Physics
Section-A

1. (b)
Conceptual

2. (d)

$L = 2\pi R$

$$I = \frac{MR^2}{2} = \frac{\lambda \times L}{2} \times \left(\frac{L}{2\pi}\right)^2 = \frac{\lambda L^3}{8\pi^2}$$

3. (a)

$$\frac{50\,g}{\pi r^2} = y \cdot \frac{\Delta \ell}{\ell}$$

$$\frac{50 \times 3\pi}{\pi \times (3 \times 10^{-3})^2} = P \times 10^{11} \times \frac{0.1 \times 10^{-3}}{3}$$

$$\Rightarrow P = \frac{50 \times 3 \times 3}{3^2 \times 10^{-6} \times 10^{11} \times 0.1 \times 10^{-3}}$$

$P = 5$

4. (a)

$$\sigma \propto \frac{1}{ROC}$$

$(ROC)_1 < (ROC)_3 < (ROC)_2 = (ROC)_4$

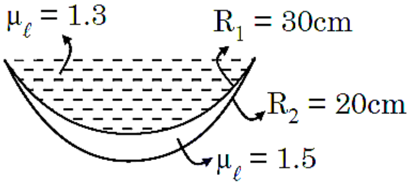
$\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4$

5. (d)

$mgh = ms\Delta T$

$$\Delta T = \frac{gh}{s} = \frac{10 \times 200}{4200}\,K = \frac{10}{21}\,K$$

6. (d)



$$\frac{1}{f} = \left(\frac{1.3-1}{1}\right)\left(\frac{1}{\infty} - \frac{1}{-30}\right)$$

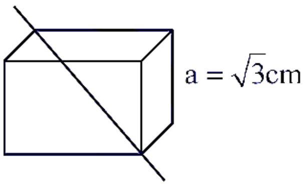
$$= \left(\frac{1.5-1}{1}\right)\left(\frac{1}{-30} - \frac{1}{-30}\right)$$

$$= \frac{0.3}{30} + \frac{0.5}{60} = \frac{1}{100} + \frac{1}{120}$$

$$= \frac{6+5}{600} = \frac{11}{600}$$

$$f = \frac{600}{11}\text{ cm}$$

7. (d)



$$\phi = \frac{q_{enc}}{\varepsilon_0} = \frac{\lambda \cdot \sqrt{3}a}{\varepsilon_0}$$

$= 2 \times 10^{-9} \times \sqrt{3} \times \sqrt{3} \times 10^{-2} \times 36\pi \times 10^9 Nm^2C^{-1}$

$= 2.16\pi Nm^2C^{-1}$

8. (b)

Here D_1 is reverse biased and D_2 is forward biased. Therefore current flow through D_Q and 5 V drop on resistor.

So, $V_{out} = 0$

9. (d)

For conducting sphere, $V = \frac{\sigma r}{\varepsilon_0}$

After contact, $V_1 = V_2$

$\sigma_1 r_1 = \sigma_2 r_2$

$\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$

$\frac{\sigma_1}{\sigma_2} = 3$

10. (c)

Fractional error $= 2\frac{\Delta X}{X} + \frac{3}{2}\frac{\Delta Y}{Y} + \frac{2}{5}\frac{\Delta Z}{Z}$

$= 2(0.1) + \frac{3}{2}(0.2) + \frac{2}{5}(0.5)$

$= 0.2 + 0.3 + 0.2 = 0.7$

11. (c)

$P_i V_i^\gamma = P_f V_f^\gamma$

$\frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^\gamma = (8)^{5/3}$

$\frac{P_f}{P_i} = 32$

12. (d)

Equivalent focal length

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$= \frac{1}{30} + \frac{1}{-20} = \frac{2-3}{60} = -\frac{1}{60}$

$f = -60$ cm

Lens formula

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-20} = \frac{1}{-60}$

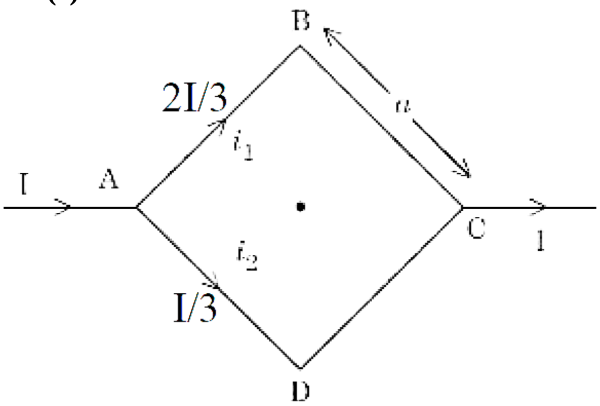
$v = -15$ cm

13. (b)

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{f\pi R^2}}$

$\frac{v_2}{v_1} = \frac{R_1}{R_2} = 2$

14. (c)



$\vec{R} = \vec{B} + \vec{B} + \vec{B} + \vec{B}$

$$\vec{B} = \left[\frac{-\mu_0(2I/3)}{4\pi(a/2)}\sqrt{2} - \frac{\mu_0(2I/3)}{4\pi(a/2)}\sqrt{2} + \frac{\mu_0(I/3)}{4\pi(a/2)}\sqrt{2} + \frac{\mu_0(I/3)}{4\pi(a/2)}\sqrt{2}\right]\hat{k}$$

$$\vec{B} = \left[\frac{-2\sqrt{2}\mu_0 I}{3\pi a} + \frac{\sqrt{2}\mu_0 I}{3\pi a}\right]\hat{k}$$

$$\vec{B} = \frac{-\sqrt{2}\mu_0 I}{3\pi a}\hat{k}$$

15. (b)

$\vec{a} = \frac{B}{2}\hat{k} = 3\hat{k}, t = \frac{5}{3}$ s

$\vec{u} = 3\hat{\imath} + 4\hat{\jmath}$

$\vec{v} = \vec{u} + \vec{a}t = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$

16. (a)

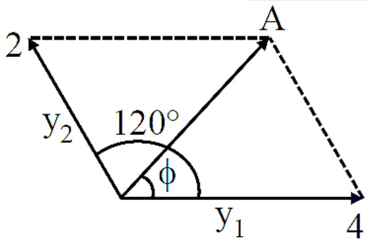
Given, $(H_{max})_1 = 8 \times (H_{max})_2$

$$\frac{u^2\sin^2\theta_1}{2g} = 8 \times \frac{u^2\sin^2\theta_2}{2g}$$

$\Rightarrow \sin\theta_1 = 2\sqrt{2}\sin\theta_2$

$$\frac{T_1}{T_2} = \frac{2u\sin\theta_1/g}{2u\sin\theta_2/g} = \frac{\sin\theta_1}{\sin\theta_2} = 2\sqrt{2}$$

17. (d)



$$A = \sqrt{2^2 + 4^2 + 2 \times 2 \times 4 \times \cos 120°}$$
$$= \sqrt{12} = 2\sqrt{3}$$

$\tan\phi$

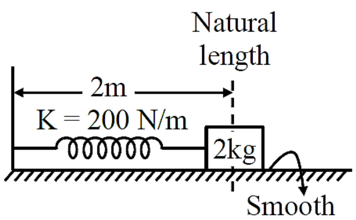
18. (b)

Different colors will have different fringe width. Within a few fringes of red, there will be several fringes of violet. Also, there will be overlapping of colors.

