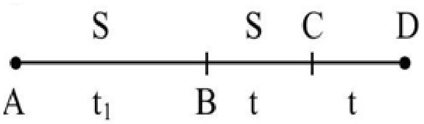
1. (b) $\lambda_{DB} = \frac{h}{p} = \frac{h}{\sqrt{2mk}}$

$\Rightarrow \lambda_{DB} \alpha \frac{1}{\sqrt{m}}$

$\Rightarrow \lambda_a < \lambda_p < \lambda_e$

2. (d)



$BD \Rightarrow S = 9t + 15t = 24t$

$AB \Rightarrow S = 6t_1 = 24t \Rightarrow t_1 = 4t$

$< \text{speed} >= \frac{\text{dist.}}{\text{time}} = \frac{48t}{2t+t_1}$

$= \frac{48t}{2t+4t} \Rightarrow \frac{48t}{6t} \Rightarrow 8 \text{ m/s}$

3. (b)

$E = BC \Rightarrow 60 = B \times 3 \times 10^8$
$\Rightarrow B = 2 \times 10^{-7}$

Also $C = f\lambda$

$\Rightarrow 3 \times 10^8 = f \times 4 \times 10^{-3}$

$\Rightarrow f = \frac{3}{4} \times 10^{11}$

$\Rightarrow \omega = 2\pi f = \frac{3}{4} \times 2\pi \times 10^{11}$

$\Rightarrow \omega = \frac{\pi}{2} \times 10^3 C$

$\Rightarrow$ Electric field $\Rightarrow$ y direction

Propagation $\Rightarrow$ x direction

Magnetic field $\Rightarrow$ z-direction

4. (a)

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-2f} = \frac{1}{-f}$

$\Rightarrow \frac{1}{v} = \frac{-1}{2f} \Rightarrow v = -2f$

$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} \Rightarrow$ Virtual image of Real object.

In statement II, it is not mentioned that object is real or virtual hence Statement II is false.

5. (c)

$= \frac{1240}{1.42} = 875$ nm (Approx)

6. (a)

weight (w) $= \frac{4}{3}\pi\left(\frac{D^3-d^3}{8}\right)\sigma g$

Buoyant force $(F_b) = 1 \times \frac{4}{3}\pi\left(\frac{D^3}{8}\right) \cdot g$

For Just Float $\Rightarrow w = F_b$

$\Rightarrow (D^3 - d^3)\sigma = D^3$

$$\Rightarrow 1 - \frac{d^3}{D^3} = \frac{1}{\sigma}$$

$$\Rightarrow 1 - \frac{1}{\sigma} = \left(\frac{d}{D}\right)^3$$

$$\Rightarrow \left(\frac{\sigma}{\sigma - 1}\right)^{\frac{1}{3}} = \left(\frac{D}{d}\right)$$

7. (a) All capacitor are in parallel combination.

Also effective area is common area only

$\Rightarrow C_{eq} = C_1 + C_2 + C_3$

$$\Rightarrow C_{eq} = \frac{A\varepsilon_0}{3\,d} + \frac{A\varepsilon_0}{3(2\,d)} + \frac{A\varepsilon_0}{3(3\,d)}$$

$$\Rightarrow C_{eq} = \frac{A\varepsilon_0}{3}\left(\frac{11}{6\,d}\right)$$

$$\Rightarrow C_{eq} = \frac{11\,A\varepsilon_0}{18\,d}$$

8. (a)

$$a_{sys} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \frac{g}{8}$$

$$\Rightarrow \frac{m_2}{m_1} = \frac{9}{7}$$

9. (c) Latent heat is specific heat

$$\Rightarrow \frac{ML^2\,T^{-2}}{M} = M^0\,L^2\,T^{-2}$$

10. (c) For Adiabatic process

$$P_iV_i = P_fV_f^{\gamma}$$
$$P_i(5)^{1.5} = P_f(4)^{1.5}$$
$$\frac{P_i}{P_f} = \left(\frac{4}{5}\right)^{\frac{3}{2}} = \frac{4}{5}\cdot\left(\frac{4}{5}\right)^{\frac{1}{2}} \Rightarrow \frac{8}{5\sqrt{5}}$$

11. (d)

$$E = mC^2$$

$$\Rightarrow E = (1\times10^{-3})\times(3\times10^{8})^2\ \mathrm{J}$$
$$\Rightarrow E = (10^{-3})(9\times10^{16})(6.241\times10^{18})\mathrm{eV}$$
$$E = 56.169\times10^{31}\mathrm{eV}$$
$$E \approx 5.6\times10^{26}\mathrm{MeV}$$

