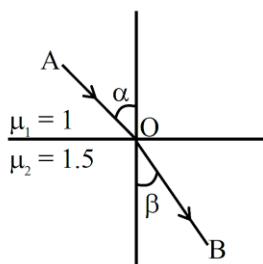


PHYSICS

SECTION-A

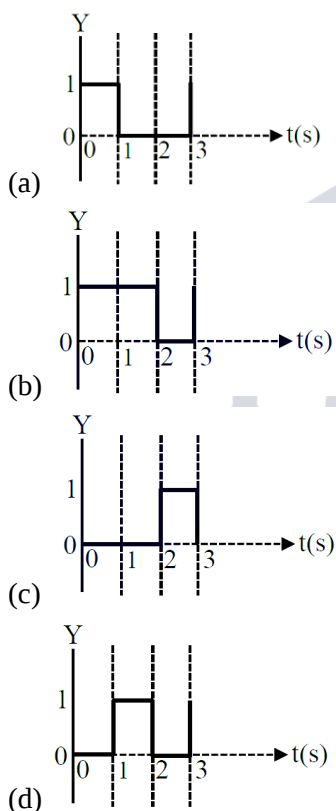
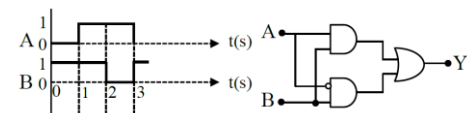
1. A new unit (α) of length is chosen such that it is equal to the speed of light in vacuum. What is the distance between Venus and Earth in terms of α units if light takes 6 min. 40 s to cover this distance?
 (a) 200α (b) 400α
 (c) 300α (d) 500α
2. Consider the equation $H = \frac{x^p \epsilon^q E^r}{t^s}$
 Where H = magnetic field; E = electric field; ϵ = permittivity, x = distance, t = time. The values of p, q, r and s respectively are :
 (a) 1,1,1,1 (b) -1,1,2,1
 (c) 1, -1, -2,1 (d) -1, -2, -2,1
3. A car moving with a speed of 54 km/h takes a turn of radius 20 m . A simple pendulum is suspended from the ceiling of the car. Determine the angle made by the string of the pendulum with the vertical during the turning. (Take $g = 10 \text{ m/s}^2$)
 (a) $\tan^{-1}(0.5)$ (b) $\tan^{-1}(0.75)$
 (c) $\tan^{-1}(1.125)$ (d) $\tan^{-1}(0.25)$
4. A gas balloon is going up with a constant velocity of 10 m/s. When this balloon reached a height of 75 m , a stone is dropped from it and balloon keeps moving up with the same velocity. The height of the balloon when the stone hits the ground is ___ m . (Take $g = 10 \text{ m/s}^2$)
 (a) 85 (b) 150
 (c) 129 (d) 125
5. A thin biconvex lens is prepared from the glass ($\mu = 1.5$) both curved surfaces of which have equal radii of 20 cm each. Left side surface of the lens is silvered from outside to make it reflect. To have the position of image and object at the same place, the object should be placed, from the lens at a distance of ___ cm.
 (a) 10 (b) 12.5
 (c) 13 (d) 13.5
6. Two identical bodies projected with the same speed at two different angles cover the same horizontal range R . If the time of flight of these bodies are 5 s and 10 s , respectively, then the value of R is ___ m. (Take $g = 10 \text{ m/s}^2$)
 (a) 250 (b) 25
 (c) 500 (d) 125
7. A solid cylinder having radius R and length L is slipping on a rough horizontal plane. At time $t = 0$ the cylinder has a translational velocity $v_0 = 49 \text{ m/s}$, perpendicular to its axis and a rotational velocity $v_0/4R$ about the centre. The time taken by the cylinder to start rolling is ___ seconds. (coefficient of kinetic friction $\mu_k = 0.25$ and $g = 9.8 \text{ m/s}^2$)
 (a) 15 (b) 5
 (c) 10 (d) 7.5
8. A liquid of density 600 kg/m^3 flowing steadily in a tube of varying cross-section. The cross-section at a point A is 1.0 cm^2 and that at B is 20 mm^2 . Both the points A and B are in same horizontal plane, the speed of the liquid at A is 10 cm/s . The difference in pressures at A and B points is ___ Pa.
 (a) 18 (b) 144
 (c) 36 (d) 72
9. A spherical liquid drop of radius R acquires the terminal velocity v_1 when falls through a gas of viscosity η . Now the drop is broken into 64 identical droplets and each droplet acquires terminal velocity v_2 falling through the same gas. The ratio of terminal velocities v_1/v_2 is ___ .
 (a) 4 (b) 0.25
 (c) 32 (d) 16

