

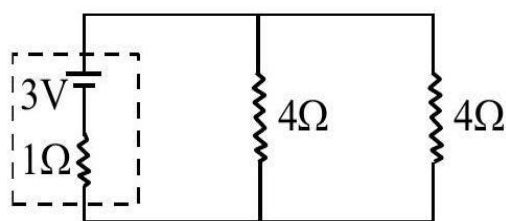
PART- I PHYSICS

SECTION- A

- Three bodies A, B and C have equal kinetic energies and their masses are 400 g, 1.2 kg and 1.6 kg respectively. The ratio of their linear momenta is:
 (a) $1:\sqrt{3}:2$ (b) $1:\sqrt{3}:\sqrt{2}$
 (c) $\sqrt{2}:\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}:1$
- Average force exerted on a non-reflecting surface at normal incidence is 2.4×10^{-4} N. If 360 W/cm^2 is the light energy flux during span of 1 hour 30 minutes. Then the area of the surface is:
 (a) 0.2 m^2 (b) 0.02 m^2
 (c) 20 m^2 (d) 0.1 m^2
- A proton and an electron are associated with same de-Broglie wavelength. The ratio of their kinetic energies is:

(Assume $h = 6.63 \times 10^{-34} \text{ J s}$, $m_e = 9.0 \times 10^{-31} \text{ kg}$ and $m_p = 1836 \text{ times } m_e$)

- (a) $1:1836$ (b) $1:\frac{1}{1836}$
 (c) $1:\frac{1}{\sqrt{1836}}$ (d) $1:\sqrt{1836}$
- A mixture of one mole of monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature (27°C). The ratio of specific heat of gases at constant volume respectively is:
 (a) $\frac{7}{5}$ (b) $\frac{3}{2}$
 (c) $\frac{3}{5}$ (d) $\frac{5}{3}$
- In an expression $a \times 10^b$:
 (a) a is order of magnitude for $b \leq 5$
 (b) b is order of magnitude for $a \leq 5$
 (c) b is order of magnitude for $5 < a \leq 10$
 (d) b is order of magnitude for $a \geq 5$
- In the given circuit, the terminal potential difference of the cell is:



- (a) 2 V (b) 4 V
 (c) 1.5 V (d) 3 V
- Binding energy of a certain nucleus is $18 \times 10^8 \text{ J}$. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus:
 (a) $0.2 \mu\text{g}$ (b) $20 \mu\text{g}$
 (c) $2 \mu\text{g}$ (d) $10 \mu\text{g}$
- Paramagnetic substances:
 (A) align themselves along the directions of external magnetic field.
 (B) attract strongly towards external magnetic field.
 (C) has susceptibility little more than zero.

(D) move from a region of strong magnetic field to weak magnetic field.

Choose the most appropriate answer from the options given below:

- (a) A, B, C, D (b) B, D Only
(c) A, B, C Only (d) A, C Only

9. A clock has 75 cm, 60 cm long second hand and minute hand respectively. In 30 minutes, duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly

(Take $\pi = 3.14$) :

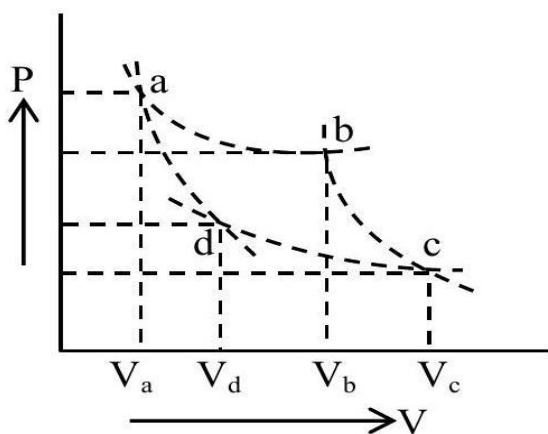
- (a) 139.4 (b) 140.5
(c) 220.0 (d) 118.9

10. Young's modulus is determined by the equation given by $Y = 49000 \frac{M}{\ell} \frac{\text{dyne}}{\text{cm}^2}$ where M is the mass and ℓ is the extension of wire used in the experiment. Now error in Young modulus (Y) is estimated by taking data from $M - \ell$ plot in graph paper. The smallest scale divisions are 5 g and 0.02 cm along load axis and extension axis respectively. If the value of M and ℓ are 500 g and 2 cm respectively then percentage error of Y is:

- (a) 0.2% (b) 0.02%
(c) 2% (d) 0.5%

11. Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram.

