

Physics

Section-A

1. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Work done in moving a test charge between two points inside a uniformly charged spherical shell is zero, no matter which path is chosen.

Reason R : Electrostatic potential inside a uniformly charged spherical shell is constant and is same as that on the surface of the shell.

In the light of the above statements, choose the correct answer from the options given below

- (a) A is true but R is false
- (b) Both A and R are true and R is the correct explanation of A
- (c) A is false but R is true
- (d) Both A and R are true but R is NOT the correct explanation of A

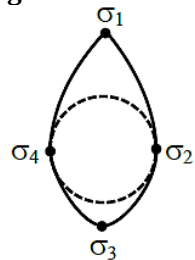
2. A rod of linear mass density ' λ ' and length ' L ' is bent to form a ring of radius ' R '. Moment of inertia of ring about any of its diameter is :

- (a) $\frac{\lambda L^3}{16\pi^2}$
- (b) $\frac{\lambda L^3}{12}$
- (c) $\frac{\lambda L^3}{4\pi^2}$
- (d) $\frac{\lambda L^3}{8\pi^2}$

3. A 3 m long wire of radius 3 mm shows an extension of 0.1 mm when loaded vertically by a mass of 50 kg in an experiment to determine Young's modulus. The value of Young's modulus of the wire as per this experiment is $P \times 10^{11} \text{ Nm}^{-2}$, where the value of P is: (Take $g = 3\pi \text{ m/s}^2$)

- (a) 5
- (b) 10
- (c) 25
- (d) 2.5

4. Electric charge is transferred to an irregular metallic disk as shown in figure. If $\sigma_1, \sigma_2, \sigma_3$ and σ_4 are charge densities at given points then, choose the correct answer from the options given below:



- (A) $\sigma_1 > \sigma_3; \sigma_2 = \sigma_4$
- (B) $\sigma_1 > \sigma_2; \sigma_3 > \sigma_4$
- (C) $\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4$
- (D) $\sigma_1 < \sigma_3 < \sigma_2 = \sigma_4$
- (E) $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$

- (a) A, B and C Only
- (b) A and C Only
- (c) D and E Only
- (d) B and C Only

5. Water falls from a height of 200 m into a pool. Calculate the rise in temperature of the water assuming no heat dissipation from the water in the pool.

(Take $g = 10 \text{ m/s}^2$, specific heat of water = 4200 J/(kgK))

- (a) 0.23 K
- (b) 0.36 K
- (c) 0.14 K
- (d) 0.48 K

6. A concave-convex lens of refractive index 1.5 and the radii of curvature of its surfaces are 30 cm and 20 cm, respectively. The concave surface is upwards and is filled with a liquid of refractive index 1.3. The focal length of the liquid-glass combination will be.

- (a) $\frac{500}{11} \text{ cm}$
- (b) $\frac{800}{11} \text{ cm}$
- (c) $\frac{700}{11} \text{ cm}$
- (d) $\frac{600}{11} \text{ cm}$

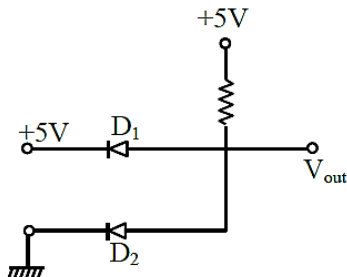
7. An infinitely long wire has uniform linear charge density $\lambda = 2 \text{ nC/m}$.

The net flux through a Gaussian cube of side length $\sqrt{3}$ cm, if the wire passes through any two corners of the cube, that are maximally displaced from each other, would be $x\text{Nm}^2\text{C}^{-1}$, where x is .

[Neglect any edge effects and use $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ SI units]

- (a) 0.72π (b) 1.44π
(c) 6.48π (d) 2.16π

8. The output voltage in the following circuit is (Consider ideal diode case.)



- (a) 10 V (b) 0 V
(c) +5 V (d) -5 V

9. Two metal spheres of radius R and $3R$ have same surface charge density σ . If they are brought in contact and then separated, the surface charge density on smaller and bigger sphere becomes σ_1 and σ_2 , respectively. The ratio $\frac{\sigma_1}{\sigma_2}$ is.

- (a) $\frac{1}{9}$ (b) 9
(c) $\frac{1}{2}$ (d) 3

10. A quantity Q is formulated as $X^{-2}Y^{+\frac{3}{2}}Z^{-\frac{2}{5}}$. X , Y and Z are independent parameters which have fractional errors of 0.1, 0.2 and 0.5, respectively in measurement. The maximum fractional error of Q is.