10. One mole of diatomic gas having rotational modes only is kept in a cylinder with a piston system. The cross-section area of the cylinder is 4 cm^2 . The gas is heated slowly to raise the temperature by 1.2°C during which the piston moves by 25 mm . The amount of heat supplied to the gas is ____ J. (Atmospheric pressure = 100 kPa , $R = 8.3 \text{ J/mol} \cdot \text{K}$) (Neglect mass of the piston)
- (a) 24.8 (b) 25
(c) 15.04 (d) 29.98
11. Initial pressure and volume of a monoatomic ideal gas are P and V . The change in internal energy of this gas in adiabatic expansion to volume $V_{\text{final}} = 27V$ is ____ J.
- (a) $-2PV(3\sqrt{3} - 1)$ (b) $\frac{4}{3}PV$
(c) $-\frac{4}{3}PV$ (d) $\frac{3}{4}PV$
12. The frequency of oscillation of a mass m suspended by spring is ν_1 . If the length of the spring is cut to half, the same mass oscillates with frequency ν_2 . The value of $\frac{\nu_2}{\nu_1}$ is ____.
- (a) 1 (b) 2
(c) $\sqrt{2}$ (d) $\sqrt{3}$
13. A monochromatic source of light operating at 15 kW emits 2.5×10^{22} photons/s. The region of an electromagnetic spectrum to which the emitted electromagnetic radiation belongs to ____.
- (Take $h = 6.6 \times 10^{-34} \text{ J}\cdot\text{s}$ and $c = 3 \times 10^8 \text{ m/s}$).
- (a) Microwave (b) Infrared
(c) Visible (d) Ultraviolet
14. A current carrying circular loop of radius 2 cm with unit normal $\hat{n} = \frac{\hat{k} + \hat{i}}{\sqrt{2}}$ is placed in a magnetic field, $\vec{B} = B_0(3\hat{i} + 2\hat{k})$. If $B_0 = 4 \times 10^{-3} \text{ T}$ and current $I = 100\sqrt{2} \text{ A}$, the torque experienced by the loop is ____ $\text{Wb}\cdot\text{A}$. ($\pi = 3.14$)
- (a) $16 \times 10^{-5}\hat{k}$ (b) $5024 \times 10^{-7}\hat{k}$
(c) $5024 \times 10^{-7}\hat{i}$ (d) $5024 \times 10^{-7}\hat{j}$
15. A 30 cm long solenoid has 10 turns per cm and area of 5 cm^2 . The current through the solenoid coil varies from 2 A to 4 A in 3.14 s . The e.m.f. induced in the coil is $\alpha \times 10^{-5} \text{ V}$. The value α is ____.
- (a) 60 (b) 12
(c) 120 (d) 34
16. Two point charges $q_1 = 3\mu\text{C}$ and $q_2 = -4\mu\text{C}$ are placed at points $(2\hat{i} + 3\hat{j} + 3\hat{k})$ and $(\hat{i} + \hat{j} + \hat{k})$ respectively. Force on charge q_2 is ____ N. (Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ SI Units}$)
- (a) $(12\hat{i} + 24\hat{j} + 24\hat{k}) \times 10^{-3}$
(b) $(4\hat{i} + 8\hat{j} + 8\hat{k}) \times 10^{-3}$
(c) $(3\hat{i} + 6\hat{j} + 6\hat{k}) \times 10^{-3}$
(d) $(-4\hat{i} - 8\hat{j} - 8\hat{k}) \times 10^{-3}$
17. Light ray incident along a vector $\vec{AO}(\vec{AO} = 2\hat{i} - 3\hat{j})$ emerges out along vector $\vec{OB}(\vec{OB} = C\hat{i} - 4\hat{j})$ as shown in the figure below. The value of C is ____.