12. (a)

$$h = 318.5 \approx \left(\frac{R_e}{20}\right)$$

$$T\cdot E_i = \frac{-GM_em}{R_e}$$

$$T\cdot E_f = \frac{-GM_em}{2(R_e+h)} = \frac{-GM_em}{2\left(R_e+\frac{R_e}{20}\right)}$$

$$\Rightarrow T\cdot E_f = \frac{-10GM_em}{21R_e}$$

Change in total mechanical energy

$$= TE_f - TE_i$$

$$= \frac{GM_em}{Re}\left[1-\frac{10}{21}\right] = \frac{11GM_em}{21Re}$$

13. (b)

14. (c)

$$v = \alpha\sqrt{x}$$

at $x = 0: v = 0$

& at $x = d; v = \alpha\sqrt{d}$

$\mathrm{W.D} = K_f - K_i$

$$W.D = \frac{1}{2}m(\alpha\sqrt{d})^2 - \frac{1}{2}m(0)^2$$

$$\Rightarrow \mathrm{W.D} = \frac{m\alpha^2 d}{2}$$

15. (b)

$G = 200\Omega$

$i_g = 20\mu A$

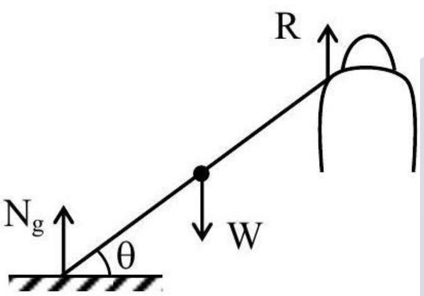
$i = i_g\left(\frac{G}{S} + 1\right)$

$\Rightarrow 20 \times 10^{-3} = 20 \times 10^{-6}\left(\frac{200}{S} + 1\right)$

$\Rightarrow \frac{200}{S} = 999$

$\Rightarrow S \approx 0.2\Omega$

16. (a)



R = net reaction force by shoulder

Balancing torque about pt of contact on ground:

$W\left(\frac{L}{2}\cos\theta\right) = R(L\cos\theta)$

$\Rightarrow R = \frac{W}{2}$

17. (b) $nMSD = (n + 1)VSD$

$\Rightarrow 1VSD = \frac{n}{n + 1}MSD$

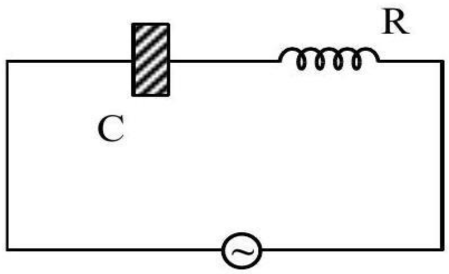
$L \cdot C = 1MSD - 1VSD$

$L \cdot C = m - m\left(\frac{n}{n + 1}\right)$

$L \cdot C = m\left(\frac{n + 1 - n}{n + 1}\right)$

$\Rightarrow L \cdot C = \left(\frac{m}{n + 1}\right)$

18. (a)



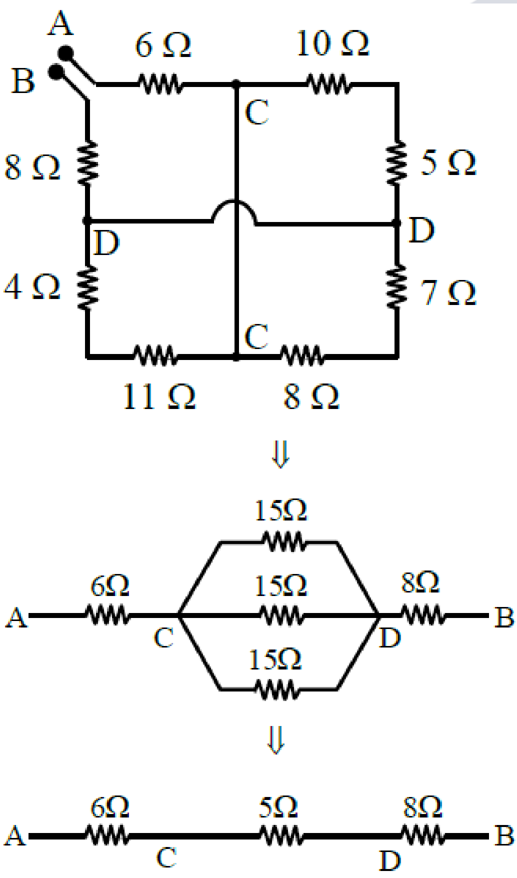
$Z = \sqrt{R^2 + X_C^2} \& X_C = \frac{1}{WC}$

due to dielectric

$C \uparrow \Rightarrow X_C \downarrow \Rightarrow Z \downarrow$

So, current increases & thus bulb will glow more -brighter.