- (a) 0.1 (b) 0.8
(c) 0.7 (d) 0.6

11. A monoatomic gas having $\gamma = \frac{5}{3}$ is stored in a thermally insulated container and the gas is suddenly compressed to $\left(\frac{1}{8}\right)^{\text{th}}$ of its initial volume. The ratio of final pressure and initial pressure is: (γ is the ratio of specific heats of the gas at constant pressure and at constant volume)

- (a) 16 (b) 40
(c) 32 (d) 28

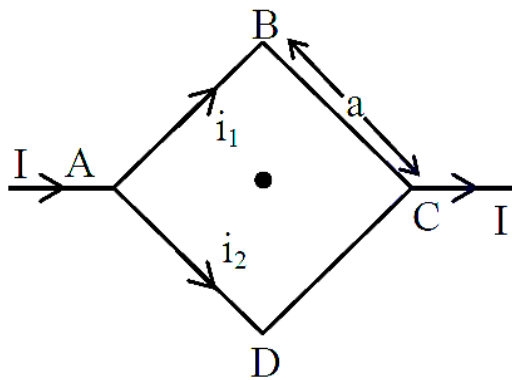
12. A convex lens of focal length 30 cm is placed in contact with a concave lens of focal length 20 cm. An object is placed at 20 cm to the left of this lens system. The distance of the image from the lens in cm is_____.

- (a) 30 (b) 45
(c) $\frac{60}{7}$ (d) 15

13. Two strings with circular cross section and made of same material, are stretched to have same amount of tension. A transverse wave is then made to pass through both the strings. The velocity of the wave in the first string having the radius of cross section R is v_1 , and that in the other string having radius of cross section $R/2$ is v_2 . Then $\frac{v_2}{v_1} =$

- (a) $\sqrt{2}$ (b) 2
(c) 8 (d) 4

14. Figure shows a current carrying square loop ABCD of edge length is ' a ' lying in a plane. If the resistance of the ABC part is r and that of ADC part is $2r$, then the magnitude of the resultant magnetic field at center of the square loop is.



(a) $\frac{3\pi\mu_0 I}{\sqrt{2}a}$
(c) $\frac{\sqrt{2}\mu_0 I}{3\pi a}$

(b) $\frac{\mu_0 I}{2\pi a}$
(d) $\frac{2\mu_0 I}{3\pi a}$

15. A body of mass 2 kg moving with velocity of $\vec{v}_{in} = 3\hat{i} + 4\hat{j}\text{ms}^{-1}$ enters

into a constant force field of 6 N directed along positive z-axis. If the body remains in the field for a period of $\frac{5}{3}$ seconds, then velocity of the body when it emerges from force field is.

- (a) $4\hat{i} + 3\hat{j} + 5\hat{k}$
(b) $3\hat{i} + 4\hat{j} + 5\hat{k}$
(c) $3\hat{i} + 4\hat{j} - 5\hat{k}$
(d) $3\hat{i} + 4\hat{j} + \sqrt{5}\hat{k}$

16. Two balls with same mass and initial velocity, are projected at different angles in such a way that maximum height reached by first ball is 8 times higher than that of the second ball. T_1 and T_2 are the total flying times of first and second ball, respectively, then the ratio of T_1 and T_2 is:

- (a) $2\sqrt{2}:1$ (b) $2:1$
(c) $\sqrt{2}:1$ (d) $4:1$

17. The amplitude and phase of a wave that is formed by the

superposition of two harmonic travelling waves, $y_1(x, t) = 4\sin(kx - \omega t)$ and $y_2(x, t) = 2\sin\left(kx - \omega t + \frac{2\pi}{3}\right)$, are :

(Take the angular frequency of initial waves same as ω)

- (a) $\left[6, \frac{2\pi}{3}\right]$ (b) $\left[6, \frac{\pi}{3}\right]$
(c) $\left[\sqrt{3}, \frac{\pi}{6}\right]$ (d) $\left[2\sqrt{3}, \frac{\pi}{6}\right]$

18. In a Young's double slit experiment, the source is white light. One of the slits is covered by red filter and another by a green filter. In this case.

- (a) There shall be an interference pattern for red distinct from that for green.
(b) There shall be no interference fringes.
(c) There shall be alternate interference fringes of red and green.
(d) There shall be an interference pattern, where each fringe's pattern center is green and outer edges is red.