- (a) 1.6 (b) 0.16
(c) 11.6 (d) 16

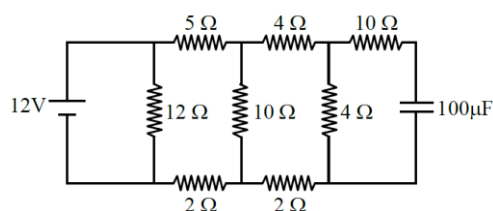
18. K_1 and K_2 be the maximum kinetic energies of photoelectrons emitted from a surface of a given material for the light of wavelength λ_1 and λ_2 , respectively. If $\lambda_1 = 2\lambda_2$ then the work function of material is given by :
- (a) $K_2 + 2K_1$ (b) $2K_2 - K_1$
 (c) $K_1 - 2K_2$ (d) $K_2 - 2K_1$
19. Two radioactive substances A and B of mass numbers 200 and 212 respectively, shows spontaneous α -decay with same Q value of 1 MeV. The ratio of energies of α -rays produced by A and B is ____.
- (a) $\frac{2548}{2650}$ (b) $\frac{2706}{2646}$
 (c) $\frac{2597}{2600}$ (d) $\frac{2862}{2499}$
20. The output Y for the given inputs A and B to the circuit is :



SECTION-B

21. A parallel plate capacitor is having separation between plates 0.885 mm. It has a capacitance of $1\mu\text{F}$ when the space between the plates is filled with an insulating material of resistivity $1 \times 10^{13}\Omega\text{m}$ and resistance $17.7 \times 10^{14}\Omega$. Relative permittivity of the insulating material is $\alpha \times 10^7$. The value of α is ____ . (Take permittivity of free space = $8.85 \times 10^{-12}\text{F/m}$)
22. Some distance star is to be observed by some telescope of diameter of objective lens a, at an angular resolution of 3.0×10^{-7} radian. If the wavelength of light from the star reaching the telescope is 500 nm, the minimum diameter of the objective lens of the telescope is ____ cm. (Nearest integer)
23. A 5 mg particle carrying a charge of $5\pi \times 10^{-6}\text{C}$ is moving with velocity of $(3\hat{i} + 2\hat{k}) \times 10^{-2}\text{m/s}$ in a region having magnetic field $\vec{B} = 0.1\hat{k}\text{Wb/m}^2$. It moves a distance of α meter along $\hat{k}$ when it completes 5 revolutions. The value of α is ____ .

24. The stored charge in the capacitor in steady state of the following circuit is ____ μC .



25. Two masses of 3.4 kg and 2.5 kg are accelerated from an initial speed of 5 m/s and 12 m/s, respectively. The distances traversed by the masses in the 5th second are 104 m and 129 m, respectively. The ratio of their momenta after 10 s is $\frac{x}{8}$. The value of x is ____.

CHEMISTRY

SECTION-A

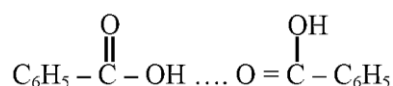
26. Match List-I with List-II.

List-I		List-II	
Mass of substance		Number of atoms	
(A)	1.8mg water	(I)	$2 \times 10^{-4} \times N_A$
(B)	9.8mg sulphuric acid	(II)	$1.5 \times 10^{-4} \times N_A$
(C)	1.8 mg carbon	(III)	$3 \times 10^{-4} \times N_A$
(D)	5.85 mg salt (NaCl)	(IV)	$7 \times 10^{-4} \times N_A$

Choose the correct answer from the options given below:

- (a) A-IV, B-III, C-I, D-II
 (b) A-III, B-II, C-IV, D-I
 (c) A-III, B-IV, C-II, D-I
 (d) A-III, B-IV, C-I, D-II
27. Given below are two statements :
- Given :** Molar mass of C, H, O, Cl are 12, 1, 16 and 35.5 g mol^{-1} , respectively.
- Statement I :** In 30%(w/w) solution of methanol in CCl_4 (at T K), the mole fraction of CCl_4 is equal to 0.33.
- Statement II :** Mixture of methanol and CCl_4 shows positive deviation from Raoult's law.
- In the light of the above statements, choose the correct answer from the option given below :**
- (a) Both Statement I and Statement II are true
 (b) Both Statement I and Statement II are false
 (c) Statement I is true but Statement II are false
 (d) Statement I is false but Statement II are true
28. Bromine trifluoride autoionizes to form BrF_2^+ and BrF_4^- . The shapes of the cation and anion are respectively ____, and ____.
- (a) bent, square planar
 (b) linear, square planar
 (c) bent, see-saw
 (d) linear, tetrahedral
29. Which of the following are not correct?
- (A) For water, magnitude of K_b is more than the magnitude of K_f .
 (B) The elevation in boiling point of water when a non-volatile solute is added to it is larger in magnitude than its depression in freezing point.
 (C) Osmotic pressure measurement is preferred over any other colligative property to determine molar mass of proteins and polymers.