19. (d)

Conceptual

20. (b)



Given, Natural length of spring = 2 m

Initial compression in spring $(x_i) = 1$ m

Final compression in spring $(x_f) = (2 - x)m$

Using energy conservation

$K_i + U_i = K_f + U_f$

$0 + \frac{1}{2}Kx_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}Kx_f^2$

$\frac{1}{2}mv^2 = \frac{1}{2}K(x_i^2 - x_f^2)$

$\frac{1}{2} \times 2 \times v^2 = \frac{1}{2} \times 200 \times (1^2 - (2-x)^2)$

$v^2 = 100[1 - (2-x)^2]$

$v = 10[1 - (2-x)^2]^{1/2}$

Section-A

21. (b)

Let the momentum of e^- at any time t is p and its de-broglie wavelength is λ.

Then, $p = \frac{h}{\lambda}$

$\frac{dp}{dt} = \frac{-h}{\lambda^2}\frac{d\lambda}{dt}$

$ma = F = -\frac{h}{\lambda}\frac{d\lambda}{dt}$ [m = mass of e^-]

Where, - ve sign represents decrease in λ with time $ma = \frac{-h}{(h/p)^2}\frac{d\lambda}{dt}$

$a = -\frac{p^2}{mh}\frac{d\lambda}{dt}$

$a = -\frac{mv^2}{h}\frac{d\lambda}{dt}$

$\frac{d\lambda}{dt} = -\frac{ah}{mv^2}$

here, $a = \frac{qE}{m} = \frac{e}{m}\frac{\sigma}{2\varepsilon_0}$

$a = \frac{\sigma e}{2\,m\varepsilon_0}$

and $v = u + at$

$v = at$

Substituting values of a & v in equation (1)

$\frac{d\lambda}{dt} = -\frac{2\,h\varepsilon_0}{\sigma e t^2}$

$\Rightarrow \frac{d\lambda}{dt} \propto \frac{1}{t^2}$

$\Rightarrow n = 2$

22. (4)

Given, Initial pressure of liquid $(P_i) = 1$ atm Final pressure of liquid $(P_f) = 5$ atm

Change in pressure $(dP) = P_f - P_i = 4 \text{ atm} = 4 \times 10^5$ Pa

Change in volume $(dV) = -0.8\ \text{cm}^3$

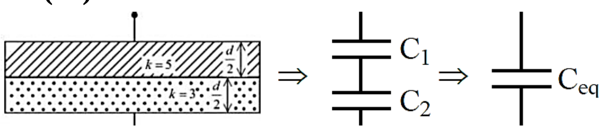
Bulk modulus $(B) = 2 \times 10^9$ Pa

Now, $B = \frac{-dP}{(dV/V)} \Rightarrow V = -B\left(\frac{dV}{dP}\right)$

$\Rightarrow V = -2 \times 10^9 \times \frac{(-0.8 \times 10^{-6})}{4 \times 10^5}$

$= 4 \times 10^{-3}\ \text{m}^3 = 4$ litre

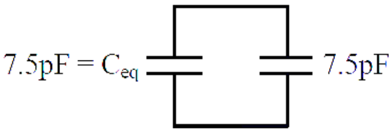
23. (15)



$$C_1 = \frac{5 \times 4 \times 10^{-4} \times 8.85 \times 10^{-12}}{\frac{1.77}{2} \times 10^{-3}} = 20\text{pF}$$

$$C_2 = \frac{3 \times 4 \times 10^{-4} \times 8.85 \times 10^{-12}}{\frac{1.77}{2} \times 10^{-3}} = 12\text{pF}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{12 \times 20}{12 + 20} = 7.5\text{pF}$$



Finally equivalent capacitance

$$(C_{eq})_{final} = 7.5 + 7.5 = 15\text{pF}$$

24. (40)

Given, m = 1 kg

$$\omega_i = 1800\text{rpm} = 1800 \times \frac{2\pi}{60} = 60\pi \frac{\text{rad}}{\text{sec}}$$

$$\omega_f = 2100\text{rpm} = 2100 \times \frac{2\pi}{60} = 70\pi \frac{\text{rad}}{\text{sec}}$$

$$\tau_{ext} = 25\pi\text{Nm}$$

$$t = 40\text{sec}$$

Using equation of motion

$$\omega_f = \omega_i + \alpha t$$

$$70\pi = 60\pi + \alpha(40)$$

$$\alpha = \frac{\pi}{4}\text{rad/sec}^2$$

Also, $\tau = I\alpha$

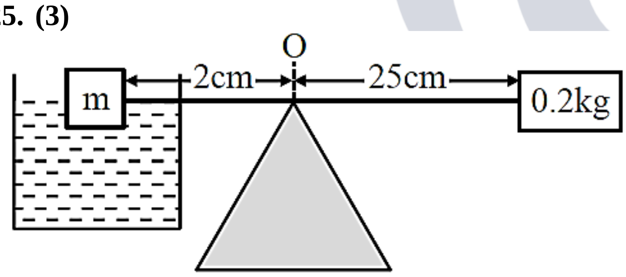
$$\tau = \frac{mR^2}{4}\alpha$$

$$25\pi = \frac{1 \times R^2}{4} \times \frac{\pi}{4}$$

$$R = 20 \text{ m}$$

Hence, diameter of disk $= 2R = 2 \times 20 = 40$ m

25. (3)



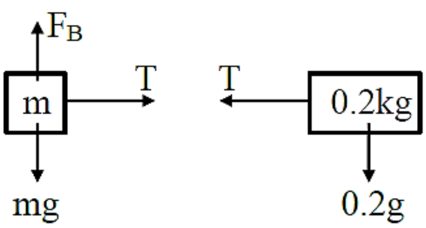
Given, volume of block $= (10 \times 10^{-2})^3 = 10^{-3}$ m^3

Let density of block $= \rho$ kg/m^3

mass of block $= \rho \times 10^{-3}$ kg

Buoyant Force $(F_B) = 1000 \times \frac{10^{-3}}{2} \times 10 = 5$ N

F.B.D. of blocks



Balancing torque about point O , we get

$$mg(2\times10^{-2})-F_B(2\times10^{-2})=0.2\,g(25\times10^{-2})$$

$$\rho\times10^{-3}\times10\times2-10=50$$

$$\rho=3000\ \mathrm{kg/m^3}$$

Hence, mass of block $=\rho\times10^{-3}$

$=3000\times10^{-3}=3\ \mathrm{kg}$

Chemistry

Section-A

26. (d)

For Ist order reaction

When $C_t=Co/4$

$t_1=2t_{50\%}$.

