

1. (a) $KE = \frac{p^2}{2m}$

$$P \propto \sqrt{m}$$

Hence, $P_A : P_B : P_C$

$$= \sqrt{400} : \sqrt{1200} : \sqrt{1600} = 1 : \sqrt{3} : 2$$

2. (b) Pressure = $\frac{I}{C} = \frac{F}{A}$

$$\Rightarrow \frac{360}{10^{-4} \times 3 \times 10^8} = \frac{2.4 \times 10^{-4}}{A}$$

$$\Rightarrow A = 2 \times 10^{-2} \text{ m}^2 = 0.02 \text{ m}^2$$

3. (a) λ is same for both

$$P = \frac{h}{\lambda} \text{ same for both}$$

$$P = \sqrt{2mK}$$

Hence,

$$K \propto \frac{1}{m}$$

$$\Rightarrow \frac{KE_p}{KE_e} = \frac{m_e}{m_p} = \frac{1}{1836}$$

4. (c)

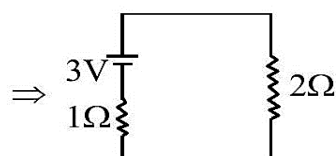
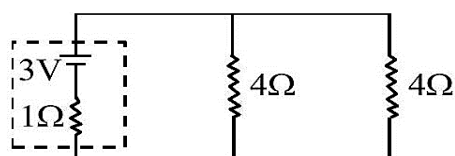
$$\frac{(C_V)_{\text{mono}}}{(C_V)_{\text{dia}}} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}$$

5. (b) $a \times 10^b$

if $a \leq 5$ order is b

$a > 5$ order is $b + 1$

6. (a)



$$i = \frac{3}{1+2} = 1 \text{ A}$$

$$v = E - ir$$

$$= 3 - 1 \times 1 = 2 \text{ V}$$

7. (b)

$$\Delta mc^2 = 18 \times 10^8$$

$$\Delta m \times 9 \times 10^{16} = 18 \times 10^8$$

$$\Delta m = 2 \times 10^{-8} \text{ kg} = 20 \mu\text{g}$$

8. (d)

9. (a)

$$x_{\min} = \pi \times r_{\min}$$

$$= \pi \times \frac{60}{100} \text{ m}$$

$$x_{\text{second}} = 30 \times 2\pi \times r_{\text{second}}$$

$$= 30 \times 2\pi \times \frac{75}{100}$$

$$x = x_{\text{second}} - x_{\min}$$

$$= 139.4 \text{ m}$$

10. (c)

$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta \ell}{\ell}$$

$$= \frac{5}{500} + \frac{0.02}{2} = 0.01 + 0.01$$

$$\frac{\Delta Y}{Y} = 0.02 \Rightarrow \% \frac{\Delta Y}{Y} = 2\%$$

11. (c) For adiabatic process

$$TV^{Y-1} = \text{constant}$$

$$T_a \cdot V_a^{Y-1} = T_d \cdot V_d^{Y-1}$$

$$\left(\frac{V_a}{V_d}\right)^{Y-1} = \frac{T_d}{T_a}$$

$$T_b \cdot V_b^{Y-1} = T_c \cdot V_c^{Y-1}$$

$$\left(\frac{V_b}{V_c}\right)^{Y-1} = \frac{T_c}{T_b}$$

$$\frac{V_a}{V_d} = \frac{V_b}{V_c} \left(\begin{array}{l} \because T_d = T_c \\ T_a = T_b \end{array} \right)$$

12. (c)

$$\frac{\pi r_1^2}{T_A} = \frac{L}{2 m_1}$$

$$\frac{\pi r_2^2}{T_B} = \frac{3 L}{2 m_2}$$

$$\Rightarrow \frac{T_A}{T_B} = 3 \cdot \frac{m_1}{m_2} \cdot \left(\frac{r_1}{r_2} \right)^2$$

$$\left(\frac{T_A}{T_B} \right)^2 = \left(\frac{r_1}{r_2} \right)^3 \Rightarrow \left(\frac{r_1}{r_2} \right)^2 = \left(\frac{T_A}{T_B} \right)^{\frac{4}{3}}$$

$$\Rightarrow \frac{1}{27} \cdot \left(\frac{m_2}{m_1} \right)^3 = \left(\frac{T_A}{T_B} \right)$$

13. (b) In resonance $Z = R$

$$I = \frac{V}{R}$$

$R \rightarrow$ halved

$$\Rightarrow I \rightarrow 2I$$

I becomes doubled.

14. (c) By truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

15. (c) Potential at surface will be same

$$\frac{Kq_1}{a} = \frac{Kq_2}{b}$$

$$\frac{q_1}{q_2} = \frac{a}{b}$$

16. (b)

$$P + \rho g h + \frac{1}{2} \rho V^2 = \text{constant}$$

17. (c)

$$F = \frac{\Delta P}{\Delta t} = \frac{mv - 0}{0.1}$$

$$= \frac{150 \times 10^{-3} \times 20}{0.1} = 30 \text{ N}$$

18. (a) Initial momentum is zero.

$$\text{Hence } |P_A| = |P_B|$$

$$\Rightarrow m_A v_B = m_B v_B$$

$$\frac{(KE)_A}{(KE)_B} = \frac{\frac{1}{2} m_A v_A^2}{\frac{1}{2} m_B v_B^2} = \frac{v_A}{v_B}$$

$$\frac{(KE)_B}{(KE)_A} = \frac{v_B}{v_A}$$

19. (a) $\sin \theta_c = \frac{\mu_R}{\mu_d} = \frac{\mu_2}{\mu_1}$

$$\sin 45^\circ = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{\sqrt{2}}{1}$$

20. (d) Given 9MSD = 10VSD

$$\text{mass} = 8.635 \text{ g}$$

$$LC = 1\text{MSD} - 1\text{VSD}$$

$$LC = 1\text{MSD} - \frac{9}{10} \text{MSD}$$

$$LC = \frac{1}{10} \text{MSD}$$

$$LC = 0.01 \text{ cm}$$

$$\text{Reading of diameter} = \text{MSR} + LC \times \text{VSR}$$

$$= 2 \text{ cm} + (0.01) \times (b)$$

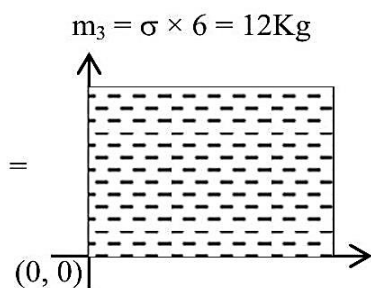
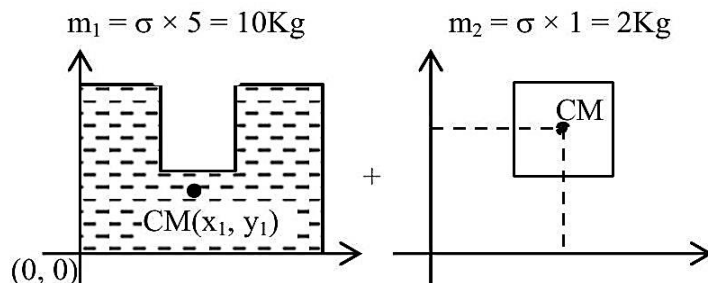
$$= 2.02 \text{ cm}$$

$$\text{Volume of sphere} = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 = \frac{4}{3} \pi \left(\frac{2.02}{2}\right)^3$$

$$= 4.32 \text{ cm}^3$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{8.635}{4.32} = 1.998 \sim 2.00 \text{ g}$$