19. (d)



$\Rightarrow R_{eq} = 6\Omega + 5\Omega + 8\Omega = 19\Omega$

20. (a)

$TV^{\gamma-1}$ = constant

$\Rightarrow T(V)^{\frac{3}{2}-1} = T_f(2\,V)^{\frac{3}{2}-1}$

$\Rightarrow TV^{\frac{1}{2}} = T_f(2)^{\frac{1}{2}}(\,V)^{\frac{1}{2}}$

$\Rightarrow T_f = \left(\frac{T}{\sqrt{2}}\right)$

Now, W.D. $= \frac{nR\Delta T}{1-\gamma} = \frac{1\cdot R\left[\frac{T}{\sqrt{2}}-T\right]}{1-\frac{3}{2}}$

$\Rightarrow$ W.D. $= 2RT\left[1-\frac{1}{\sqrt{2}}\right]$

$\Rightarrow$ W.D. $= RT\,[2-\sqrt{2}]$

21. (3) $\cos\theta = \frac{5}{9}$

$\frac{\vec{a}\cdot\vec{b}}{ab} = \frac{5}{9}$

$|\vec{a}+\vec{b}| = \sqrt{2}|\vec{a}-\vec{b}|$

$a^2 + b^2 + 2\vec{a}\cdot\vec{b} = 2a^2 + 2b^2 - 4\vec{a}\cdot\vec{b}$

$6\vec{a}\cdot\vec{b} = a^2 + b^2$

$6 \times \frac{5}{9}ab = a^2 + b^2$

$\frac{10}{3}ab = a^2 + b^2 \;\&\; a = nb$

$\frac{10}{3}nb^2 = n^2\,b^2 + b^2$

$3n^2 - 10n + 3 = 0$

$n = \frac{1}{3}$ and $n = 3$

integer value $n = 3$

22. (36) Potential at centre of half ring

$V = \frac{KQ}{R}$

$V = \frac{K\lambda\pi R}{R}$

$V = K\lambda\pi \Rightarrow V = 9\times10^9\times4\times10^{-9}\pi$

$V = 36\pi$

23. (5)

${}^4He + {}^4He + {}^4He \rightarrow {}^{12}C + Q$

power generated $= \frac{N}{t} Q$

where, N → No. of reaction/sec.

$Q = (3\,m_{He} - m_C)C^2$

$Q = (3 \times 4.0026 - 12)(3 \times 10^8)^2$

$Q = 7.266\text{MeV}\ \frac{N}{t} = \frac{\text{power}}{Q} = \frac{5.808 \times 10^{30}}{7.266 \times 10^6 \times 1.6 \times 10^{-19}}$

$\frac{N}{t} = 5 \times 10^{42}$

rate of conversion of ^{4}He into ${}^{12}C = 15 \times 10^{42}$

Hence, $n = 15$

24. (200)

$I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

$\frac{I_0}{4} = \cos^2\left(\frac{\Delta\phi}{2}\right)$

$\Delta\phi = \frac{2\pi}{3}$

$\frac{2\pi}{\lambda}\left(\frac{yd}{D}\right) = \frac{2\pi}{3}$

$y = \frac{\lambda D}{3\,d} = \frac{600 \times 10^{-9} \times 1}{3 \times 10^{-3}} = 2 \times 10^{-4}\ \text{m}$

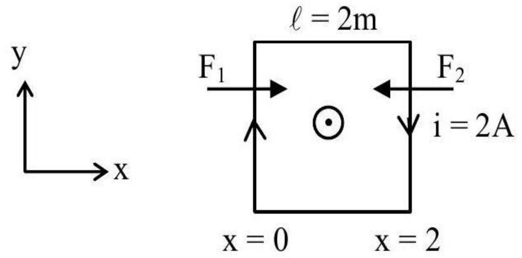
25. (100)

$\tau = FR = I\alpha \Rightarrow 40 \times 0.1 = 0.4\alpha$

$\alpha = 10\text{rad/s}^2$

$W_f = 10 \times 10 = 100\text{rad/s}$

26. (160)



$B(x = 0) = B_0,\ \ B(x = 2) = 9\,B_0$

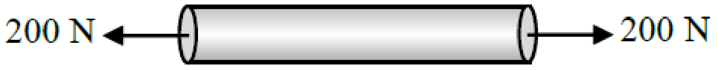
Also, $F = i\ell B$

$\Rightarrow F_1 = i\ell B_0$ & $F_2 = 9i\ell B_0$

$F = F_2 - F_1 = 8i\ell B_0 = 8 \times 2 \times 2 \times 5$

$F = 160\text{ N}$

27. (20)