19. For a nucleus of mass number A and radius R , the mass density of nucleus can be represented as.

- (a) A^3 (b) $A^{\frac{1}{3}}$
(c) $A^{\frac{2}{3}}$ (d) Independent of A

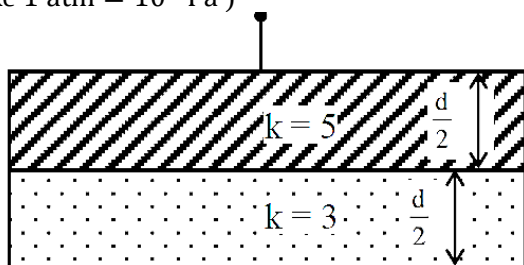
20. A block of mass 2 kg is attached to one end of a massless spring whose other end is fixed at a wall. The spring-mass system moves on a frictionless horizontal table. The spring's natural length is 2 m and spring constant is 200 N/m. The block is pushed such that the length of the spring becomes 1 m and then released. At distance $x\text{m}$ ($x < 2$) from the wall. the speed of the block will be :

- (a) $10[1 - (2 - x)]^{3/2} \text{ m/s}$
(b) $10[1 - (2 - x)^2]^{1/2} \text{ m/s}$
(c) $10[1 - (2 - x)^2] \text{ m/s}$
(d) $10[1 - (2 - x)^2]^2 \text{ m/s}$

Section-A

21. An electron is released from rest near an infinite non-conducting sheet of uniform charge density $-\sigma$. The rate of change of de-Broglie wave length associated with the electron varies inversely as n^{th} power of time. The numerical value of n is .
22. A sample of a liquid is kept at 1 atm . It is compressed to 5 atm which leads to change of volume of 0.8 cm^3 . If the bulk modulus of the liquid is 2 GPa , the initial volume of the liquid was litre. (Take $1 \text{ atm} = 10^5 \text{ Pa}$)

23.



Space between the plates of a parallel plate capacitor of plate area 4 cm^2 and separation of $(d) 1.77 \text{ mm}$, is filled with uniform dielectric materials with dielectric constants (3 and 5) as shown in figure. Another capacitor of capacitance 7.5 pF is connected in parallel with it. The effective capacitance of this combination is ____ pF . (Given $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$)

24. A thin solid disk of 1 kg is rotating along its diameter axis at the speed of 1800 rpm . By applying an external torque of $25\pi \text{ Nm}$ for 40 s , the speed increases to 2100 rpm . The diameter of the disk is ____m

25. A cube having a side of 10 cm with unknown mass and 200 gm mass were hung at two ends of an uniform rigid rod of 27 cm long. The rod along with masses was placed on a wedge keeping the distance between wedge point and 200 gm weight as 25 cm . Initially the masses were not at balance. A beaker is placed beneath the unknown mass and water is added slowly to it. At given point the masses were in balance and half volume of the unknown mass was inside the water. (Take the density of unknown mass is more than that of the water, the mass did not absorb water and water density is 1 gm/cm^3 .) The unknown mass is ____ kg .

Chemistry Section-A

26. In a first order decomposition reaction, the time taken for the decomposition of reactant to one fourth and one eighth of its initial concentration are t_1 and $t_2(s)$, respectively. The ratio t_1/t_2 will :

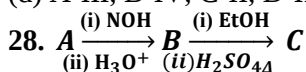
- (a) $\frac{4}{3}$ (b) $\frac{3}{2}$
(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

27. Match the LIST-I with LIST-II.

LIST-I		LIST-II	
A.	Carbocation	I.	Species that can supply a pair of electrons.
B.	C-Free radical	II.	Species that can receive a pair of electrons.
C.	Nucleophile	III.	sp^2 hybridized carbon with empty p-orbital.
D.	Electrophile	IV	sp^2/sp^3 hybridized carbon with one unpaired electron.