21. (15) $m_1 = \sigma \times 5 = 10\text{Kg}$



$$\Rightarrow m_1 x_1 + m_2 x_2 = m_3 x_3$$

$$10x_1 + 2(1.5) = 12(1.5) \Rightarrow x_1 = 1.5 \text{ cm}$$

$$\Rightarrow m_1 y_1 + m_2 y_2 = m_3 y_3$$

$$10y_1 + 2(1.5) = 12 \times 1 \Rightarrow y_1 = 0.9 \text{ cm}$$

$$\frac{x_1}{y_1} = \frac{1.5}{0.9} = \frac{15}{9}$$

$$n = 15$$

22. (4) For the given condition of moving undeflected, net force should be zero.

$$qE = qVB$$

$$E = VB$$

$$= \sqrt{\frac{2 \times KE}{m}} \times B$$

$$= \sqrt{\frac{2 \times 5 \times 1.6 \times 10^{-19}}{9 \times 10^{-31}}} \times 3 \times 10^{-6}$$

$$= 4 \text{ N/C}$$

23. (3)

$$\epsilon = NB\ell v$$

$$i = \frac{\epsilon}{R} = \frac{NB\ell v}{R}$$

$$F = N(i\ell B) = \frac{N^2 B^2 \ell^2 v}{R}$$

$$W = F \times \ell = \frac{N^2 B^2 \ell^3}{R} \left(\frac{\ell}{t} \right)$$

$$A = \ell^2$$

$$W = \frac{(10 \times 10)(0.5)^2 \times (3.6 \times 10^{-3})^2}{100 \times 1}$$

$$W = 3.24 \times 10^{-6} \text{ J}$$

24. (748)

$$R = R_0(1 + \alpha \Delta T)$$

$$\frac{\Delta R}{R_0} = \alpha \Delta T$$

Case-I

$$0^\circ\text{C} \rightarrow 100^\circ\text{C}$$

$$\frac{10.2-10}{10} = \alpha(100-0) \dots\dots\dots (1)$$

Case-II

$$0^\circ\text{C} \rightarrow t^\circ\text{C}$$

$$\frac{10.95-10}{10} = \alpha(t-0) \dots\dots\dots (2)$$

$$\Rightarrow \frac{t}{100} = \frac{0.95}{0.2} = 475^\circ\text{C}$$

$$t = 475 + 273 = 748 \text{ K}$$

25. (12)

$$\phi = \vec{E} \cdot \vec{A}$$

$$= \left(\frac{2\hat{i} + 6\hat{j} + 8\hat{k}}{\sqrt{6}} \right) \cdot 4 \left(\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}} \right)$$

$$= \frac{4}{6} \times (4 + 6 + 8) = 12 \text{Vm}$$

26. (7) $\rho gh = \frac{4S}{R}$

$$\Rightarrow R = \frac{4 \times 0.28}{8 \times 10^3 \times 10 \times 4 \times 10^{-4}}$$

$$\Rightarrow \frac{0.28}{8} \text{ m} = \frac{28}{8} \text{ cm}$$

$$\Rightarrow R = 3.5 \text{ cm}$$

$$\text{Diameter} = 7 \text{ cm}$$

27. (16)

For closed organ pipe

$$f_c = (2n + 1) \frac{v}{4\ell} = \frac{15v}{4\ell}$$

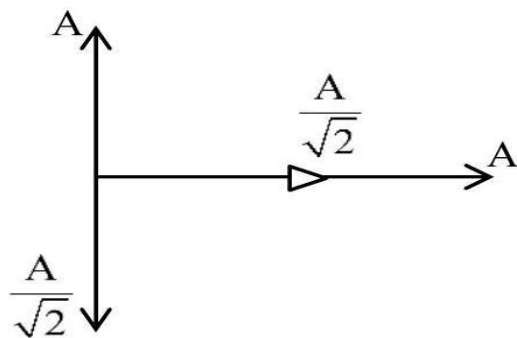
For open organ pipe

$$f_o = (n + 1) \frac{v}{2\ell} = \frac{8v}{2\ell}$$

$$\frac{f_c}{f_o} = \frac{15}{16} = \frac{a - 1}{a}$$

$$\Rightarrow a = 16$$

28. (3)



$$\vec{R} = \left(A + \frac{A}{\sqrt{2}}\right) \hat{i} + \left(A - \frac{A}{\sqrt{2}}\right) \hat{j}$$

$$|\vec{R}| = \sqrt{\left(A + \frac{A}{\sqrt{2}}\right)^2 + \left(A - \frac{A}{\sqrt{2}}\right)^2} = \sqrt{3} A$$

29. (60) $\sin \theta \simeq \theta \simeq \frac{2\lambda}{b}$

$$= \frac{2 \times 600 \times 10^{-9}}{4 \times 10^{-4}} = 3 \times 10^{-3} \text{ rad}$$

$$\text{Total divergence} = (3 + 3) \times 10^{-3} = 6 \times 10^{-3} \text{ rad}$$

30. (156)

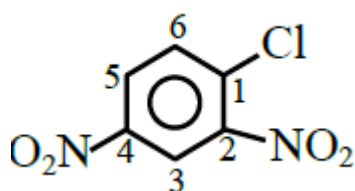
$$v = \sqrt{\frac{4KZe^2}{mr_{\min}}}$$

$$= \sqrt{\frac{4 \times 9 \times 10^9 \times 80}{6.72 \times 10^{-27} \times 4.5 \times 10^{-14}}} \times 1.6 \times 10^{-19}$$

$$= 9.759 \times 10^{25} \times 1.6 \times 10^{-19}$$

$$= 156 \times 10^5 \text{ m/s}$$

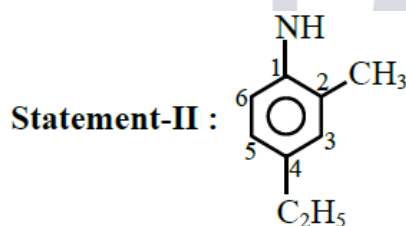
31. (b) Statement I:



IUPAC name

⇒ 1-chloro-2, 4-dinitrobenzene

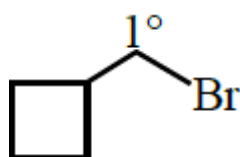
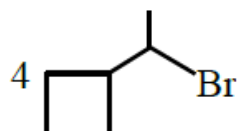
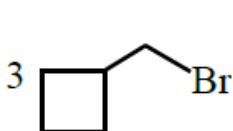
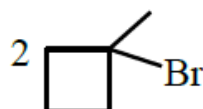
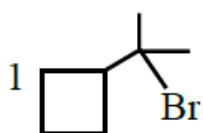
⇒ statement-I is incorrect



⇒ 4-ethyl-2-methylaniline

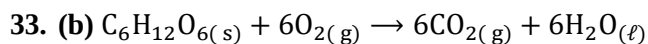
⇒ statement-II is correct

32. (c)



fastest SN^2 reaction give

Rate of SN^2 is $\text{Me} - \text{x} > 1^\circ - \text{x} > 2^\circ - \text{x} > 3^\circ - \text{x}$