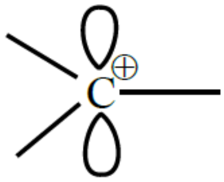
when $C_t=Co/8$

$t_2=3t_{50\%}$

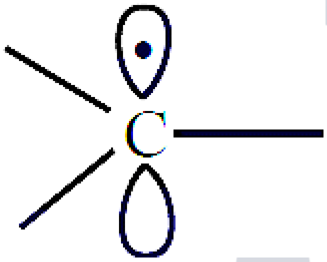
so $\frac{t_1}{t_2}=\frac{2}{3}$

27. (a)

(A) Carbocation → sp^2 hybridised carbon with empty P-orbital



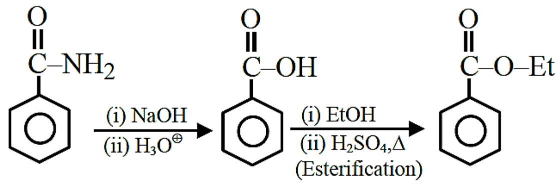
(B) Carbon free radical → sp^2/sp^3 hybridised carbon with one unpaired electron.



(C) Nuecleophile → species of that can supply a pair of electron.

(D) Electrophile → species that can receive a pair of electron.

28. (a)



(A) (B) (C)

Molar mass = 121

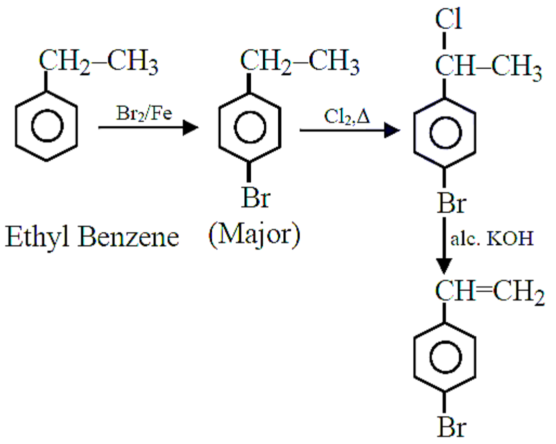
Molar mass $=121$

A → Benzamide Shows positive Lassaigh's test.

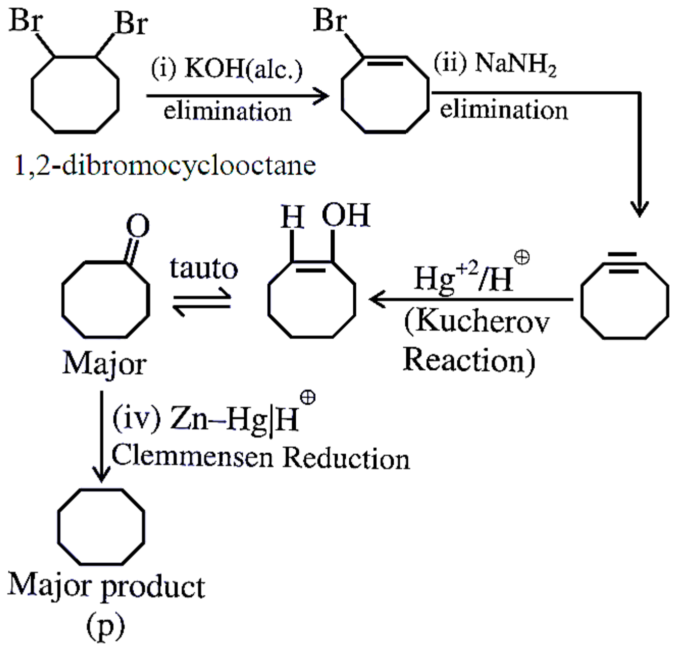
B → Benzoic acid gives effervescence with aq. $NaHCO_3$.

C → Ester gives fruity smell.

29. (a)



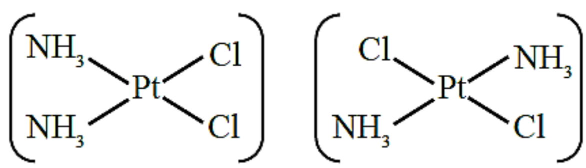
30. (b)



31. (d)

Homoleptic complex of type $[Ma_6]$ (Where a ⇒ monodentate ligand) cannot show geometrical as well as optical isomerism.

Cis-platin and trans-platin has formula $[Pt(NH_3)_2Cl_2]$ which is a heteroleptic complex of platinum.



Cis-platin Trans-platin

32.(c)

Atomic no. 32 ⇒ Ge

Atomic no. 35 ⇒ Br

Atomic no. 87 ⇒ Fr

Atomic no. 19 ⇒ K

Lowest first I.E. among the given element will be of Fr [87].

Fr – $[Rn]7s^1$

33. (b)

Binary mixture of C_6H_5OH and $C_6H_5NH_2$ shows negative deviation from Raoult's law

So vapour pressure of solution is less than V.P of pure C_6H_5OH&$C_6H_5NH_2$

So B.P. of solution is greater than boiling point of pure $C_6H_5OH \& C_6H_5NH_2$
So shows maximum Boiling azeotrope

34.(a)

$\Delta T_f = ik_f\, m$

$0.2 = i \times 1.8 \times 0.1$

$i = \frac{20}{18} = \frac{10}{9}$

For $HA_{(aq)} \rightleftharpoons H^+_{(aq)} + A^-_{(aq)}$

$t = 0 \quad 1$

$t = t_{eq} \quad 1-\alpha \quad \alpha \quad \alpha$

$i = 1 + \alpha$

$\frac{10}{9} = 1 + \alpha$

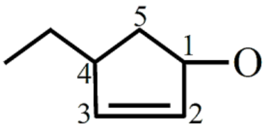
$= \frac{1}{9}$

$K_{eq} = \frac{[H^+][A^-]}{[HA]} = \frac{Ca^2}{1-\alpha}$

$= \frac{0.1\left(\frac{1}{9}\right)^2}{1-\frac{1}{9}} = \frac{1}{720}$

$K_{eq} = 1.38 \times 10^{-3}$

35. (d)



4-Ethylclopent-2-en-1-ol

36. (c)

Cu^+: $[Ar]3\,d^{10}$, Spin only magnetic moment $= 0$ B.M.

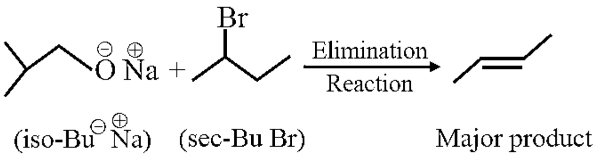
Cu^{+2}: $[Ar]3\,d^9$, Spin only magnetic moment $= \sqrt{3}$ B.M.

Cr^{+2}: $[Ar]3\,d^4$, Spin only magnetic moment $= \sqrt{24}$ B.M.

Cr^{+3}: $[Ar]3\,d^3$, Spin only magnetic moment $= \sqrt{15}$ B.M.