The relation between the ratio $\frac{V_a}{V_d}$ and the ratio $\frac{V_b}{V_c}$ is:



- (a) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$ (b) $\frac{V_a}{V_d} \neq \frac{V_b}{V_c}$
(c) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$ (d) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^2$

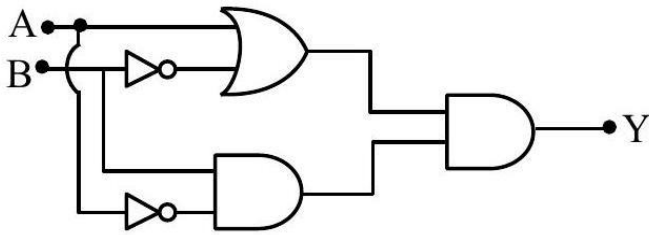
12. Two planets A and B having masses m_1 and m_2 move around the sun in circular orbits of r_1 and r_2 radii respectively. If angular momentum of A is L and that of B is $3L$, the ratio of time period $\left(\frac{T_A}{T_B}\right)$ is:

- (a) $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$ (b) $\left(\frac{r_1}{r_2}\right)^3$
(c) $\frac{1}{27} \left(\frac{m_2}{m_1}\right)^3$ (d) $27 \left(\frac{m_1}{m_2}\right)^3$

13. A LCR circuit is at resonance for a capacitor C , inductance L and resistance R . Now the value of resistance is halved keeping all other parameters same. The current amplitude at resonance will be now:

- (a) Zero (b) double
(c) same (d) halved

14. The output Y of following circuit for given inputs is:



- (a) $A \cdot B(A + B)$ (b) $A \cdot B$
(c) 0 (d) $\bar{A} \cdot B$

15. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is:

- (a) $\sqrt{ab}$ (b) ab
(c) $\frac{a}{b}$ (d) $\frac{b}{a}$

16. Correct Bernoulli's equation is (symbols have their usual meaning):

- (a) $P + mgh + \frac{1}{2}mv^2 = \text{constant}$
(b) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$
(c) $P + \rho g h + \rho v^2 = \text{constant}$
(d) $P + \frac{1}{2}\rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

17. A player caught a cricket ball of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is:

- (a) 150 N (b) 3 N
(c) 30 N (d) 300 N

18. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies ($K_B:K_A$) is :

- (a) $v_B:v_A$ (b) $m_B:m_A$
(c) $m_B v_B:m_A v_A$ (d) 1:1

19. Critical angle of incidence for a pair of optical media is 45° . The refractive indices of first and second media are in the ratio:

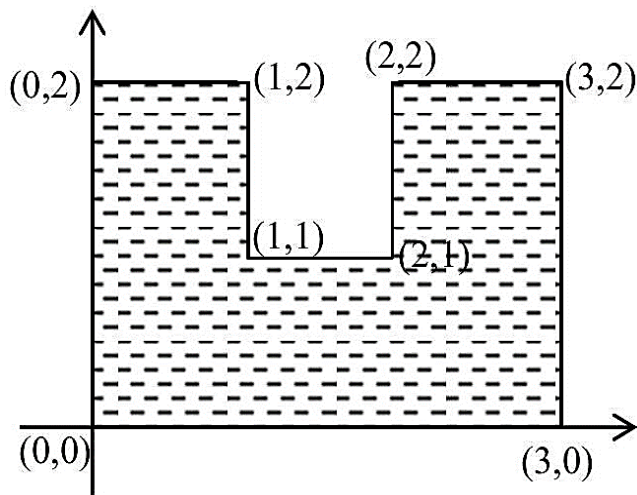
- (a) $\sqrt{2}:1$ (b) 1:2
(c) $1:\sqrt{2}$ (d) 2:1

20. The diameter of a sphere is measured using a vernier caliper whose 9 divisions of main scale are equal to 10 divisions of vernier scale. The shortest division on the main scale is equal to 1 mm. The main scale reading is 2 cm and second division of vernier scale coincides with a division on main scale. If mass of the sphere is 8.635 g, the density of the sphere is:

- (a) 2.5 g/cm^3 (b) 1.7 g/cm^3
(c) 2.2 g/cm^3 (d) 2.0 g/cm^3

SECTION- B

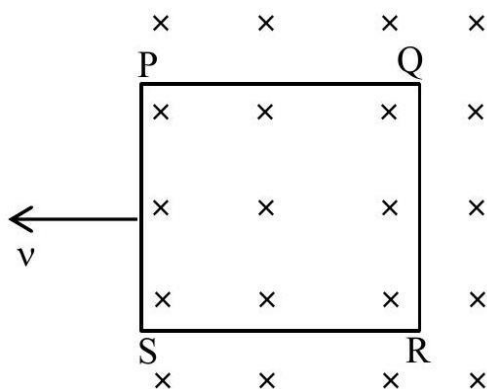
21. A uniform thin metal plate of mass 10 kg with dimensions is shown. The ratio of x and y coordinates of center of mass of plate in $\frac{n}{9}$. The value of n is _____



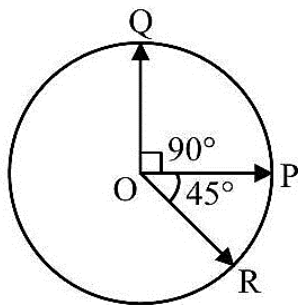
22. An electron with kinetic energy 5eV enters a region of uniform magnetic field of $3\mu\text{T}$ perpendicular to its direction. An electric field E is applied perpendicular to the direction of velocity and magnetic field. The value of E , so that electron moves along the same path, is _____ NC^{-1} .

(Given, mass of electron = $9 \times 10^{-31} \text{ kg}$, electric charge = $1.6 \times 10^{-19} \text{ C}$)

23. A square loop PQRS having 10 turns, area $3.6 \times 10^{-3} \text{ m}^2$ and resistance 100Ω is slowly and uniformly being pulled out of a uniform magnetic field of magnitude $B = 0.5 \text{ T}$ as shown. Work done in pulling the loop out of the field in 1.0 s is _____ $\times 10^{-6} \text{ J}$.



24. Resistance of a wire at 0°C , 100°C and $t^\circ\text{C}$ is found to be 10Ω , 10.2Ω and 10.95Ω respectively. The temperature t in Kelvin scale is _____.
25. An electric field, $\vec{E} = \frac{2\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{6}}$ passes through the surface of 4 m^2 area having unit vector $\hat{n} = \left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$. The electric flux for that surface is _____ V m .
26. A liquid column of height 0.04 cm balances excess pressure of soap bubble of certain radius. If density of liquid is $8 \times 10^3 \text{ kg m}^{-3}$ and surface tension of soap solution is 0.28 Nm^{-1} , then diameter of the soap bubble is _____ cm . (if $g = 10 \text{ ms}^{-2}$)
27. A closed and an open organ pipe have same lengths. If the ratio of frequencies of their seventh overtones is $\left(\frac{a-1}{a}\right)$ then the value of a is _____.
28. Three vectors $\vec{OP}$, $\vec{OQ}$ and $\vec{OR}$ each of magnitude A are acting as shown in figure. The resultant of the three vectors is $A\sqrt{x}$. The value of x is _____.



29. A parallel beam of monochromatic light of wavelength 600 nm passes through single slit of 0.4 mm width. Angular divergence corresponding to second order minima would be $____ \times 10^{-3}$ rad.
30. In an alpha particle scattering experiment distance of closest approach for the α particle is 4.5×10^{-14} m. If target nucleus has atomic number 80, then maximum velocity of α -particle is $___ \times 10^5$ m/s approximately.

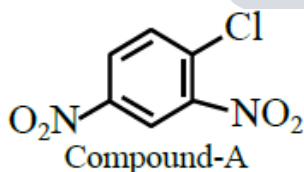
$$\left(\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ SI unit, mass of } \alpha \text{ particle} = 6.72 \times 10^{-27} \text{ kg}\right)$$

PART- II CHEMISTRY

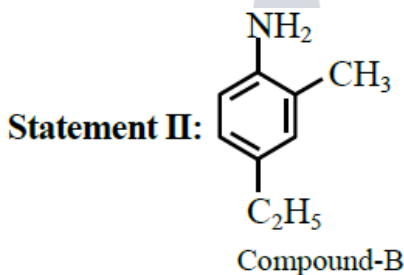
SECTION- A

31. Given below are two statements:

Statement I:



IUPAC name of Compound A is 4-chloro-1, 3-dinitrobenzene:



Statement II:

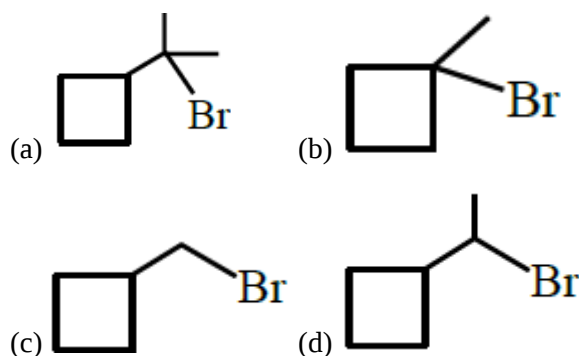
IUPAC name of Compound B is

4-ethyl-2-methylaniline.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Both Statement I and Statement II are correct
- (b) Statement I is incorrect but Statement II is correct
- (c) Statement I is correct but Statement II is incorrect
- (d) Both Statement I and Statement II are incorrect

32. Which among the following compounds will undergo fastest S_N2 reaction?

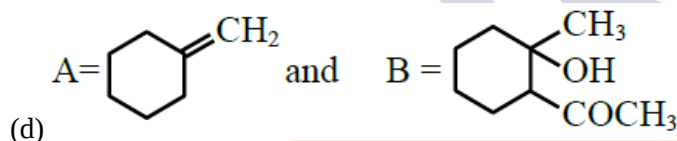
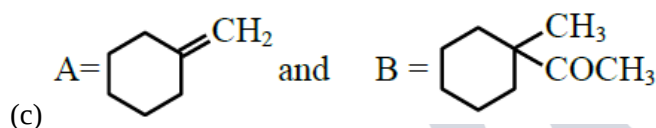
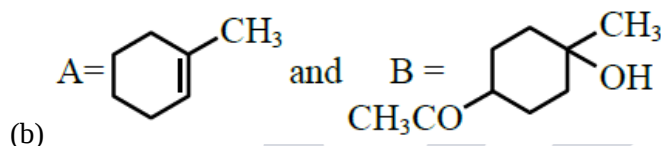
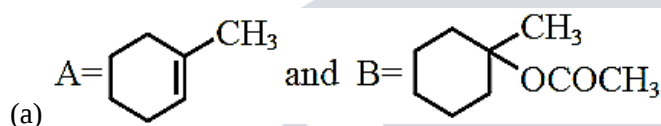
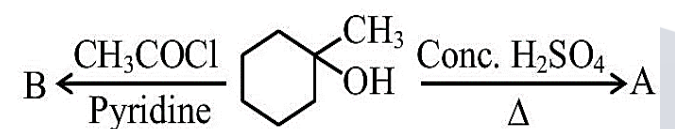


33. Combustion of glucose ($C_6H_{12}O_6$) produces CO_2 and water. The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is:

[Molar mass of glucose in $gmol^{-1} = 180$]

- (a) 480 (b) 960
(c) 800 (d) 32

34. Identify the major products A and B respectively in the following set of reactions.



35. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: The stability order of +1 oxidation state of Ga, In and Tl is $Ga < In < Tl$.

Reason R : The inert pair effect stabilizes the lower oxidation state down the group.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both A and R are true and R is the correct explanation of A.
(b) A is true but R is false.
(c) Both A and R are true but R is NOT the correct explanation of A.
(d) A is false but R is true.

36. Match List I with List-II

List-I (Name of the test)		List-II (Reaction sequence involved) [M is metal]	
A	Borax bead test	I.	$MCO_3 \rightarrow MO$ $\xrightarrow{+\Delta} Co(NO_3)_2 \rightarrow CoO \cdot MO$
B.	Charcoal cavity test	II.	$MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$
C.	Cobalt nitrate test	III	$MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7}$ $M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$
D.	Flame test	IV	$MSO_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow$ $MO \rightarrow M$

Choose the correct answer from the option below:

- (a) A-III, B-I, C-IV, D-II (b) A-III, B-II, C-IV, D-I
(c) A-III, B-I, C-II, D-IV (d) A-III, B-IV, C-I, D-II

37. Match List I and with List II

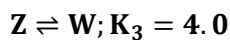
List-I (Molecule)		List-II(Shape)	
A	NH ₃	I.	Square pyramid
B.	BrF ₅	II.	Tetrahedral
C.	PCl ₅	III	Trigonal pyramidal
D.	CH ₄	IV	Trigonal bipyramidal

Choose the correct answer from the option below:

- (a) A-IV, B-III, C-I, D-II (b) A-II, B-IV, C-I, D-III
(c) A-III, B-I, C-IV, D-II (d) A-III, B-IV, C-I, D-II

38. For the given hypothetical reactions, the equilibrium constants are as follows:



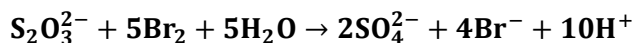
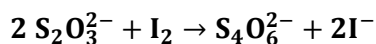


The equilibrium constant for the reaction

$X \rightleftharpoons W$ is

- (a) 6.0 (b) 12.0
(c) 8.0 (d) 7.0

39. Thiosulphate reacts differently with iodine and bromine in the reaction given below:



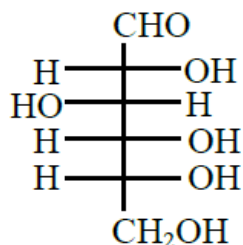
Which of the following statement justifies the above dual behaviour of thiosulphate?