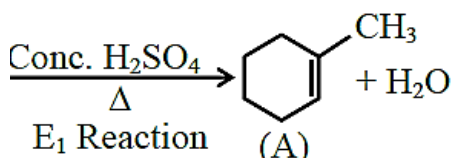
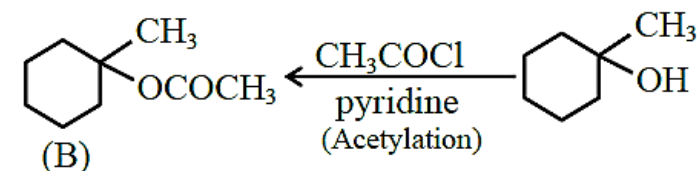


$$\frac{900}{180}$$

$$= 5 \text{ mol } 30 \text{ mol}$$

$$\text{Mass of } O_2 \text{ required} = 30 \times 32 = 960 \text{ gm}$$

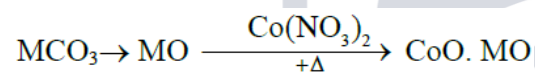
34. (a)



35. (a) The relative stability of +1 oxidation state progressively increases for heavier elements due to inert pair effect.

$$\therefore \text{Stability of } Al^{+1} < Ga^{+1} < In^{+1} < Tl^{+1}$$

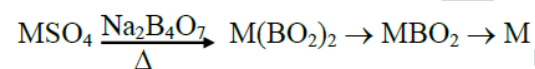
36. (d) Cobalt nitrate test



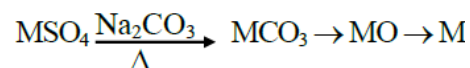
Flame test



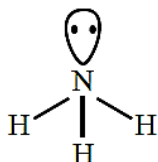
Borax Bead test



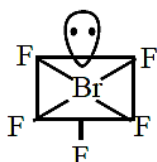
Charcoal cavity test



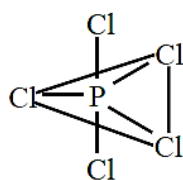
37. (c)



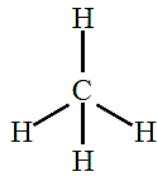
Trigonal pyramidal



Square pyramidal

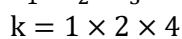
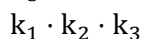


Trigonal bipyramidal

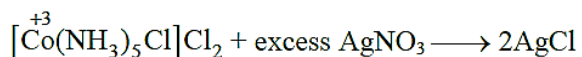


Tetrahedral

38. (c)


39. (c) In the reaction of $S_2O_3^{2-}$ with I_2 , oxidation state of sulphur changes to +2 to +2.5

In the reaction of $S_2O_3^{2-}$ with Br_2 , oxidation state of sulphur changes from +2 to +6.

 $\therefore$ Both I_2 and Br_2 are oxidant (oxidising agent) and Br_2 is stronger oxidant than I_2 .


(2 moles)

40. (c)

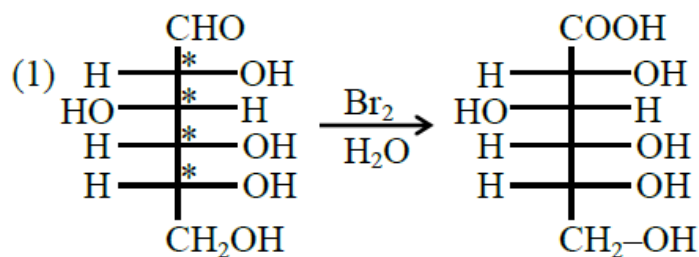
$$x + 0 - 1 - 2 = 0$$

$$x = +3$$

$$n = 5$$

$$\therefore x + n = 8$$

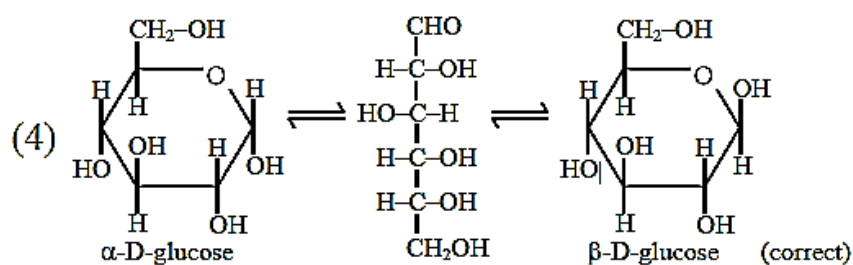
41. (a)



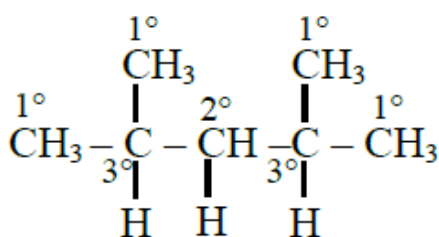
statement 1 is incorrect (monocarboxylic acid)

(b) correct

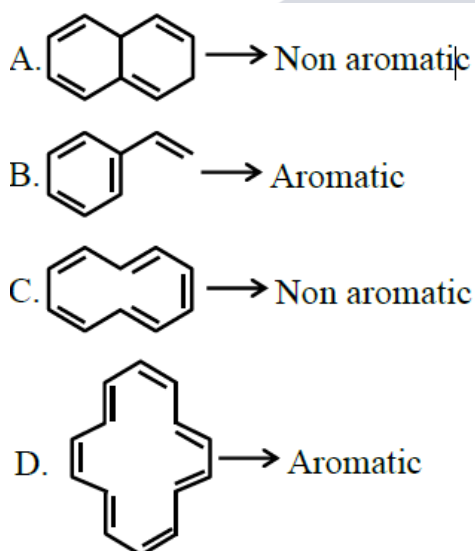
(c) c.c. is 4 (correct)



42. (b)



43. (a)



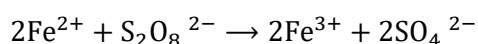
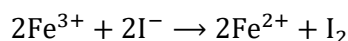
44. (b) F_2 do not disproportionate because fluorine do not exist in positive oxidation state however Cl_2 , Br_2 & I_2 undergoes disproportionation.

45. (c) $N(CH_3)_3$ and $P(CH_3)_3$ both are Lewis base and acts as ligand, However, $P(CH_3)_3$ has a π -acceptor character.

46. (c) Elements with highest electronegativity $\rightarrow$ F, O Elements with largest atomic size $\rightarrow$ Fr, Ra Elements which shows properties of both metal and non-metals i.e. metalloids $\rightarrow$ Ge, As

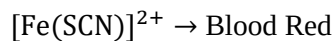
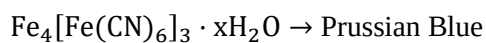
Elements with highest negative electron gain enthalpy $\rightarrow$ Cl, S

47. (d)