Choose the correct answer from the options given below :

- (a) A-IV, B-II, C-III, D-I
(b) A-II, B-III, C-I, D-IV
(c) A-III, B-IV, C-II, D-I
(d) A-III, B-IV, C-II, D-II

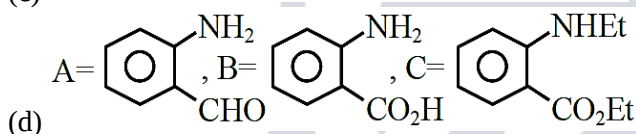
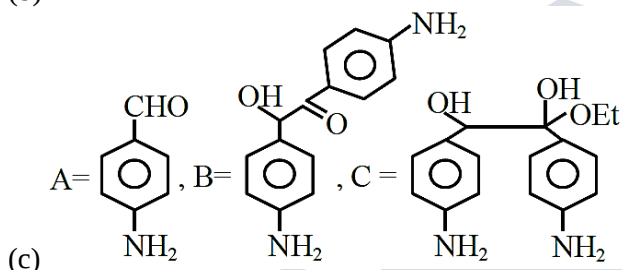
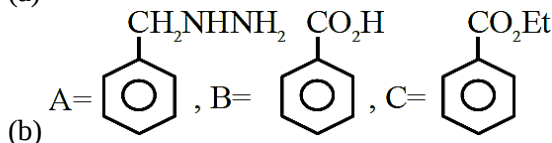
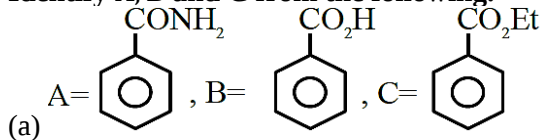


'A' shows positive Lassaing's test for N and its molar mass is 121 .

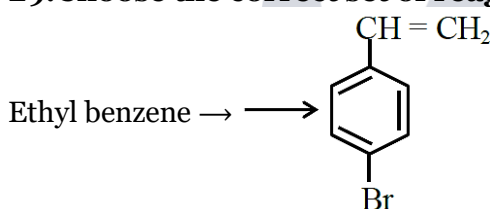
'B' gives effervescence with aq. $NaHCO_3$.

'C' gives fruity smell.

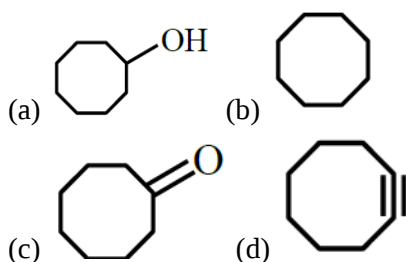
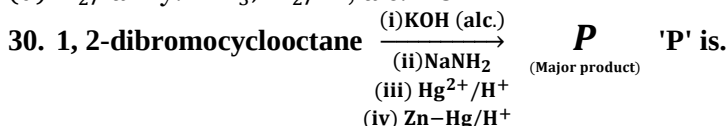
Identify A, B and C from the following.



29. Choose the correct set of reagents for the following conversion.



- (a) $Br_2/Fe; Cl_2, \Delta; alc. KOH$
(b) $Cl_2/Fe; Br_2/ anhy. AlCl_3; aq. KOH$
(c) $Br_2/ anhy. AlCl_3; Cl_2, \Delta; aq. KOH$
(d) $Cl_2/ anhy. AlCl_3; Br_2/Fe; alc. KOH$



31. Given below are two statements :

Statement I : A homoleptic octahedral complex, formed using monodentate ligands, will not show stereoisomerism.

Statement II : cis- and trans- platin are heteroleptic complexes of Pd.

In the light of the above statements, choose the correct answer from the options given below.

- (a) Both statement I and Statement II are false.
 (b) Statement I is false but Statement II is true.
 (c) Both statement I and Statement II are true.
 (d) Statement I is true but Statement II is false.

32. The atomic number of the element from the following with lowest 1st ionisation enthalpy is :

- (a) 32 (b) 35
 (c) 87 (d) 19

33. Which of the following binary mixture does not show the behaviour of minimum boiling azeotropes ?

- (a) $\text{H}_2\text{O} + \text{CH}_3\text{COC}_2\text{H}_5$
 (b) $\text{C}_6\text{H}_5\text{OH} + \text{C}_6\text{H}_5\text{NH}_2$
 (c) $\text{CS}_2 + \text{CH}_3\text{COCH}_3$
 (d) $\text{CH}_3\text{OH} + \text{CHCl}_3$

34. $\text{HA}(\text{aq}) \rightleftharpoons \text{H}^+(\text{aq}) + \text{A}^-(\text{aq})$

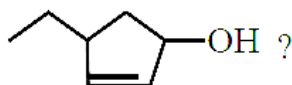
The freezing point depression of a 0.1 m aqueous solution of a monobasic weak acid HA is 0.20°C. The dissociation constant for the acid is The freezing point depression of a 0.1 m aqueous solution of a monobasic weak acid

Given :

