1. **(b)** $\alpha = 3 \times 10^8$ m

Distance $= (400) \times 3 \times 10^8$ m $= 400\alpha$

2. **(a)** As per the question, considering H as magnetic field

$[\mathrm{H}] = \frac{\mathrm{MLT^{-2}}}{\mathrm{LT^{-1}IT}} = \mathrm{MI^{-1}\,T^{-2}}$

$[\mathrm{E}] = \frac{\mathrm{MLT^{-2}}}{\mathrm{IT}} = \mathrm{MLI^{-1}\,T^{-3}}$

$[\epsilon] = \frac{\mathrm{I^2\,T^2}}{\mathrm{MLT^{-2}\,L^2}} = \mathrm{M^{-1}\,L^{-3}I^2\,T^4}$

$\mathrm{MI^{-1}\,T^{-2} = M^{-q+r}L^{p-3q+r}I^{2q-r}T^{4q-3r-s}}$

$-\mathrm{q} + \mathrm{r} = 1$

$\mathrm{P} - 3\mathrm{q} + \mathrm{r} = 0$

$2\mathrm{q} - \mathrm{r} = -1$

$4q - 3r - S = -2$

Solving we get

$\mathrm{q} = 0, \mathrm{r} = 1$

$\mathrm{p} = -1, \mathrm{S} = -1$

No option matching

Considering H as magnetic field intensity

$\mathrm{H} = \frac{\mathrm{B}}{\mu_0} = \mathrm{Ni} \Rightarrow$ current per unit length $\Rightarrow \mathrm{IL^{-1}}$

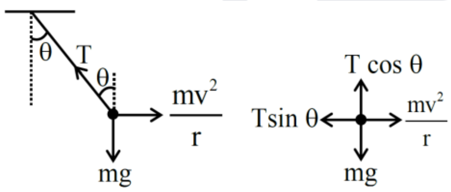
$-\mathrm{q} + \mathrm{r} = 0 \quad \mathrm{q} = 1$

$\mathrm{p} - 3\mathrm{q} + \mathrm{r} = -1 \quad \mathrm{r} = 1$

$2\mathrm{q} - \mathrm{r} = 1 \quad \mathrm{p} = +1$

$4\mathrm{q} - 3\mathrm{r} - \mathrm{s} = 0 \quad \mathrm{s} = +1$

3. **(c)**



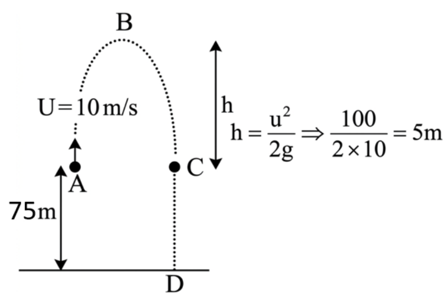
$\tan\theta = \frac{\mathrm{v^2}}{\mathrm{rg}} \left(\mathrm{v} = 54 \times \frac{5}{18} = 15\ \mathrm{m/s}\right)$

$\tan\theta = \frac{225}{20 \times 10}$

$\tan\theta = \frac{9}{8}$

$\theta = \tan^{-1}(1.125)$

4. **(d)**



$-75 = 10 \times \mathrm{t} - \frac{1}{2} \times 10 \times \mathrm{t}^2$

$\mathrm{t}^2 - 2\mathrm{t} - 15 = 0$

$(\mathrm{t} - 5)(\mathrm{t} + 3) = 0 \Rightarrow \mathrm{t} = 5$

Height $= 75 + 10 \times 5 = 125$ m

5. **(a)** Combination of lens + mirror

$f_m = \frac{R}{2} = -10\text{ cm}; f_L = \frac{R}{2(\mu-1)} = 20\text{ cm}$

$\frac{1}{f} = \frac{1}{f_m} - \frac{2}{f_L}$

$\frac{1}{f} = \frac{-1}{10} - \frac{2}{20} = -\frac{1}{5}$

$f = -5\text{ cm}$

To form image at the position of object, it should be placed at center of curvature of mirror + lens combination.

$\therefore u = 2f = 2 \times 5 = 10\text{ cm}$

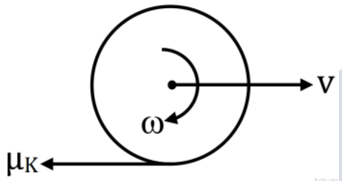
6. **(a)** For range to be same, angle of projection must be complementary angles.

$T_1 = \frac{2u\sin\theta}{g} = 10,\ T_2 = \frac{2u\cos\theta}{g} = 5$

Range $= \frac{2(u\sin\theta)(u\cos\theta)}{g}$

$= \frac{2(50)(25)}{10}\text{ m} = 250\text{ m}$

7. **(b)**



$a = \frac{\mu_K mg}{m} = \mu_K g$

$\tau = I\alpha \Rightarrow \mu_K mgR = \frac{MR^2}{2} \times \alpha$

$\Rightarrow \alpha = \frac{2\mu_K g}{R}$

$\therefore v = v_0 - \mu_K gt$ ...(i)

Also, $\omega = \omega_0 + \frac{2\mu_K gt}{R}$ ...(ii)

$\frac{v}{R} = \frac{v_0}{4R} + \frac{2\mu_k gt}{R}$

Solving Eq. (i) & (ii)

$\frac{v_0}{4} + 2\mu gt = v_0 - \mu_k gt$

$3\mu gt = v_0 - \frac{v_0}{4}$

$3\mu gt = \frac{3v_0}{4}$

$\Rightarrow t = \frac{v_0}{4\mu_K g} = \frac{49}{4\times\frac{1}{4}\times 9.8} = 5\text{sec.}$

8. **(d)** Using continuity equation,

$A_1V_1 = A_2V_2$

$1 \times 10 = \frac{1}{5} \times V_2$

$\Rightarrow V_2 = 50\text{ cm/s}$

Using Bernoulli's theorem,

$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$

$P_1 - P_2 = \frac{1}{2}\rho[V_2^2 - V_1^2]$

$= \frac{1}{2} \times 600[(50)^2 - (10)^2] \times 10^{-4} = 72\text{ Pa.}$

9. **(d)** Using volume conservation,

$$\frac{4}{3}\pi R_1^3 = 64 \times \frac{4}{3}\pi R_2^3 \Rightarrow R_2 = \frac{R_1}{4}$$