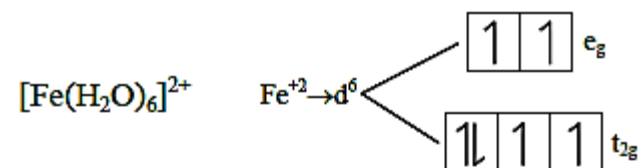
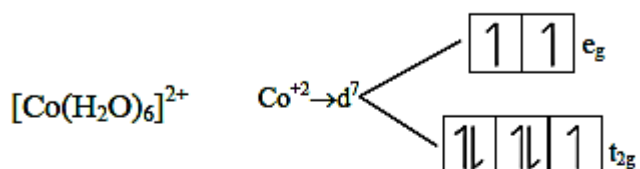
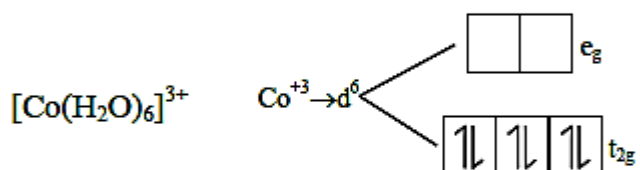
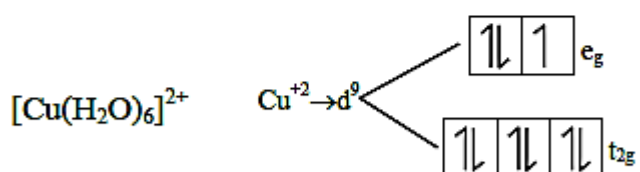
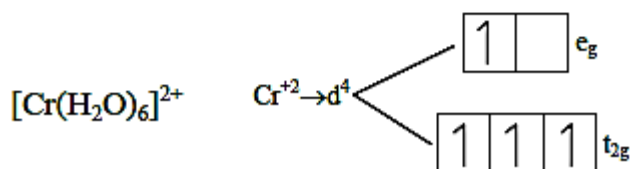


Fe^{+3} oxidises I^- to I_2 and convert itself into Fe^{+2} . This Fe^{+2} reduces $S_2O_8^{2-}$ to SO_4^{2-} and converts itself into Fe^{+3} .

48. (a)

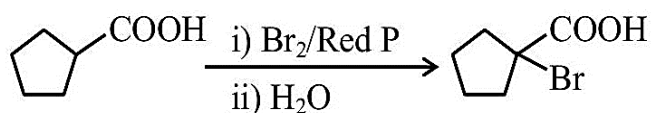


49. (b)


Electron in $t_{2g} = 4(\text{even})$

Electron in $t_{2g} = 5(\text{odd})$

Electron in $t_{2g} = 6(\text{even})$

Electron in $t_{2g} = 6(\text{even})$

Electron in $t_{2g} = 3(\text{odd})$

50. (a)

HVZ Reaction



51. (5) $\lambda = 1.5 \times 4\text{pm}$

$$= 6 \times 10^{-12} \text{ meter}$$

$$\lambda v = C$$

$$6 \times 10^{-12} \times v = 3 \times 10^8$$

$$v = 5 \times 10^{19} \text{ Hz}$$

52. (55)

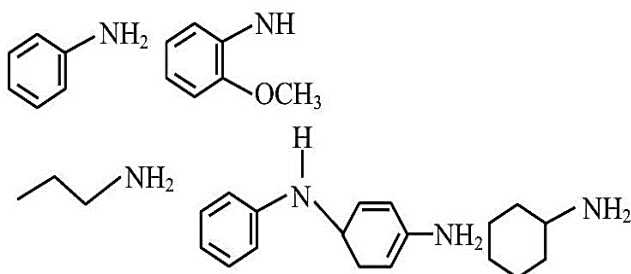
$$\omega = -nRT \ln \left(\frac{V_2}{V_1} \right)$$

$$= -1 \times .08206 \times 291 \cdot 15 \ln \left(\frac{100}{10} \right)$$

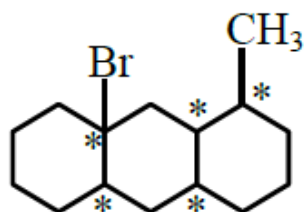
$$= -55.0128$$

Work done by system $\approx 55 \text{ atm lit.}$

53. (5) Primary amine give an ionic solid upon reaction with Hinsberg reagent which is soluble in NaOH.



54. (32)



Total chiral centre = 5

No. of optical isomers = $2^5 = 32$.

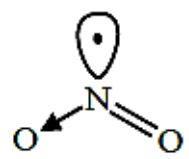
55. (0) Metal having least metallic radii among Sc, Ti, V, Cr, Mn&Zn is Cr.

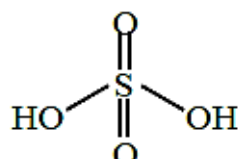
Spin only magnetic moment of CrO_4^{2-} .

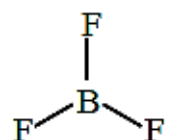
Here Cr^{+6} is in d^0 configuration (diamagnetic).

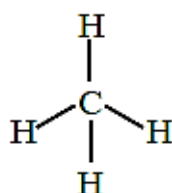
56. (6)

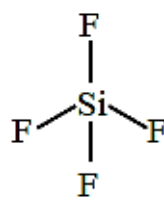
 $\text{O}=\text{C}=\text{O}$
complete
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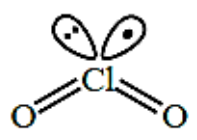

exception to
octet rule

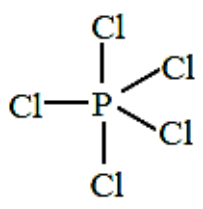

exception to
octet rule


exception to
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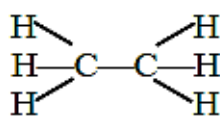

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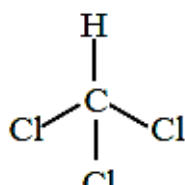

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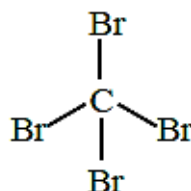

exception
to octet rule


exception to
octet rule

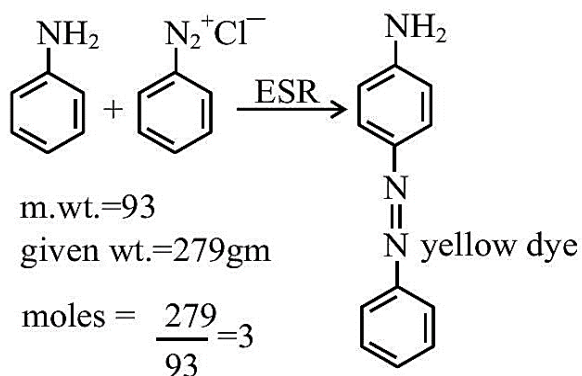
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octet


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57. (591)



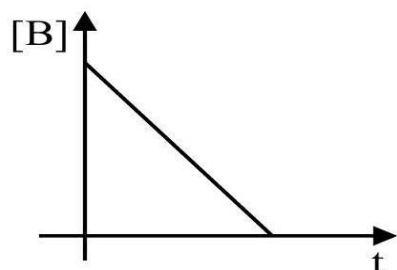
moles formed = 3
 m.wt. = 197
 amount formed
 = $197 \times 3 = 591$ gm

58. (1)

For 1st order reaction

$$75\% \text{ life} = 2 \times 50\% \text{ life}$$

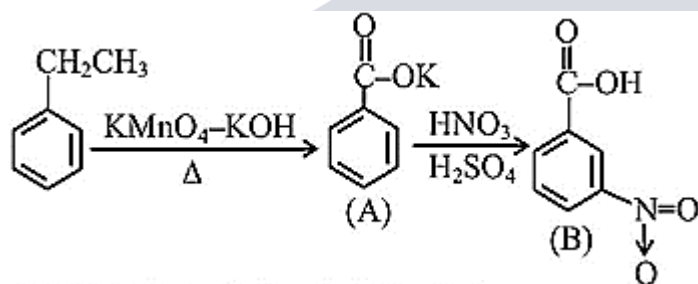
So order with respect to A will be first order.



