1. **(B)** $w_g + w_{Fr} + w_s = \Delta KE$

$$5 \times 10 \times 5 - 0.5 \times 5 \times 10 \times x - \frac{1}{2}Kx^2 = 0 - 0$$

$$250 = 25x + 50x^2$$

$$2x^2 + x - 10 = 0$$

$$x = 2$$

2. **(c)**

Z in LHS $= 92$

Z in RHS $= 56 + 36 = 92$

A in LHS $= 236$

A in RHS $= 141 + 92 = 233$

So 3 neutrons are released.

3. **(b)** $E = \frac{KQ}{R^2}$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$

$$\varepsilon_0 = \frac{Q}{4\pi R^2 E}$$

Now, $\varepsilon_0 E^2 = \frac{Q}{4\pi R^2 E}.E^2 = \frac{Q}{4\pi R^2}.E$

$$[\varepsilon_0 E^2] = \left[\frac{QE}{R^2}\right] = \frac{[Q][E]}{[R^2]} = \frac{[Q]}{[R^2]}\frac{[W]}{[Q][R]}$$

$$= \frac{[W]}{[R^3]} = \frac{ML^2\,T^{-2}}{L^3} = ML^{-1}\,T^{-2}$$

4. **(a)** For lens 1: $f_1 = 10, u = -30, v = ?$

$$v = \frac{uf}{u+f} = \frac{-30 \times 10}{-30 + 10} = 15$$

For lens 2: $f_1 = -10, u = 10, v = ?$

$$v = \frac{uf}{u+f} = \frac{10 \times -10}{10 - 10} = \infty$$

For lens 3: $f = 30, u = -\infty, v = ?$

So v will be 30.

5. **(b)** $y = 2\cos 2\pi(330t - x)m$

$y = A\cos(\omega t - kx)$

by comparing $\omega = 2\pi \times 330$

$2\pi f = 2\pi \times 330$

$f = 330$

6. **(c)** $I_1\omega = I_2\omega_2$

$\frac{MR^2}{2}\omega = \frac{3}{2}\left(\frac{MR^2}{2}\right)\omega_2$

$\omega_2 = \frac{2}{3}\omega$

7. **(d)** v_1 = volume immersed in water.

v_2 = volume immersed in oil.

$v_1\rho_w g + v_2\rho_o g = (v_1 + v_2)\rho_c g$

$v_1 + \frac{v_2\rho_o}{\rho_w} = (v_1 + v_2)\frac{\rho_c}{\rho_w}$

$= v_1 + 0.8v_2 = 0.9v_1 + 0.9v_2$

$= 0.1v_1 = 0.1v_2$

$v_1 : v_2 = 1 : 1$

8. **(d)**

$\lambda = \frac{RT}{\sqrt{2}\pi d^2\, N_A P}$

$KE = \frac{f}{2}nRT$

9. **(c)**

$\mathbf{v} = \sqrt{\frac{GM}{R}}$

$\frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$

$v_B = 2v_A = 6v$

10. **(c)** $B_1 2\pi\frac{a}{2} = \mu_o\frac{I}{4}$

$B_1 = \frac{\mu_o I}{4\pi a}$

$B_2 2\pi 2a = \mu_o I$

$B_2 = \dfrac{\mu_o I}{4\pi a}$

11. (b) $\dfrac{u^2 \sin 2\theta}{g} = \dfrac{u^2 \sin^2 \theta}{2\,g}$

$4\sin\theta\cos\theta = \sin^2\theta$

$4 = \tan\theta$

12. (c) $P = \frac{v^2}{R}, R = \frac{\rho\ell}{A}$

$P \propto \dfrac{1}{\ell}$

$\dfrac{P_1}{P_2} = \dfrac{t_2}{t_1} = \dfrac{15}{20} = \dfrac{\ell_2}{\ell_1}$

$\ell_2 = \dfrac{3}{4}\ell_1$

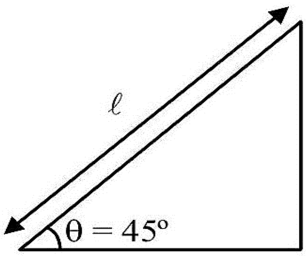
13. (a)

$\dfrac{A\varepsilon_o}{d} = \dfrac{A\varepsilon_o}{\left(0.2 + \frac{d}{k}\right)}$

$0.6 = 0.2 + \dfrac{0.6}{k}$

$k = \dfrac{3}{2}$

14. (a)



Case-1 : No friction

$a = g\sin\theta$

$\ell = \dfrac{1}{2}(g\sin\theta)t_1^2$

$t_1 = \sqrt{\dfrac{2\ell}{g\sin\theta}}$

Case-2 : With friction

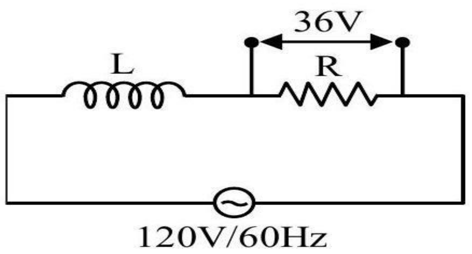
$a = g\sin\theta - \mu g\cos\theta$

$\ell = \frac{1}{2}(g\sin\theta - \mu g\cos\theta)t_2^2$

$\sqrt{\frac{2\ell}{g\sin\theta - \mu g\cos\theta}} = n\sqrt{\frac{2\ell}{g\sin\theta}}$

$\mu = 1 - \frac{1}{n^2}$

15. (a)



$36 = I_{rms}R$

$36 = \frac{120}{\sqrt{X_L^2 + R^2}} \times R$

$R = 90\Omega \Rightarrow 36 = \frac{120 \times 90}{\sqrt{X_L^2 + 90^2}}$

$\sqrt{X_L^2 + 90^2} = 300$

$X_L^2 = 81900$

$X_L = 286.18$

$\omega L = 286.18$

$L = \frac{286.18}{376.8}$

$L = 0.76H$

16. (b) Least count $= \frac{1}{100}$ mm $= 0.01$ mm

zero error $= +0.05$ mm

Reading $= 4 \times 1$ mm $+ 60 \times 0.01$ mm $- 0.05$ mm $= 4.55$ mm

17. (d) De Broglie wavelength of proton & electron $= \lambda$

$\because \lambda = \frac{h}{p}$

$\therefore p_{proton} = p_{electron}$

$\because KE = \frac{p^2}{2\,m}$

$\therefore KE_{proton} < KE_{electron}$

$[K_p < K_e]$

18. (b) Least count of vernier calipers $= \frac{1}{20\,N}$ cm

$\because$ Least count $= 1MSD - 1VSD$

let x no. of divisions of main scale coincides with N division of vernier scale, then