- (a) Bromine undergoes oxidation and iodine undergoes reduction by iodine in these reactions
(b) Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reaction
(c) Bromine is a stronger oxidant than iodine
(d) Bromine is a weaker oxidant than iodine

40. An octahedral complex with the formula $CoCl_3 \cdot nNH_3$ upon reaction with excess of $AgNO_3$ solution given 2 moles of $AgCl$. Consider the oxidation state of Co in the complex is 'x'. The value of "x + n" is _____.

- (a) 3 (b) 6
(c) 8 (d) 5

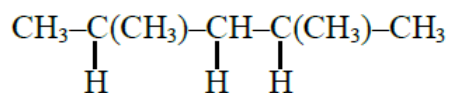
41.



The incorrect statement regarding the given structure is

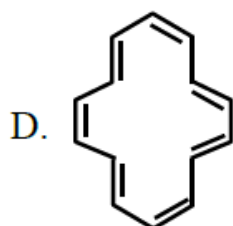
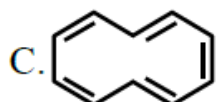
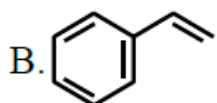
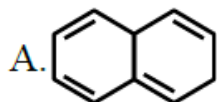
- (a) Can be oxidized to a dicarboxylic acid with Br_2 water
(b) despite the presence of $-CHO$ does not give Schiff's test
(c) has 4-asymmetric carbon atom
(d) will coexist in equilibrium with 2 other cyclic structure

42. In the given compound, the number of 2° carbon atom/s is _____



- (a) Three (b) One
(c) Two (d) Four

43. Which of the following are aromatic?



- (a) B and D only (b) A and C only
(c) A and B only (d) C and D only

44. Among the following halogens

F_2 , Cl_2 , Br_2 and I_2

Which can undergo disproportionation reaction?

- (a) Only I_2 (b) Cl_2 , Br_2 and I_2
(c) F_2 , Cl_2 and Br_2 (d) F_2 and Cl_2

45. Given below are two statements:

Statement I: $N(CH_3)_3$ and $P(CH_3)_3$ can act as ligands to form transition metal complexes.

Statement II: As N and P are from same group, the nature of bonding of $N(CH_3)_3$ and $P(CH_3)_3$ is always same with transition metals.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Statement I is incorrect but Statement II is correct
(b) Both Statement I and Statement II are correct
(c) Statement I is correct but Statement II is incorrect
(d) Both Statement I and Statement II are incorrect

46. Match List I with List II

List-I (Elements)		List-II (Properties in their respective groups)	
A	Cl, S	I.	Elements with highest electronegativity
B.	Ge, As	II.	Elements with largest atomic size
C.	Fr, Ra	III	Elements which show properties of both metals and non metal
D.	F, O	IV	Elements with highest negative electron gain enthalpy

Choose the correct answer from the options given below:

- (a) A-II, B-III, C-IV, D-I (b) A-III, B-II, C-I, D-IV
(c) A-IV, B-III, C-II, D-I (d) A-II, B-I, C-IV, D-III

47. Iron (III) catalyses the reaction between iodide and persulphate ions, in which

- (A) Fe^{3+} oxidises the iodide ion
(B) Fe^{3+} oxidises the persulphate ion
(C) Fe^{2+} reduces the iodide ion
(D) Fe^{2+} reduces the persulphate ion

Choose the most appropriate answer from the options given below:

- (a) B and C only (b) B only
(c) A only (d) A and D only

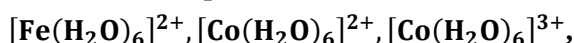
48. Match List I with List II

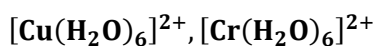
List-I (Compound)		List-II (Colour)	
A	$\text{Fe}_4[\text{Fe}(\text{CN})_6]_3 \cdot x\text{H}_2\text{O}$	I.	Violet
B.	$[\text{Fe}(\text{CN})_5\text{NOS}]^{4-}$	II.	Blood Red
C.	$[\text{Fe}(\text{SCN})]^{2+}$	III.	Prussian Blue
D.	$(\text{NH}_4)_3\text{PO}_4 \cdot 12\text{MoO}_3$	IV.	Yellow