Terminal velocity $v \propto R^2$

$$\frac{v_1}{v_2} \Rightarrow \frac{R_1^2}{R_2^2} \Rightarrow (4)^2 = 16$$

10. **(b) Method-1**

$Q = \Delta U + W$

$Q = \frac{f}{2}nR\Delta T + nR\Delta T = \left(\frac{f}{2} + 1\right)nR\Delta T$

if $f = 2 \Rightarrow Q = 2nR\Delta T \Rightarrow 2 \times 1 \times 8.3 \times 1.2 = 19.92$

(No option is matching)

if $f = 5 \Rightarrow Q = 3.5nR\Delta T$

$\Rightarrow 3.5 \times 1 \times 8.3 \times 1.2 = 34.86$

(No option is matching)

Method-2

$Q = \Delta U + W$

$Q = \frac{f}{2}nR\Delta T + P\Delta V$

if $f = 2$

$\Rightarrow Q = nR\Delta T + P\Delta V$

$= 1 \times 8.3 \times 1.2 + 10^5 \times 25 \times 10^{-3} \times 4 \times 10^{-4}$

$= 10.96$

(No option is matching)

if $f = 5$

$\Rightarrow Q = 2.5nR\Delta T + P\Delta V$

$2.5 \times 1 \times 8.3 \times 1.2 + 10^5 \times 25 \times 10^{-3} \times 4 \times 10^{-4}$

$= 25.9$

11. **(c)** $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$\gamma = \frac{5}{3}$

$PV^{5/3} = P_2 \times (27\,V)^{5/3}$

$P_2 = \frac{P}{(27)^{5/3}} = \frac{P}{3^5}$

$\Delta U \Rightarrow nC_v\Delta T = \frac{nR\Delta T}{(\gamma - 1)}$

$= \frac{P_2V_2 - P_1V_1}{(\gamma - 1)}$

$\Rightarrow \frac{\frac{P}{3^5} \times 3^3.V - PV}{\frac{5}{3} - 1} \Rightarrow \frac{-PV\left\{1 - \frac{1}{9}\right\}}{\frac{2}{3}}$

$= -PV \times \frac{8}{9} \times \frac{3}{2}$

$\Rightarrow -\frac{4}{3}PV$

12. **(c)** For a spring, $k\ell$ = constant

$k_1\ell = k_2\frac{\ell}{2} \Rightarrow k_2 = 2k_1$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{k_1}{k_2}}$

$\Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{1}{2}} \Rightarrow \frac{f_2}{f_1} = \sqrt{2}$

13. (d) $\frac{hC}{\lambda} = \frac{15\times10^3}{2.5\times10^{22}}$

$\therefore \lambda = \frac{6.6\times10^{-34}\times3\times10^8\times2.5\times10^{22}}{15\times10^3}$

$\lambda = 3.3\times10^{-7}$

14. (d) $\vec{\tau} = \vec{M}\times\vec{B}$

$\Rightarrow 100\sqrt{2}\times3.14\times4\times10^{-4}\left[\left(\frac{\hat{k}+\hat{i}}{\sqrt{2}}\right)\times(3\hat{i}+2\hat{k})\right]B_0$

$\Rightarrow 16\times3.14\times[+3\hat{j}-2\hat{j}]\times10^{-5}$

$\Rightarrow 50.24\times10^{-5}\hat{j}$

15. (b) $\varepsilon = -\frac{Ldi}{dt} = -\mu_0 n^2 A\ell\frac{di}{dt}$

$= -4\pi\times10^{-7}\times10^6\times5\times10^{-4}\times0.3\times\frac{2}{\pi}$

$\Rightarrow -12\times10^{-5}$

16. (b) $\vec{r} = \vec{r}_2-\vec{r}_1 \Rightarrow -\hat{i}-2\hat{j}-2\hat{k}$

Force $\Rightarrow \frac{kq_1q_2}{r^3}\vec{r}$

$\Rightarrow \frac{9\times10^9\times(3)(+4)\times10^{-12}}{27}(\hat{i}+2\hat{j}+2\hat{k})$

$\Rightarrow 4\times10^{-3}[\hat{i}+2\hat{j}+2\hat{k}]$

17. (a) $1(AO\times\hat{j}) \Rightarrow (OB\times\hat{j})\times1.5$

$\frac{2\hat{k}}{\sqrt{13}} = \frac{3}{2}\left[\frac{C\hat{k}}{\sqrt{C^2+16}}\right]$

$\therefore \frac{4}{13} = \frac{9C^2}{4(C^2+16)} \Rightarrow 16C^2+256 = 117C^2$

$256 = 101C^2$

$C = 1.59 \approx 1.6$

18. (d) $2\,K_1 = \frac{2hC}{\lambda_1} - 2\phi$ ...(i)

$K_2 = \frac{2hC}{\lambda_1} - \phi$ ...(ii)

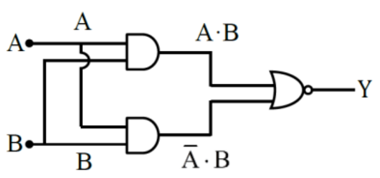
Solving (i) & (ii), we get

$K_2 - 2\,K_1 = \phi$

19. (c) $k_\alpha = \left(\frac{A-4}{A}\right)Q$

$\therefore \frac{k_{\alpha_1}}{k_{\alpha_2}} = \frac{196}{200}\times\frac{212}{208} = \frac{2597}{2600}$

20. (b)



$A\cdot B+\bar{A}\cdot B \Rightarrow (A+\bar{A})\cdot B = B$

21. (2) $RC = \rho K \in_0$

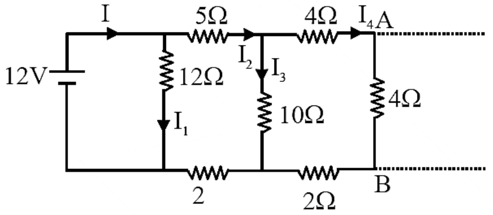
$$K=\frac{RC}{\rho\varepsilon_0}=\frac{17.7\times10^{14}\times10^{-6}}{10^{13}\times8.85\times10^{-12}}=2\times10^{7}$$

22.(203) $\theta=\frac{1.22\lambda}{b}$

$$3\times10^{-7}=\frac{1.22\times5\times10^{-7}}{b}\Rightarrow\ b=203.33\text{ cm}$$

23.(2) $5\left(\frac{2\pi\text{ m}}{\text{qB}}\right)\text{v}\Rightarrow\frac{5\times2\times\pi\times5\times10^{-6}\times2\times10^{-2}}{5\pi\times10^{-6}\times0.1}=2\text{ m}$