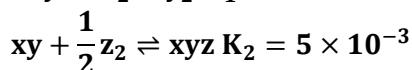
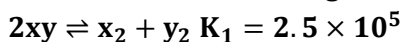
(D) The dimerized form of benzoic acid in benzene is



Choose the correct answer from the options given below :

- (a) A and B only (b) A and D only
(c) A, B and D only (d) A, C and D only

30. Consider the following reactions in which all the reactants and products are present in gaseous state



The value of K_3 for the equilibrium $\frac{1}{2}x_2 + \frac{1}{2}y_2 + \frac{1}{2}z_2 \rightleftharpoons xyz$ is :

- (a) 2.5×10^{-3} (b) 2.5×10^3
(c) 1.0×10^{-5} (d) 5×10^{-3}

31. Given at 298 K: $E_{\text{Fe}^{2+}/\text{Fe}}^\ominus = X$ Volt

$$E_{\text{Fe}^{3+}/\text{Fe}}^\ominus = Y \text{ Volt}$$

The $E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^\ominus$ in Volt at 298 K is given by :

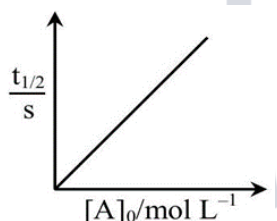
- (a) $2X - 3Y$ (b) $3Y - 2X$
(c) $3Y + 2X$ (d) $Y + X$

32. Given below are two statements :

$$R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1} \text{ and } 1 \text{ cal} = 4.2 \text{ J}$$

Statement I : When $E_a = 12.6 \text{ kcal/mol}$, the room temperature rate constant is doubled by a 10°C increase in temperature (298 K to 308 K)

Statement II : For a first order reaction $A \rightarrow B$,



Here $[A]_0$ is the initial concentration of A and $t_{1/2}$ is half-life of reaction.

In the light of the above statements, choose the correct answer from the option given below :

- (a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false
(c) Statement I is true but Statement II are false
(d) Statement I is false but Statement II are true

33. Match List-I with List-II.

List-I Electronic configuration of neutral atom (where $n = 2$)		List-II 1 st Ionization Energy (kJ mol^{-1})	
(A)	ns^2	(I)	2080
(B)	ns^2np^1	(II)	899
(C)	ns^2np^3	(III)	800
(D)	ns^2np^6	(IV)	1402

Choose the correct answer from the options given below :

- (a) A-II, B-III, C-IV, D-I
(b) A-IV, B-III, C-II, D-I
(c) A-III, B-II, C-IV, D-I
(d) A-III, B-II, C-I, D-IV

34. Find the correct statements related to group 15 hydrides.

- (A) Reducing nature increases from NH_3 to BiH_3
 (B) Tendency to donate lone pair of electrons decreases from NH_3 to BiH_3
 (C) The stability of hydrides decreases from NH_3 to BiH_3
 (D) HEH bond angle decreases from NH_3 to SbH_3 (E = Elements of group 15)

Choose the correct answer from the options given below :

- (a) A and B only
 (b) B and C only
 (c) A, B, C and D
 (d) A, C and D only

35. Given below are two statements :

Statement I : The number of pairs among $[\text{Ti}^{4+}, \text{V}^{2+}]$, $[\text{V}^{2+}, \text{Mn}^{2+}]$, $[\text{Mn}^{2+}, \text{Fe}^{3+}]$ and $[\text{V}^{2+}, \text{Cr}^{2+}]$ in which both ions are coloured is 3 .

Statement II : The number pairs among $[\text{La}^{3+}, \text{Yb}^{2+}]$, $[\text{Lu}^{3+}, \text{Ce}^{4+}]$ and $[\text{Ac}^{3+}, \text{Lr}^{3+}]$ ions in which both are diamagnetic is 3 .

In the light of the above statements, choose the correct answer from the option given below :

- (a) Both Statement I and Statement II are correct
 (b) Both Statement I and Statement II are incorrect
 (c) Statement I is correct but Statement II are incorrect
 (d) Statement I is incorrect but Statement II are correct

36. Given below are two statements for catalytic properties of transition metals :

Statement I : First row transition metals which act as catalyst utilise their 3d electrons only for formation of bonds between reactant molecules and atoms on the surface of catalyst.

Statement II : There is increase in the concentration of reactants on the surface of catalyst which strengthens the bonds in reacting molecules.