$1VSD = \frac{x \times 1\text{ mm}}{N}$

$\therefore \frac{1}{20\,N}\text{ cm} = 1\text{ mm} - \frac{x \times 1\text{ mm}}{N}$

$\frac{1}{2\,N}\text{ mm} = 1\text{ mm} - \frac{x}{N}\text{mm}$

$x = \left(1 - \frac{1}{2\,N}\right)N$

$x = \frac{2\,N - 1}{2}$

19. (a) $B.E. = \Delta mC^2$

$(5M_p + 7M_n - M_o)C^2$

20. (a) For Isobaric process

$w = P\Delta v = nR\Delta T = 100\text{ J}$

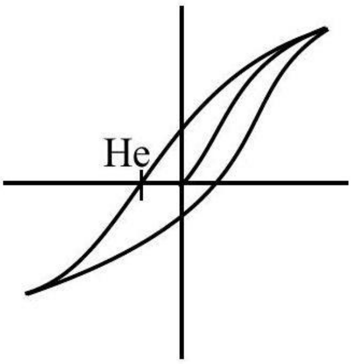
$Q = \Delta u + w$

$\Delta Q = \frac{F}{2}nR\Delta T + nR\Delta T$

$\left(\frac{f}{2} + 1\right)nR\Delta T$

$\left(\frac{5}{2} + 1\right)100 = 350\text{ J}$

21. (10)



$H_c = \frac{\mu_o ni}{\mu_o}$

$5 \times 10^3 = \frac{150}{30} \times 100 \times i$

$\frac{50}{5} = i$

$I = 10$

22. (40) m = mass of small drop

M = mass of bigger drop

$V_t = \frac{2}{9} \frac{R^2(\rho - \sigma)g}{\eta}$

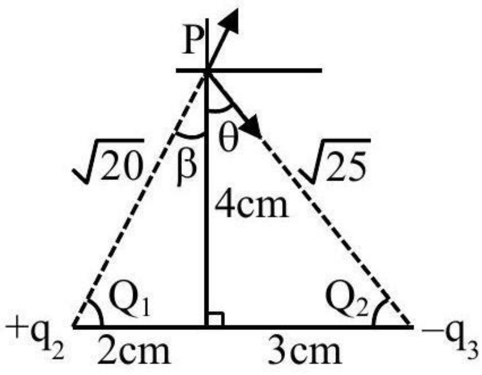
$8 \propto m = M$

$8r^3 = R^3 \Rightarrow R = 2R$

as $V_t \times R^2$ $\because$ Radius double so V_t becomes 4 time

$\therefore 4 \times 10 = 40$ cm/s

23. (5)



$\frac{Kq_2}{20} \cos\beta = \frac{Kq_3}{25} \cos\theta$

$\frac{Kq_2}{20} \frac{4}{\sqrt{20}} = \frac{Kq_3}{25} \frac{4}{\sqrt{25}}$

$$\frac{q_2}{q_3} = \frac{20}{25}\sqrt{\frac{20}{25}} = \frac{8}{5\sqrt{x}}$$

$$\Rightarrow \sqrt{x} = \frac{8 \times 25\sqrt{25}}{5 \times 20\sqrt{20}}$$

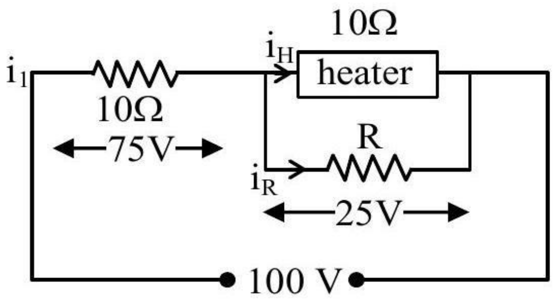
$x = 5$

24. (5) $R_{heater} = \frac{V^2}{P} = \frac{(100)^2}{1000} = 10\Omega$

For heater $P = \frac{V^2}{R} \Rightarrow V = \sqrt{PR}$

$V = \sqrt{62.5 \times 10}$

$V = 25v$



$i_1 = \frac{75}{10} = 7.5\ A,\ i_H = \frac{25}{10} = 2.5\ A.$

$i_R = i_1 - i_H = 5$

$V = IR$

$$R = \frac{25}{5} = 5\Omega$$

25. (22)

$C = 2\mu f;\ E = 110\sqrt{2}\sin(100t)$

$$X_C = \frac{1}{\omega c} = \frac{1}{100 \times 2 \times 10^6}$$

$$= \frac{10000}{2} = 5000\Omega$$

$$i_o = \frac{110\sqrt{2}}{5000}$$

$$i_{rms} = \frac{110\sqrt{2}}{5000\sqrt{2}}$$

$$= \frac{110}{5}\ mA$$

$= 22\text{ mA}$

26. (2) $\mathrm{d} = 1\text{ mm}, \mathrm{D} = 1\text{ m}, \lambda = 500\text{ nm}$

$$10\left(\frac{\lambda D}{d}\right) = \frac{2\lambda D}{a}$$

$$\mathbf{a} = \frac{\mathrm{d}}{5}$$

$$= \frac{10 \times 10^{-4}\text{ m}}{5}$$

$$= 2 \times 10^{-4}$$

27. (6) Total energy = K.E. + P.E.

at $x = 0.04$ m, T.E. $= 0.5 + 0.4 = 0.9$ J

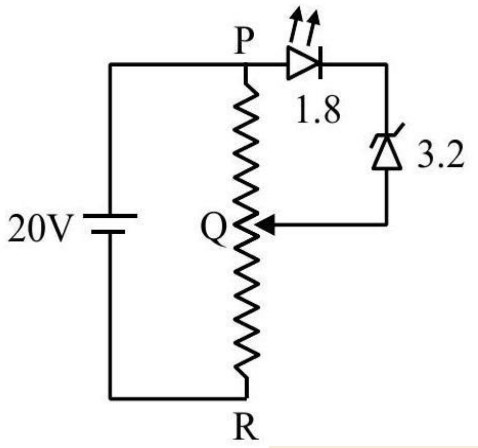
$\mathrm{T.E} = 1\ \mathrm{m}\omega^2\ \mathrm{A}^2 = 0.9$

$$= \frac{1}{2} \times 0.2\left(2\pi \times \frac{25}{\pi}\right)^2 \times \mathrm{A}^2 = 0.9$$

$\Rightarrow \mathrm{A} = 0.06$ m

$\mathrm{A} = 6$ cm

28. (5)