$K_f(\text{H}_2\text{O}) = 1.8 \text{ K kg mol}^{-1}$, molality $\equiv$ molarity

- (a) 1.38×10^{-3}
 (b) 1.1×10^{-2}
 (c) 1.90×10^{-3}
 (d) 1.89×10^{-1}

35. What is the correct IUPAC name of

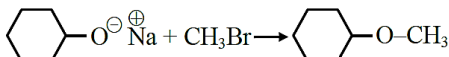
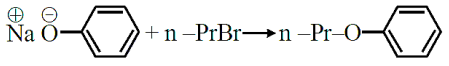


- (a) 4-Ethyl-1-hydroxycyclopent-2-ene
 (b) 1-Ethyl-3-hydroxycyclopent-2-ene
 (c) 1-Ethylcyclopent-2-en-3-ol
 (d) 4-Ethylcyclopent-2-en-1-ol

36. The correct decreasing order of spin only magnetic moment values (BM) of Cu^+ , Cu^{2+} , Cr^{2+} and Cr^{3+} ions is:

- (a) $\text{Cu}^+ > \text{Cu}^{2+} > \text{Cr}^{3+} > \text{Cr}^{2+}$
 (b) $\text{Cu}^{2+} > \text{Cu}^+ > \text{Cr}^{2+} > \text{Cr}^{3+}$
 (c) $\text{Cr}^{2+} > \text{Cr}^{3+} > \text{Cu}^{2+} > \text{Cu}^+$
 (d) $\text{Cr}^{3+} > \text{Cr}^{2+} > \text{Cu}^+ > \text{Cu}^{2+}$

37. Which one of the following reactions will not lead to the desired ether formation in major proportion? (iso- Bu $\Rightarrow$ isobutyl, sec- Bu $\Rightarrow$ sec-butyl, nPr $\Rightarrow$ n-propyl, ^tBu $\Rightarrow$ tert-butyl, Et $\Rightarrow$ ethyl)

- (a) $\text{tBuO}^-\text{Na}^+ + \text{EtBr} \rightarrow \text{tBu-O-Et}$
 (b) 
 (c) 

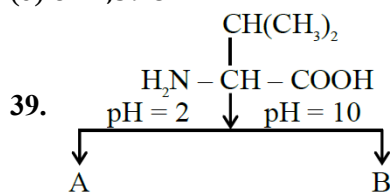
- (d) $\text{iso-BuO}^-\text{Na}^+ + \text{sec-BuBr} \rightarrow \text{Sec-Bu-O-iso-Bu}$

38. On combustion 0.210 g of an organic compound containing C, H and O

gave 0.127 g H_2O and 0.307 g CO_2 . The percentagels of hydrogen and oxygen in the given organic compound respectively are:

- (a) 53.41, 39.6
 (b) 6.72, 53.41

- (c) 7.55,43.85
(d) 6.72,39.87



Choose the correct option for structures of A and B, respectively

- (a) $\text{H}_3\text{N}^+-\text{CH}(\text{CH}_3)_2-\text{COOH}$ And $\text{H}_2\text{N}-\text{CH}(\text{CH}_3)_2-\text{COO}^-$
(b) $\text{H}_2\text{N}-\text{CH}(\text{CH}_3)_2-\text{COO}^-$ And $\text{H}_3\text{N}^+-\text{CH}(\text{CH}_3)_2-\text{COOH}$
(c) $\text{H}_2\text{N}-\text{CH}(\text{CH}_3)_2-\text{COO}^\ominus$ And $\text{H}_3\text{N}^+-\text{CH}(\text{CH}_3)_2-\text{COO}^-$
(d) $\text{H}_3\text{N}^+-\text{CH}(\text{CH}_3)_2-\text{COO}^-$ And $\text{H}_3\text{N}^+-\text{CH}(\text{CH}_3)_2-\text{COOH}$

40. Correct statements for an element with atomic number 9 are

- A. There can be 5 electrons for which $m_s = +\frac{1}{2}$ and 4 electrons for which $m_s = -\frac{1}{2}$
B. There is only one electron in p_z orbital
C. The last electron goes to orbital with $n = 2$ and $l = 1$
D. The sum of angular nodes of all the atomic orbitals is 1.