So order with respect to B will be zero.

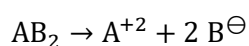
$$\text{Overall order of reaction} = 1 + 0 = 1$$

59. (5) Major product B is →



Total number of π bonds in B are 5

60. (5)



$$i = 1 + (3 - 1)\alpha$$

$$i = 1 + 2\alpha$$

$$\Delta T_b = k_b i m$$

$$0.52 = 0.52(1 + 2\alpha) \frac{\frac{10}{200}}{\frac{100}{1000}}$$

$$1 = (1 + 2\alpha) \frac{10}{20}$$

$$2 = 1 + 2\alpha$$

$$\alpha = 0.5$$

$$\text{Ans. } \alpha = 5 \times 10^{-1}$$

61. (d)

$$I_n = \int_0^1 (1 - x^k)^n \cdot 1 dx$$

$$I_n = (1 - x^k)^n \cdot x - nk \int_0^1 (1 - x^k)^{n-1} \cdot x^{k-1} \cdot dx$$

$$I_n = nk \int_0^1 \left[(1-x^k)^n - (1-x^k)^{n-1} \right] dx$$

$$I_n = nkI_n - nkI_n$$

$$\frac{I_n}{I_{n-1}} = \frac{nk}{nk+1}$$

$$\frac{I_{21}}{I_{20}} = \frac{21k}{1+21k}$$

$$= \frac{147}{148} \Rightarrow k = 7$$

62. (c)

$$(8)^{2x} - 16 \cdot (8)^x + 48 = 0$$

$$\text{Put } 8^x = t$$

$$t^2 - 16 + 48 = 0$$

$$\Rightarrow t = 4 \text{ or } t = 12$$

$$\Rightarrow 8^x = 4 \text{ or } 8^x = 12$$

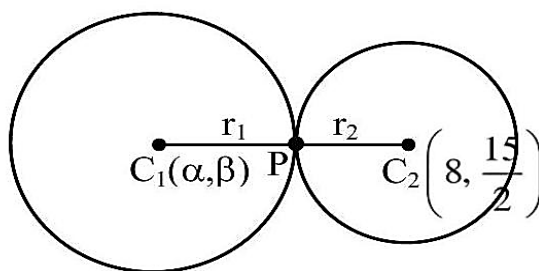
$$\Rightarrow x = \log_8 4 \text{ or } x = \log_8 12$$

$$\text{sum of solution} = \log_8 4 + \log_8 12$$

$$= \log_8 48 = \log_8 (6 \cdot 8)$$

$$= 1 + \log_8 6$$

63. (b)



$$\begin{array}{c} 2 : 1 \\ (\alpha, \beta) \quad C_1 \quad P \quad C_2 \left(8, \frac{15}{2} \right) \\ (6, 6) \end{array}$$

$$\therefore \frac{16+\alpha}{3} = 6 \text{ and } \frac{15+\beta}{3} = 6$$

$$\Rightarrow (\alpha, \beta) \equiv (2, 3)$$

$$\text{Also, } C_1C_2 = r_1 + r_2$$

$$\Rightarrow \sqrt{(2-8)^2 + \left(3 - \frac{15}{2}\right)^2} = 2r_2 + r_2$$

$$\Rightarrow r_2 = \frac{5}{2} \Rightarrow r_1 = 2r_2 = 5$$

$$\therefore (\alpha + \beta) + 4(r_1^2 + r_2^2)$$

$$= 5 + 4\left(\frac{25}{4} + 25\right) = 130$$

64. (a)

$$P(x, y, z), Q(x, y, 0); x^2 + y^2 + z^2 = \gamma^2$$

$$\overline{OQ} = x\hat{i} + y\hat{j}$$

$$\cos \theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\Rightarrow \sin^2 \phi = \frac{x^2 + y^2}{x^2 + y^2 + z^2}$$

$$\text{distance of P from x-axis } \sqrt{y^2 + z^2}$$

$$\Rightarrow \sqrt{\gamma^2 - x^2} \Rightarrow \gamma \sqrt{1 - \frac{x^2}{\gamma^2}}$$

$$= \gamma \sqrt{1 - \cos^2 \theta \sin^2 \phi}$$

65. (a)

$$f(x) = (x-2)^{2/3}(2x+1)$$

$$f'(x) = \frac{2}{3}(x-2)^{-1/3}(2x+1) + (x-2)^{2/3}$$

$$f'(x) = 2 \times \frac{(2x+1) + (x-2)}{3(x-2)^{1/3}}$$

$$\frac{3x-1}{(x-2)^{1/3}} = 0$$

$$\text{Critical points } x = \frac{1}{3} \text{ and } x = 2$$

66. (c)

$$\int_0^a f(x) dx = e^{-a} + 4a^2 + a - 1$$

$$f(a) = -e^{-a} + 8a + 1$$

$$f(x) = -e^{-x} + 8x + 1$$

$$\text{Now } y = C_1 f(x) + C_2$$

$$\frac{dy}{dx} = C_1 f'(x) = C_1(e^{-x} + 8) \dots (1)$$

$$\frac{d^2y}{dx^2} = -C_1 e^{-x} \Rightarrow -e^x \frac{d^2y}{dx^2}$$

Put in equation (1)

$$\frac{dy}{dx} = -e^x \frac{d^2y}{dx^2} (e^{-x} + 8)$$

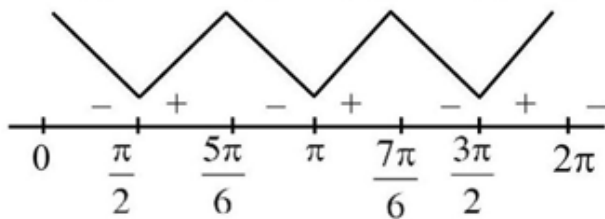
$$(8e^x + 1) \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

67. (b)

$$f(x) = 4\cos^3(x) + 3\sqrt{3}\cos^2(x) - 10; x \in (0, 2\pi)$$

$$\Rightarrow f'(x) = 12\cos^2 x [-\sin(x)] + 3\sqrt{3}(2\cos(x))[-\sin(x)]$$

$$\Rightarrow f'(x) = -6\sin(x)\cos(x)[2\cos(x) + \sqrt{3}]$$



$$\text{local maxima at } x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

68. (b)

$$A^3 - 4A^2 + A + 21I = 0$$

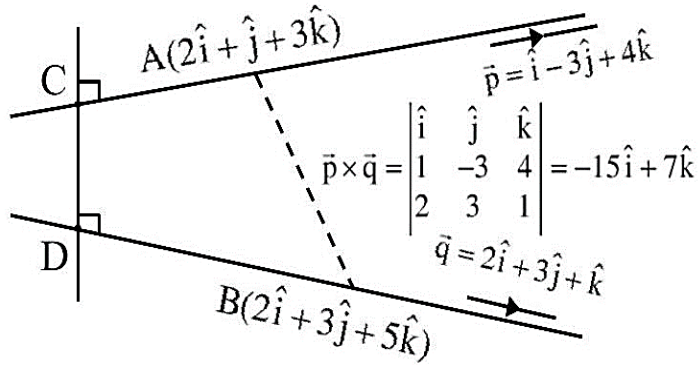
$$\text{tr}(A) = 4 = 5 + 6 \Rightarrow b = -1$$