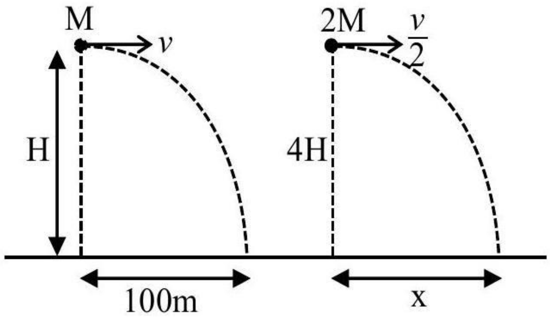
$\mathrm{PR} = 20$ cm

$$\mathrm{V_{PQ}} = \frac{1}{4} \times \mathrm{R_{PR}}$$

$$\ell_{\min}\ (\mathrm{PQ}) = \frac{1}{4} \times 20$$

$= 5$ cm

29. (100)



$$100 = v\sqrt{\frac{2H}{g}};\ x = \frac{v}{2}\sqrt{\frac{2(4H)}{g}} = v\sqrt{\frac{2H}{g}}$$

$$\Rightarrow x = 100$$

30. (2)

$$a_c = \omega^2 x$$

$$\frac{vdv}{dx} = \omega^2 x$$

$$\int_0^v vdv = \int_1^3 \omega^2 x dx$$

$$\frac{v^2}{2} = \omega^2\left[\frac{x^2}{2}\right]$$

$$\frac{v^2}{2} = \frac{\omega^2}{2}[3^2 - 1^2]$$

$$v = 2\sqrt{2}\omega$$

$$x = 2$$

31. (c)

$$\underset{\text{Or } HPO_4^{-}}{PO_4^{3-}} + \underset{\substack{\text{Ammonium}\\ \text{Molybdate}}}{(NH_4)_2MoO_4} \xrightarrow{H^+} \underset{\substack{\text{Canary yellow ppt.}\\ \text{(Ammonium phopho}\\ \text{molybdate)}}}{(NH_4)_3PO_4.12MoO_3 \downarrow}$$

32. (d)

Rate of formation of B is

$$\frac{d[B]}{dt} = k_1[A] - k_2[B]$$

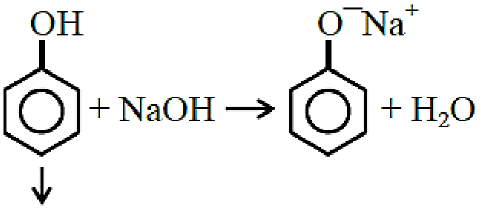
$$0 = k_1[A] - k_2[B]$$

$$\left(\frac{k_1}{k_2}\right)[A] = [B]$$

33. (b) Antibonding molecular orbitals are formed by destructive interference of wave functions.

(ABMO) $\sigma^* = \psi_A - \psi_B$

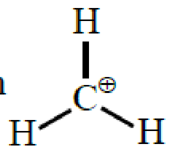
34. (d)



Stronger ACID than H_2O

35. (a)

Carbocation



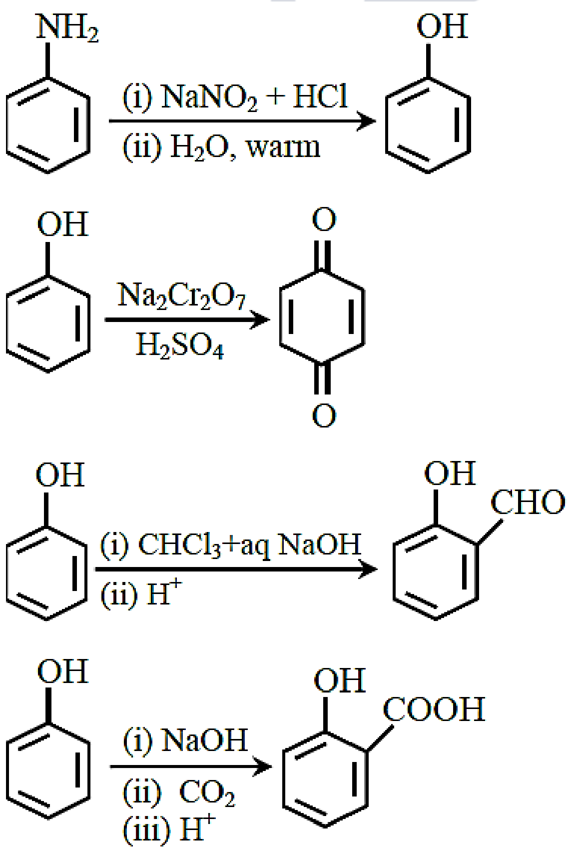
Trigonal planar

36. (c)

$SN^2 \rightarrow$ Inversion

$SN^1 \rightarrow$ Racemisation

37. (d)



38. (d)

(A) Bayer's test → Unsaturation

(B) Ceric ammonium nitrate test → Alcoholic-OH group

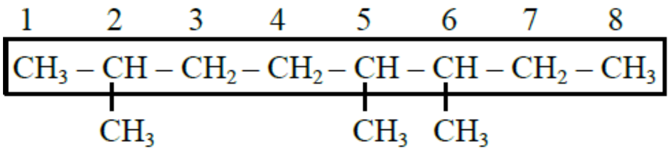
(C) Phthalein dye test → Phenol

(D) Schiff's test → Aldehyde

39. (d) (D) Due to inert pair effect lower oxidation state is more stable.

(E) Nitrogen belongs to 2^{nd} period and cannot expand its octet.

40. (c)



2,5,6-Trimethyloctane

$$Cr_2O_7^{2-} \underset{H^+}{\overset{OH^-}{\rightleftharpoons}} 2CrO_4^{2-}$$

41. (b)

42. (a) Buffer solution is a mixture of either weak acid weak base and its respective conjugate.

Blood is a buffer solution of carbonic acid H_2CO_3 and bicarbonate HCO_3^-

Statement 1 is false but Statement II is true.

43. (a) $CH_3CH_2COOH, CH_3COOH, CH_3CH_2CH_2COOH, HCOOH$

The correct order is :

$HCOOH > CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH$

44. (a) Hinsberg test given by 1° amine only.

45. (d)

46. (b) As electronegativity increases non-metallic nature increases

Along the period ionisation energy increases.

High electronegativity difference results in ionic bond formation.

Oxides of metals are generally basic and that of non-metals are acidic in nature.

47. (a) Nitrogen present in pyridine can -not be estimated by Kjeldahl method as the nitrogen present in pyridine can-not be easily converted into ammonium sulphate.