Choose the correct answer from the options given below:

- (a) C and D Only
(b) A and C Only
(c) A, C and D Only
(d) A and B Only

41. The number of species from the following that are involved in $sp^3 d^2$

hybridization is $[\text{Co}(\text{NH}_3)_6]^{3+}$, SF_6 , $[\text{CrF}_6]^{3-}$, $[\text{CoF}_6]^{3-}$, $[\text{Mn}(\text{CN})_6]^{3-}$ and $[\text{MnCl}_6]^{3-}$

- (a) 5 (b) 6
(c) 4 (d) 3

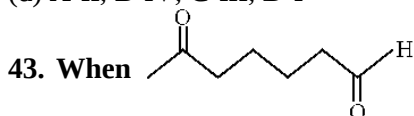
42. Match the LIST-I with LIST-II

LIST-I (Reagent)		LIST-II (Functional Group detected)	
A.	Sodium bicarbonate solution	I.	double bond/unsaturation
B.	Neutral ferric chloride	II.	carboxylic acid

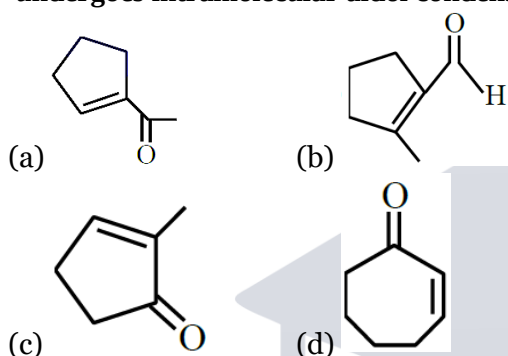
C.	ceric ammonium nitrate	III.	phenolic-OH
D.	alkaline KMnO_4	IV	alcoholic - OH

Choose the correct answer from the options given below.

- (a) A-II, B-III, C-IV, D-I
 (b) A-II, B-III, C-I, D-IV
 (c) A-III, B-II, C-IV, D-I
 (d) A-II, B-IV, C-III, D-I



undergoes intramolecular aldol condensation, the major product formed is



44. Match the LIST-I with LIST-II

LIST-I (Complex/Species)		LIST-II (Shape & magnetic moment)	
A.	$[\text{Ni}(\text{CO})_4]$	I.	Tetrahedral, 2.8 BM
B.	$[\text{Ni}(\text{CN})_4]^{2-}$	II.	Square planar, 0 BM
C.	$[\text{NiCl}_4]^{2-}$	III.	Tetrahedral, 0 BM
D.	$[\text{MnBr}_4]^{2-}$	IV	Tetrahedral, 5.9 BM

Choose the correct answer from the options given below:

- (a) A-III, B-IV, C-II, D-I
 (b) A-I, B-II, C-III, D-IV
 (c) A-III, B-II, C-I, D-IV
 (d) A-IV, B-I, C-III, D-II

45. Given below are two statements:

Statement I: H_2Se is more acidic than H_2Te .

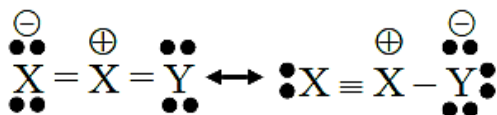
Statement II : H_2Se has higher bond enthalpy for dissociation than H_2Te .

In the light of the above statements, choose the correct answer from the options given below.

- (a) Both statement I and Statement II are false.
 (b) Both statement I and Statement II are true.
 (c) Statement I is true but Statement II is false.
 (d) Statement I is false but Statement II is true.

Section-B

46. Resonance in X_2Y can be represented as



The enthalpy of formation of X_2Y ($X \equiv X(g) + \frac{1}{2}Y = Y(g) \rightarrow X_2Y(g)$) is 80 kJ mol^{-1} .

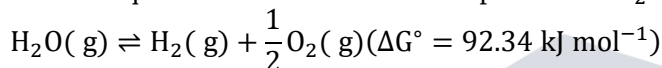
The magnitude of resonance energy of X_2Y is _____ kJ mol^{-1} (nearest integer value)

Given : Bond energies of $X \equiv X$, $X = X$, $Y = Y$ and $X = Y$ are 940, 410, 500 and 602 kJ mol^{-1} respectively. valence X: 3, Y: 2

47. The energy of an electron in first Bohr orbit of H -atom is -13.6 eV . The magnitude of energy value of electron in the first excited state of Be^{3+} is _____ eV . (nearest integer value)

48. 20 mL of sodium iodide solution gave 4.74 g silver iodide when treated with excess of silver nitrate solution. The molarity of the sodium iodide solution is _____ M. (Nearest Integer value) (Given : Na = 23, I = 127, Ag = 108, N = 14, O = 16 g mol^{-1})

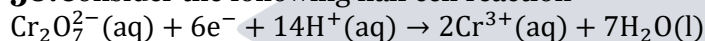
49. The equilibrium constant for decomposition of $\text{H}_2\text{O}(g)$



is 8.0×10^{-3} at 2300 K and total pressure at equilibrium is 1 bar. Under this condition, the degree of dissociation (α) of water is _____ $\times 10^{-2}$ (nearest integer value).

