24. The electric field in a region is given by $\vec{E} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3 \text{ N/C}$. The flux of the field through a rectangular surface parallel to $x - z$ plane is $6.0 \text{ Nm}^2 \text{ C}^{-1}$. The area of the surface is _____ cm^2 .
25. M and R be the mass and radius of a disc. A small disc of radius $R/3$ is removed from the bigger disc as shown in figure. The moment of inertia of remaining part of bigger disc about an axis AB passing through the center O and perpendicular to the plane of disc is $\frac{4}{x} MR^2$. The value of x is.



Chemistry Section-A

26. Given below are two statements :

Statement (I) : On hydrolysis, oligo peptides give rise to fewer number of α -amino acids while proteins give rise to a large number of β -amino acids.

Statement (II) : Natural proteins are denatured by acids which convert the water soluble form of fibrous proteins to their water insoluble form.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (a) Both statement I and statement II are correct
 (b) Statement I is incorrect but Statement II is correct
 (c) Both statement I and statement II are incorrect
 (d) Statement I is correct but Statement II is incorrect

27. Mixture of 1 g each of chlorobenzene, aniline and benzoic acid is dissolved in 50 mL ethyl acetate and placed in a separating funnel, 5 M NaOH (30 mL) was added in the same funnel. The funnel was shaken vigorously and then kept aside. The ethyl acetate layer in the funnel contains

- (a) benzoic acid
 (b) benzoic acid and aniline

- (c) benzoic acid and chlorobenzene
(d) chlorobenzene and aniline

28. The hydration energies of K^+ and Cl^- are $-x$ and $-y$ kJ/mol respectively. If lattice energy of KCl is $-z$ kJ/mol, then the heat of solution of KCl is :

- (a) $+x - y - z$ (b) $x + y + z$
(c) $z - (x + y)$ (d) $-z - (x + y)$

29. $A(g) \rightarrow B(g) + C(g)$ is a first order reaction.

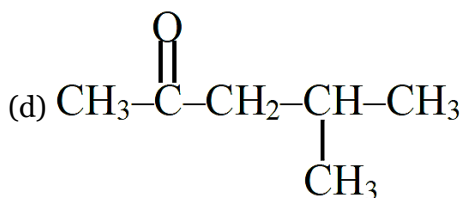
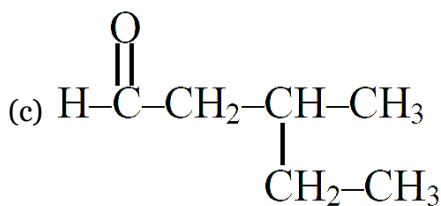
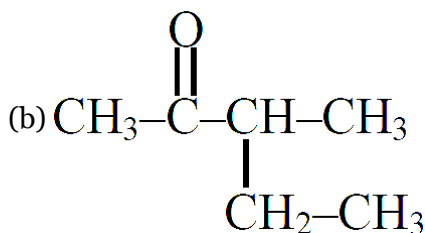
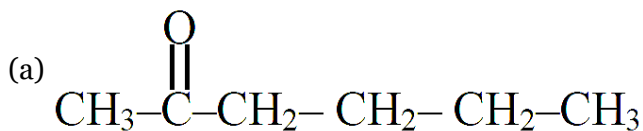
Time	T	∞
P_{system}	P_t	P_{∞}

The reaction was started with reactant A only. Which of the following expression is correct for rate constant k ?

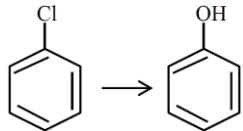
- (a) $k = \frac{1}{t} \ln \frac{2(p_{\infty} - P_t)}{P_t}$
(b) $k = \frac{1}{t} \ln \frac{p_{\infty}}{P_t}$
(c) $k = \frac{1}{t} \ln \frac{p_{\infty}}{2(p_{\infty} - P_t)}$
(d) $k = \frac{1}{t} \ln \frac{p_{\infty}}{(p_{\infty} - P_t)}$