48. (c) Anodic half cell

Gas - gas ion electrode

$$\frac{1}{2}H_{2(g)} \rightarrow H^+_{(aq)} + e^-$$

Cathodic Reaction

Metal-metal insoluble salt anion electrode

$Ag^{+}_{(aq)} + e^{-} \rightarrow Ag_{(s)}$

$\underline{AgCl_{(s)} \rightleftharpoons Ag^{+}_{(aq)} + Cl^{-}_{(aq)}}$

$AgCl_{(s)} + e^{-} \rightarrow Ag_{(s)} + Cl^{-}_{(aq)}$

Overall redox reaction

$\frac{1}{2} H_{2(g)} + AgCl_{(s)} \rightarrow H^{+}{}_{(aq)} + Cl^{-}_{(aq)} + Ag_{(s)}$

Cell Representation

$Pt|H_{2(g)}|kCl_{(sol)}\ |AgCl_{(s)}|Ag$

49. (a) $MnO_2 + 4KOH + O_2 \xrightarrow{\text{fused}} 2\ K_2MnO_4 + 2H_2O$

Dark green

Electrolytic oxidation in alkaline medium:

At anode:

$MnO_4^{2-} \rightarrow MnO_4^{-} + e^{-}$

50. (c)

(A)

$[Cr(NH_3)_6]^{3+}$
$Cr^{3+}: 3\ d^3$
$n = 3$ (unpaired electrons)
$\mu \simeq 3.87$ B.M. (II)

(B)

$[NiCl_4]^{2-}$
$Ni^{2+}: 3\ d^8$
$n = 2$
$\mu \simeq 2.83$ B.M. (IV)

(C)

$[CoF_6]^{3-}$
$Co^{3+}: 3\ d^6$
$n = 4$
$\mu \simeq 4.90$ B.M. (I)

(D)

$[Ni(CN)_4]^{2-}$
$Ni^{2+}: 3\,d^8$
$n = 0$
$\mu = 0$ B.M. (III)

51. (38)

$H_2O(\ell) \rightleftharpoons H_2O(g)\ \Delta H^0_{vap} = 40.79$ kJ/ mole $\Delta H^0_{vap} = \Delta U^0_{vap} + \Delta n_g RT$

$$40.79 = \Delta U^0_{vap} + \frac{1 \times 8.3 \times 373.15}{1000}$$

$\Delta U^0_{vap} = 40.79 - 3.0971$

$= 37.6929$

$\Delta U^0_{vap} \simeq 38$

52. (2)

C_2
$(12e^-): \sigma 1\,s^2, \sigma^* 1\,s^2, \sigma 2\,s^2, \sigma^* 2\,s^2 [\pi 2p_x^2 = \pi 2p_y^2]$
B.O. $= \frac{8-4}{2} = 2$

O_2
$(16e^-): \sigma 1\,s^2, \sigma^* 1\,s^2, \sigma 2\,s^2, \sigma^* 2\,s^2, \sigma 2pz^2$
$[\pi 2p_x^2 = \pi 2p_y^2][\pi * 2p_x^1 = \pi^* 2p_y^1]$

$$\text{B.O.} = \frac{10-6}{2} = 2$$

Be_2
$(8e^-): \sigma 1\,s^2, \sigma^* 1\,s^2, \sigma 2\,s^2, \sigma^* 2\,s^2$
B.O. $= \frac{4-4}{2} = 0$
Li_2
$(6e^-): \sigma 1\,s^2, \sigma^* 1\,s^2, \sigma 2\,s^2$

B.O. $= \frac{4-2}{2} = 1$
Ne_2

$(20e^-): \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2pz^2$
$[\pi 2p_x^2 = \pi 2p_y^2][\pi * 2p_x^2 = \pi * 2p_y^2] \sigma * 2p_z^2$
B.O. $= \frac{10-10}{2} = 0$
N_2

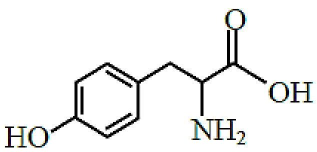
$(14e^-): \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2 \left[\pi 2p_x^2 = \pi 2p_y^2\right] \sigma 2p_z^2$

B.O. $= \frac{10-4}{2} = 6$
He_2
$(4e^-): \sigma 1\,s^2, \sigma^* 1\,s^2$
B.O. $= \frac{2-2}{2} = 0$

53. (1)

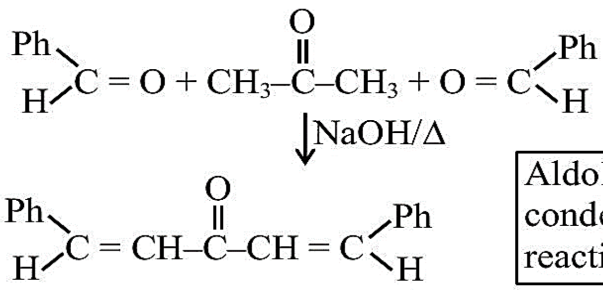
$$\underset{\displaystyle\ |\atop\displaystyle\ Cl}{CH_3-CH_2-CH-CH_3}$$

54. (9) Tyrosine



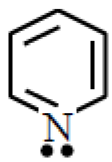
Number of carbon atoms = 9

55. (9)



Aldol condensation reaction

56. (1)



57. (74) Molality of urea is 4.44 m, that means 4.44 moles of urea present in 1000gm of water.

$$\therefore X_{urea} = \frac{4.44}{4.44 + \frac{1000}{18}}$$

$= 0.0740$

OR

74×10^{-3}

$X = 74$

58. (2)

$Co^{+3}: 3\ d^6\ t_{2\,g}^{2,2,2} e_g^{0,0}$

Unpaired $e^- = 0$

$Ni^{+2}: 3\ d^8\ e^{2,2} t_2^{2,1,1}$

Unpaired $e^- = 2$

59. (1724) $\bar{\nu}$ (wave no.) $= \frac{1}{\lambda} = \frac{1}{5800\times10^{-8}\ \text{cm}} = 17241$

OR

$1724 \times 10\ \text{cm}^{-1} \Rightarrow x = 1724$

60. (22) Mass percent of Alcohol

$$= \frac{\text{Mass of ethyl alcohol}}{\text{Total mass of solution}} \times 100$$

$$= \frac{1 \times 46}{1 \times 46 + 9 \times 18} \times 100 = \frac{4600}{208}$$

$= 22.11$ Or 22

61. (b) Image of point $(-4,5)$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\left(\frac{ax_1 + b_1 + c}{a^2 + b^2}\right)$$