Order of μ: $Cr^{+2} > Cr^{+3} > Cu^{+2} > Cu^+$

37. (d)



38. (b)

In the combustion of organic compound, all "C" in CO_2 and all " H " in H_2O comes from organic compound

$\underset{0.210\text{gm}}{C_yO_z{}_x} + O_2 \longrightarrow \underset{0.307\text{gm}}{CO_2} + \underset{0.127\text{gm}}{H_2O}$

Weight of "C" in $CO_2 = \frac{12}{44} \times 0.307$

$= 0.0837$gm

Weight of " H " in $H_2O = \frac{2}{18} \times 0.127 = 0.0141$ g

% ' H ' in compound $= \frac{0.0141}{0.21} \times 100 = 6.719\%$

$= 6.72\%$

Weight of "O" in compound

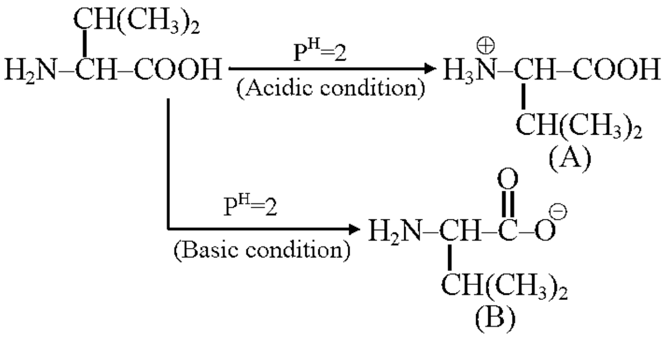
$= 0.210 - (0.0837 + 0.0141)$

$= 0.1122$

% of "O" in compound $= \dfrac{0.1122}{0.21} \times 100$

$= 53.41\%$

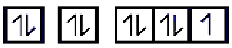
39. (a)



40. (b)

Element with atomic number 9 is Fluorine

$F(9) = 1s^2 2s^2 2p^5$



(A) 5 electrons can be up-spin $\left[m_s = +\frac{1}{2}\right]$ and 4 electrons can be down spin $\left[m_s = -\frac{1}{2}\right]$

(B) Unpaired electron can be in anyone of p_x, p_y or p_z orbital

(C) Last electron is in 2 p subshell with $n = 2, \ell = 1$

(D) Angular node for s -orbital $= 0$ while of each p-orbital $= 1$

Sum of all angular node $= 3$

41. (d)

In $[Co(NH_3)_6]^{3+}, Co^{+3}: [Ar]3\ d^6, NH_3$ is S.F.L

Hybridisation state of Co^{3+} is d^2sp^3

In SF_6, Hybridisation state of sulphur is $sp^3\ d^2$

In $[CrF_6]^{3-}, Cr^{+3}: [Ar]3\ d^3$

Hybridisation state of Cr^{3+} is d^2sp^3

$[CoF_6]^{3-}, Co^{+3}: [Ar]3\ d^6\ F^-$is W.F.L

Hybridisation state of Co^{3+} is $sp^3\ d^2$

$[Mn(CN)_6]^{3-}, Mn^{+3}: [Ar]3\ d^4 CN^-$is S.F.L

Hybridisation state of Mn^{3+} is d^2sp^3

$[MnCl_6]^{3-}, Mn^{+3}: [Ar]3\ d^4 Cl^-$is W.F.L

Hybridisation state of $Cl^-$is $sp^3\ d^2$

Total number of $sp^3\ d^2$ hybridized molecules is 3

42. (a)

(1) Carboxylic acid gives efferve scence with sodium bicarbonate solution

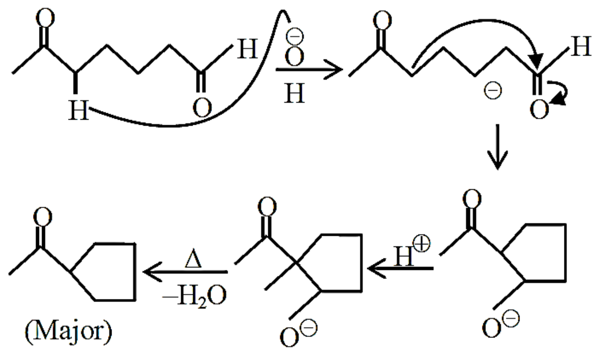
(2) Phenolic-OH gives voilet coloured complex with Neutral $FeCl_3$.

(3) Alcoholic-OH gives Red colour with cerric ammonium Nitrate.

(4) When alkaline $KMnO_4$ reacts with an unsaturated compound (Alkene or alkyne) the purple colour of $KMnO_4$ solution disappears, indicating positive test for unsaturation.

43. (a)

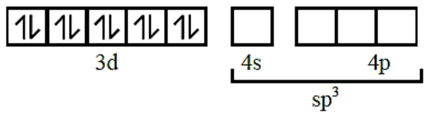
Aldol condensation reaction



44. (c)

(A) $[Ni(CO)_4], Ni^0: [Ar]3\ d^8 4\ s^2$

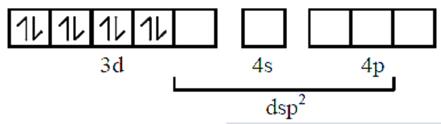
Valence orbitals of Ni^0 in pre-hybridisation state :



Tetrahedral, Diamagnetic, $\mu = 0$ B.M.

(B) $[Ni(CN)_4]^{2-}, Ni^{+2}: [Ar]3\ d^8 4\ s^0$

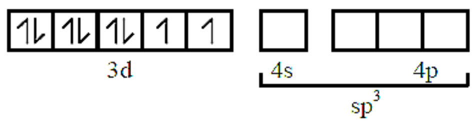
Valence orbitals of Ni^{+2} in pre-hybridisation state :



Square planar, Diamagnetic, $\mu = 0$ B.M.

(C) $[NiCl_4]^{2-}, Ni^{+2}: [Ar]3\ d^8 4\ s^0$

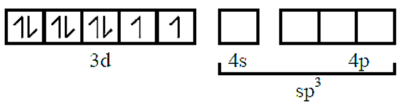
Valence orbitals of Ni^{+2} in ground state :



Tetrahedral, paramagnetic, $\mu = \sqrt{8} = 2.8$ B.M.

(D) $[MnBr_4]^{2-}, Mn^{+2}: [Ar]3\ d^5$

Valence orbitals of Mn^{+2} in ground state :



Tetrahedral, paramagnetic, $\mu = \sqrt{35} = 5.9$ B.M.

45. (d)

Acidic strength: $H_2Se < H_2Te$

$\Delta_{dis}H: H_2Se > H_2Te$

[276KJ/mol] [238KJ/mol]

Section-B

46.(98)

$\Delta H_{R.E} = \Delta H_{f(exp)} - \Delta H_{f(\text{ Theo })}$

$\Delta H_{f(exp)}$ for $X_2Y_{(g)} = 80$ kJ/mole

for $\Delta H_{f(\text{ Theo })}$

$X_{2(g)} + \frac{1}{2}Y_{2(g)} \rightarrow X_2Y_{(g)} \Delta H_f = ?$

$\Delta H_{f(\text{ Theo })} = \left(BE_{X\equiv X} + \frac{1}{2}BE_{Y=Y}\right) - (BE_{X=X} + BE_{X=Y})$

$= \left(940 + \frac{1}{2} \times 500\right) - (410 + 602)$

$= 178$ kJ/mole

$\Delta H_{R.E} = 80 - 178$

$= -98$ kJ/mol

$|\Delta H_{R.E}| = 98$

47. (54)

$E_T = -13.6\frac{z^2}{n^2}ev$

For energy of H -atom, energy of 1st Bohr orbit $E_1 = -13.6eV[z = 1, n = 1]$

For Be^{+3} ion, energy of Ist E.S. $[z = 4, n = 2]$

$\frac{E_H}{E_{Be^{+3}}} = \frac{z_1^2}{n_1^2} \times \frac{n_2^2}{z_2^2}$

$\frac{E_H}{E_{Be^{+3}}} = \frac{1}{1} \times \frac{4}{16}$

$E_{Be^{+3}} = -13.6 \times 4 = -54.4eV$

$|E_{Be^{+3}}| = 54.4eV$

48. (1)