In the light of the above statements, choose the correct answer from the option given below :

- (a) Both Statement I and Statement II are correct
 (b) Both Statement I and Statement II are incorrect
 (c) Statement I is correct but Statement II are incorrect
 (d) Statement I is incorrect but Statement II are correct

37. Given below are two statements :

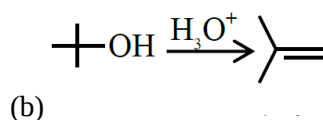
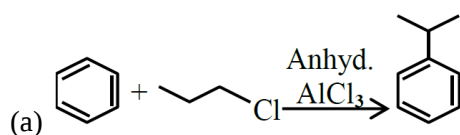
Statement I : Vapors of the liquid with higher boiling point condense before vapours of the liquid with lower boiling points in fractional distillation.

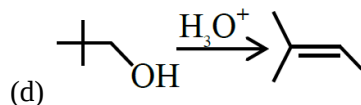
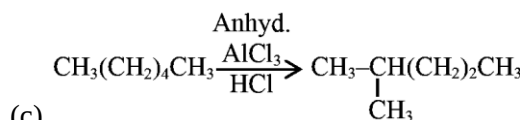
Statement II : The vapors rising up in the fractionating column become richer in high boiling component of the mixture.

In the light of the above statements, choose the correct answer from the option given below :

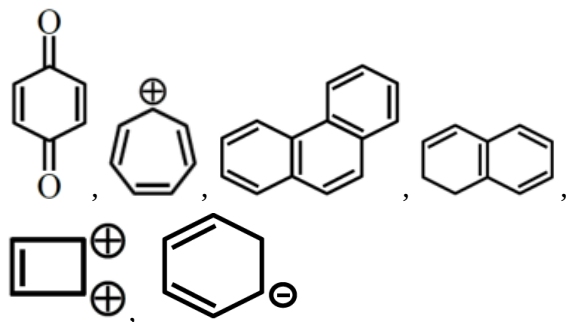
- (a) Both Statement I and Statement II are true
 (b) Both Statement I and Statement II are false
 (c) Statement I is true but Statement II are false
 (d) Statement I is false but Statement II are true

38. The major product of which of the following reaction is not obtained by rearrangement reaction?





39. The total number of aromatic compounds/species from the following is



- (a) 6 (b) 4
(c) 3 (d) 5

40. n-Butane on monochlorination under photochemical conditions gives an optically active compound "P". "P" on further chlorination gives dichlorvos compounds.

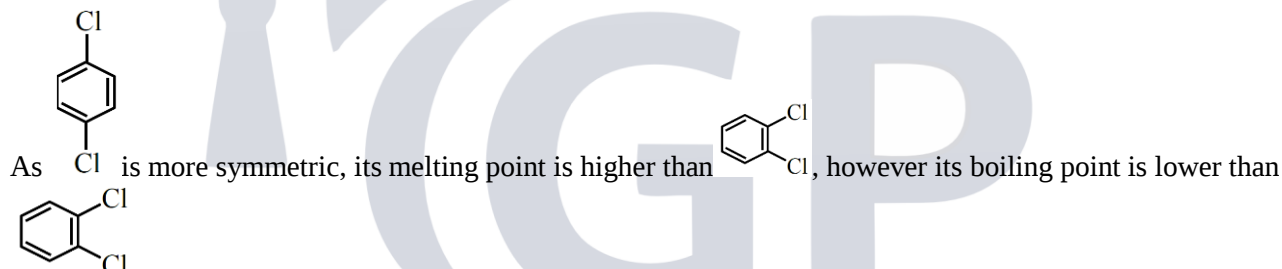
The number of dichlorvos compounds obtained (ignore stereoisomers) is :

- (a) 3 (b) 4
(c) 5 (d) 6

41. Given below are two statements :

Statement-I : Due to increase in van der Waals forces, the order of boiling points is $\text{CH}_3\text{CH}_2\text{CH}_2\text{I} > \text{CH}_3\text{CH}_2\text{I} > \text{CH}_3\text{I}$.