Line: $x + 2y - 2 = 0$

$$\frac{x + 4}{1} = \frac{y - 5}{2} = -2\left(\frac{-4 + 10 - 2}{1^2 + 2^2}\right)$$

$$= \frac{-8}{5}$$

$$x = -4 - \frac{8}{5} = -\frac{28}{5}$$

$$y = -\frac{16}{5} + 5 = \frac{9}{5}$$

Point lies on circle $(x + 4)^2 + (y - 3)^2 = r^2$

$$\frac{64}{25} + \left(\frac{9}{5} - 3\right)^2 = r^2$$

$$\frac{100}{25} = r^2, r = 2$$

62. (b)

$\vec{r} = k(\vec{b} + \vec{c})$

$\vec{r} \cdot \vec{a} = 3$

$\vec{r} \cdot \vec{a} = k(\vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a})$

$3 = k(2 + 6 - 15 + 3 - 2 + 3\lambda)$

$3 = k(-6 + 3\lambda)$……. (1)

$\vec{r} = k(5\hat{i} + 2\hat{j} - (5 - \lambda)\hat{k})$

$|\vec{r}| = k\sqrt{25+4+25+\lambda^2-10\lambda} = 1 \ldots\ldots (2)$

$k = \frac{3}{-6+3\lambda} = \frac{1}{-2+\lambda}$ put in (2)

$4+\lambda^2-4\lambda = 54+\lambda^2-10\lambda$

$6\lambda = 50$

$3\lambda = 25$

63. (c) $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$\begin{vmatrix} \alpha - a & b-\beta & 0 \\ 0 & \beta - b & c-\gamma \\ a & b & \gamma \end{vmatrix} = 0$$

$(\alpha-a)(\gamma(\beta-b)-b(c-\gamma))-(b-\beta)(-a(c-\gamma)) = 0$

$\gamma(\alpha-a)(\beta-b)-b(\alpha-a)(c-\gamma)+a(b-\beta)(c-\gamma)$

$$\frac{\gamma}{\gamma-c}+\frac{b}{\beta-b}+\frac{a}{\alpha-a} = 0$$

64. (c) $T_2 + T_6 = \frac{70}{3}$

$ar + ar^5 = \frac{70}{3}$

$T_3 \cdot T_5 = 49$

$ar^2 \cdot ar^4 = 49$

$a^2r^6 = 49$

$ar^3 = +7, a = \frac{7}{r^3}$

$ar(1+r^4) = \frac{70}{3}$

$\frac{7}{r^2}(1+r^4) = \frac{70}{3}, r^2 = t$

$\frac{1}{t}(1+t^2) = \frac{10}{3}$

$3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$

Increasing G.P. $r^2 = 3, r = \sqrt{3}$

$T_4 + T_6 + T_8$

$= ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 7(1 + 3 + 9) = 91$

65. (d)

AA, MM, TT, H, I, C, S, E

(a) All distinct $^8C_5 \rightarrow 56$

(b) 2 same, 3 different $^3C_1 \times {}^7C_3 \rightarrow 105$

(c) 2 same I^{st} kind, 2 same 2^{nd} kind, 1 different $^3C_2 \times {}^6C_1 \rightarrow 18$

Total $\rightarrow 179$

66. (b) $Z = \frac{1+i\cos\theta}{1-2i\cos\theta}$

$$Z = -\bar{Z} \Rightarrow \frac{1 + i\cos\theta}{1 - 2i\cos\theta} = -\left(\overline{\frac{1 + i\cos\theta}{1 - 2i\cos\theta}}\right)$$

$(1 + i\cos\theta)(\overline{1 - 2i\cos\theta}) = -(1 - 2i\cos\theta)(\overline{1 + i\cos\theta})$

$(1 + i\cos\theta)(1 + 2i\cos\theta) = -(1 - 2i\cos\theta)(1 - i\cos\theta)$

$1 + 3i\cos\theta - 2\cos^2\theta = -(1 - 3i\cos\theta - 2\cos^2\theta)$

$2 - 4\cos^2\theta = 0$

$$\Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

sum $= 3\pi$

67. (b)

$$\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow (18 - \mu) - 4(14 - 5\mu) - (7 - 45) = 0 \Rightarrow \mu = 0$

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ (For infinite solution)

$$\Delta_x = \begin{vmatrix} \lambda & 4 & -1 \\ -3 & 9 & \mu \\ -1 & 1 & 2 \end{vmatrix} = 0$$

$\lambda(18 - \mu) - 4(-6 + \mu) - 1(-3 + 9) = 0$

$18\lambda + 24 - 6 = 0 \Rightarrow \lambda = -1$

68. (c)

$$\left.\begin{aligned}\bar{r}_1 &= (\lambda\hat{i}+4\hat{j}+3\hat{k})+\alpha(2\hat{i}+3\hat{j}+4\hat{k})\\ \bar{r}_2 &= (2\hat{i}+4\hat{j}+7\hat{k})+\beta(2\hat{i}+3\hat{j}+4\hat{k})\end{aligned}\right\}\begin{aligned}\bar{b} &= 2\hat{i}+3\hat{j}+4\hat{k}\\ \bar{a}_1 &+\lambda\hat{i}+4\hat{j}+3\hat{k}\\ \bar{a}_2 &= 2\hat{i}+4\hat{j}+7\hat{k}\end{aligned}$$

Shortest dist. $= \frac{|\bar{b}\times(\bar{a}_2-\bar{a}_1)|}{|b|} = \frac{13}{\sqrt{29}}$

$$\frac{|(2\hat{\imath}+3\hat{\jmath}+4\hat{k})\times((2-\lambda)\hat{\imath}+4\hat{k})|}{\sqrt{29}} = \frac{13}{\sqrt{29}}$$

$|-8\hat{\jmath}-3(2-\lambda)\hat{k}+12\hat{\imath}+4(2-\lambda)\hat{\jmath}| = 13$