Choose the correct answer from the options given below:

- (a) A-III, B-I, C-II, D-IV (b) A-IV, B-I, C-II, D-III
(c) A-II, B-III, C-IV, D-I (d) A-I, B-II, C-III, D-IV

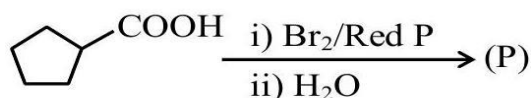
49. Number of complexes with even number of electrons in t_{2g} orbitals is -





- (a) 1 (b) 3
(c) 2 (d) 5

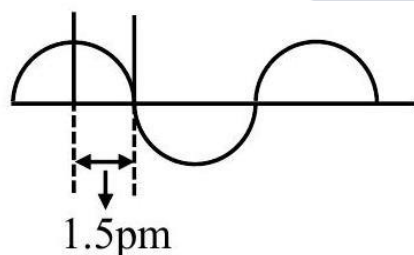
50. Identify the product (P) in the following reaction:



- (a) (b)
(c) (d)

SECTION- B

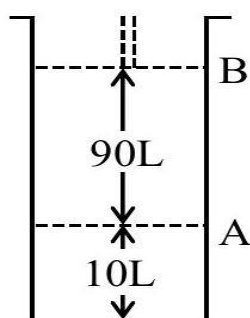
51. A hypothetical electromagnetic wave is show below.



The frequency of the wave is $x \times 10^{19}$ Hz.

$x =$ _____ (nearest integer)

52.



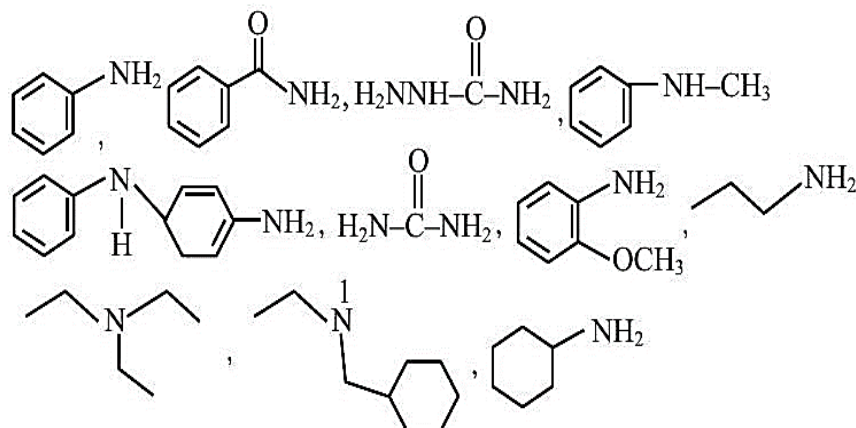
Consider the figure provided.

1 mol of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at 18°C . If the piston is moved to position B, keeping the temperature unchanged, then ' x ' L atm work is done in this reversible process.

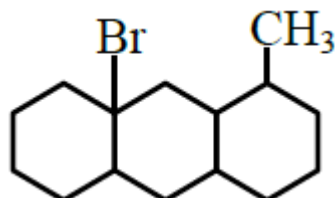
$x =$ _____ L atm. (nearest integer)

[Given: Absolute temperature = $^\circ\text{C} + 273.15$, $R = 0.08206 \text{ L atm mol}^{-1} \text{ K}^{-1}$]

53. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsberg's reagent is _____



54. The number of optical isomers in following compound is : _____.



55. The 'spin only' magnetic moment value of MO_4^{2-} is _____ BM. (Where M is a metal having least metallic radii. among Sc, Ti, V, Cr, Mn and Zn).