$$|A| = -21$$

$$-16 + a = -21 \Rightarrow a = -5$$

$$2a + 3b = -13$$

69. (b)



$$\text{Shortest distance (CD)} = \frac{|\vec{AB} \cdot \vec{p} \times \vec{q}|}{|\vec{p} \times \vec{q}|}$$

$$= \frac{|(0\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (-15\hat{i} + 7\hat{j} + 9\hat{k})|}{\sqrt{355}}$$

$$= \frac{0 + 14 + 18}{\sqrt{355}} = \frac{32}{\sqrt{355}}$$

$$\therefore m + n = 32 + 355 = 387$$

70. (d) $x + y = 24, x, y \in N$

$$AM > GM \Rightarrow xy \leq 144$$

$$xy \geq 108$$

Favorable pairs of (x, y) are

$$(13, 11), (12, 12), (14, 10), (15, 9), (16, 8),$$

$$(17, 7), (18, 6), (6, 18), (7, 17), (8, 16), (9, 15),$$

$$(10, 14), (11, 13)$$

i.e. 13 cases

Total choices for $x + y = 24$ is 23

$$\text{Probability} = \frac{13}{23} = \frac{m}{n}$$

$$n - m = 10$$

71. (a)

$$\sin x = \frac{-3}{5}, \pi < x < \frac{3\pi}{2}$$

$$\tan x = \frac{3}{4} \cos x = -\frac{4}{5}$$

$$80(\tan^2 x - \cos x)$$

$$= 80 \left(\frac{9}{16} + \frac{4}{5} \right) = 45 + 64 = 109$$

72. (b)

$$I(x) = \int \frac{6dx}{\sin^2 x (1 - \cot x)^2} = \int \frac{6 \operatorname{cosec}^2 x dx}{(1 - \cot x)^2}$$

$$\text{Put } 1 - \cot x = t$$

$$\operatorname{cosec}^2 x dx = dt$$

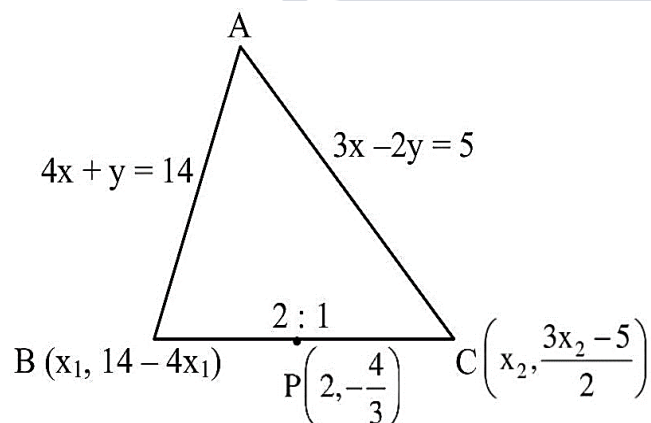
$$I = \int \frac{6dt}{t^2} = \frac{-6}{t} + c$$

$$I(x) = \frac{-6}{1 - \cot x} c, c = 3$$

$$I(x) = 3 - \frac{6}{1 - \cot x}, I\left(\frac{\pi}{12}\right) = 3 - \frac{6}{1 - (2 + \sqrt{3})}$$

$$I\left(\frac{\pi}{12}\right) = 3 + \frac{6}{\sqrt{3} + 1} = 3 + \frac{6(\sqrt{3} - 1)}{2} = 3\sqrt{3}\sqrt{2}$$

73. (c)



$$\frac{2x_2 + x_1}{3} = 2, \frac{2\left(\frac{3x_2 - 5}{2}\right) + (14 - 4x_1)}{3} = \frac{-4}{3}$$

$$2x_2 + x_1 = 6, 3x_2 - 4x_1 = -13$$

$$x_2 = 1, x_1 = 4$$

$$\text{So, } C(1, -1), B(4, -2)$$

$$m = \frac{-1}{3}$$

$$\text{Equation of BC: } y + 1 = \frac{-1}{3}(x - 1)$$

$$3y + 3 = -x + 1$$

$$x + 3y + 2 = 0$$

74. (b)

$$N = 2310 = 231 \times 10$$

$$= 3 \times 11 \times 7 \times 2 \times 5$$

$$A = \{2, 3, 5, 7, 11\}$$

$$f(x) = \left[\log_2 \left(x^2 + \left\lceil \frac{x^3}{5} \right\rceil \right) \right]$$

$$f(b) = [\log_2(5)] = 2$$

$$f(c) = [\log_2(14)] = 3$$

$$f(5) = [\log_2(25 + 25)] = 5$$

$$f(7) = [\log_2(117)] = 6$$

$$f(11) = [\log_2 387] = 8$$

$$\text{Range of } f : B = \{2, 3, 5, 6, 8\}$$

$$\text{No. of one-one functions} = 5! = 120$$

75. (d) $|z + 2| = 1, \operatorname{Im} \left(\frac{z+1}{z+2} \right) = \frac{1}{5}$

$$\text{Let } z + 2 = \cos \theta + i \sin \theta$$

$$\frac{1}{z+2} = \cos \theta - i \sin \theta$$

$$\Rightarrow \frac{z+1}{z+2} = 1 - \frac{1}{z+2} = 1 - (\cos \theta - i \sin \theta)$$

$$= (1 - \cos \theta) + i \sin \theta$$

$$\operatorname{Im} \left(\frac{z+1}{z+2} \right) = \sin \theta, \sin \theta = \frac{1}{5}$$

$$\cos \theta = \pm \sqrt{1 - \frac{1}{25}} = \pm \frac{2\sqrt{6}}{5}$$

$$|\operatorname{Re}(\overline{z+2})| = \frac{2\sqrt{6}}{5}$$

76. (b)

$$a + 5b = 42, a, b \in \mathbb{N}$$

$$a = 42 - 5b, b = 1, a = 37$$

$$b = 2, a = 32$$

$$b = 3, a = 27$$

$$b = 8, a = 2$$

R has " 8 " elements $\Rightarrow m = 8$

$$\sum_{n=1}^8 (1 - i^{n!}) = x + iy$$

$$\text{for } n \geq 4, i^{n!} = 1$$

$$\Rightarrow (1 - i) + (1 - i^{2!}) + (1 - i^{3!})$$

$$= 1 - i + 2 + 1 + 1$$

$$= 5 - i = x + iy$$

$$m + x + y = 8 + 5 - 1 = 12$$

77. (b) $f(x) = \cos x - x + 1$

$$f'(x) = -\sin x - 1$$

f is decreasing $\forall x \in \mathbb{R}$

$$f(x) = 0$$

$$f(0) = 2, f(\pi) = -\pi$$

f is strictly decreasing in $[0, \pi]$ and $f(0) \cdot f(\pi) < 0$

$\Rightarrow$ only one solution of $f(x) = 0$

S1 is correct and S2 is incorrect.