$|12\hat{\imath}-4\lambda\hat{\jmath}+(3\lambda-6)\hat{k}| = 13$

$144+16\lambda^2+(3\lambda-6)^2 = 169$

$16\lambda^2+(3\lambda-6)^2 = 25 = \lambda \Rightarrow = 1$

69. (c)

$$\frac{\frac{3(\sqrt{5}+1)}{4}+5\left(\frac{\sqrt{5}-1}{4}\right)}{5\left(\frac{\sqrt{5}+1}{4}\right)-3\left(\frac{\sqrt{5}-1}{4}\right)} = \frac{8\sqrt{5}-2}{2\sqrt{5}+8}$$

$$= \frac{4\sqrt{5}-1}{\sqrt{5}+4}\times\frac{\sqrt{5}-4}{\sqrt{5}-4}$$

$$= \frac{20-16\sqrt{5}-\sqrt{5}+4}{-11}$$

$$= \frac{17\sqrt{5}-24}{11} \Rightarrow a = 17, b = 27, c = 11$$

$a+b+c = 52$

70. (c) $\sec^2 y\frac{dy}{dx}+2x\sin y\sec y = x^3\cos y\sec y$

$\sec^2 y\frac{dy}{dx}+2x\tan y = x^3$

$\tan y = t \Rightarrow \sec^2 y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx}+2xt = x^3$, If $= e^{\int 2xdx} = e^{x^2}$

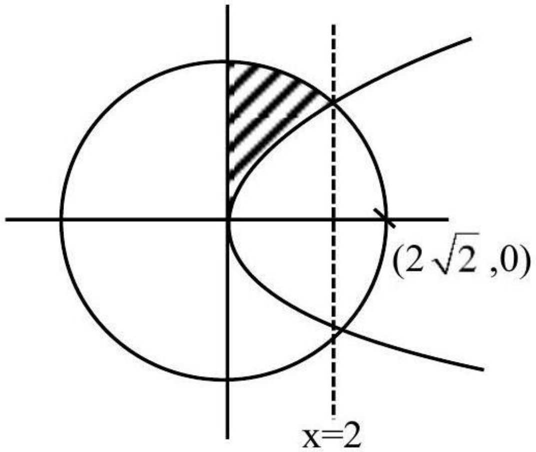
$te^{x^2} = \int x^3\cdot e^{x^2}dx+c$

$x^2 = Z \Rightarrow t\cdot e^Z = \frac{1}{2}\int e^Z\cdot ZdZ = \frac{1}{2}[e^z\cdot Z-e^Z]+c$

$2\tan y = (x^2-1)+2ce^{-x^2}$

$y(1) = 0 \Rightarrow c = 0 \Rightarrow y(\sqrt{3}) = \frac{\pi}{4}$

71. (b)



Required area = Ar(circle from 0 to 2) − ar (para from 0 to 2)

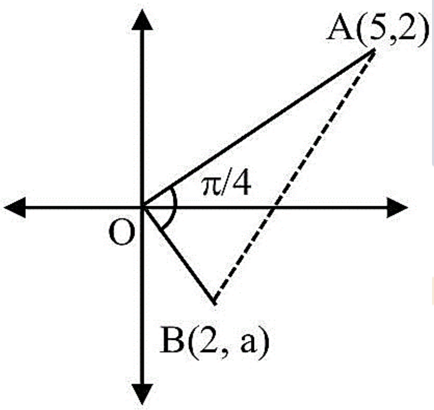
$= \int_0^2 \sqrt{8-x^2}dx - \int_0^2 \sqrt{2x}dx$

$= \left[\frac{x}{2}\sqrt{8-x^2} + \frac{8}{2}\sin^{-1}\frac{x}{2\sqrt{2}}\right]_0^2 - \sqrt{2}\left[\frac{x\sqrt{x}}{3/2}\right]_0^2$

$= \frac{2}{2}\sqrt{8-4} + \frac{8}{2}\sin^{-1}\frac{2}{2\sqrt{2}} - \frac{2\sqrt{2}}{3}(2\sqrt{2}-0)$

$\Rightarrow 2 + 4 \cdot \frac{\pi}{4} - \frac{8}{3} = \pi - \frac{2}{3}$

72. (d)



$m_{OA} = \frac{2}{5}$

$m_{OB} = \frac{a}{2} \tan\frac{\pi}{4} = \left|\frac{2}{5} - \frac{a}{2}\right|$

$1 = \left|\frac{4-5a}{10+2a}\right|$

$4-5a=\pm(10+2a)$

$4-5a=10+2a$ $\quad\quad$ $4-5a=-10-2a$

$\Rightarrow 7a+6=0 \quad 3a=14$

$\Rightarrow a=-\frac{6}{7} \quad\quad a=+\frac{14}{3}$

$-\frac{6}{7}\times\frac{14}{3}=-4$

73. (b)

$(\vec{a}+\vec{b})\times\vec{c}-\vec{c}\times(-2\vec{a}+3\vec{b})=0$

$(\vec{a}+\vec{b})\times\vec{c}+(-2\vec{a}+3\vec{b})\times\vec{c}=0$

$\Rightarrow(\vec{a}+\vec{b})-2\vec{a}+3\vec{b})\times\vec{c}=0$

$\Rightarrow\vec{c}=\lambda(4\vec{b}-\vec{a})$

$\Rightarrow=\lambda(44\hat{i}-4\hat{j}+4\hat{k}-4\hat{i}+\hat{j}-\hat{k})$

$=\lambda(40\hat{\imath}-3\hat{\jmath}+3\hat{k})$

Now

$(8\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}+33\hat{\imath}-3\hat{\jmath}+3\hat{k})\cdot\lambda(40\hat{\mathbf{i}}-3\hat{\jmath}+3\hat{k})=1670$

$\Rightarrow(41\hat{\imath}-5\hat{\jmath}+5\hat{k})\cdot(40\hat{\imath}-3\hat{\jmath}+3\hat{k})\times\lambda=1670)$

$\Rightarrow(1640+15+15)\lambda=1670\Rightarrow\lambda=1$

so $\vec{c}=40\hat{i}-3\hat{j}-3\hat{k}$

$\Rightarrow|\vec{c}|^2=1600+9+9=1618$

74. (a) $f'(x)=6x^2-18ax+12a^2=0<\begin{matrix}\alpha\\ \alpha^2\end{matrix}$

$\alpha+\alpha^2=3a\&\alpha\times\alpha^2=2a^2$

$\downarrow$

$(\alpha+\alpha^2)^3=27a^3$

$\Rightarrow 2a^2+4a^4+3(3a)(2a^2)=27a^3$

$\Rightarrow 2+4a^2+18a=27a$

$\Rightarrow 4a^2-9a+2=0$

$\Rightarrow 4a^2-8a-a+2=0$

$\Rightarrow (4a-1)(a-2)=0 \Rightarrow a=2$

so $6x^2-36x+48=0$

$\Rightarrow x^2-6x+8=0$

If we take a $=\frac{1}{4}$ then $\alpha=\frac{1}{2}$ which is not possible

75. (a)