$NaI_{(aq)} + AgNO_{3(aq)} \rightarrow AgI_{(s)} + NaNO_3(aq)$

M, 20ml excess 4.74 g

Moles of I^- in NaI = Moles of (I^-) in AgI $= \frac{4.74}{235}$

Moles of NaI $= \frac{4.74}{235}$

Molarity [NaI] $= \frac{4.74}{235 \times 0.02} = 1.008$

49.(5)

$H_2O(g) \rightleftharpoons H_{2(g)} + \frac{1}{2}O_{2(g)}$

$t = 0$ 1 mole

$t = t_{eq}$ $1-\alpha$ α $\frac{\alpha}{2}$

$n_T = 1 + \frac{\alpha}{2} \simeq 1(\alpha \ll 1)$

$k_P = \frac{P_{H_2} \cdot P_{O_2}^{1/2}}{P_{H_2O}} = \frac{(\alpha \cdot P)\left(\frac{\alpha}{2}P\right)^{\frac{1}{2}}}{(1-\alpha)P}$

$8 \times 10^{-3} = \frac{\alpha^{3/2}}{\sqrt{2}}$

$\alpha^{3/2} = 8\sqrt{2} \times 10^{-3}$

$\alpha^3 = 128 \times 10^{-6}$

$\alpha = \sqrt[3]{128} \times 10^{-2}$

$= 5.03 \times 10^{-2}$

50.(10)

$Cr_2O_{7(aq)}^{-2} + 14H_{(aq)}^+ + 6e^- \rightarrow 2Cr_{(aq)}^{+3} + 7H_2O_{(\ell)}$

$$E_R = E_R^0 - \frac{0.059}{6}\log\frac{[Cr^{+3}]^2}{[Cr_2O_7^{-2}][H^+]^{14}}$$

$$0 = 1.33 - \frac{0.059}{6}\log\frac{10^{-6}}{[H^+]^{14}}$$

$$\frac{1.33\times 6}{0.059} = \log\frac{10^{-6}}{[H]^{14}}$$

$$135.254 = -6 - 14\log[H^+]$$

$$141.254 = 14pH$$

$$pH = \frac{141.254}{14} = 10.08$$

Mathematics

Section-A

51. (a)

$$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\Rightarrow \vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$A \equiv (1,2,3) B \equiv (\lambda, 4,5)$$

$$\text{Shortest Distance } = \left|\frac{\overrightarrow{AB}\cdot(\vec{P}\times\vec{q})}{|\vec{P}\times\vec{q}|}\right|$$

$$\frac{1}{\sqrt{6}} = \left|\frac{((\lambda-1)\hat{i} + 2\hat{j} + 2\hat{k})\cdot(-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{6}}\right|$$

$$\Rightarrow |-\lambda + 1 + 4 - 2| = 1 \Rightarrow |\lambda - 3| = 1$$

$$\Rightarrow \lambda = 3 \pm 1 = 4,2$$

Radius of circle passing through points

$(0,0), (4,2) \& (2,4)$

$$= \frac{abc}{4\Delta} = \frac{\sqrt{20}\times\sqrt{20}\times\sqrt{8}}{4\times\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 0 & 4 & 2 \\ 0 & 2 & 4 \end{vmatrix}} = \frac{20\times 2\sqrt{2}}{2\times 12}$$

$$= \frac{5\sqrt{2}}{3}$$

52. (b)

$x^2 + x + 1 = 0$

α is root

$\therefore \alpha^2 + \alpha + 1 = 0$

$\Rightarrow \alpha = \omega$ as ω^2 [cube root of unity]

also

$[4-a-2b\ 64-a-14b\ 52+2a-8b]$

$= \begin{bmatrix} 0 & 0 \\ \therefore & a+2b=4 \end{bmatrix}$

$a+14b=64$

$\Rightarrow 12b=60 \Rightarrow b=5$

$\Rightarrow a=-6$

$\therefore \frac{4}{\alpha^4}+\frac{m}{\alpha^{-6}}+\frac{n}{\alpha^5}=3$

$\Rightarrow \frac{4}{\omega}+\frac{m}{1}+\frac{n}{\omega^2}=3$

$\Rightarrow 4\omega^2+m+n\omega=3$

$\Rightarrow 4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)+m+n\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=3$

$\therefore -2+m-\frac{n}{2}=3 \ldots\ldots..(\text{i})$

$\& \frac{-4\sqrt{3}}{2}+\frac{\text{n}\sqrt{3}}{2}=0$

$\therefore \text{n}=4$

$\text{m}=7$

$\therefore\ \text{m}+\text{n}=11$

53. (d)

$f(x)=\frac{x}{3}+\frac{3}{x}+3, x\neq 0$

$f'(x)=\frac{1}{3}-\frac{3}{x^2}=0 \Rightarrow x=\pm 3$

$f'(x)=\frac{x^2-3}{3x^2}$

$f'(x)>0 \forall(-\infty,-3)\cup(3,\infty) \rightarrow$ increasing

$f'(x)<0 \forall(-3,0)\cup(0,3) \rightarrow$ decreasing

$\sum_{i=1}^{5} \alpha_i^2=(-3)^2+(3)^2+(-3)^2+(0)^2+(3)^2$

$=36$

54. (a)

$P(A)=\frac{7}{10}, P(B)=\frac{4}{10}$

$P(A\cup\bar{B})=\frac{5}{10}$

$P\left(\frac{B}{A\cup\bar{B}}\right)=\frac{P(B\cap(A\cup\bar{B}))}{P(A\cup\bar{B})}$

$=\frac{P((B\cap\bar{B})\cup(B\cap A))}{P(A\cup\bar{B})}=\frac{P(A\cap B)}{P(A\cup\bar{B})}$

$=\frac{P(A)-P(A\cap\bar{B})}{P(A)+P(\bar{B})-P(A\cap\bar{B})}=\frac{\frac{7}{10}-\frac{5}{10}}{\frac{7}{10}+\left(1-\frac{4}{10}\right)-\frac{5}{10}}$

$=\frac{2}{8}=\frac{1}{4}$

55. (c)

If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots..\infty=\frac{\pi^4}{90}$ ……. (i)

$\beta=\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\cdots.$

$=\frac{1}{16}\left[\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots..\right],$

$=\frac{1}{16}\times\frac{\pi^4}{90}$ using (ii) ……. (ii)

$\alpha=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots..\infty$

$\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\frac{1}{5^4}+\cdots..\right)$

$-\left(\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\cdots..\right)$

$\alpha=\frac{\pi^4}{90}-\frac{1}{16}\times\frac{\pi^4}{90}$ [using (ii) ……. (ii)

$\alpha=\frac{16-1}{16\times 90}\times\pi^4=\frac{15}{16\times 90}\pi^4=\frac{\pi^4}{96}$

$\therefore\frac{\alpha}{\beta}=\frac{\frac{\pi^4}{96}}{\frac{\pi^4}{16\times 90}}=\frac{16\times 90}{96}=15$