24.(200) At steady state,



$$I=\frac{E}{R_{eq}}=\frac{12}{6}=2A$$

$$I_1=I_2=1\text{ A}$$

$$I_3=I_4=\frac{1}{2}\text{ A}$$

$$V_{AB}=\frac{1}{2}\times4=2\text{ V}$$

Charge on capacitor

$$Q=CV_{AB}=100\times2=200\mu C$$

25.(9) $S=u+\frac{a}{2}(2n-1)\Rightarrow104=5+\frac{a}{2}(9)\Rightarrow a=\frac{22\text{ m}}{s^2}$

$$129=12+\frac{a}{2}\times9\Rightarrow a=26\text{ m/s}^2$$

$$\frac{p_1}{p_2}=\frac{(5+22\times10)3.4}{(12+26\times10)(2.5)}=\frac{225\times(3.4)}{272\times2.5}=\frac{9}{8}$$

26. (c) (A) No. of atoms in 1.8 mg water

$$=\frac{1.8\times10^{-3}}{18}\times N_A\times3=3\times10^{-4}\ N_A$$

(B) No. of atoms in 9.8mgH_2SO_4

$$=\frac{9.8\times10^{-3}}{98}\times N_A\times7=7\times10^{-4}\times N_A$$

(C) No. of atoms in 1.8 mg carbon

$$=\frac{1.8\times10^{-3}}{12}\times N_A=1.5\times10^{-4}\times N_A$$

(D) No. of atoms in 5.85 mg NaCl

$$=\frac{5.85\times10^{-3}}{58.5}\times N_A\times2=2\times10^{-4}\ N_A$$

27. (a) (I) Let wt of solution = 100 g

wt of methanol = 30 g

wt of CCl_4 = 100 − 30 = 70 g

mole fraction of $CCl_4=\frac{n_{CCl_4}}{n_{CCl_4}+n_{CH_3OH}}=\frac{\frac{70}{154}}{\frac{70}{154}+\frac{30}{32}}=0.3267\approx0.33$

(II) CCl_4& methanol is mixture of positive deviation from Raoult's law.

So, statement (I) & statement (II) are correct.

28. (a) $BrF_2^{\oplus}\Rightarrow sp^3$

[Bond pair =2, lone pair =2]

$\Rightarrow$ bent

$BrF_2^{\oplus} \Rightarrow sp^3\ d^2$

[Bond pair =4, lone pair =2]

$\Rightarrow$ square planar

29.(c) (A) K_b for water $= 0.512 \frac{^\circ C-Kg}{mol}$

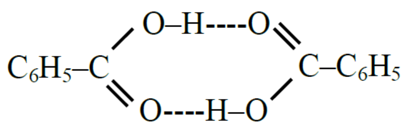
K_f for water $= 1.86 \frac{^\circ C-Kg}{mol}$

So, $K_b < K_f$ for water

(B) Since, $K_b < K_f$ so $\Delta T_b < \Delta T_f$ [same equimolal solution]

(C) Osmotic pressure is used to determine molar mass of proteins & polymers

(D) dimerized form of benzoic acid in benzene



30. (c) $x_2 + y_2 \rightleftharpoons 2xy, K' = \frac{1}{K_1} = \frac{1}{2.5\times10^5} = \frac{1}{25\times10^4}$

$\frac{1}{2}x_2 + \frac{1}{2}y_2 \rightleftharpoons xy, K'' = \left(\frac{1}{25\times10^4}\right)^{1/2} = \frac{1}{5\times10^2}$ ...(1)

$xy + \frac{1}{2}z_2 \rightleftharpoons xyz,\ K_2 = 5\times10^{-3}$ ...(2)

On adding equation (1) & (2)

$\frac{1}{2}x_2 + \frac{1}{2}y_2 + \frac{1}{2}z_2 \rightleftharpoons xyz,$

$K_3 = \frac{1}{5\times10^2} \times 5\times10^{-3} = 1\times10^{-5}$

31. (b) (i) $Fe^{+2}(aq) + 2e^- \rightarrow Fe(s)$ ... . $\Delta G_1{}^\circ$

(ii) $Fe^{+3}(aq) + 3e^- \rightarrow Fe(s)$ ... . ΔG_2°

(iii) $Fe^{+3}(aq) + e^- \rightarrow Fe^{+2}(aq)$ ... ... $\Delta G_3{}^\circ$

iii = ii -i

$-1\times F\times E^\circ{}_{Fe^{3+}/Fe^{2+}} = -3\times F\times y - (-2\times F\times x)$

$E^\circ{}_{Fe^{3+}/Fe^{2+}} = 3y - 2x$

32.(c) S-I : $\ln\left(\frac{K_2}{K_1}\right) = \frac{12.6\times10^3}{\left(\frac{8.314}{4.2}\right)}\left[\frac{10}{298\times308}\right]$

$\Rightarrow \ln\left(\frac{K_2}{K_1}\right) = 0.693$

$\Rightarrow \frac{K_2}{K_1} = 2$

S-II : For 1 st order

$\mathbf{t_{1/2}} = \frac{\ln 2}{K}; t_{1/2} \propto [A]^0 =$ constant.

Statement-I is true but statement-II is false.

33. (a) (A) $2\ s^2 \Rightarrow$ Be

(B) $2\ s^2 2p^1 \Rightarrow$ B

(C) $2s^2 2p^3 \Rightarrow$ N

(D) $2\ s^2 2p^6 \Rightarrow$ Ne

IE order : B < Be < N < Ne

34. (c) (A) Reducing nature : $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

(B) Lone pair definition tendency :

$NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

(C) Stability of Hydrides :

$NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

(D) HEH bond angle : $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

35. (a) Ti^{+4} is colourless due to d^0 configuration

∴ statement 1 is correct

$La^{+3}, Yb^{+2}, Lu^{+3}, Ce^{+4}, Al^{+3}$ & Lr^{+3} all are diamagnetic

36. (b) First row transition metals utilize 3d and 4s electrons for bonding on the catalyst surface.

∴ Statement 1 is incorrect

Adsorption increases reactant concentration on the catalyst surface and weakens bonds facilitating formation of activated complex and lowering activation energy.