(Given atomic number: Sc = 21, Ti = 22, V = 23, Cr = 24, Mn = 25 and Zn = 30)

56. Number of molecules from the following which are exceptions to octet rule is _____ .

CO_2 , NO_2 , H_2SO_4 , BF_3 , CH_4 , SiF_4 , ClO_2 , PCl_5 , BeF_2 , C_2H_6 , CHCl_3 , CBr_4

57. If 279 g of aniline is reacted with one equivalent of benzenediazonium chloride, the maximum amount of aniline yellow formed will be _____ g. (nearest integer)

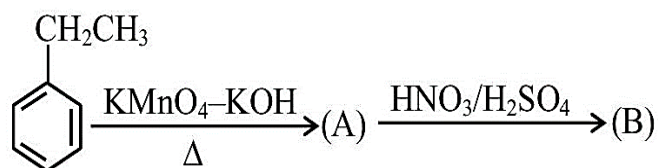
(consider complete conversion)

58. Consider the following reaction $\text{A} + \text{B} \rightarrow \text{C}$

The time taken for A to become $\frac{1}{4}$ th of its initial concentration is twice the time taken to become $\frac{1}{2}$ of the same. Also, when the change of concentration of B is plotted against time, the resulting graph gives a straight line with a negative slope and a positive intercept on the concentration axis.

The overall order of the reaction is _____.

59. Major product B of the following reaction has _____ π -bond.



60. A solution containing 10 g of an electrolyte AB_2 in 100 g of water boils at 100.52°C . The degree of ionization of the electrolyte (α) is _____ $\times 10^{-1}$. (nearest integer)

[Given : Molar mass of $AB_2 = 200 \text{ g mol}^{-1}$. K_b (molal boiling point elevation const. of water) = $0.52 \text{ K kg mol}^{-1}$, boiling point of water = 100°C ; AB_2 ionises as $AB_2 \rightarrow A^{2+} + 2 B^-$]

PART- III MATHEMATICS

SECTION- A

61. The value of $k \in \mathbb{N}$ for which the integral

$$I_n = \int_0^1 (1 - x^k)^n dx, n \in \mathbb{N}, \text{ satisfies } 147I_{20} = 148I_{21}$$

is:

- (a) 10 (b) 8
(c) 14 (d) 7

62. The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$ is :

- (a) $1 + \log_6(8)$ (b) $\log_8(6)$
(c) $1 + \log_8(6)$ (d) $\log_8(d)$

63. Let the circles $C_1: (x - \alpha)^2 + (y - \beta)^2 = r_1^2$ and $C_2: (x - 8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point $(6, 6)$. If the point $(6, 6)$ divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio 2: 1, then $(\alpha + \beta) + 4(r_1^2 + r_2^2)$ equals

- (a) 110 (b) 130
(c) 125 (d) 145

64. Let $P(x, y, z)$ be a point in the first octant, whose projection in the xy - plane is the point Q . Let $OP = \gamma$; the angle between OQ and the positive x -axis be θ ; and the angle between OP and the positive z -axis be ϕ , where O is the origin. Then the distance of P from the x -axis is :

- (a) $\gamma\sqrt{1 - \sin^2 \phi \cos^2 \theta}$
(b) $\gamma\sqrt{1 + \cos^2 \theta \sin^2 \phi}$
(c) $\gamma\sqrt{1 - \sin^2 \theta \cos^2 \phi}$
(d) $\gamma\sqrt{1 + \cos^2 \phi \sin^2 \theta}$

65. The number of critical points of the function $f(x) = (x - 2)^{2/3}(2x + 1)$ is:

- (a) 2 (b) 0
(c) 1 (d) 3

66. Let $f(x)$ be a positive function such that the area bounded by $y = f(x)$, $y = 0$ from $x = 0$ to $x = a > 0$ is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is:

(a) $(8e^x - 1) \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(b) $(8e^x + 1) \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

(c) $(8e^x + 1) \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(d) $(8e^x - 1) \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

67. Let $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is:

- (a) 1 (b) 2
(c) 3 (d) 4

68. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where

I is the identity matrix of order 3×3 , then $2a + 3b$ is equal to:

- (a) -10 (b) -13
(c) -9 (d) -12

69. If the shortest distance between the lines

$$L_1: \vec{r} = (2 + \lambda)\hat{i} + (1 - 3\lambda)\hat{j} + (3 + 4\lambda)\hat{k}, \lambda \in \mathbb{R}$$

$$L_2: \vec{r} = 2(1 + \mu)\hat{i} + 3(1 + \mu)\hat{j} + (5 + \mu)\hat{k}, \mu \in \mathbb{R}$$

is $\frac{m}{\sqrt{n}}$, where $\gcd(m, n) = 1$, then the value of $m + n$ equals.