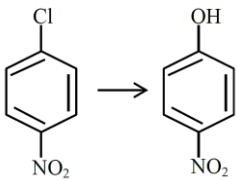
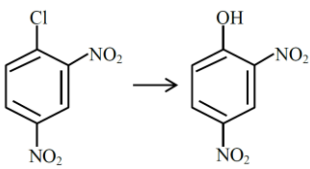
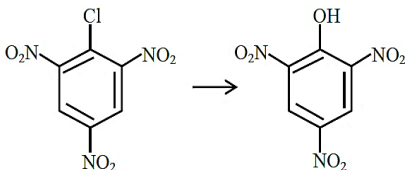
30. "P" is an optically active compound with molecular formula $C_6H_{12}O$.

When "P" is treated with 2,4-dinitrophenylhydrazine, it gives a positive test. However, in presence of Tollens reagent, "P" gives a negative test. Predict the structure of "P".



31. Match List-I with List-II

List-I Conversion		List-II Reagents, Conditions used	
(A)		(I)	Warm, H_2O

(B)		(II)	(a) NaOH, 368 K; (b) H_3O^+
(C)		(III)	(a) NaOH, 443 K; (b) H_3O^+
(D)		(IV)	(a) NaOH, 623 K, 300 atm ; (b) H_3O^+

Choose the correct answer from the options given below :

- (a) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
 (b) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
 (c) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
 (d) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

32. The correct statement amongst the following is :

- (a) The term 'standard state' implies that the temperature is 0°C
 (b) The standard state of pure gas is the pure gas at a pressure of 1 bar and temperature 273 K
 (c) $\Delta_f H_{298}^\theta$ is zero for $\text{O}(\text{g})$
 (d) $\Delta_f H_{500}^\theta$ is zero for $\text{O}_2(\text{g})$

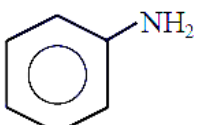
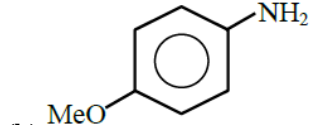
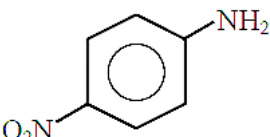
33. Liquid A and B form an ideal solution. The vapour pressure of pure liquids A and B are 350 and 750 mm Hg respectively at the same temperature. If x_A and x_B are the mole fraction of A and B in solution while y_A and y_B are the mole fraction of A and B in vapour phase then :

- (a) $\frac{x_A}{x_B} < \frac{y_A}{y_B}$
 (b) $\frac{x_A}{x_B} = \frac{y_A}{y_B}$
 (c) $\frac{x_A}{x_B} > \frac{y_A}{y_B}$
 (d) $(x_A - y_A) < (x_B - y_B)$

34. 'X' is the number of acidic oxides among VO_2 , V_2O_3 , CrO_3 , V_2O_5 and Mn_2O_7 . The primary valency of cobalt in $[\text{Co}(\text{H}_2\text{NCH}_2\text{CH}_2\text{NH}_2)_3]_2(\text{SO}_4)_3$ is Y. The value of X + Y is :

- (a) 5 (b) 4
 (c) 2 (d) 3

35. The descending order of basicity of following amines is :

- (a)  (b) 
 (c)  (d) CH_3NH_2

(E) $(\text{CH}_3)_2\text{NH}$

Choose the correct answer from the options given Below

- (a) $\text{B} > \text{E} > \text{D} > \text{A} > \text{C}$
 (b) $\text{E} > \text{D} > \text{B} > \text{A} > \text{C}$

(c) $E > D > A > B > C$

(d) $E > A > D > C > B$
36. Match List-I with List-II

List-I Complex		List-II Primary valency and Secondary valency		
(A)	$[\text{Co}(\text{en})_2\text{Cl}_2]\text{Cl}$	(I)	3	6
(B)	$[\text{Pt}(\text{NH}_3)_2\text{Cl}(\text{NO}_2)]$	(II)	3	4
(C)	$\text{Hg}[\text{Co}(\text{SCN})_4]$	(III)	2	6
(D)	$[\text{Mg}(\text{EDTA})]^{2-}$	(IV)	2	4