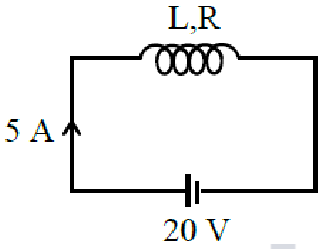


$$\frac{F}{A} = Y\frac{\Delta\ell}{\ell} \Rightarrow \Delta\ell = \frac{F\ell}{AY}$$

$$\Delta\ell = \frac{200 \times 2}{2 \times 10^{-4} \times 10^{11}} = 2 \times 10^{-5} = 20\mu m$$

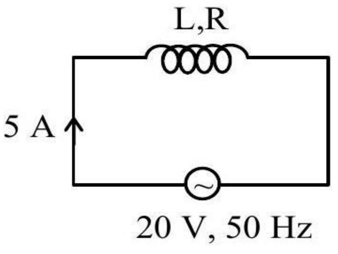
28. (10)

CASE – I



$$i = \frac{20}{R} \Rightarrow R = 4\Omega$$

Case-II:



$$i = \frac{20}{Z}$$

$$4 = \frac{20}{\sqrt{R^2 + X_L^2}} \Rightarrow \sqrt{R^2 + X_L^2} = 5$$

$$R^2 + X_L^2 = 25 \Rightarrow X_L = 3\Omega$$

$$L = \frac{3}{2\pi f} = \frac{1}{2 \times 50} = \frac{1000}{100}\text{ mH}$$

$$L = 10\text{mH}$$

29. (17) $x = 4\text{ m}, V = 2\text{ m/s}, a = 16\text{ m/s}^2$

$|a| = \omega^2 x$

$\Rightarrow 16 = \omega^2(\text{d})$

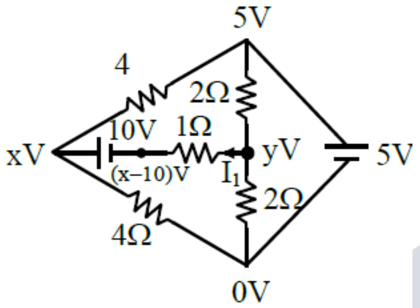
$\omega = 2\text{rad/s}$

$v = \omega\sqrt{A^2 - x^2}$

$$A = \sqrt{\frac{v^2}{\omega^2} + x^2} \Rightarrow A = \sqrt{\frac{4}{4} + 16}$$

$A = \sqrt{17}$ m

30. (25)



$$\frac{y-5}{2} + \frac{y-0}{2} + \frac{y-x+10}{1} = 0$$

$y - 5 + y + 2y - 2x + 20 = 0$

$4y - 2x + 15 = 0 \quad \dots .(i)$

$$\frac{x-5}{4} + \frac{x-0}{4} + \frac{x-10-y}{1} = 0$$

$x - 5 + x + 4x - 40 - 4y = 0$

$6x - 4y - 45 = 0 \,..(i)$

$\underline{-2x + 4y + 15 = 0 \,..(ii)}$

$4x - 30 = 0$

$x = \frac{15}{2} \,\&\, 4y - 15 + 15 = 0$

$y = 0$

$$i = \frac{y-x+10}{1}$$

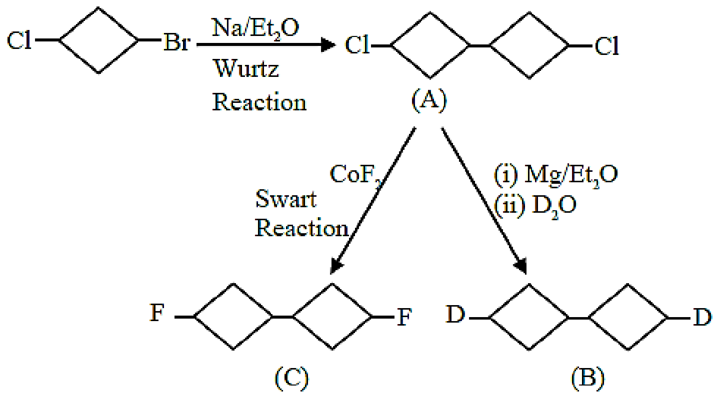
$$i = \frac{0-7.5+10}{1}$$

$$i = 2.5A = \frac{n}{10} = n = 25$$

31. **(c)** A → Weak electrolyte
B → Strong electrolyte

32. **(c)** Organic compounds are purified based on their nature and impruity present in it.