56. (b)

$|x-2|^2+2|x-2|-|x-2|-2=0$

$\Rightarrow(|x-2|+2)(|x-2|-1)=0$

$\Rightarrow|x-2|=1$

$\Rightarrow x=2\pm 1=3,1$

$\Rightarrow$ sum of square of roots $=9+1=10$

$x^2-2|x-3|-5=0$

Case-I $x-3\geq 0$

$\Rightarrow x^2-2x+1=0$

$\Rightarrow(x-1)^2=0$

$\Rightarrow x=1$

But $x\geq 3$

$\Rightarrow x\in\phi$

Case-II x $-3<0$ $x^2+2x-11=0, D>0\Rightarrow$ Real & distinct roots $f(x)=x^2+2x-11$

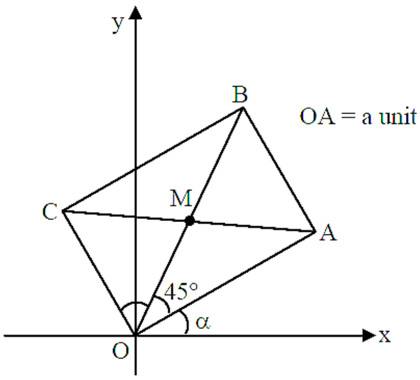
$f(3)>0,\frac{-p}{2a}=-1<3$

$\Rightarrow$ both roots <3, both roots acceptable Sum of square of roots $=(\alpha+\beta)^2-2\alpha\beta$

$=4+22=26$

$\Rightarrow$ Final sum $=10+26=36$

57. (a)



Slope of diagonal OB $=\dfrac{\sqrt{3}+1}{1-\sqrt{3}}$

$= \tan 105°$

$\therefore \alpha = 60°$

$\therefore \mathrm{A}(\mathrm{a}\cos 60°, \mathrm{a}\sin 60°)$

$\therefore \mathrm{A}\left(\dfrac{\mathrm{a}}{2}, \dfrac{\sqrt{3}\mathrm{a}}{2}\right)$

A Lies on other diagonal

$\therefore \left(\dfrac{\sqrt{3}-1}{2}\right)a - \left(\dfrac{\sqrt{3}+1}{2}\right)\cdot\sqrt{3}a + 8\sqrt{3} = 0$

$\mathrm{a}\left[\dfrac{\sqrt{3}-1-3-\sqrt{3}}{2}\right] = -8\sqrt{3}$

$\mathrm{a} = 4\sqrt{3}$

$\therefore \mathrm{a}^2 = 48$

58.(d)

$I_1 = \int_{-\frac{1}{2}}^{1} 2xf(2x(1-2x))dx$

$\Rightarrow 2x = t \Rightarrow 2dx = dt \;\Rightarrow I_1 = \frac{1}{2}\int_{-1}^{2} tf(t(1-t))dt$

$\Rightarrow 2I_1 = \int_{-1}^{2}(1-t)f(1-t)(1-(1-t))dt$

$\Rightarrow 2I_1 = \int_{-1}^{2} f\left(t(1-t)dt - \int_{-1}^{2} tf(t(1-t)dt\right.$

$\Rightarrow 2I_1 = I_2 - 2I_1$

$\Rightarrow 4I_1 = I_2$

$\Rightarrow \frac{I_2}{I_1} = 4$

59. (d)

Let vector $\vec{p}$ in plane of $\vec{a}\&\vec{b} = K(\vec{a}+\lambda\vec{b})$

$\vec{p}\perp\vec{a} = \vec{p}\cdot\vec{a} = 0$

$\Rightarrow \mathrm{K}(\vec{a}+\lambda\vec{b})\cdot\vec{a} = 0$

$\Rightarrow \vec{a}\cdot\vec{a}+\lambda\vec{b}\cdot\vec{a} = 0$

$\Rightarrow 6+\lambda(3) = 0$

$\Rightarrow \lambda = -2$

$\Rightarrow \vec{p} = (-3\hat{\imath}+3\hat{k})$

Unit vector $\rightarrow \pm\frac{(-\hat{\imath}+\hat{k})}{\sqrt{2}}$

60. (c)

$E: \frac{x^2}{4/3}+\frac{y^2}{4/P} = 1$

Centre of circle (1,2), radius

$\mathrm{r} = \sqrt{1+4+11}$

$\mathrm{r} = 4$

$\because$ E pass from centre (1,2)

$\therefore \dfrac{3}{4}+P = 1$

$P = \frac{1}{4}$ $\therefore$ vertical ellipse

$e = \sqrt{1-\dfrac{4/3}{16}} = \sqrt{1-\dfrac{1}{12}} = \sqrt{\dfrac{11}{12}}$

$\therefore$ Focal distance of C (h, k)

$= \mathrm{b}\pm\mathrm{ek}$

$F_1 = 4 + \sqrt{\frac{11}{12}} \times 2$

$F_2 = 4 - \sqrt{\frac{11}{12}} \times 2$

$\therefore F_1 F_2 = 16 - \frac{11}{3} = \frac{37}{3}$

$\therefore 6 F_1 F_2 - r = 74 - 4 = 70$

61. (c)

Let, $I = \pi^2 \int_{-1}^{3/2} |x\sin \pi x| dx$

$= \pi^2 \left\{ \int_{-1}^{1} x\sin \pi x dx - \int_{1}^{3/2} x\sin \pi x dx \right\}$

$= \pi^2 \left\{ 2\int_{0}^{1} x\sin \pi x dx - \int_{-1}^{3/2} x\sin \pi x dx \right\}$

Consider

Jx $\sin \pi x dx$

$-x \cdot \frac{1}{\pi} \cos \pi x + \int 1 \cdot \frac{1}{\pi} \cos \pi x dx$

$= -\frac{x}{\pi} \cos \pi x + \frac{\sin \pi x}{\pi^2}$

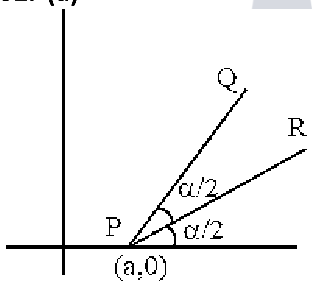
$I = \pi^2 \left\{ 2\left(-\frac{x}{\pi} \cos \pi x + \frac{\sin \pi x}{\pi^2} \right)_0^1 - \left(-\frac{x}{\pi} \cos \pi x + \frac{\sin \pi x}{\pi^2} \right)_1^{3/2} \right\}$

$= \pi^2 \left\{ \frac{2}{\pi} - \left(-\frac{1}{\pi^2} - \frac{1}{\pi} \right) \right\}$

$= \pi^2 \left\{ \frac{3}{\pi} + \frac{1}{\pi^2} \right\}$

$= 3\pi + 1$

62. (a)



$m_{PR} = 2 - \sqrt{3} = \tan 15°$

$\therefore \frac{\alpha}{2} = 15° \Rightarrow \alpha = 30°$

equation of PR :