X	Y	Z
5 one &4 five	4 one &5 five	3 one &6 five

$$P=\frac{4/9}{5/9+4/9+3/9}=\frac{4}{12}=\frac{1}{3}$$

76. (a)

$$\int_{\alpha}^{\log_e 4}\frac{dx}{\sqrt{e^x-1}}=\frac{\pi}{6}$$

Let $e^x-1=t^2$

$e^x dx=2tdt$

$$=\int\frac{2dt}{t^2+1}$$

$=2\tan^{-1}t$

$$=2\tan^{-1}\left(\sqrt{e^x-1}\right)\Big|_{\alpha}^{\log_e^4}$$

$$=2\left[\tan^{-1}\sqrt{3}-\tan^{-1}\sqrt{e^\alpha-1}\right]=\frac{\pi}{6}$$

$$=\frac{\pi}{3}-\tan^{-1}\sqrt{e^\alpha-1}=\frac{\pi}{12}$$

$$\Rightarrow \tan^{-1}\sqrt{e^\alpha-1}=\frac{\pi}{4}$$

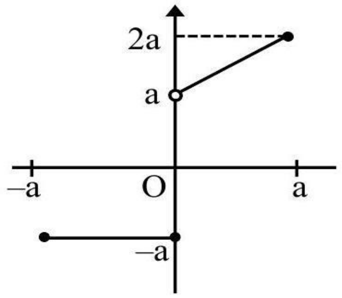
$$e^\alpha=2\ e^{-\alpha}=\frac{1}{2}$$

$$x^2-\left(2+\frac{1}{2}\right)x+1=0$$

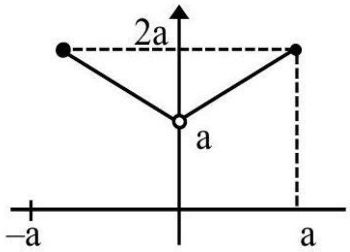
$2x^2-5x+2=0$

77. (a)

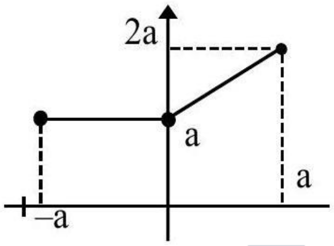
$y=f(x)$



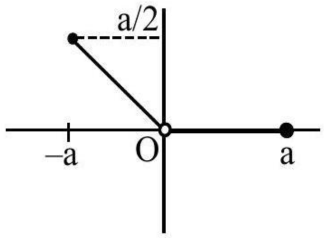
$y = f|x|$



$y = |f(x)|$



$g(x) = \dfrac{f(|x|) - |f(x)|}{2}$



78. (c) $A = \{2,3,6,8,9,11\}$ $(a, b)R(c, d)$

$B = \{1,4,5,10,15\}$ $3ad - 7bc$

Reflexive: (a, b) R(a, b) $\Rightarrow$ $3ab - 7ba = -4ab$ always even so it is reflexive.

Symmetric: If $3ad - 7bc =$ Even

Case-I: odd odd

Case-II: even even

(c, d) R(a, b) $\Rightarrow$ $3bc - 3ab$

Case-I: odd odd

Case-II: even even

so symmetric relation

Transitive:

Set (3,4)R(6,4) Satisfy relation

Set (6,4)R(3,1) Satisfy relation

but (3,4)R(3,1) does not satisfy relation

so not transitive.

79. (d) $\lim_{x\to 0} f(x) = f(0) = 3$

$$\lim_{x\to 0^+} \frac{\sqrt{ax+b^2x^2}-\sqrt{ax}}{b\sqrt{a}x\sqrt{x}} = 3$$

$$\lim_{x\to 0^+} \frac{ax+b^2x^2-ax}{b\sqrt{a}x^{3/2}\left(\sqrt{ax+b^2x^2}+\sqrt{ax}\right)}$$

$$\lim_{x\to 0^+} \frac{b^2}{b\sqrt{a}\left(\sqrt{a+b^2x}+\sqrt{a}\right)}$$

$$\frac{b}{\sqrt{a}\cdot 2\sqrt{a}} \Rightarrow \frac{b}{2a} = 3 \Rightarrow \frac{b}{a} = 6$$

80. (a) $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$

General term $= {}^{10}C_r\left(\sqrt{a}x^2\right)^{10-r}\left(\frac{1}{2x^3}\right)^r$

$20 - 2r - 3r = 0$

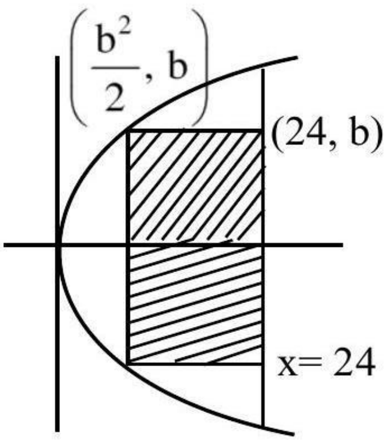
$r = 4$

$${}^{10}C_4 a^3 \cdot \frac{1}{16} = 105$$

$a^3 = 8$

$a^2 = 4$

81. (128)



$A = 2\left(24 - \frac{b^2}{2}\right) \cdot b$

$\frac{dA}{db} = 0 \Rightarrow b = 4$

$A = 2(24 - 8)4$

$= 128$

82. (6) $\alpha = \lim_{x \to 0^+} e^{\sqrt{x}} \frac{\left(e^{\sqrt{\tan x} - \sqrt{x}} - 1\right)}{\sqrt{\tan x} - \sqrt{x}}$

$= 1$

$\beta = \lim_{x \to 0} (1 + \sin x)^{\frac{1}{2}\cot x}$

$= e^{1/2}$

$x^2 - (1 + \sqrt{e}) + \sqrt{e} = 0$

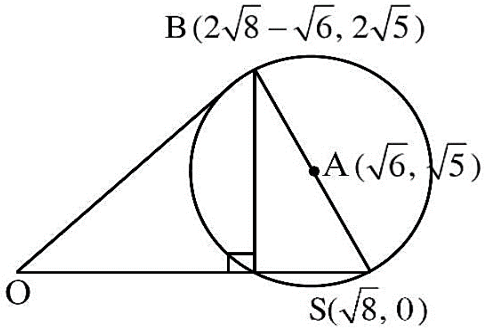
$ax^2 + bx - \sqrt{e} = 0$

On comparing

$a = -1, b = \sqrt{e} + 1$

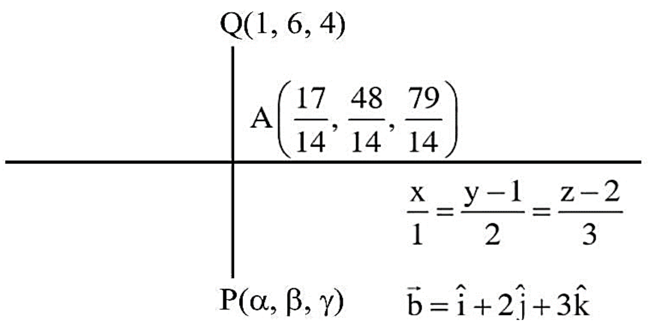
$12\ln(a + b) = 12 \times \frac{1}{2} = 6$