33. **(a)**



34. (a) Order of basic strength is $N(sp^3$, localized lone pair $) > N(sp^2$, localized lone pair $) > N(sp^2$, delocalized lone pair, aromatic)

$\therefore$ Piperidine $>$ Pyridine $>$ Pyrrole

35. (a)

$BF_3 \rightarrow sp^2$

$NO_2^- \rightarrow sp^2$
$H_2O \rightarrow sp^3$
$NO_2 \rightarrow sp^2$
$NH_2^- \rightarrow sp^3$

36. (a) Fluoroapatite $\Rightarrow [3Ca_3(PO_4)_2 \cdot CaF_2]$

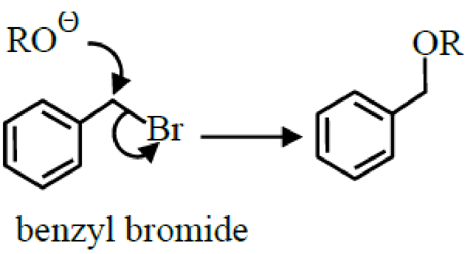
37. (b) (1) Neutral structures are more stable than charged ones. Therefore, I is more stable than II and III.

(2) + ve charge on less electronegative atom is more stable i.e., $C^{\oplus}$ is more stable than $O^{\oplus}$

$\therefore$ Order is I > II > III

38. (d) Oxidation state of an element in a particular compound is defined by the charge acquired by its atom on the basis of electronegativity consideration from other atoms in molecule.

39. (c) The benzyl group acts in much the same way using the π-system of the benzene ring for conjugation with the p-orbital in the transition state.



40. (d) Acidic strength order: -

$B > D > C > A > E$

Correct pKa Order:

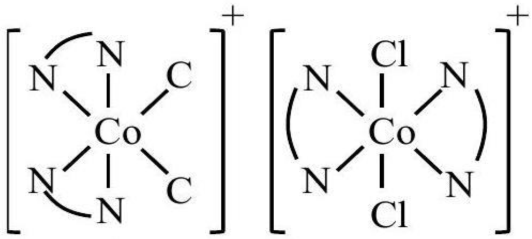
$B < D < C < A < E$

41. **(c)** Oxygen can form $2p\pi - 2p\pi$ multiple bond with itself due to its small size while sulphur cannot form multiple bond with itself as $3p\pi - 3p\pi$ bond will be unstable due to large size of sulphur, but sulphur can form multiple bond with small size atom like C and N.

eg. $S = C = S$

$S = C = N^{-} \leftrightarrow S^{\ominus} - C \equiv N$

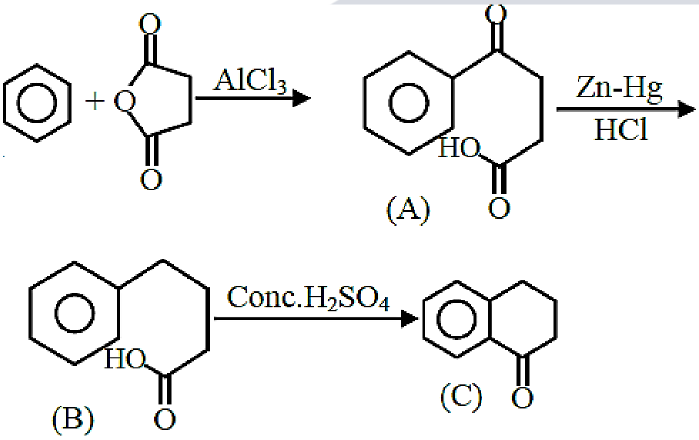
42. **(c)** $[Co(en)_2Cl_2]^{+}$ has octahedral geometry with two geometrical isomers.



cis trans

43. **(c)** Cu(II) is more stable than Cu(I) because hydration energy of Cu^{+2} ion compensate IE_2 of Cu.

44. **(a)**



45. **(d)** Energy level can be determined by comparing $(n + \ell)$ values

(A) $n = 4, \ell = 1 \Rightarrow (n + \ell) = 5$

(B) $n = 4, \ell = 2 \Rightarrow (n + \ell) = 6$

(C) $n = 3, \ell = 1 \Rightarrow (n + \ell) = 4$

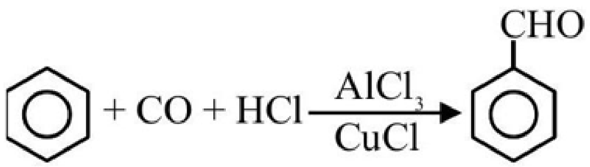
(D) $n = 3, \ell = 2 \Rightarrow (n + \ell) = 5$

(E) $n = 4, \ell = 0 \Rightarrow (n + \ell) = 4$

For same value of $(n + \ell)$, orbital having higher value of n, will have more energy.