Choose the correct answer from the options given below:

(a) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(b) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(c) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(d) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

37. Match List-I with List-II

List-I		List-II	
(A)	Solution of chloroform and acetone	(I)	Minimum boiling azeotrope
(B)	Solution of ethanol and water	(II)	Dimerizes
(C)	Solution of benzene and toluene	(III)	Maximum boiling azeotrope
(D)	Solution of acetic acid in benzene	(IV)	$\Delta V_{\text{mix}} = 0$

Choose the correct answer from the options given below:

(a) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(b) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(c) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(d) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

38. In SO_2 , NO_2^- and N_3^- the hybridizations at the central atom are respectively :

(a) sp^2 , sp^2 and sp

(b) sp^2 , sp and sp

(c) sp^2 , sp^2 and sp^2

(d) sp , sp^2 and sp

39. The number of unpaired electrons responsible for the paramagnetic nature of the following complex species are respectively :

 $[\text{Fe}(\text{CN})_6]^{3-}$, $[\text{FeF}_6]^{3-}$, $[\text{CoF}_6]^{3-}$, $[\text{Mn}(\text{CN})_6]^{3-}$

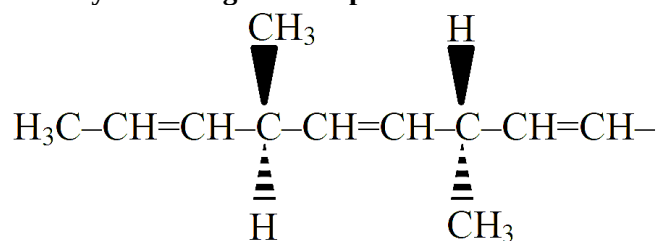
(a) 1,5,4,2

(b) 1,5,5,2

(c) 1,1,4,2

(d) 1,4,4,2

40. The number of optically active products obtained from the complete ozonolysis of the given compound is :



- (a) 2 (b) 0
(c) 1 (d) 4

41. Given below are two statements :

Statement (I) : is more polar than

Statement (II) : Boiling point of

lower than but it is more polar than

In the light of the above statements, choose the most appropriate answer from the options given below :

- (a) Statement I is correct but statement II is incorrect
(b) Statement I is incorrect but statement II is correct
(c) Both statement I and statement II are incorrect
(d) Both statement I and statement II are correct

42. The extra stability of half-filled subshell is due to

- (A) Symmetrical distribution of electrons
(B) Smaller coulombic repulsion energy
(C) The presence of electrons with the same spin in non-degenerate orbitals
(D) Larger exchange energy
(E) Relatively smaller shielding of electrons by one another

Identify the correct statements

- (a) (B), (D) and (E) only
(b) (A), (B), (D) and (E) only
(c) (B), (C) and (D) only
(d) (A), (B) and (D) only

43. The correct statements from the following are :

- (A) Ti^{3+} is a powerful oxidising agent
(B) Al^{3+} does not get reduced easily
(C) Both Al^{3+} and Ti^{3+} are very stable in solution
(D) Ti^{+} is more stable than Ti^{3+}
(E) Al^{3+} and Ti^{+} are highly stable

Choose the correct answer from the options given below :

- (a) (A), (B), (C), (D) and (E)
(b) (A), (B), (D) and (E) only
(c) (B), (D) and (E) only
(d) (A), (C) and (D) only

44. Given below are two statements :

1 M aqueous solution of each of $\text{Cu}(\text{NO}_3)_2$, AgNO_3 , $\text{Hg}_2(\text{NO}_3)_2$; $\text{Mg}(\text{NO}_3)_2$ are electrolysed using inert electrodes,

Given : $E_{\text{Ag}^+/\text{Ag}}^\theta = 0.80 \text{ V}$, $E_{\text{Hg}_2^{2+}/\text{Hg}}^\theta = 0.79 \text{ V}$,

$E_{\text{Cu}^{2+}/\text{Cu}}^\theta = 0.24 \text{ V}$ and $E_{\text{Mg}^{2+}/\text{Mg}}^\theta = -2.37 \text{ V}$

Statement (I) : With increasing voltage, the sequence of deposition of metals on the cathode will be Ag, Hg and Cu

Statement (II) : Magnesium will not be deposited at cathode instead oxygen gas will be evolved at the cathode.