$y = \tan 15°(x - a)$

$y = (2 - \sqrt{3})(x - a)$

$\perp$ distance from origin $= \frac{1}{\sqrt{2}}$

$$\left|\frac{\sqrt{3}a-2a}{\sqrt{4+3-4\sqrt{3}+1}}\right|=\frac{1}{\sqrt{2}}$$

$$\frac{|a|(2-\sqrt{3})}{2\sqrt{(2-\sqrt{3})}}=\frac{1}{\sqrt{2}}$$

$$|a|=\frac{\sqrt{2}}{\sqrt{2-\sqrt{3}}}=\sqrt{2}(\sqrt{2+\sqrt{3}})$$

$$a^2=2(2+\sqrt{3})$$

$$3a^2\tan^2\alpha-2\sqrt{3}$$

$$3\times(4+2\sqrt{3})\cdot\frac{1}{3}-2\sqrt{3}=4$$

63.(d)

$^{12}C_3-{}^{5}C_3=210$

64.(a)

$$1+10\operatorname{Re}\left(\frac{2\cos\theta+i\sin\theta}{\cos\theta-3i\sin\theta}\right)=0$$

$\therefore z+\bar{z}=2\operatorname{Re}(z)$

$$\frac{2\cos\theta+i\sin\theta}{\cos\theta-3i\sin\theta}+\frac{2\cos\theta-i\sin\theta}{\cos\theta+3i\sin\theta}=2\times\left(\frac{-1}{10}\right)$$

$$\frac{(2\cos^2\theta-3\sin^2\theta)+(2\cos^2\theta)-(3\sin^2\theta)}{\cos^2\theta+9\sin^2\theta}=\frac{-2}{10}$$

$$\Rightarrow\frac{2\cos^2\theta-3\sin^2\theta}{\cos^2\theta+9\sin^2\theta}=\frac{-1}{10}$$

$\Rightarrow 20\cos^2\theta-30\sin^2\theta=-\cos^2\theta-9\sin^2\theta$

$21\cos^2\theta-21\sin^2\theta=0$

$\Rightarrow\cos 2\theta=0$

$$2\theta=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}$$

$$\Rightarrow\sum\theta^2=\frac{\pi^2}{16}+\frac{9\pi^2}{16}+\frac{25\pi^2}{16}+\frac{49\pi^2}{16}$$

$$=\frac{84\pi^2}{16}=\frac{21\pi^2}{4}$$

65. (c)

$A=\{0,1,2,3,4,5\}$

$R\equiv\{(0,3),(3,0),(0,4),(4,0),(1,3),(3,1),(1,4),(4,1),(2,3),(3,2),(2,4),(4,2),(3,3),(3,4),(4,3),(4,4)\}$

Total 16 elements

Not reflexive as $(0,0),\ldots\ldots,(2,2)\notin R$

Symmetric $\because\forall$ all a,b

$(a,b)\&(b,a)\in R$

Not transitive $\because(0,3),(3,1)\in R$

but $(0,1)\notin R$

$\Rightarrow$ Only S_2 correct

66. (d)

$$T_r={}^{1016}C_r(5)^{\frac{1016-r}{2}}7^{\frac{r}{8}}$$

$\Rightarrow r=0,8,16,24,\ldots\ldots,1016$

$1016=0+(n-1)8$

$$\Rightarrow n-1=\frac{1016}{8}=127$$

So, $n=128$

67. (c)

$f(x)=x-1$

$f(f(x)) = f(x) - 1 = x - 1 - 1 = x - 2$

$g(f(f(x))) = e^{x-2}$

$\therefore \frac{dy}{dx} = e^{-2\sqrt{x}} \times e^{x-2} - \frac{1}{\sqrt{x}}y$

$\frac{dy}{dx} + \frac{1}{\sqrt{x}}y = e^{x-2\sqrt{x}-2}$ which is L.D.E

I.F. $= e^{\int \frac{dy}{\sqrt{x}}} = e^{2\sqrt{x}}$

Its solution is

$y \times e^{2\sqrt{x}} = \int e^{2\sqrt{x}} \times e^{x-2\sqrt{x}-2} dx + c$

$y \times e^{2\sqrt{x}} = \int e^{x-2} dx + c$

$y \times e^{2\sqrt{x}} = e^{x-2} + c$

Given $x = 0, y = 0 \Rightarrow 0 = e^{-2} + c; c = -e^{-2}$

$\therefore y \times e^{2\sqrt{x}} = e^{x-2} - e^{-2}$

when $x = 1, y \times e^2 = e^{-1} - e^{-2}$

$y = \frac{e^{-1} - e^{-2}}{e^2} = \frac{\frac{1}{e} - \frac{1}{e^2}}{e^2} = \frac{e^2 - e}{e^5} = \frac{e-1}{e^4}$

68. (a)

$\cot^{-1}\left(\frac{|\sec 2| - 1}{\tan 2}\right) - \cot^{-1}\left(\frac{\left|\sec\frac{1}{2}\right| + 1}{\tan\frac{1}{2}}\right)$

$= \cot^{-1}\left(\frac{-1 - \cos 2}{\sin 2}\right) - \cot^{-1}\left(\frac{1 + \cos\frac{1}{2}}{\sin\frac{1}{2}}\right)$

$= \pi - \cot^{-1}(\cot 1) - \cot^{-1}\left(\cot\frac{1}{4}\right)$

$= \pi - 1 - \frac{1}{4} = \pi - \frac{5}{4}$

69. (2)

$|A| = \begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$

$C_3 \to C_3 - C_2 - C_1 \times \frac{q}{2}$

Then $C_3 \to C_2 - C_1 x\left(1 + \frac{p}{2}\right)$

$\Rightarrow |A| = \begin{vmatrix} 2 & 0 & 0 \\ 4 & 2 & 2+p \\ 6 & 6 & 8+3p \end{vmatrix}$

$\Rightarrow |A| = 2(16 + 6p - 12 - 6p) = 8 = 2^3$

$|\text{adj}(\text{adj}(3A))| = |3A|^{(3-1)^2} = |3A|^4$

$= (3^3|A|)^4 = (3^3 \times 2^3)^4 = 2^{12} \times 3^{12}$

$\Rightarrow m + n = 24$

70. (d)

$\lim_{x\to 0} \frac{\tan^{-1} x + \frac{1}{2}[\ell\text{n}(1+x) - \ell\text{n}(1-x)] - 2x}{x^5}$

$= \lim_{x\to 0} \frac{\left(x - \frac{x^3}{3} + \frac{x^5}{5} \ldots\right) + \frac{1}{2}\left[x - \frac{x^2}{2} + \frac{x^3}{3} \ldots - \left(-x - \frac{x^2}{2} - \frac{x^3}{3} \ldots\right)\right] - 2x}{x^5}$

$= \lim_{x\to 0} \dfrac{2x + \dfrac{2x^5}{5} \ldots - 2x}{x^5} = \dfrac{2}{5}$

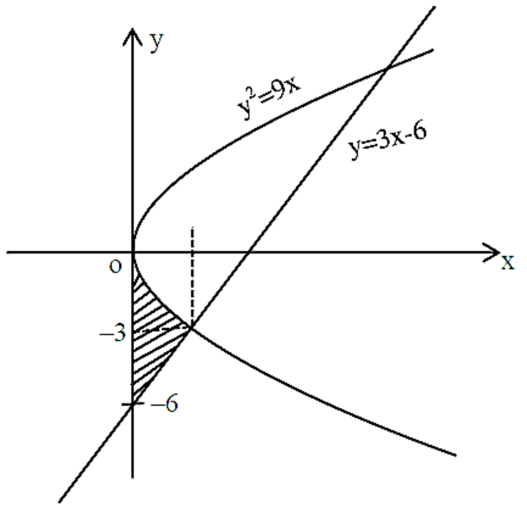
$\lim_{x\to 1} x^{\frac{2}{(1-x)}} = e^{\lim_{x\to 1}\left(\frac{2}{1-x}\right)(x-1)} = e^{-2}$