(B) > (A) > (D) > (E) > (C)

46. **(c)** This is Gattermann-Koch reaction



47. (c)

(1) Hydration Reaction:

$$CH_3-CH=CH_2+H^+\longrightarrow \underset{\text{(More stable)}}{CH_3-\overset{+}{C}H-CH_3}$$

$$CH_3-\overset{+}{C}H-CH_3+H_2O\longrightarrow (CH_3-\underset{OH}{\underset{|}{C}H}-CH_3)+H^+$$

(2) Hydroboration Oxidation Reaction :

$$3CH_3-CH=CH_2+B_2H_6\xrightarrow{THF}$$

$$2(CH_3CH_2CH_2)_3B$$

$$(CH_3CH_2CH_2)_3B+3H_2O_2\xrightarrow{OH^-}$$

$$\underset{(B)}{3CH_3CH_2CH_2OH}+H_3BO_3$$

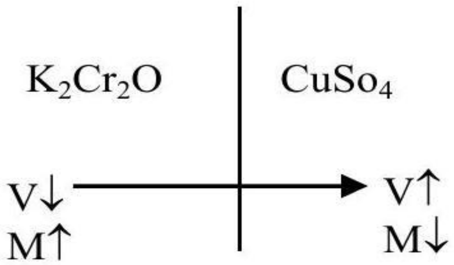
48. (d) $PbS+HNO_3\rightarrow Pb(NO_3)_2+NO+S+H_2O$

Nitrous oxide (N_2O) is not formed during the reaction.

49. (d) $KMnO_4$&$NaOH\rightarrow$ Secondary standard.

Primary standard should not be Hygroscopic.

50. (d) Only solvent Molecules are allowed to pass through the SPM.



51. (82)

$$\begin{array}{ll}(1) & CuSO_4(s)+5H_2O\xrightarrow{x}CuSO_4.5H_2O\\ (2) & CuSO_4.5H_2O+H_2O\xrightarrow{12}CuSO_4(aq)\\ \hline & CuSO_4+H_2O\xrightarrow{-70}CuSO_4(aq)\end{array}$$

from (1) & (2)

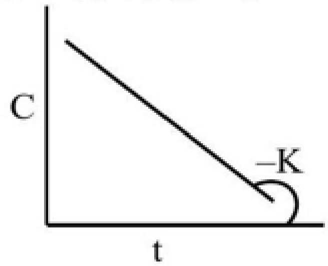
$-70 = x + 12$
$x = -82$

52. (8) $r = K[A]^2|B|$

if conc. are doubled

$r' = K[2A]^2[2B]^1$

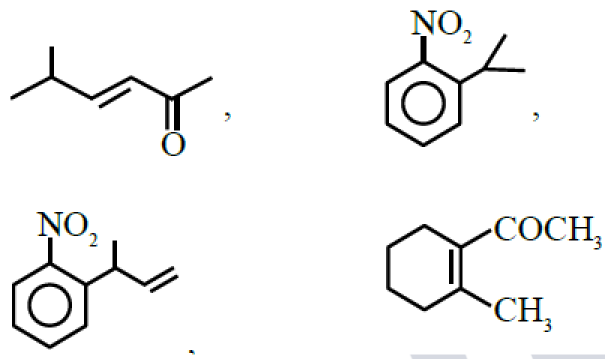
$r' = 8r \Rightarrow x = 8$



$\Rightarrow$ Zero order, $y = 0$

$x + y = 8$

53. (4)



54. (3) Fe, Ni, Ag will be oxidized due to lower S.R.P.

55. (2) $H^+ + OH^- \rightarrow H_2O \Rightarrow x$

$2H^+ + 2OH^- \rightarrow 2H_2O \Rightarrow 2x = y$

$y/x = 2$

56. (164)

$M_{sol^n} = v_{sol^n} \times d_{sol^n}$
$= 500 \times 1.25 = 625$ g

Mass of solute (x) $= 0.2 \times 0.5 \times 159.5$

$= 15.95$

$n_{solute} = 0.1,$

Mass of solvent = Mass of solution - Mass of solute

$= 625 - 15.95$

$= 609.05$

$$m = \frac{0.1}{\frac{609.05}{1000}}$$

$m = 0.164 = 164 \times 10^{-3}$

57. **(4)** One unpaired e^- is present in : $C_2^-; O_2^-; H_2^+; He_2^+$

58. **(c)** Ligands which have two different donor sites but at a time connects with only one donor site to central metal are ambidentate ligands.