In the light of the above statement, choose the most appropriate answer from the options given below

- (a) Both statement I and statement II are incorrect
 (b) Statement I is correct but statement II is incorrect
 (c) Both statement I and statement II are correct
 (d) Statement I is incorrect but statement II is correct

45. Choose the incorrect trend in the atomic radii (r) of the elements:

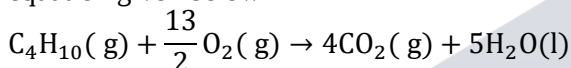
- (a) $r_{Br} < r_K$
 (b) $r_{Mg} < r_{Al}$
 (c) $r_{Rb} < r_{Cs}$
 (d) $r_{At} < r_{Cs}$

Section-B

46. Only litre buffer solution was prepared by adding 0.10 mol each of NH_3 and NH_4Cl in deionised water. The change in pH on addition of 0.05 mol of HCl to the above solution is $\times 10^{-2}$, (Nearest integer) (Given : pK_b of $NH_3 = 4.745$ and $\log_{10} 3 = 0.477$).

47. In Dumas' method 292 mg of an organic compound released 50 mL of nitrogen gas (N_2) at 300 K temperature and 715 mm Hg pressure. The percentage composition of 'N' in the organic compound is % (Nearest integer)
 (Aqueous tension at 300 K = 15 mmHg)

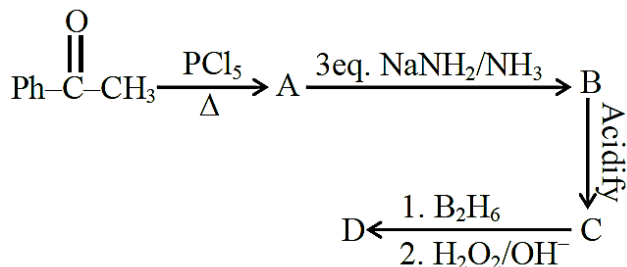
48. Butane reacts with oxygen to produce carbon dioxide and water following the equation given below



If 174.0 kg of butane is mixed with 320.0 kg of O_2 , the volume of water formed in litres is . (Nearest integer)
 [Given : (a) Molar mass of C, H, O are 12, 1, 16 g mol⁻¹ respectively, (b) Density of water = 1 g mL⁻¹]

49. The number of paramagnetic metal complex species among $[Co(NH_3)_6]^{3+}$, $[Co(C_2O_4)_3]^{3-}$, $[MnCl_6]^{3-}$, $[Mn(CN)_6]^{3-}$, $[CoF_6]^{3-}$, $[Fe(CN)_6]^{3-}$ and $[FeF_6]^{3-}$ with same number of unpaired electrons is.

50. Identify the structure of the final product (D) in the following sequence of the reactions:



Total number of sp^2 hybridised carbon atoms in product D is.

Mathematics

Section-A

51. If the orthocenter of the triangle formed by the lines $y = x + 1$, $y = 4x - 8$ and $y = mx + c$ is at $(3, -1)$, then $m - c$ is :

- (a) 0 (b) -2
 (c) 4 (d) 2

52. Let $\vec{a}$ and $\vec{b}$ be the vectors of the same magnitude such that $\frac{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|} =$

$\sqrt{2} + 1$. Then $\frac{|\vec{a}+\vec{b}|^2}{|\vec{a}|^2}$ is:

- (a) $2 + 4\sqrt{2}$ (b) $1 + \sqrt{2}$
 (c) $2 + \sqrt{2}$ (d) $4 + 2\sqrt{2}$

53. Let

$$A = \{(\alpha, \beta) \in \mathbb{R} \times \mathbb{R} : |\alpha - 1| \leq 4 \text{ and } |\beta - 5| \leq 6\}$$

and

$$B = \{(\alpha, \beta) \in \mathbb{R} \times \mathbb{R} : 16(\alpha - 2)^2 + 9(\beta - 6)^2 \leq 144\}.$$

Then

- (a) $B \subset A$
 (b) $A \cup B = \{(x, y) : -4 \leq x \leq 4, -1 \leq y \leq 11\}$
 (c) neither $A \subset B$ nor $B \subset A$
 (d) $A \subset B$

54. If the range of the function $f(x) = \frac{5-x}{x^2-3x+2}$, $x \neq 1, 2$, is $(-\infty, \alpha) \cup [\beta, \infty)$,

then $\alpha^2 + \beta^2$ is equal to :

- (a) 190 (b) 192
 (c) 188 (d) 194

55. A bag contains 19 unbiased coins and one coin with head on both sides. One coin drawn at random is tossed and head turns up. If the probability that the drawn coin was unbiased, is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $n^2 - m^2$ is equal to:

- (a) 80 (b) 60
 (c) 72 (d) 64

56. Let a random variable X take values 0, 1, 2, 3 with $P(X = 0) = P(X = 1) = p$, $P(X = 2) = P(X = 3)$ and $E(X^2) = 2E(X)$. Then the value of $8p - 1$ is :

- (a) 0 (b) 2
 (c) 1 (d) 3

57. If the area of the region $\{(x, y) : 1 + x^2 \leq y \leq \min\{x + 7, 11 - 3x\}\}$ is A , then $3A$ is equal to.

- (a) 50 (b) 49
 (c) 46 (d) 47

58. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a polynomial function of degree four having extreme values at $x = 4$ and $x = 5$. If $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 5$, then $f(2)$ is equal to :

- (a) 12 (b) 10
 (c) 8 (d) 14

59. The number of solutions of the equation $\cos 2\theta \cos \frac{\theta}{2} + \cos \frac{5\theta}{2} = 2\cos^3 \frac{5\theta}{2}$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is :

- (a) 7 (b) 5
 (c) 6 (d) 9

60. Let a_n be the n^{th} term of an A.P.

If $S_n = a_1 + a_2 + a_3 + \dots + a_n = 700$, $a_6 = 7$ and $S_7 = 7$, then a_n is equal to :

- (a) 56 (b) 65
 (c) 64 (d) 70

61. If the locus of $z \in \mathbb{C}$, such that

$$\operatorname{Re}\left(\frac{z-1}{2z+i}\right) + \operatorname{Re}\left(\frac{\bar{z}-1}{2\bar{z}-i}\right) = 2,$$

is a circle of radius r and center (a, b) then $\frac{15ab}{r^2}$ is equal to:

- (a) 24 (b) 12
 (c) 18 (d) 16

62. Let the length of a latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be 10. If its eccentricity is the minimum value of the function $f(t) = t^2 + t + \frac{11}{12}$, $t \in \mathbb{R}$, then $a^2 + b^2$ is equal to :

- (a) 125 (b) 126
 (c) 120 (d) 115

63. Let $y = y(x)$ be the solution of the differential equation $(x^2 + 1)y' - 2xy = (x^4 + 2x^2 + 1)\cos x$, $y(0) = 1$. Then $\int_{-3}^3 y(x)dx$ is :

- (a) 24 (b) 36
 (c) 30 (d) 18

64. If the equation of the line passing through the point $(0, -\frac{1}{2}, 0)$ and

perpendicular to the lines $\vec{r} = \lambda(\hat{i} + \hat{a} + \hat{b}\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} - 6\hat{k}) + \mu(-\hat{b}\hat{i} + \hat{a}\hat{j} + 5\hat{k})$ is $\frac{x-1}{-2} = \frac{y+4}{d} = \frac{z-c}{-4}$, then $a + b + c + d$ is equal to:

- (a) 10 (b) 14
(c) 13 (d) 12

65. Let p be the number of all triangles that can be formed by joining the

vertices of a regular polygon P of n sides and q be the number of all quadrilaterals that can be formed by joining the vertices of P . If $p + q = 126$, then the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{n} = 1$ is

- (a) $\frac{3}{4}$ (b) $\frac{1}{2}$
(c) $\frac{\sqrt{7}}{4}$ (d) $\frac{1}{\sqrt{2}}$

66. Consider the lines $L_1: x - 1 = y - 2 = z$ and $L_2: x - 2 = y = z - 1$. Let the feet of the perpendiculars from the point $P(5, 1, -3)$ on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A , then $4A^2$ is equal to :

- (a) 139 (b) 147
(c) 151 (d) 143

67. The number of real roots of the equation $x|x - 2| + 3|x - 3| + 1 = 0$ is :

- (a) 4 (b) 2
(c) 1 (d) 3

68. Let e_1 and e_2 be the eccentricities of the ellipse $\frac{x^2}{b^2} + \frac{y^2}{25} = 1$ and the hyperbola $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$, respectively. If $b < 5$ and $e_1 e_2 = 1$, then the eccentricity of the ellipse having its axes along the coordinate axes and passing through all four foci (two of the ellipse and two of the hyperbola) is:

- (a) $\frac{4}{5}$ (b) $\frac{3}{5}$
(c) $\frac{\sqrt{7}}{4}$ (d) $\frac{\sqrt{3}}{2}$

69. Let the system of equations

$$x + 5y - z = 1$$

$$4x + 3y - 3z = 7$$

$$24x + y + \lambda z = \mu$$

$\lambda, \mu \in \mathbb{R}$, have infinitely many solutions. Then the number of the solutions of this system, If x, y, z are integers and satisfy $7 \leq x + y + z \leq 77$, is

- (a) 3 (b) 6
(c) 5 (d) 4

70. If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is:

- (a) 760 (b) 755
(c) 750 (d) 757

Section-B

71. If the function $f(x) = \frac{\tan(\tan x) - \sin(\sin x)}{\tan x - \sin x}$ is continuous at $x = 0$, then $f(0)$ is equal to

$$\int \left(\frac{1}{x} + \frac{1}{x^3} \right) ({}^{23}\sqrt{3x^{-24} + x^{-26}}) dx$$

72. If $\int \left(\frac{1}{x} + \frac{1}{x^3} \right) ({}^{23}\sqrt{3x^{-24} + x^{-26}}) dx = -\frac{\alpha}{3(\alpha+1)} (3x^\beta + x^\gamma)^{\frac{\alpha+1}{\alpha}} + C, x > 0$

$(\alpha, \beta, \gamma \in \mathbb{Z})$, where C is the constant of integration, then $\alpha + \beta + \gamma$ is equal to .

73. For $t > -1$, let α_t and β_t be the roots of the equation

$$\left((t+2)^{\frac{1}{7}} - 1 \right) x^2 + \left((t+2)^{\frac{1}{6}} - 1 \right) x + \left((t+2)^{\frac{1}{21}} - 1 \right) = 0$$

If $\lim_{t \rightarrow -1^+} \alpha_t = a$ and $\lim_{t \rightarrow -1^+} \beta_t = b$, then $72(a+b)^2$ is equal to_____ .

74. Let the lengths of the transverse and conjugate axes of a hyperbola in standard

form be $2a$ and $2b$, respectively, and one focus and the corresponding directrix of this hyperbola be $(-5, 0)$ and $5x + 9 = 0$, respectively. If the product of the focal distances of a point $(\alpha, 2\sqrt{5})$ on the hyperbola is p , then $4p$ is equal to .

75. The sum of the series

$2 \times 1 \times {}^{20}C_4 - 3 \times 2 \times {}^{20}C_5 + 4 \times 3 \times {}^{20}C_6 - 5 \times 4 \times {}^{20}C_7 + \dots + 18 \times 17 \times {}^{20}C_{20}$, is equal to .