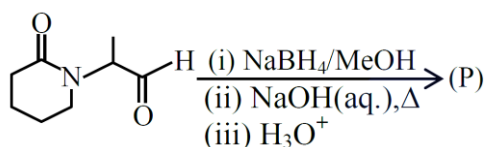
Statement-II :



In the light of the above statements, choose the correct answer from the options given below :

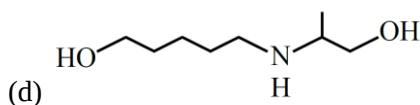
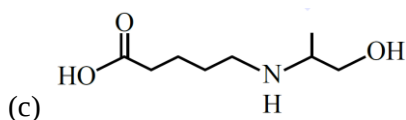
- (a) Both Statement-I and Statement-II are true.
(b) Both Statement-I and Statement-II are false.
(c) Statement-I is true but Statement-II is false.
(d) Statement-I is false but Statement-II is true.

42. Consider the following reaction.



The major product (P) formed is :

- (a)
(b)



43. Which statements are True?

- (A) In Hoffmann bromamide degradation, 4 moles of NaOH and 2 moles of Br₂ are consumed per mole of an amide
 (B) Hoffmann bromamide reaction is not given by alkyl amides.
 (C) Primary amines can be synthesized by Hoffmann bromamide degradation.
 (D) Secondary amide on reaction with Br₂ and NaOH will give secondary amine.
 (E) The by-products of Hoffmann degradation are Na₂CO₃, NaBr and H₂O.

Choose the correct answer from the options given below :

- (a) A, C and E only (b) B, C and D only
 (c) C and E only (d) C, D and E only

44. The incorrect statement from the following with respect to carbohydrates is :

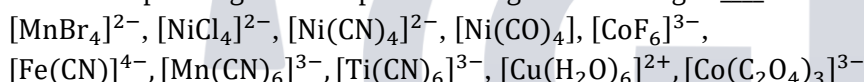
- (a) All monosaccharides are reducing sugars.
 (b) The monosaccharide units obtained from hydrolysis of oligosaccharides are always the same.
 (c) Starch and cellulose are typical examples of polysaccharides, which are very high molecular weight compounds of more than ten monosaccharide units.
 (d) Open chain and cyclic structures co-exist at equilibrium that are responsible for certain properties as in the case of D-(+)-glucose.

45. Which of the following amino acid will give violet coloured complex with neutral ferric chloride solution?

- (a) Threonine (b) Serine
 (c) Tyrosine (d) Cysteine

SECTION-B

46. Number of paramagnetic complexes among the following is ____ .



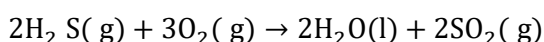
47. 'x' is the product which is obtained from benzene by reacting it with carbon monoxide and hydrogen chloride in the presence of cuprous chloride. 'y' is the major product obtained from the benzene by reacting it with ethanoyl chloride in the presence of anhydrous AlCl₃. Product (major) obtained by heating x and y in the presence of alkali is z. Total number of π (pi) electrons in z is ____ .

48. Consider two radiations of wavelengths :

1. λ₁ = 2000 Å
 2. λ₂ = 6000 Å

The ratio of the energies of these two radiations ($\frac{E_1}{E_2}$) is ____ . (Nearest integer)

49. Consider the reaction :



The magnitude of enthalpy change for the reaction in kJmol⁻¹ is ____ . (Nearest integer)

Given : Δ_fH[⊖](H₂S) = -20.1 kJ mol⁻¹

Δ_fH[⊖](H₂O) = -286.0 kJ mol⁻¹

Δ_fH[⊖](SO₂) = -297.0 kJ mol⁻¹

50. Solid carbon, CaO and CaCO₃ are mixed and allowed to attain equilibrium at T K .



The partial pressure of CO is ____ × 10⁻¹ atm.