Ambidentate ligands are $NO_2^-; SCN^-; CN^-$

59. **(4)** Essential Amino acids are: -

Arginine, Phenylalanine, Histidine, Valine

60. **(b)**

$La^{+3} - [Xe]4f^0$

$Nd^{+3} - [Xe]4f^3$

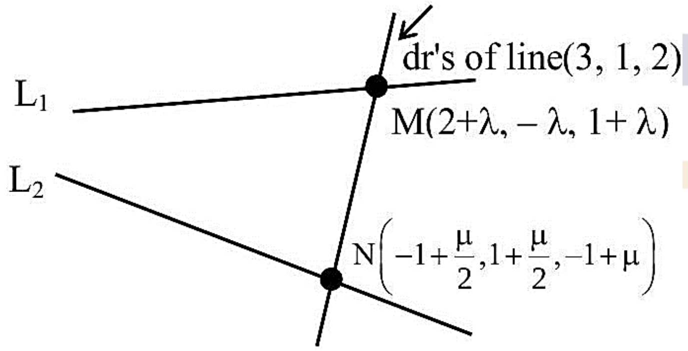
$Sm^{+3} - [Xe]4f^5$

$Eu^{+3} - [Xe]4f^6$

$Lu^{+3} - [Xe]4f^{14}$

La^{+3} and Lu^{+3} do not show any colour because no unpaired electron is present.

61. **(b)**



$$L_1: \frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{1} = \lambda$$

$$L_2: \frac{x+1}{\frac{1}{2}} = \frac{y-1}{\frac{1}{2}} = \frac{z+1}{1} = \mu$$

dr of line MN will be

$< 3+\lambda-\frac{\mu}{2}, -1-\lambda-\frac{\mu}{2}, 2+\lambda-\mu >$ & it will be proportional to $< 3,1,2 >$

$$\therefore \frac{3+\lambda-\frac{\mu}{2}}{3} = \frac{-1-\lambda-\frac{\mu}{2}}{1} = \frac{2+\lambda-\mu}{2}$$

$4\lambda+\mu=-6$ $\quad$ $4+3\lambda=0$

$$\Rightarrow \lambda = -\frac{4}{3} \& \mu = -\frac{2}{3}$$

$\therefore$ Coordinate of M will be $< \left(\frac{2}{3}, \frac{4}{3}, -\frac{1}{3}\right)$

and equation of required line will be.

$$\frac{x-\frac{2}{3}}{3} = \frac{y-\frac{4}{3}}{1} = \frac{z+\frac{1}{3}}{2} = k$$

So any point on this line will be

$$\left(\frac{2}{3}+3k, \frac{4}{3}+k, -\frac{1}{3}+2k\right)$$

$$\because \frac{2}{3}+3k = -\frac{1}{3} \Rightarrow k = -\frac{1}{3}$$

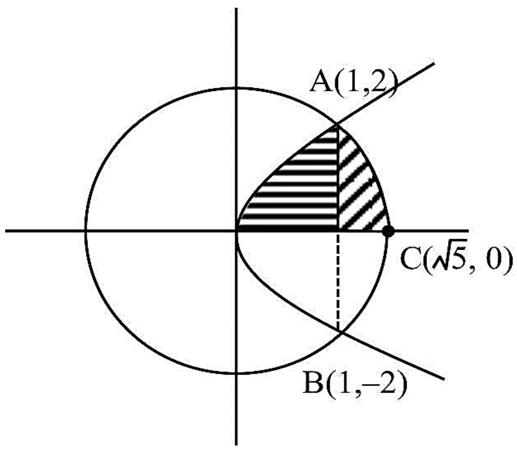
$\therefore$ Point lie on the line for

$k = -\frac{1}{3}$ is $\left(-\frac{1}{3}, 1, -1\right)$

62. (a) $y^2 = 4x$

$x^2+y^2=5$

$\therefore$ Area of shaded region as shown in the figure will be



$A_1 = \int_0^1 \sqrt{4x}dx + \int_1^{\sqrt{5}} \sqrt{5-x^2}dx$

$$=\frac{4}{3}\cdot\left[x^{\frac{3}{2}}\right]_0^1+\left[\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_1^{\sqrt{5}}$$

$$=\frac{1}{3}+\frac{5\pi}{4}-\frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$\therefore$ Required Area $= 2\ A_1$

$$=\frac{2}{3}+\frac{5\pi}{2}-5\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$$=\frac{2}{3}+5\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{\sqrt{5}}\right)$$

$$=\frac{2}{3}+5\cos^{-1}\frac{1}{\sqrt{5}}$$

$$=\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

63. (a) $(2y-5)\frac{dy}{dx}=-3$

$(2y-5)dy=-3dx$

$2\cdot\frac{y^2}{2}-5y=-3x+\lambda \because$ Curve passes through $(0,1)$

$\Rightarrow \lambda=-4$

$\because$ Curve will be

$$\left(y-\frac{5}{2}\right)^2=-3\left(x-\frac{3}{4}\right)$$

$\therefore$ Vertex of parabola will be $\left(\frac{3}{4},\frac{5}{2}\right)$

$\because 2x+3y=9$

64. (d)