[Assume α is negligible with respect to 1]

50. Consider the following half cell reaction



The reaction was conducted with the ratio of $\frac{[\text{Cr}^{3+}]^2}{[\text{Cr}_2\text{O}_7^{2-}]}$ = 10^{-6} . The pH value at which the EMF of the half cell will become zero is _____ (nearest integer value)

[Given : standard half cell reduction potential

$$E^\circ_{\text{Cr}_2\text{O}_7^{2-}, \text{H}^+ / \text{Cr}^{3+}} = 1.33 \text{ V}, \frac{2.303RT}{F} = 0.059 \text{ V}]$$

Mathematics

Section-A

51. Let the values of λ for which the shortest distance between the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-\lambda}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be λ_1 and λ_2 . Then the radius of the circle passing through the points $(0, 0)$, (λ_1, λ_2) and (λ_2, λ_1) is

- (a) $\frac{5\sqrt{2}}{3}$ (b) 4
(c) $\frac{\sqrt{2}}{3}$ (d) 3

52. Let α be a solution of $x^2 + x + 1 = 0$, and for some a and b in

$$\mathbb{R}, [4 \ a \ b] \begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix} = [0 \ 0 \ 0]. \text{ If } \frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3, \text{ then } m + n \text{ is equal to}$$

- (a) 3 (b) 11
(c) 7 (d) 8

53. Let the function $f(x) = \frac{x}{3} + \frac{3}{x} + 3$, $x \neq 0$ be strictly increasing in (

$-\infty, \alpha_1)$ $U(\alpha_2, \infty)$ and strictly decreasing in $(\alpha_3, \alpha_4) U(\alpha_4, \alpha_5)$. Then $\sum_{i=1}^5 \alpha_i^2$ is equal to :-

- (a) 48 (b) 28
(c) 40 (d) 36

54. If A and B are two events such that $P(A) = 0.7$, $P(B) = 0.4$ and

$P(A \cap \bar{B}) = 0.5$, where $\bar{B}$ denotes the complement of B, then $P(B | (A \cup \bar{B}))$ is equal:-

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

If $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \infty = \frac{\pi^4}{90}$,

55. $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots \infty = \alpha$, then $\frac{\alpha}{\beta}$ is equal to

$\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \dots \infty = \beta$,

- (a) 23 (b) 18
(c) 15 (d) 14

56. The sum of the squares of the roots of $|x + 2|^2 + |x - 2| - 2 = 0$ and the squares of the roots of $x^2 - 2|x - 3| - 5 = 0$, is

- (a) 26 (b) 36
(c) 30 (d) 24

57. Let a be the length of a side of a square OABC with O being the origin.

Its side OA makes an acute angle α with the positive x -axis and the equations of its diagonals are $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 0$ and $(\sqrt{3} - 1)x - (\sqrt{3} + 1)y + 8\sqrt{3} = 0$. Then a^2 is equal to.

- (a) 48 (b) 32
(c) 16 (d) 24

58. Let $f(x)$ be a positive function and $I_1 = \int_{-\frac{1}{2}}^1 2xf(2x(1 - 2x))dx$ and $I_2 =$

$\int_{-1}^2 f(x(1 - x))dx$. Then the value of $\frac{I_2}{I_1}$ is equal to _____.

- (a) 9 (b) 6
(c) 12 (d) 4

59. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Let $\hat{c}$ be a unit vector in the plane of the vectors $\vec{a}$ and $\vec{b}$ and be perpendicular to $\vec{a}$. Then such a vector $\hat{c}$ is :

- (a) $\frac{1}{\sqrt{5}}(\hat{j} - 2\hat{k})$ (b) $\frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$
(c) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (d) $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$

60. Let the ellipse $3x^2 + py^2 = 4$ pass through the centre C of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ of radius r . Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6f_1f_2 - r$ is equal to.