$\Rightarrow$ Both statements correct

Section-B

71. (15)

$0 \le 9x \le y^2 \& y \ge 3x - 6$



$A =$ Required Area $= \left[\int_0^1 (-3\sqrt{x})dx - \int_0^1 (3x-6)dx\right]$

$A = -3\left(\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_0^1 - \left(\dfrac{3x^2}{2} - 6x\right)\Bigg|_0^1$

$A = -2[1-0]\left[\dfrac{3}{2} - 6\right]$

$A = -2 - \dfrac{3}{2} + 6 = \dfrac{5}{2}$ Sq. unit

$\therefore 6A = 6 \times \dfrac{5}{2} = 15$

72. (96)

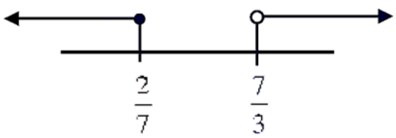
$f(x) = \cos^{-1}\left(\dfrac{4x+5}{3x-7}\right)$

$\Rightarrow -1 \le \left(\dfrac{4x+5}{3x-7}\right) \le 1$

$\left(\dfrac{4x+5}{3x-7}\right) \ge -1$

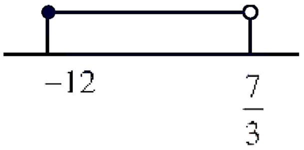
$\dfrac{4x+5+3x-7}{3x-7} \ge 0$

$\Rightarrow \dfrac{7x-2}{3x-7} \ge 0$



$x \in \left(-\infty, \dfrac{2}{7}\right] \cup \left(\dfrac{7}{3}, \infty\right)$

$\& \dfrac{4x+5}{3x-7} \le 1 \Rightarrow \dfrac{x+12}{3x-7} \le 0$



$\therefore$ Domain of f(x) is

$\left[-12,\frac{2}{7}\right]\alpha=-12,\beta=\frac{2}{7}$

$g(x)=\log_2(2-6\log_{27}(2x+5))$

Domain

$2-6\log_{27}(2x+5)>0$

$6\log_{27}(2x+5)<2$

$\log_{27}(2x+5)<\frac{1}{3}$

$2x+5<3$

$x<-1$

$\&2x+5>0\Rightarrow x>-\frac{5}{2}$

Domain is $x\in\left(-\frac{5}{2},-1\right)$

$\gamma=-\frac{5}{2},\delta=-1$

$|7(\alpha+\beta)+4(\gamma+\delta)|=\left|7\left(-12+\frac{2}{7}+4\left(-\frac{5}{2}-1\right)\right|\right.\ |-82-14|=96$

73. (56)

$L_1: x+2=y-1=z=\ell$

$L_2:\frac{x-3}{5}=\frac{y}{-1}=\frac{z-1}{1}=m$

$L_3:\frac{x}{-3}=\frac{y-3}{5}=\frac{z-2}{1}=n$

Point of intersection of L_1 and L_2

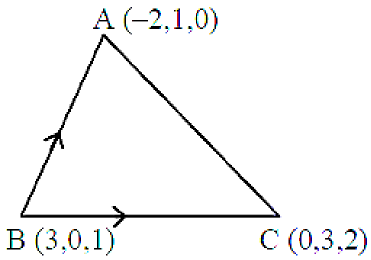
$\left.\begin{array}{r}\ell-2=5m+3\\ \ell+1=-m\\ \ell=m+1\end{array}\right\}\ell=0, m=-1\ \ A(-2,1,0)$

Point of intersection of L_2 and L_3

$\left.\begin{array}{r}5m+3=-3n\\ -m=3n+3\\ m+1=n+2\end{array}\right\}m=0, n=-1, B(3,0,1)$

Point of intersection L_3 and L_4

$\left.\begin{array}{r}-3n=\ell-2\\ 3n+3=\ell+1\\ n+2=\ell\end{array}\right\}\ell=2, n=0, C(0,3,2$



$$\text{Ar}(\triangle \text{ABC}) = \left|\frac{1}{2}\right| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 1 & -1 \\ -3 & 3 & 1 \end{vmatrix}$$

$$A = \frac{1}{2}|\hat{i}(4) - \hat{j}(-8) + \hat{k}(-12)|$$

$$A = \frac{1}{2}\sqrt{16 + 64 + 144} = \sqrt{56}$$

$$A^2 = 56$$

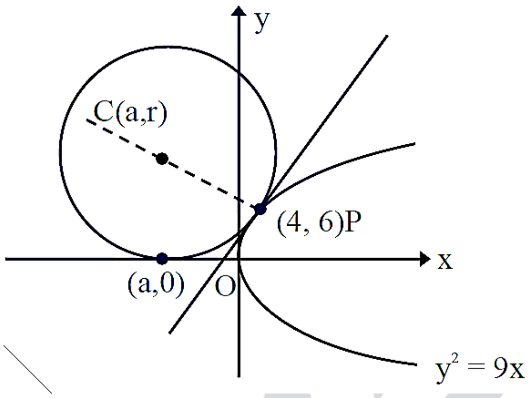
74. (63)

$$(1919)^{1919} = (1920 - 1)^{1919}$$
$$= {}^{1919}C_0(1920)^{1919} - {}^{1919}C_1(1920)^{1918} + \cdots \ldots$$
$$+ {}^{1919}C_{1918}(1920)^1 - {}^{1919}C_{1919}$$
$$= 100\lambda + 1919 \times 1920 - 1$$
$$= 100\lambda + 3684480 - 1$$
$= 100\lambda + \cdots \ldots \ldots ..79$ (last two digit)

$\Rightarrow$ Number having last two digit 79

$\therefore$ Product of last two digit 63

75. (30)



$$(x-a)^2 + (y-r)^2 = r^2$$
$$(4-a)^2 + (6-r)^2 = r^2$$
$$16 + a^2 - 8a + 36 + r^2 - 12r = r^2$$
$$a^2 - 8a - 12r + 52 = 0$$

Tangent to parabola at (4,6) is

$6.4 = 9.\left(\frac{x+4}{2}\right)$ i.e. $3x - 4y + 12 = 0$

This is also tangent to the circle

$\therefore\ CP = r$

$$\frac{3a - 4r + 12}{5} = \pm r$$

$3a + 12 = 4r \pm 5r\{\{ {}^{ar}_{-r}$

equation of circle is

$$(x-a)^2 + (y-r)^2 = r^2$$

satsty $P(4,6) \Rightarrow a^2 - 8a - 12r + 52 = 0$

From equation (1)

If a + 4 = 3r then a = +6 (rejected)

If $3a + 12 = -r$ then $a = -14$ and $r = 30$