MATHEMATICS

SECTION-A

51. Consider the relation R on the set $\{-2, -1, 0, 1, 2\}$ defined by $(a, b) \in R$ if and only if $1 + ab > 0$. Then among the statements :
- (I) The number of elements in R is 17
 (II) R is an equivalence relation
- (a) Only I is true
 (b) Only II is true
 (c) Both I and II are true
 (d) Neither I nor II is true
52. The number of values of $z \in \mathbb{C}$, satisfying the equations $|z - (4 + 8i)| = \sqrt{10}$ and $|z - (3 + 5i)| + |z - (5 + 11i)| = 4\sqrt{5}$, is :
- (a) 0 (b) 2
 (c) 1 (d) 4
53. If the system of linear equations :
- $$x + y + z = 6,$$
- $$x + 2y + 5z = 10,$$
- $$2x + 3y + \lambda z = \mu.$$
- has infinitely many solutions, then the value of $\lambda + \mu$ equals.
- (a) 12 (b) 16
 (c) 22 (d) 28
54. Let $A = \begin{bmatrix} \alpha & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5\alpha & 0 \\ 0 & 4\alpha & -2\alpha \end{bmatrix} + \text{adj}(A)$. If $\det(B) = 66$, then $\det(\text{adj}(A))$ equals :
- (a) 289 (b) 361
 (c) 441 (d) 529
55. Let $\alpha = 3 + 4 + 8 + 9 + 13 + 14 + \dots$ upto 40 terms. If $(\tan \beta)^{\frac{\alpha}{1020}}$ is a root of the equation $x^2 + x - 2 = 0$, $\beta \in \left(0, \frac{\pi}{2}\right)$, then $\sin^2 \beta + 3\cos^2 \beta$ is equal to :
- (a) 2 (b) $\frac{7}{4}$
 (c) $\frac{5}{2}$ (d) $\frac{3}{2}$
56. A candidate has to go to the examination center to appear in an examination. The candidate uses only one means of transportation for the entire distance out of bus, scooter and car. The probabilities of the candidate going by bus, scooter and car, respectively, are $\frac{2}{5}, \frac{1}{5}$ and $\frac{2}{5}$. The probabilities that the candidate reaches late at the examination center are $\frac{1}{5}, \frac{1}{3}$ and $\frac{1}{4}$ if the candidate uses bus, scooter and car, respectively. Given that the candidate reached late at the examination centre, the probability that the candidate travelled by bus is :
- (a) $\frac{11}{37}$ (b) $\frac{12}{37}$
 (c) $\frac{13}{37}$ (d) $\frac{14}{37}$
57. A set of four observations has mean 1 and variance 13. Another set of six observations has mean 2 and variance 1. Then, the variance of all these 10 observations is equal to :
- (a) 5.96 (b) 6.14
 (c) 6.04 (d) 6.24
58. If $26 \left(\frac{2^3}{3} ({}^{13}C_2) + \frac{2^5}{5} ({}^{12}C_4) + \frac{2^7}{7} ({}^{12}C_6) + \dots + \frac{2^{13}}{13} ({}^{12}C_{12}) \right) = 3^{13} - \alpha$, then α is equal to :
- (a) 45 (b) 48
 (c) 51 (d) 54

59. A person has three different bags and four different books. The number of ways, in which he can put these books in the bags so that no bag is empty, is

- (a) 18 (b) 36
(c) 39 (d) 72

60. If a straight line drawn through the point of intersection of the lines $4x + 3y - 1 = 0$ and $3x + 4y - 1 = 0$, meet the co-ordinate axes at the points P and Q, then the locus of the mid point of PQ is:

- (a) $x + y - 7 = 0$
(b) $x + y - 14xy = 0$
(c) $2x + y + 14xy = 0$
(d) $x + 2y - 14xy = 0$

61. Let O be the vertex of the parabola $y^2 = 4x$ and its chords OP and OQ are perpendicular to each other. If the locus of the mid-point of the line segment PQ is a conic C, then the length of its latus rectum is :

- (a) 1 (b) 2
(c) 4 (d) 8

62. Let $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where inverse trigonometric functions take only the principal values.

Given below are two statements:

Statement I : $\cos(\alpha + \beta) > 0$

Statement II : $\cos(\alpha) < 0$

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

63. For the function $f(x) = e^{\sin x} - |x|$, $x \in \mathbb{R}$, consider the following statements:

Statement I : f is differentiable for all $x \in \mathbb{R}$.

Statement II : f is increasing in $\left(-\pi, -\frac{\pi}{2}\right)$.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

64. Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 10\hat{i} + 2\hat{j} - \hat{k}$ and a vector $\vec{c}$ be such that $2(\vec{a} \times \vec{b}) + 3(\vec{b} \times \vec{c}) = \vec{0}$. If $\vec{a} \cdot \vec{c} = 15$, then $\vec{c} \cdot (\hat{i} + \hat{j} - 3\hat{k})$ is equal to :

- (a) -6 (b) -5
(c) -4 (d) -3

65. Let the foot of perpendicular from the point $(\lambda, 2, 3)$ on the line $\frac{x-4}{1} = \frac{y-9}{2} = \frac{z-5}{1}$ be the point $(1, \mu, 2)$.