- (a) 74 (b) 68
(c) 70 (d) 78

61. The integral $\int_{-1}^{\frac{3}{2}} (|\pi^2 x \sin(\pi x)|) dx$ is equal to :

- (a) $3 + 2\pi$
(b) $4 + \pi$
(c) $1 + 3\pi$
(d) $2 + 3\pi$

62. A line passing through the point $P(a, \theta)$ makes an acute angle α with the positive x -axis. Let this line be rotated about the point P through an angle $\frac{\alpha}{2}$ in the clock-wise direction. If in the new position, the slope of the line is $2 - \sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2 \tan^2 \alpha - 2\sqrt{3}$ is.

- (a) 4 (b) 6
(c) 5 (d) 8

63. There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is.

- (a) 230 (b) 220
(c) 200 (d) 210

64. Let $A =$

$\left\{ \theta \in [0, 2\pi] : 1 + 10 \operatorname{Re} \left(\frac{2 \cos \theta + i \sin \theta}{\cos \theta - 3i \sin \theta} \right) = 0 \right\}.$

Then $\sum_{\theta \in A} \theta^2$ is equal to

- (a) $\frac{21}{4}\pi^2$ (b) $8\pi^2$
(c) $\frac{27}{4}\pi^2$ (d) $6\pi^2$

65. Let $A = \{0, 1, 2, 3, 4, 5\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $\max\{x, y\} \in \{3, 4\}$. Then among the statements

(S_1) : The number of elements in R is 18, and

(S_2) : The relation R is symmetric but neither reflexive nor transitive

- (a) both are true
(b) both are false
(c) only (S_2) is true
(d) only (S_1) is true

66. The number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1016}$ is.

- (a) 127 (b) 130
(c) 129 (d) 128

67. Let $f(x) = x - 1$ and $g(x) = e^x$ for $x \in \mathbb{R}$. If $\frac{dy}{dx} = \left(e^{-2\sqrt{x}} g(f(f(x))) - \frac{y}{\sqrt{x}}\right)$, $y(0) = 0$, then $y(1)$ is :-

- (a) $\frac{1-e^2}{e^4}$ (b) $\frac{2e-1}{e^3}$
(c) $\frac{e-1}{e^4}$ (d) $\frac{1-e^3}{e^4}$

68. The value of $\cot^{-1}\left(\frac{\sqrt{1+\tan^2(2)}-1}{\tan(2)}\right) - \cot^{-1}\left(\frac{\sqrt{1+\tan^2(\frac{1}{2})}+1}{\tan(\frac{1}{2})}\right)$ is equal to.

- (a) $\pi - \frac{5}{4}$ (b) $\pi - \frac{3}{2}$
(c) $\pi + \frac{3}{2}$ (d) $\pi + \frac{5}{2}$

69. Let $A = \begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$

If $\det(\text{adj}(\text{adj}(3A))) = 2^m \cdot 3^n$, $m, n \in \mathbb{N}$, then $m + n$ is equal to.

- (a) 22 (b) 24
(c) 26 (d) 20

70. Given below are two statements:

Statement I :

$$\lim_{x \rightarrow 0} \left(\frac{\tan^{-1} x + \log_e \sqrt{\frac{1+x}{1-x}} - 2x}{x^5} \right) = \frac{2}{5}$$

Statement II : $\lim_{x \rightarrow 1} \left(x^{\frac{2}{1-x}} \right) = \frac{1}{e^2}$

In the light of the above statements, choose the correct answer from the options given below.

- (a) Statement I is false but Statement II is true
(b) Statement I is true but Statement II is false
(c) Both Statement I and Statement II are false
(d) Both Statement I and Statement II are true

Section-B

71. Let the area of the bounded region $\{(x, y) : 0 \leq 9x \leq y^2, y \geq 3x - 6\}$ be A .

Then $6A$ is equal to.

72. Let the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$ be $[\alpha, \beta]$ and the domain of $g(x) = \log_2(2 - 6\log_{27}(2x+5))$ be (γ, δ) . Then $|7(\alpha + \beta) + 4(\gamma + \delta)|$ is equal to .

73. Let the area of the triangle formed by the lines $x + 2 = y - 1 = z$, $\frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1}$ and $\frac{x}{-3} = \frac{y-3}{3} = \frac{z-2}{1}$ be A . Then A^2 is equal to.

74. The product of the last two digits of $(1919)^{1919}$ is.

75. Let r be the radius of the circle, which touches x -axis at point $(a, 0)$, $a < 0$ and the parabola $y^2 = 9x$ at the point $(4, 6)$. Then r is equal to.