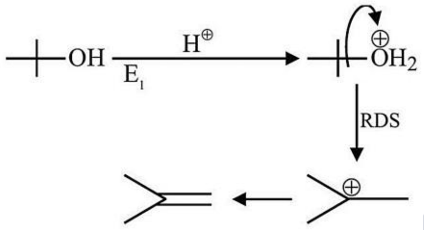
Statement 2 is incorrect

37. (c) Statement I : True

Statement II : False

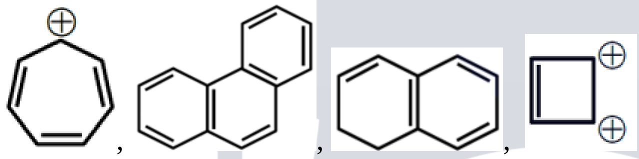
Reason - As vapors rise, they become richer in the lower-boiling component, not high-boiling component.

38. (b)

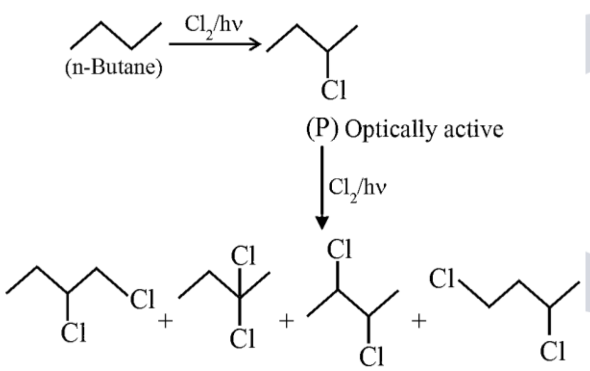


Product obtained without rearrangement.

39. (b) Aromatic compounds are



40. (b)



Total dichlorvos products (excluding stereoisomers) = 4

41. (a) Statement I : True

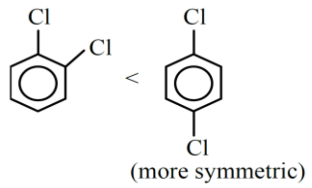
Order of boiling point :

$CH_3CH_2CH_2I > CH_3CH_2I > CH_3I$

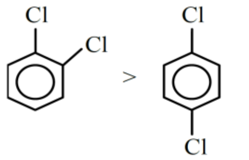
Vander Waals forces increases due to increase in molecular weight

Statement 2: True

Order of melting point :-

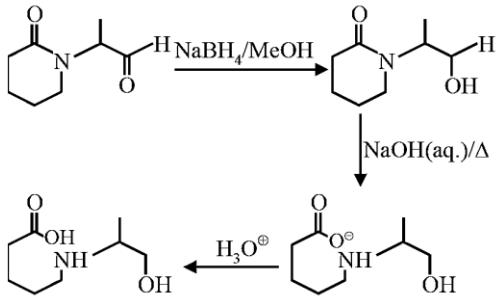


Boiling point order :-
(Boiling point $\propto \mu$)



$(\mu \neq 0)$ $(\mu = 0)$

42. (c)



43. (c) Hoffmann bromamide degradation reaction -

$$\underset{1^\circ \text{ Amide}}{R-\overset{O}{\overset{\|}{C}}-NH_2} + 4NaOH + Br_2 \longrightarrow \underset{1^\circ \text{ Amine}}{R-NH_2} + Na_2CO_3 + 2NaBr + 2H_2O$$

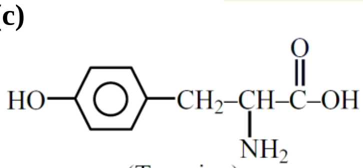
Statement (A) - False
Statement (B) - False
Statement (C) - True
Statement (D) - False
Statement (E) - True

44. (b) The monosaccharide units obtained from hydrolysis of oligosaccharides may be or may not be same.
Example :

$$\text{Lactose} \xrightarrow{H_3O^{\oplus}} \underbrace{\text{Glucose} + \text{Galactose}}_{\text{different monosaccharide units}}$$

$$\text{Maltose} \xrightarrow{H_3O^{\oplus}} \underbrace{\text{Glucose} + \text{Glucose}}_{\text{same monosaccharide units}}$$

45. (c)

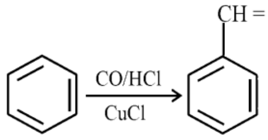


(Tyrosine)

Tyrosine amino acid will give violet coloured complex with neutral $FeCl_3$ solution because it contains phenolilc group.

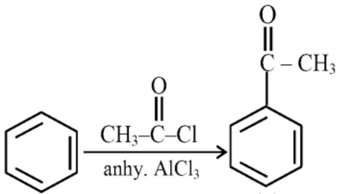
46.(6) $[MnBr_4]^{2-}$, $[NiCl_4]^{2-}$, $[CoF_6]^{3-}$, $[Mn(CN)_6]^{3-}$, $[Ti(CN)_6]^{3-}$, $[Cu(H_2O)_6]^{2+}$are paramagnetic.

47. (16)



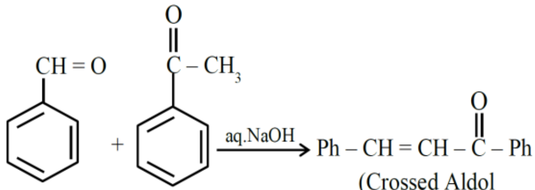
(x)

(Gattermann Koch reaction)



(y)

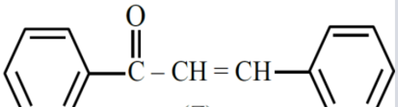
(Friedal Craft acylation)



(x) (y)

(Crossed Aldol Condensation product)
(Z)

Total number of pi (π) electrons in ' z ' $= 16\pi$ electrons



(Z)

48. **(3)** $E_{photon} = \frac{hc}{\lambda}$

$\Rightarrow \frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1}$

$\Rightarrow \frac{E_1}{E_2} = \frac{6000}{2000}$

$\Rightarrow \frac{E_1}{E_2} = 3$

49. (1126) $\Delta_r H^\circ = \Sigma \Delta H_f{}^\circ$ (product) $- \Sigma \Delta H_f{}^\circ$ (Reactant)

$\Delta_r H^\circ = 2 \times \Delta H_f^\circ(SO_2, g) + 2 \times \Delta H_f^\circ(H_2O, \ell) - 2 \times \Delta H_f^\circ(H_2 S, g)$

$\Delta_r H^\circ = 2 \times (-297) + 2 \times (-286) - 2(-20.1)$
$= -594 - 572 + 40.2$
$\Delta_r H^\circ = -1125.8$ kJ/mol.
$|\Delta_r H^\circ| = 1125.8$ kJ/mol.