78. (c) $\vec{a} = \alpha \hat{i} + 6\hat{j} - 3\hat{k}$

$$\vec{b} = \hat{i} - 2\hat{j} - 2\alpha \hat{k}$$

$$\text{so } \vec{a} \cdot \vec{b} < 0, \forall t \in \mathbb{R}$$

$$\alpha t^2 - 12 + 6\alpha t < 0$$

$$\alpha t^2 + 6\alpha t - 12 < 0, \forall t \in \mathbb{R}$$

$$\alpha < 0, \text{ and } D < 0$$

$$36\alpha^2 + 48\alpha < 0$$

$$12\alpha(3\alpha + 4) < 0$$

$$-\frac{4}{3} < \alpha < 0$$

$$\text{also for } a = 0, \vec{a} \cdot \vec{b} < 0$$

$$\text{hence } \alpha \in \left(-\frac{4}{3}, 0\right]$$

79. (c)

$$(1 + y^2)e^{\tan x} dx + \cos^2 x(1 + e^{2\tan x})dy = 0$$

$$\int \frac{\sec^2 x e^{\tan x}}{1 + e^{2\tan x}} dx + \int \frac{dy}{1 + y^2} = C$$

$$\Rightarrow \tan^{-1}(e^{\tan x}) + \tan^{-1} y = C$$

$$\text{for } x = 0, y = 1, \tan^{-1}(a) + \tan^{-1} 1 = C$$

$$C = \frac{\pi}{2}$$

$$\tan^{-1}(e^{\tan x}) + \tan^{-1} y = \frac{\pi}{2}$$

$$\text{Put } x = \pi, \tan^{-1} e + \tan^{-1} y = \frac{\pi}{2}$$

$$\tan^{-1} y = \cot^{-1} e$$

$$y = \frac{1}{e}$$

80. (b)

$$\text{H: } \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1, e = \sqrt{3}$$

$$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{3} \Rightarrow \frac{a^2}{b^2} = 2$$

$$a^2 = 2b^2$$

$$\text{length of L.R.} = \frac{2a^2}{b} = 4\sqrt{3}$$

$$a = \sqrt{6}$$

$$P(\alpha, 6) \text{ lie on } \frac{y^2}{3} - \frac{x^2}{6} = 1$$

$$12 - \frac{\alpha^2}{6} = 1 \Rightarrow \alpha^2 = 66$$

$$\text{Foci} = (0, \pm be) = (0, 3) \& (0, -3)$$

Let d_1 & d_2 be focal distances of $P(\alpha, 6)$

$$d_1 = \sqrt{\alpha^2 + (6 + be)^2}, d_2 = \sqrt{\alpha^2 + (6 - be)^2}$$

$$d_1 = \sqrt{66 + 81}, d_2 = \sqrt{66 + 9}$$

$$\beta = d_1 d_2 = \sqrt{147 \times 75} = 105$$

$$\alpha^2 + \beta = 66 + 105 = 171$$

81. (7)

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$$

$$A^7 = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix} \begin{bmatrix} -54 & 27 \\ -27 & -27 \end{bmatrix} = \begin{bmatrix} 3^6 \times 2 & -27^2 \\ 27^2 & 3^6 \end{bmatrix}$$

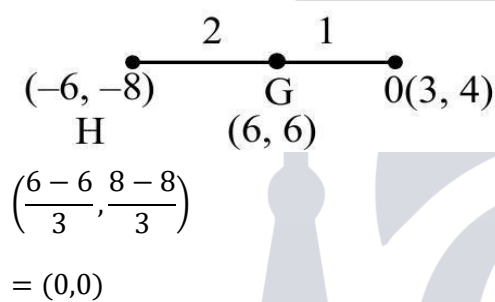
$$3^7 = 3^n \Rightarrow n = 7$$

82. (16)

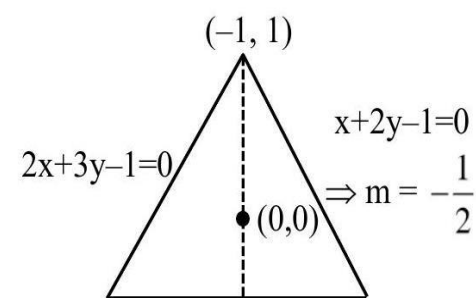
$$2x + 3y - 1 = 0$$

$$x + 2y - 1 = 0$$

$$ax + by - 1 = 0$$



$$\begin{aligned} & \text{Point G is the intersection of the line segment HO and the line } x + 2y - 1 = 0. \\ & \text{Coordinates of G: } \left(\frac{6 - 6}{3}, \frac{8 - 8}{3} \right) \\ & = (0, 0) \end{aligned}$$



$$\begin{aligned} & \text{Line } 2x + 3y - 1 = 0 \text{ is perpendicular to the line } x + 2y - 1 = 0. \\ & \Rightarrow m = -\frac{1}{2} \end{aligned}$$

$$ax + by - 1 = 0$$

$$\left(\frac{1 - 0}{-1 - 0} \right) \left(\frac{-a}{b} \right) = -1$$

$$\Rightarrow -a = b$$

$$\Rightarrow ax - ay - 1 = 0$$

$$ax - a \left(1 - \frac{2x}{3} \right) - 1 = 0$$

$$x \left(a + \frac{2a}{3} \right) = \frac{a}{3}$$

$$x = \frac{a+3}{5a}$$

$$2\left(\frac{a+3}{5a}\right) + 3y - 1 = 0$$

$$y = \frac{1 - \frac{2a+6}{5a}}{3} = \frac{3a-6}{3 \times 5a}$$

$$y = \frac{a-2}{5a}$$

$$\frac{\left(\frac{a-2}{5a}\right)}{\left(\frac{a+3}{5a}\right)} = 2 \Rightarrow a-2 = 2a+6$$

$$a = -8$$

$$b = 8$$

$$-8x + 8y - 1 = 0$$

$$|a-b| = 16$$

83. (17)

Blue balls	0	1	2	3	4	5
Prob.	$\frac{{}^5C_0 \cdot {}^4C_1}{{}^9C_3}$	$\frac{{}^5C_1 \cdot {}^4C_2}{{}^9C_3}$	$\frac{{}^5C_2 \cdot {}^4C_1}{{}^9C_3}$	$\frac{{}^5C_3 \cdot {}^4C_0}{{}^9C_3}$	0	0

$$7\bar{x} = \frac{{}^5C_1 \cdot {}^4C_2 + {}^5C_2 \cdot {}^4C_1 \times 2 + {}^5C_3 \cdot {}^4C_0 \times 3}{{}^9C_3} \times 7$$

$$\frac{30 + 80 + 30}{84} \times 7$$

$$= \frac{140}{12} = \frac{70}{6} = \frac{35}{3}$$

yellow	0	1	2	3	4
		${}^5C_2 \cdot {}^4C_1$	${}^5C_1 \cdot {}^4C_2$	${}^5C_0 \cdot {}^4C_3$	0

$$4\bar{y} = \frac{40 + 60 + 12}{84} \times 4 = \frac{112}{21} = \frac{16}{3}$$

84. (36) 2, 3, 4, 5, 7

total number of three - digit numbers not divisible by 3 will be formed by using the digits

(4,5,7)

(3,4,7)

(2,5,7)

(2,4,7)

(2,4,5)