83. (40)



Area $= \frac{1}{2}(OS)h = \frac{1}{2}\sqrt{8}2\sqrt{5} = \sqrt{40}$

84. (11)



$A(t, 2t+1, 3t+2)$

$\overrightarrow{QA} = (t-1)\hat{\imath} + (2t-5)\hat{\jmath} + (3t-2)\hat{k}$

$\overrightarrow{QA} \cdot \vec{b} = 0$

$(t-1) + 2(2t-5) + 3(3t-2) = 0$

$14t = 17$

$\alpha = \frac{20}{14}\ \beta = \frac{12}{14}\ \gamma = \frac{102}{14}$

$2\alpha + \beta + \gamma = \frac{154}{14} = 11$

85. (1505)

2,5,11,20, ...

General term $= \frac{3n^2 - 3n + 4}{2}$

$$T_{10} = \frac{3(100) - 3(10) + 4}{2}$$
$$= 137$$

10 terms with c.d. = 3

$$\text{sum} = \frac{10}{2}(2(137) + 9(c))$$
$$= 1505$$

86. (2)

$|x+1||x+3| - 4|x+2| + 5 = 0$

case-1

$x \leq -3$

$(x+1)(x+3) + 4(x+2) + 5 = 0$

$x^2 + 4x + 3 + 4x + 8 + 5 = 0$

$x^2 + 8x + 16 = 0$

$(x + 4)^2 = 0$

$x = -4$

case-2

$-3 \leq x \leq -2$

$-x^2 - 4x - 3 + 4x + 8 + 5 = 0$

$-x^2 + 10 = 0$

$x = \pm\sqrt{10}$

case-3

$-2 \leq x \leq -1$

$-x^2 - 4x - 3 - 4x - 8 + 5 = 0$

$-x^2 - 8x - 6 = 0$

$x^2 + 8x + 6 = 0$

$x = \frac{-8 \pm 2\sqrt{10}}{2} = -4 \pm \sqrt{10}$

case-4

$x \geq -1$

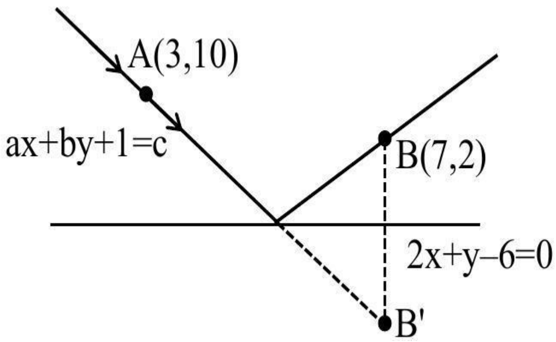
$x^2 + 4x + 3 - 4x - 8 + 5 = 0$

$x^2 = 0$

$x = 0$

No. of solution = 2

87. (1)



For B' $\frac{x-7}{2}=\frac{y-2}{1}=-2\left(\frac{14+2-6}{5}\right)$

$$\frac{x-7}{2}=\frac{y-2}{1}=-4$$
$$x=-1\ y=-2\ B'(-1,-2)$$

incident ray AB'

$M_{AB'}=3$

$y+2=3(x+1)$

$3x-y+1=0$

$a=3\ b=-1$

$a^2+b^2+3ab=9+1-9=1$

88. (33)

$a,b,c\in N$

$a<b<c$

$\bar{x}=\text{mean}=\frac{9+25+a+b+c}{5}=18$

$a+b+c=56$

Mean deviation $=\frac{\sum|x_i-\bar{x}|}{n}=4$

$=9+7+|18-a|+|18-b|+|18-c|=20$

$=|18-a|+|18-b|+|18-c|=4$

Variance $=\frac{\Sigma|x_i-\bar{x}|^2}{n}=\frac{136}{5}$

$=81+49+|18-a|^2+|18-b|^2+|18-c|^2=136$

$=(18-a)^2+(18-b)^2+(18-c)^2=6$

Possible values $(18-a)^2=1,\ (18-b)^2=1\ (18-c)^2=4\ a<b<c$

so $\begin{array}{lll} 18-a=1 & 18-b=-1 & 18-c=-2 \\ a=17 & b=19 & c=20 \end{array}$

$a+b+c=56$

$2a+b-c34=19-20=33$

89. (4)

$a|x|=|y|e^{yx-\beta}, a,b\in N$

$xdy-ydx+xy(xdy+ydx)=0$

$$\frac{dy}{y}-\frac{dx}{x}+(xdy+ydx)=0$$

$$\ell n|y|-\ell n|x|+xy=c$$

$$y(a)=2$$

$$\ell n|2|-0+2=c$$

$$c=2+\ln 2$$

$$\ell n|y|-\ell n|x|+xy=2+\ell n2$$

$$\ell n|x|=\ln\left|\frac{y}{2}\right|-2+xy$$

$$|x|=\left|\frac{y}{2}\right|e^{xy-2}$$

$$2|x|=|y|e^{xy-2}$$

$$\alpha=2\ \beta=2\ \alpha+\beta=4$$

90. (7)

$$\int\frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}}dx=A\left(\frac{\alpha x-1}{\beta x+3}\right)^B+C$$

$$I=\int\frac{1}{(x-1)^{4/5}(x+3)^{6/5}}dx$$

$$I=\int\frac{1}{\left(\frac{x-1}{x+3}\right)^{4/5}(x+3)^2}dx$$

$$\left(\frac{x-1}{x+3}\right)=t\ \Rightarrow\frac{4}{(x+3)^2}dx=dt\,t^{-4/5+1}$$

$$I=\frac{1}{4}\int\frac{1}{t^{4/5}}dt=\frac{1}{4}\frac{t^{1/5}}{1/5}+c$$

$$I=\frac{5}{4}\left(\frac{x-1}{x+3}\right)^{1/5}+C$$

$$A=\frac{5}{4}\ \alpha=\beta=1\ \ B=\frac{1}{5}$$

$$\alpha+\beta+20AB=2+20\times\frac{5}{4}\times\frac{1}{5}=7$$