50. (4) $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g), K_{P_1} = 0.08$ atm

$K_{P_1} = P_{CO_2} = 0.08$ atm

$C(s) + CO_2(g) \rightleftharpoons 2CO(g), K_{P_2} = 2$

$K_{P_2} = \frac{P_{CO}^2}{P_{CO_2}} = 2$

$\frac{P_{CO}^2}{0.08} = 2$

$P_{CO}^2 = 16 \times 10^{-2}$

$P_{CO} = 4 \times 10^{-1}$

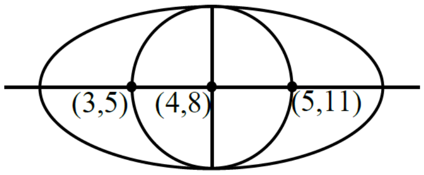
51. (a) Reflexive If $(a, a) \in R$

$\Rightarrow 1 + a^2 > 0$

$\Rightarrow$ It is reflexive relation

Symmetric If $1 + ab > 0 \Rightarrow 1 + ba > 0$
i.e. If $(a, b) \in R$ and $(b, a) \in R$
Therefore it is symmetric relation
Transitive $\because (-2,0) \in R \& (0,2) \in R$
But $(-2,2) \notin R$
Hence it is not transitive
$\Rightarrow$ It is not an equivalence relation.
Check Number of elements in R
$\because 1 + ab > 0$
a& b each can be filled by 5 ways
$\therefore$ total elements = 25 But out of these
$(-2,-1), (-2,2), (-1,2), (1,-2), (2,-2), (2,-1)$
$(-1,1) \& (1,-1)$ are not in relation.

52. (b) $|z - (3 + 5i)| + |z - (5 + 11i)| = 4\sqrt{5}$
So, $S_1(3,5)$ & $S_2(5,11)$ are two foci of ellipse, where $2ae = S_1 S_2 = 2\sqrt{10} \& 2a = 4\sqrt{5}$
$\Rightarrow a = 2\sqrt{5}, e = \frac{1}{\sqrt{2}} \& b^2 = a^2(1 - e^2)$ gives $b^2 = 10$
& center of an ellipse is mid point of both focus
$\therefore$ center $(4,8)$
Equation of circle is $|Z - (4 + 8i)| = \sqrt{10}$
$\Rightarrow$ Ellipse & given circle are concentric & radius of given circle is equal to length of minor axis of given ellipse



$\Rightarrow$ Two points which are common to both.

53. (c) $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 2 & 3 & \lambda \end{vmatrix} = \lambda - 6$

$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 5 \\ \mu & 3 & \lambda \end{vmatrix} = 2\lambda + 3\mu - 60$

$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 5 \\ 2 & \mu & \lambda \end{vmatrix} = 4(\lambda - \mu + 10)$

$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 2 & 3 & \mu \end{vmatrix} = \mu - 16$

For infinite many solution
$D = D_1 = D_2 = D_3 = 0$
$\Rightarrow \lambda = 6 \& \mu = 16$
$\Rightarrow \lambda + \mu = 22$

54. (c) $A = \begin{bmatrix} \alpha & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \& |A| = 5\alpha + 6$

Now $\text{adj}A = \begin{bmatrix} 15 & 3 & -6 \\ -10 & 5\alpha & 4 \\ 8 & -4\alpha & 3\alpha - 2 \end{bmatrix}$

$\therefore B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5\alpha & 0 \\ 0 & 4\alpha & -2\alpha \end{bmatrix} + \text{adj}(A)$

$$B=\begin{bmatrix}16 & 3 & -6\\ -10 & 0 & 4\\ 8 & 0 & \alpha-2\end{bmatrix}$$

$\&|B|=6(5\alpha+6)=66$ (given in question)

$\Rightarrow \alpha=1$

$\Rightarrow |\mathrm{A}|=21$

$\therefore \det(\mathrm{adjA})=|\mathrm{A}|^{(\mathrm{n}-1)^2}$

Here $\mathrm{n}=3$

$=21^2$

$=441$

55. (a) a $=(3+8+13$ ... upto 20 terms $)+(4+9+14+$ ... upto 20 terms$)$

$=\frac{20}{2}[6+19\times 5]+\frac{20}{2}(8+19\times 5)$

$=2040$

$(\tan\beta)^{\frac{2040}{1020}}=\tan^2\beta$

$\mathrm{x}^2+\mathrm{x}-2=0<{}^{1}_{-2}$

$\Rightarrow \tan^2\beta=1; \sin^2\beta=\frac{1}{2}\&\cos^2\beta=\frac{1}{2}$

$\sin^2\beta+3\cos^2\beta=\frac{1}{2}+\frac{3}{2}=\frac{4}{2}=2$

56. (b) $\mathrm{P}(\text{ Bus})=\mathrm{P}(\text{ B})=\frac{2}{5}\mathrm{P}\left(\frac{\text{late}}{\mathrm{B}}\right)=\frac{1}{5}$

$\mathrm{P}(\text{ Scooter })=\mathrm{P}(\text{ S})=\frac{1}{5}\mathrm{P}\left(\frac{\text{late}}{\mathrm{S}}\right)=\frac{1}{3}$

$\mathrm{P}(\text{ Car })=\mathrm{P}(\mathrm{C})=\frac{2}{5}\mathrm{P}\left(\frac{\text{late}}{\mathrm{C}}\right)=\frac{1}{4}$

$\mathrm{P}\left(\frac{\mathrm{B}}{\text{late}}\right)=\frac{\frac{2}{5}\times\frac{1}{5}}{\frac{2}{5}\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{3}\times\frac{2}{5}\times\frac{1}{4}}=\frac{12}{37}$

57. (c) Given $\overline{\mathrm{x}}=1, \sigma_1^2=13$

$\overline{\mathrm{y}}=2, \sigma_2^2=1$

Combined variance $=\frac{\mathrm{n}_1\sigma_1^2+\mathrm{n}_2\sigma_2^2}{\mathrm{n}_1+\mathrm{n}_2}+\frac{\mathrm{n}_1\mathrm{n}_2}{(\mathrm{n}_1+\mathrm{n}_2)^2}(\overline{\mathrm{x}}-\overline{\mathrm{y}})^2$

$=\frac{4\times 13+6\times 1}{10}+\frac{4\times 6}{10^2}(2-1)^2$

$\frac{58}{10}+\frac{24}{100}=\frac{580+24}{100}=\frac{604}{100}=6.04$

58. (c) $(1+\mathrm{x})^{12}={}^{12}\mathrm{C}_0+{}^{12}\mathrm{C}_1\mathrm{x}+{}^{12}\mathrm{C}_2\mathrm{x}^2+\ldots+{}^{12}\mathrm{C}_{12}\mathrm{x}^{12}$ integrating both sides