- (a) 384 (b) 387
(c) 377 (d) 390

70. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest positive product, is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $n - m$ equals:

- (a) 9 (b) 11
(c) 8 (d) 10

71. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to:

- (a) 109 (b) 108
(c) 18 (d) 19

72. Let $I(x) = \int \frac{6}{\sin^2 x (1 - \cot x)^2} dx$. If $I(0) = 3$, then $I\left(\frac{\pi}{12}\right)$ is equal to:

- (a) $\sqrt{3}$ (b) $3\sqrt{3}$
(c) $6\sqrt{3}$ (d) $2\sqrt{3}$

73. The equations of two sides AB and AC of a triangle ABC are $4x + y = 14$ and $3x - 2y = 5$, respectively.

The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio 2: 1. The equation of the side BC is:

- (a) $x - 6y - 10 = 0$ (b) $x - 3y - 6 = 0$
(c) $x + 3y + 2 = 0$ (d) $x + 6y + 6 = 0$

74. Let $[t]$ be the greatest integer less than or equal to

t . Let A be the set of all prime factors of 2310 and $f: A \rightarrow \mathbb{Z}$ be the function $f(x) = \left[\log_2 \left(x^2 + \left[\frac{x^3}{5} \right] \right) \right]$.

The number of one-to-one functions from A to the range of f is:

- (a) 20 (b) 120
(c) 25 (d) 24

75. Let z be a complex number such that $|z + 2| = 1$ and $\operatorname{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $|\operatorname{Re}(z + 2)|$ is:

- (a) $\frac{\sqrt{6}}{5}$ (b) $\frac{1+\sqrt{6}}{5}$
(c) $\frac{24}{5}$ (d) $\frac{2\sqrt{6}}{5}$

76. If the set $R = \{(a, b); a + 5b = 42, a, b \in \mathbb{N}\}$ has m elements and $\sum_{n=1}^m (1 + i^{n!}) = x + iy$, where $I = \sqrt{-1}$, then the value of $m + x + y$ is:

- (a) 8 (b) 12
(c) 4 (d) 5

77. For the function $f(x) = (\cos x) - x + 1, x \in \mathbb{R}$, between the following two statements

(S1) $f(x) = 0$ for only one value of x is $[0, \pi]$.

(S2) $f(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$.

- (a) Both (S1) and (S2) are correct
 (b) Only (S1) is correct
 (c) Both (S1) and (S2) are incorrect
 (d) Only (S2) is correct

78. The set of all α , for which the vector $\vec{a} = \alpha\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 2\alpha\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$ is :

- (a) $[0, 1)$ (b) $(-2, 0]$
 (c) $(-\frac{4}{3}, 0]$ (d) $(-\frac{4}{3}, 1)$

79. Let $y = y(x)$ be the solution of the differential equation $(1 + y^2)e^{\tan x}dx + \cos^2 x(1 + e^{2\tan x})dy = 0$, $y(0) = 1$. Then $y(\frac{\pi}{4})$ is equal to:

- (a) $\frac{2}{e}$ (b) $\frac{1}{e^2}$
 (c) $\frac{1}{e}$ (d) $\frac{2}{e^2}$

80. Let $H: \frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$.

Suppose the point $(\alpha, 6)$, $\alpha > 0$ lies on H . If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to:

- (a) 170 (b) 171
 (c) 169 (d) 172

SECTION -B

81. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n , then n is equal to ____.

82. If the ortho centre of the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumcentre and orthocentre respectively are $(3, 4)$ and $(-6, -8)$, then the value of $|a - b|$ is ____.

83. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and Yellow balls. If $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively, then $7\bar{X} + 4\bar{Y}$ is equal to ____.

84. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to ____.

85. Let the positive integers be written in the form:

```

      1
    2   3
  4   5   6
. 7   8   9   10 .
. . . . .
    
```

If the k^{th} row contains exactly k numbers for every natural number k , then the row in which the number 5310 will be, is ____.

86. If the range of $f(\theta) = \frac{\sin^4 \theta + 3\cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$, $\theta \in \mathbb{R}$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to ____.

87. Let $\alpha = \sum_{r=0}^n (4r^2 + 2r + 1) {}^nC_r$

and $\beta = \left(\sum_{r=0}^n \frac{{}^nC_r}{r+1} \right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$, then the value of n is _____.

88. Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$, $\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then $\frac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$ is equal to _____

89. Let the area of the region enclosed by the curve $y = \min\{\sin x, \cos x\}$ and the x-axis between $x = -\pi$ to $x = \pi$ be A . Then A^2 is equal to _____

90. The value of

$\lim_{x \rightarrow 0} 2 \left(\frac{1 - \cos x \sqrt{\cos 2x} \sqrt[3]{\cos 3x} \dots \sqrt[10]{\cos 10x}}{x^2} \right)$ is _____