Then the distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ and $\frac{x-\lambda}{2} = \frac{y-\mu}{3} = \frac{z+5}{6}$ is equal to

- (a) $\frac{12}{7}$ (b) $\frac{\sqrt{145}}{7}$
(c) $\frac{\sqrt{146}}{7}$ (d) $\frac{\sqrt{143}}{7}$

66. The value of the integral $\int_0^2 \frac{\sqrt{x(x^2+x+1)}}{(\sqrt{x+1})(\sqrt{x^4+x^2+1})} dx$ is equal to :

- (a) $\frac{1}{3} \log_e(3 - 2\sqrt{2})$
(b) $\frac{2}{3} \log_e(4 + \sqrt{2})$
(c) $\frac{2}{3} \log_e(3 + 2\sqrt{2})$
(d) $\frac{1}{3} \log_e(1 + 6\sqrt{2})$

67. Let $y = y(x)$ be the solution of the differential equation $x\sqrt{1-x^2}dy + (y\sqrt{1-x^2} - x\cos^{-1}x)dx = 0$, $x \in (0, 1)$, $\lim_{x \rightarrow 1^-} y(x) = 1$. Then $y\left(\frac{1}{2}\right)$ equals :

- (a) $3 - \frac{\pi}{\sqrt{3}}$ (b) $4 - \sqrt{3}\pi$
(c) $4 - \frac{2\pi}{\sqrt{3}}$ (d) $3 - \frac{\pi}{2\sqrt{3}}$

68. Let $f: (1, \infty) \rightarrow \mathbb{R}$ be function defined as $f(x) = \frac{x-1}{x+1}$. Let $f^{i+1}(x) = f(f^i(x))$, $i = 1, 2, \dots, 25$, where $f^1(x) = f(x)$. If $g(x) + f^{26}(x) = 0$, $x \in (1, \infty)$, then the area of the region bounded by the curves $y = g(x)$, $2y = 2x - 3$, $y = 0$ and $x = 4$ is:

- (a) $\frac{1}{8} + \log_e 2$ (b) $\frac{1}{4} + \log_e 2$
(c) $\frac{5}{6} + 3\log_e 2$ (d) $\frac{5}{6} + \log_e 2$

69. Let $f(x) = \begin{cases} \frac{1}{3} & , x \leq \pi/2 \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & , x > \pi/2 \end{cases}$, If f is continuous at $x = \pi/2$, then the value of $\int_0^{3b-6} |x^2 + 2x - 3| dx$ is :

- (a) 5 (b) 2
(c) 3 (d) 4

70. Let $\frac{x^2}{f(a^2+7a+3)} + \frac{y^2}{f(3a+15)} = 1$ represent an ellipse with major axis along y-axis, where f is a strictly decreasing positive function on $\mathbb{R}$. If the set of all possible values of a is $\mathbb{R} - [\alpha, \beta]$, then $\alpha^2 + \beta^2$ is equal to :

- (a) 28 (b) 40
(c) 61 (d) 24

SECTION-B

71. The sum of squares of all real solution of the equation $\log_{(x+1)}(2x^2 + 5x + 3) = 4 - \log_{(2x+3)}(x^2 + 2x + 1)$ is equal to ____.

72. If $\int_{\pi/6}^{\pi/4} \left(\cot\left(x - \frac{\pi}{3}\right) \cot\left(x + \frac{\pi}{3}\right) + 1 \right) dx = \alpha \log_e(\sqrt{3} - 1)$, then $9\alpha^2$ is equal to

73. Let a line L_1 pass through the origin and be perpendicular to the lines $L_2: \vec{r} = (3+t)\hat{i} + (2t-1)\hat{j} + (2t+4)\hat{k}$ and $L_3: \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$, $t, s \in \mathbb{R}$. If (a, b, c) , $a \in \mathbb{Z}$, is the point on L_3 at a distance of $\sqrt{17}$ from the point of intersection of L_1 and L_2 , then $(a+b+c)^2$ is equal to ____.

74. Consider the circle $C: x^2 + y^2 - 6x - 8y - 11 = 0$. Let a variable chord AB of the circle C subtend a right angle at the origin. If the locus of the foot of the perpendicular drawn from the origin on the chord AB is the circle $x^2 + y^2 - \alpha x - \beta y - \gamma = 0$, then $\alpha + \beta + 2\gamma$ is equal to ____.

75. Let f be a polynomial function such that $\log_2(f(x)) = \left(\log_2\left(2 + \frac{2}{3} + \frac{2}{9} + \dots \infty\right) \right)$.

$\log_3\left(1 + \frac{f(x)}{f(1/x)}\right)$, $x > 0$ and $f(6) = 37$. Then $\sum_{n=1}^{10} f(n)$ is equal to ____.