$\left(\frac{(1+x)^{13}}{13}\right)_{-2}^{2}=\left({}^{12}C_0x+{}^{12}C_1\frac{x^2}{2}+{}^{12}C_2\frac{x^3}{3}+\ldots+{}^{12}C_{12}\frac{x^{13}}{13}\right)_{-2}^{2}$

$\frac{3^{13}}{13}+\frac{1}{13}=\left({}^{12}\mathrm{C}_0 2+{}^{12}\mathrm{C}_1\frac{2^2}{2}+{}^{12}\mathrm{C}_2\frac{2^3}{3}+\ldots+{}^{12}\mathrm{C}_{12}\frac{2^{13}}{13}\right)$

$-\left(-{}^{12}\mathrm{C}_0\cdot 2+{}^{12}\mathrm{C}_1\frac{2^2}{2}-{}^{12}\mathrm{C}_2\cdot\frac{2^3}{3}+\ldots+{}^{12}\mathrm{C}_{12}\cdot\frac{2^{13}}{13}\right)$

$=2\left(2.\ {}^{12}\mathrm{C}_0+{}^{12}\mathrm{C}_2\cdot\frac{2^3}{3}+{}^{12}\mathrm{C}_4\cdot\frac{2^5}{5}+\ldots+{}^{12}\mathrm{C}_{12}\cdot\frac{2^{13}}{13}\right)$

$3^{13}+1=26\left(2+{}^{12}\mathrm{C}_{12}\cdot\frac{2^3}{3}+{}^{12}\mathrm{C}_4\cdot\frac{2^5}{5}+\ldots+{}^{12}\mathrm{C}_{12}\cdot\frac{2^{13}}{13}\right)$

$3^{13}-51=26\left({}^{12}\mathrm{C}_2\cdot\frac{2^3}{3}+{}^{12}\mathrm{C}_4\cdot\frac{2^5}{5}+\ldots+{}^{12}\mathrm{C}_{12}\cdot\frac{2^{13}}{13}\right)$

$\therefore \alpha = 51$

59. (b)

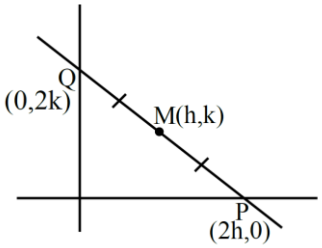
B_1 B_1 B_1

1 1 2

No. of ways $= \frac{4!}{2!2!} \times 3! = 6 \times 6 = 36$

60. (b) Point of intersection of the lines

$3x + 4y = 1$ and $4x + 3y = 1$ is $\left(\frac{1}{7}, \frac{1}{7}\right)$



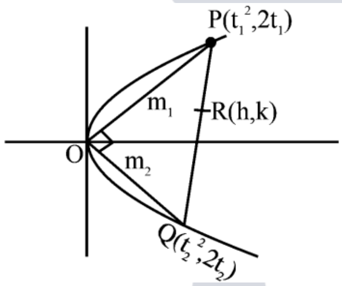
Let the line be $\frac{x}{2h} + \frac{y}{2k} = 1$

Satisfy $\left(\frac{1}{7}, \frac{1}{7}\right)$

$$\frac{1}{14h} + \frac{1}{14k} = 1$$

$$\frac{1}{x} + \frac{1}{y} = 14$$

61. (b)



$t_1 t_2 = -4$

$2h = t_1^{\ 2} + t_2^{\ 2}$

$2k = 2(t_1 + t_2)$

$2h = k^2 - 2t_1 t_2$

$2h = k^2 + 8$

$k^2 = 2h - 8$

$= 2((h - 4)$

L.R. = 2

62. (a) $\frac{1}{2} < \frac{6}{11} < \frac{1}{\sqrt{2}}$

$$\sin^{-1}\left(\frac{1}{2}\right) < \sin^{-1}\left(\frac{6}{11}\right) < \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{\pi}{6} < 3\sin^{-1}\left(\frac{6}{11}\right) < \frac{\pi}{4}$$

$$\frac{\pi}{2} < \alpha < \frac{3\pi}{4} \quad \therefore \cos\alpha < 0$$

$$0 < \frac{4}{9} < \frac{1}{2}$$

$$\frac{\pi}{3} < \cos^{-1}\left(\frac{4}{9}\right) < \frac{\pi}{2}$$

$\pi < 3\cos^{-1}\left(\frac{4}{9}\right) < \frac{3\pi}{2}$

$\pi < \beta < \frac{3\pi}{2}$

Now $\frac{3\pi}{2} < \alpha + \beta < \frac{9\pi}{4}$

$\therefore \cos(\alpha+\beta) > 0$

63.(d) $f(x) = e^{|\sin x|} - |x|$ is non-differentiable at $x = \pi$ and for $x \in \left(-\pi, \frac{-\pi}{2}\right)$

$f(x) = e^{-\sin x} + x$

$f'(x) = -e^{-\sin x}\cos x + 1$

for $x \in \left(-\pi, \frac{-\pi}{2}\right)$ $\cos x < 0$

$\therefore f'(x) > 0$

$\therefore$ f(x) is increasing

64. (b) $2(\vec{a}\times\vec{b}) - 3(\vec{c}\times\vec{b}) = \vec{0}$

$(2\vec{a} - 3\vec{c})\times\vec{b} = \vec{0}$

$\vec{b} \| (2\vec{a} - 3\vec{c})$

$2\vec{a} - 3\vec{c} = \lambda\vec{b}$

$\vec{c} = \frac{2\vec{a} - \lambda\vec{b}}{3}$ ...(1)

$\vec{a}.\vec{c} = 15$

$\left(\frac{2\vec{a} - \lambda\vec{b}}{3}\right)\cdot\vec{a} = 15$

$\frac{2|a|^2 - \lambda\vec{b}\cdot\vec{a}}{3} = 15$

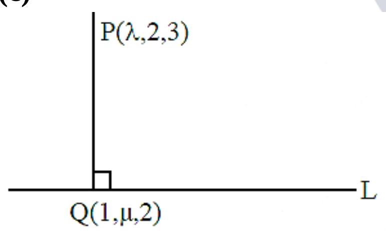
$2(26) - \lambda(40 - 2 - 3) = 45$

$\lambda = \frac{1}{5}$

$\Rightarrow \vec{c}\cdot(\hat{\imath} + \hat{\jmath} - 3\hat{k}) = \frac{\left(2(4\hat{\imath} - \hat{\jmath} + 3k) - \frac{1}{5}(10\hat{\imath} + 2\hat{\jmath} - k)\cdot(\hat{\imath} + \hat{\jmath} - 3\hat{k})\right)}{3}$