(2,3,5)

number of ways = $6 \times 3! = 36$

85. (103)

$$S = 1 + 2 + 4 + 7 + \dots + T_n$$

$$S = 1 + 2 + 4 + \dots$$

$$T_n = 1 + 1 + 2 + 3 + \dots + (T_n - T_{n-1})$$

$$T_n = 1 + \left(\frac{n-1}{2}\right)[2 + (n-2) \times 1]$$

$$T_n = 1 + 1 + \frac{n(n-1)}{2}$$

$$n = 100 \quad T_n = 1 + \frac{100 \times 99}{2} = 4950 + 1$$

$$n = 101 \quad T_n = 1 + \frac{101 \times 100}{2} = 5050 + 1 = 5051$$

$$n = 102 \quad T_n = 1 + \frac{102 \times 101}{2} = 5151 + 1 = 5152$$

$$n = 103 \quad T_n = 1 + \frac{103 \times 102}{2} = 5254$$

$$n = 104 \quad T_n = 1 + \frac{104 \times 103}{2} = 5357$$

86. (96)

$$f(\theta) = \frac{\sin^4 \theta + 3\cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$$

$$f(\theta) = 1 + \frac{2\cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$$

$$f(\theta) = \frac{2\cos^2 \theta}{\cos^4 \theta - \cos^2 \theta + 1} + 1$$

$$f(\theta) = \frac{2}{\cos^2 \theta + \sec^2 \theta - 1} + 1$$

$$f(\theta)_{\min.} = 1$$

$$f(\theta)_{\max.} = 3$$

$$S = \frac{64}{1 - 1/3} = 96$$

87. (5)

$$\alpha = \sum_{r=0}^n (4r^2 + 2r + 1) \cdot {}^nC_r$$

$$\alpha = 4 \sum_{r=0}^n r^2 \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1} + 2 \sum_{r=0}^n r \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1} + \sum_{r=0}^n {}^nC_r$$

$$+ 4n \sum_{r=0}^n {}^{n-1}C_{r-1} + 2n \sum_{r=0}^n {}^{n-1}C_{r-1} + \sum_{r=0}^n {}^nC_r$$

$$\alpha = 4n(n-1) \cdot 2^{n-2} + 4n \cdot 2^{n-1} + 2n \cdot 2^{n-1} + 2^n$$

$$\alpha = 2^{n-2} [4n(n-1) + 8n + 4n + 4]$$

$$\alpha = 2^{n-2} [4n^2 + 8n + 4]$$

$$\alpha = 2n(n+1)^2$$

$$\beta = \sum_{r=0}^n \frac{{}^nC_r}{r+1} + \frac{1}{n+1}$$

$$= \sum_{r=0}^n \frac{{}^{n+1}C_{r+1}}{n+1} + \frac{1}{n+1}$$

$$= \frac{1}{n+1} (1 + {}^{n+1}C_1 + \dots + {}^{n+1}C_{n+1})$$

$$= \frac{2^{n+1}}{n+1}$$

$$\frac{2\alpha}{\beta} = \frac{2^{n+1}(n+1)^2}{2^{n+1}} \cdot (n+1) = (n+1)^3$$

$$140 < (n+1)^3 < 281$$

$$n = 4 \Rightarrow (n+1)^3 = 125$$

$$n = 5 \Rightarrow (n+1)^3 = 216$$

$$n = 6 \Rightarrow (n+1)^3 = 343$$

$$\therefore n = 5$$

88. (569)

$$\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$$

$$\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$$

$$\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b} + \vec{c} = 20\hat{i} + 5\hat{j} - 12\hat{k}$$

$$\vec{b} - \vec{c} = -14\hat{i} + 9\hat{j} - 14\hat{k}$$

$$(\vec{r} - (\vec{b} + \vec{c})) \times \vec{a} = 0$$

$$\vec{r} - (\vec{b} + \vec{c}) = \lambda \vec{a}$$

$$\vec{r} = \lambda \vec{a} + \vec{b} + \vec{c}$$

$$\text{But } \vec{r} \cdot (\vec{b} - \vec{c}) = 0$$

$$\Rightarrow (\lambda \vec{a} + \vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c}) = 0$$

$$\Rightarrow \lambda \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b} - \lambda \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} - \vec{c} \cdot \vec{c} = 0$$

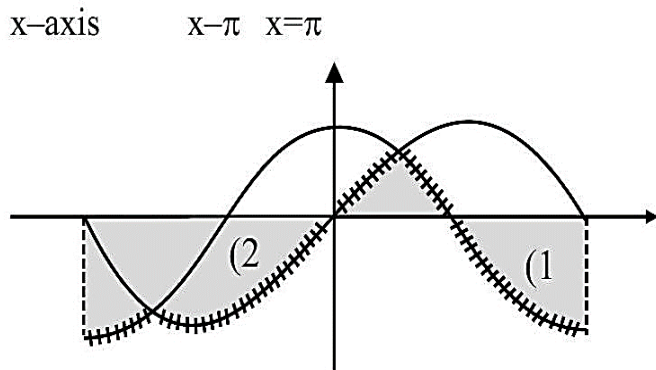
$$\lambda = \frac{\vec{c} \cdot \vec{c} - \vec{b} \cdot \vec{b}}{\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}} = \frac{294 - 227}{-389} = \frac{-67}{593}$$

$$\therefore \vec{r} = \vec{b} + \vec{c} - \frac{67}{593} \vec{a}$$

$$\Rightarrow 593\vec{r} + 67\vec{a} = 593(\vec{b} + \vec{c})$$

$$\Rightarrow |\vec{b} + \vec{c}|^2 = 569$$

89. (16) $y = \min\{\sin x, \cos x\}$



$$\int_0^{\pi/4} \sin x = (\cos x)_{\pi/4}^0 = 1 - \frac{1}{\sqrt{2}}$$

$$\int_{-\pi}^{-3\pi/4} (\sin x - \cos x) = (-\cos x - \sin x)_{-\pi}^{-3\pi/4}$$

$$= (\cos x + \sin x)_{-3\pi/4}^{-\pi}$$

$$= (-1 + 0) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= -1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$\int_{\pi/4}^{\pi/2} \cos x dx = (\sin x)_{\pi/4}^{\pi/2} = 1 - \frac{1}{\sqrt{2}}$$

$$A = 4$$

$$A^2 = 16$$

90. (55)

$$\lim_{x \rightarrow 0} 2 \left(\frac{\left(1 - \left(1 - \frac{x^2}{2!}\right)\right) \left(1 - \frac{4x^2}{2!}\right) \left(1 - \frac{9x^2}{2!}\right) \cdots \left(1 - \frac{100x^2}{2!}\right)}{x^2} \right)$$

By expansion

$$\lim_{x \rightarrow 0} \frac{2 \left(1 - \left(1 - \frac{x^2}{2}\right)\right) \left(1 - \frac{1}{2} \cdot \frac{4x^2}{2}\right) \left(1 - \frac{1}{3} \cdot \frac{9x^2}{2}\right) \cdots \left(1 - \frac{1}{10} \cdot \frac{100x^2}{2}\right)}{x^2}$$

$$\lim_{x \rightarrow 0} 2 \left(\frac{\left(1 - \left(1 - \frac{x^2}{2}\right)\right) \left(1 - \frac{2x^2}{2}\right) \left(1 - \frac{3x^2}{2}\right) \cdots \left(1 - \frac{10x^2}{2}\right)}{x^2} \right)$$

$$\lim_{x \rightarrow 0} \frac{2 \left(1 - 1 + x^2 \left(\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \cdots + \frac{10}{2}\right)\right)}{x^2}$$

$$2 \left(\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \cdots + \frac{10}{2}\right)$$

$$1 + 2 + \cdots + 10 = \frac{10 \times 11}{2} = 55$$