$= \frac{2(4 - 1 - 9) - \frac{1}{5}(10 + 2 + 3)}{3} = -5$

65. (c)



$\frac{x-4}{1} = \frac{y-9}{2} = \frac{z-5}{1}$

$Q(1, \mu, 2)$ satisfies the line L

$\therefore \frac{1-4}{1} = \frac{\mu-9}{2} = \frac{2-5}{1}$

$\Rightarrow -3 = \frac{\mu-9}{2} = -3 \Rightarrow \mu = 3$

D.R. of PQ : $\lambda - 1, 2 - \mu, 1$

Since $PQ \perp L \Rightarrow (\lambda - 1).1 + 2(2 - \mu) + 1.1 = 0$

$\Rightarrow \lambda - 2\mu + 4 = 0$

$\therefore \lambda = 2$

Now lines : $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$ &

$$\frac{x-2}{2}=\frac{y-3}{3}=\frac{z+5}{6}$$

i.e. $\vec{r}=(\hat{\imath}+2\hat{\jmath}-4\hat{k})+t_1(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$

$\&\vec{r}=(2\hat{\imath}+3\hat{\jmath}-5\hat{k})+t_2(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$

Distance

$$=\frac{|((\hat{\imath}+2\hat{\jmath}-4\hat{k})-(2\hat{\imath}+3\hat{\jmath}-5\hat{k}))\times(2\hat{\imath}+3\hat{\jmath}+6\hat{k})|}{|2\hat{\imath}+3\hat{\jmath}+6\hat{k}|}$$

$$=\frac{|-9\hat{\imath}+8\hat{\jmath}-\hat{k}|}{7}=\frac{\sqrt{146}}{7}$$

66. (c) $I=\int_0^2\sqrt{\frac{x(x^2+x+1)}{(x+1)(x^2-x+1)(x^2+x+1)}}dx$

$$=\int_0^2\sqrt{\frac{x}{x^3+1}}dx$$

$\left(x^{3/2}=t\Rightarrow\frac{3}{2}x^{1/2}dx=dt\right)$

$$=\frac{2}{3}\int_0^{2^{3/2}}\frac{dt}{\sqrt{t^2+1}}$$

$$=\frac{2}{3}\left(\ln\left(t+\sqrt{t^2+1}\right)\right)_0^{2^{3/2}}$$

$$=\frac{2}{3}\ln\left(2^{3/2}+3\right)$$

67. (a) $x\sqrt{1-x^2}dy+\left(y\sqrt{1-x^2}-x\cos^{-1}x\right)dx=0$

$$\Rightarrow\frac{dy}{dx}+\frac{y}{x}=\frac{\cos^{-1}x}{\sqrt{1-x^2}}$$

it is L.D.E.

$\therefore$ I.F. $=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$

$\therefore$ solution is $y\cdot x=\int\frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx+C$

$y\cdot x=I_1+C$ ...(1)

$$I_1=\int\frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$$

Let $\cos^{-1}x=t\Rightarrow x=\cos t$

$$-\frac{1}{\sqrt{1-x^2}}dx=dt$$

$I_1=\int -t\cdot\cot dt$

$=-(t\sin t+\cos t)$

$I_1=-\left(\sqrt{1-x^2}\cdot\cos^{-1}x+x\right)$ ...(2)

$\therefore y\cdot x=-\left(\sqrt{1-x^2}\cdot\cos^{-1}x+x\right)+C$

$\because\lim_{x\to1^-}y(x)=1$

$\therefore 1=-(0+1)+C$

$C=2$

$\therefore y\cdot x=-\sqrt{1-x^2}\cdot\cos^{-1}x-x+2$

(put $x=\frac{1}{2}$)

$$y\left(\frac{1}{2}\right)\times\frac{1}{2}=-\frac{\sqrt{3}}{2}\times\frac{\pi}{3}-\frac{1}{2}+2$$

$$\Rightarrow y\left(\frac{1}{2}\right)=3-\frac{\pi}{\sqrt{3}}$$

68. (a) $f^2(x) = f(f(x)) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = \frac{-1}{x}$

$f^3(x) = f(f^2(x)) = f\left(\frac{-1}{x}\right) = \frac{1+x}{1-x}$

$f^4(x) = f(f^3(x)) = f\left(f\left(\frac{-1}{x}\right)\right) = x$

$f^5(x) = f(f^4(x)) = f(x)$

$f^6(x) = f(f^5(x)) = f(f(x)) = \frac{-1}{x}$

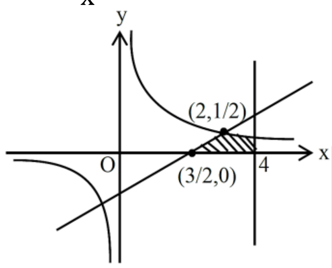
.

.

.

$f^{26}(x) = \frac{-1}{x}$

$g(x) = \frac{1}{x}$



$x - 3/2 = 1/x \Rightarrow 2x^2 - 3x - 2 = 0$
$\Rightarrow x = -1/2, 2$

Req. area $= \int_2^4 \frac{1}{x}dx + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8} + (\ell n x)_2^4 = \frac{1}{8} + \ell n 2$

69. (d) $\lim_{x\to\frac{\pi}{2}} \frac{b\left(1-\cos\left(\frac{\pi}{2}-x\right)\right)}{4\left(\frac{\pi}{2}-x\right)^2} = \frac{1}{3} \Rightarrow b = \frac{8}{3}$

$\int_0^2 |x^2 + 2x - 3|dx = \int_0^2 |(x+3)(x-1)|dx$

put $x - 1 = t$

$\int_{-1}^1 |(t+4)t|dt = \int_{-1}^0 (-t^2 - 4t)dt + \int_0^1 (t^2 + 4t)dt$

$\left(\frac{-t^3}{3} - 2t^2\right)_{-1}^0 + \left(\frac{t^3}{3} + 2t^2\right)_0^1 = 4$

70. (b) Given ellipse is vertical

$\therefore f(3a+15) > f(a^2+7a+3)$

$\because f(x)$ is decreasing $\forall x \in R$

$\Rightarrow 3a + 15 < a^2 + 7a + 3$

$\Rightarrow a^2 + 4a - 12 > 0$

$\Rightarrow (a+2)^2 > 16$

$\Rightarrow |(a+2)| > 4$

$\Rightarrow a > 2$ or $a < -6$

$\Rightarrow a \in R - [-6, 2]$

$\therefore \alpha = -6, \beta = 2$

$\therefore \alpha^2 + \beta^2 = 36 + 4 = 40$

71. (2) $\log_{(x+1)}(2x+3)(x+1) + \log_{(2x+3)}(x+1)^2 = 4$

$1 + \log_{(x+1)}(2x+3) + 2\log_{(2x+3)}(x+1) = 4$

$\log_{x+1}(2x+3) = t\ t + \frac{2}{t} = 3$
$t^2 - 3t + 2 = 0 \Rightarrow t = 1,2$
for $t = 1 \Rightarrow \log_{(x+1)}(2x+3) = 1 \Rightarrow 2x + 3 = x + 1$
$\Rightarrow x = -2$ (rejected)
for $t = 2 \Rightarrow 2x + 3 = x^2 + 2x + 1 \Rightarrow x^2 = 2$
$\Rightarrow x = \pm\sqrt{2}$
rejecting $x = -\sqrt{2}$, we get $x = \sqrt{2}$
$\therefore$ sum of squares of all the roots $= 2$

72. (12) $\int_{\pi/6}^{\pi/4}\left(\left(\cot\left(x - \frac{\pi}{3}\right)\cot\left(x + \frac{\pi}{3}\right) + 1\right)dx\right.$

using $\cot(A - B) = \frac{\cot B\cot A + 1}{\cot B - \cot A}$

$\Rightarrow \int_{\pi/6}^{\pi/4} -\frac{1}{\sqrt{3}}\left[\cot\left(x - \frac{\pi}{3}\right) - \cot\left(x + \frac{\pi}{3}\right)\right]dx$

$\Rightarrow \frac{-1}{\sqrt{3}}\left[\ln\left|\frac{\sin\left(x - \frac{\pi}{3}\right)}{\sin\left(x + \frac{\pi}{3}\right)}\right|\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= \frac{-1}{\sqrt{3}}\left[\ln\left|\left(\frac{\sin 15^\circ}{\sin 105^\circ}\right)\right| - \ln\left|\frac{\sin 30^\circ}{\sin 90^\circ}\right|\right]$

$= \frac{-1}{\sqrt{3}}\left[\ln(\tan 15^\circ) - \ln\left(\frac{1}{2}\right)\right.$

$= \frac{-1}{\sqrt{3}}[\ln(2 - \sqrt{3}) + \ln 2]$

$\Rightarrow \frac{-1}{\sqrt{3}}\ln(4 - 2\sqrt{3}) = \frac{-2}{\sqrt{3}}\ln(\sqrt{3} - 1)$

$\therefore \alpha = \frac{-2}{\sqrt{3}} \therefore 9\alpha^2 = 12$

73. (4) $L_2 : \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2} = t$

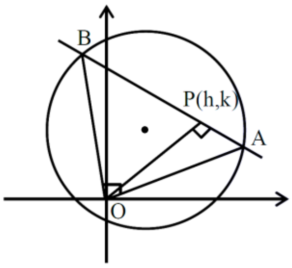
$L_5 : \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = s$

vector $\perp^{ar}$ to L_2 & L_3

$\Rightarrow \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = \hat{\imath}(-2) - \hat{\jmath}(-3) + \hat{k}(-2)$

$\Rightarrow -2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$
$\Rightarrow -(2\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$
Now, $L_1 : \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = \ell$
intersection of L_1 & L_2
$\Rightarrow t + 3 = 2\ell, 2t - 1 = -3\ell, 2t + 4 = 2\ell$
$t + 3 = 2t + 4$
$t = -1$
$\Rightarrow P \equiv (2, -3, 2)$
A point on L_3 is $Q(2s + 3, 2s + 3, s + 2)$
$\Rightarrow PQ^2 = (2s + 1)^2 + (2s + 6)^2 + s^2 = 17$
$\Rightarrow 9s^2 + 28s + 20 = 0$
$\Rightarrow (9s + 10)(s + 2) = 0 \Rightarrow s = -2$
$\therefore Q \equiv (-1, -1, 0) \equiv (a, b, c) \Rightarrow (a + b + c)^2 = 4$

74. (18)



$C: x^2 + y^2 - 6x - 8y - 11 = 0$

E^{qn} of chord AB

$$y - k = -\frac{h}{k}(x - h)$$

$hx + ky = h^2 + k^2 \qquad \ldots(2)$

Homogenizing the eq ^{n} of the circle

$$x^2 + y^2 - 6x\left(\frac{hx + ky}{h^2 + k^2}\right) - 8y\left(\frac{hx + ky}{h^2 + k^2}\right) - 11\left(\frac{hx + ky}{h^2 + k^2}\right)^2 = 0$$

coef. of x^2 + coeff of $y^2 = 0$

$$1 - \frac{6h}{h^2 + k^2} - 11 \cdot \frac{h^2}{(h^2 + k^2)^2} + 1 - \frac{8k}{h^2 + k^2} - 11 \cdot \frac{k^2}{(h^2 + k^2)^2} = 0$$

$2(h^2 + k^2) - 6h - 8k - 11 = 0$

$$x^2 + y^2 - 3x - 4y - \frac{11}{2} = 0$$

$\therefore \alpha + \beta + 2\gamma = 3 + 4 + 11 = 18$

75. (395) $\log_2 f(x) = \log_2\left(\frac{2}{1-\frac{1}{3}}\right) \cdot \log_3\left(1 + \frac{f(x)}{f\left(\frac{1}{x}\right)}\right)$

$$\log_2 f(x) = \log_2 3 . \log_3\left(1 + \frac{f(x)}{f\left(\frac{1}{x}\right)}\right)$$

$$\Rightarrow f(x) = 1 + \frac{f(x)}{f\left(\frac{1}{x}\right)}$$

$$\Rightarrow f(x) f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$$

$\Rightarrow f(x) = 1 \pm x^n \quad \Rightarrow f(6) = 37$

$\Rightarrow 1 \pm 6^n = 37 \quad \Rightarrow 6^n = 36$

$\Rightarrow n = 2$

$\therefore f(x) = 1 + x^2$

$$\sum_{n=1}^{10}(1 + n^2) = 10 + \frac{10 \cdot 11 \cdot 21}{6} = 395$$