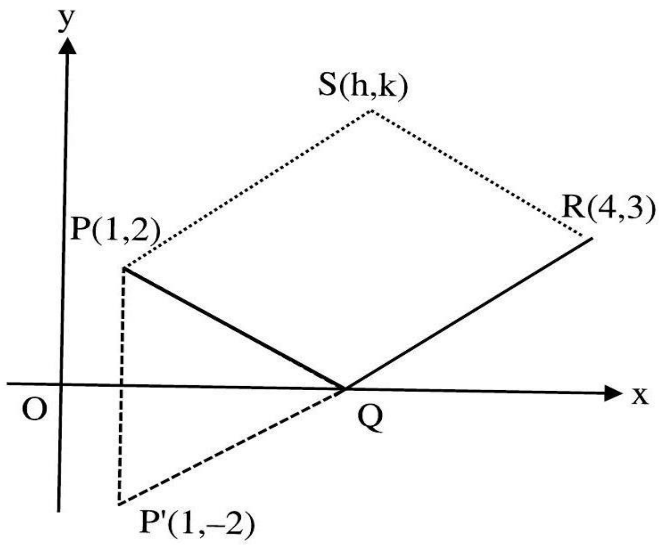


Image of P wrt x-axis will be $P'(1,-2)$ equation of line joining P'R will be

$$y-3=\frac{5}{3}(x-4)$$

Above line will meet x-axis at Q where

$$y=0 \Rightarrow x=\frac{11}{5}$$

$$\therefore Q\left(\frac{11}{5},0\right)$$

$\because$ PQRS is parallelogram so their diagonals will bisects each other

$$\Rightarrow \frac{4+1}{2}=\frac{\frac{11}{5}+h}{2} \& \frac{2+3}{2}=\frac{k+0}{2}$$

$$\Rightarrow h=\frac{14}{5} \& k=5$$

$$\therefore hk^2=\frac{14}{5}\times 5^2=70$$

65. (a) $3x+5y+\lambda z=3$

$$7x+11y-9z=2$$
$$97x+155y-189z=\mu$$
$$93x+155y+31\lambda z=93$$
$$97x+155y-189z=\mu$$
$$-\quad -\quad +\quad -$$
$$-4x+(31\lambda+189)z=93-\mu$$
$$1085x+1705y-1395z=310$$
$$1067x+1705y-2079z=11\mu$$
$$-\quad -\quad +\quad -$$
$$18x+684z=310-11\mu$$

$$-36x+9(31\lambda+189)z=9(93-\mu)$$

$36x + 1368z = 2(310 - 11\mu)$

$(279\lambda + 3069)z = 1457 - 31\mu$

for infinite solutions -

$\lambda = \frac{-3069}{279} = \frac{-341}{31}$

$\mu = \frac{1457}{31}\ \mu + 2\lambda = \frac{1457-682}{31} = \frac{775}{31} = 25$

66. (d) $x^2(1+x)^{98} + x^3(1+x^{97}) + x^4(1+x)^{96} + \cdots ..... x^{54}(1+x)^{46}$

Coeff. of x^{70}: ${}^{98}C_{68} + {}^{97}C_{67} + {}^{96}C_{66} +$
${}^{47}C_{17} + {}^{46}C_{16}$

$= {}^{46}C_{30} + {}^{47}C_{30} + \quad {}^{98}C_{30}$

$= ({}^{46}C_{31} + {}^{46}C_{30}) + {}^{47}C_{30} + \cdots \ldots \ldots .. {}^{98}C_{30} - {}^{46}C_{31}$

$= {}^{47}C_{31} + {}^{47}C_{30} + \cdots \ldots \ldots .. {}^{98}C_{30} - {}^{46}C_{31}$

......

$= {}^{99}C_{31} - {}^{46}C_{31} = {}^{99}C_p - {}^{46}C_q$

Possible values of $(p+q)$ are 62, 83, 99, 46

$\Rightarrow p + q = 83$

67. (c)

$\int \frac{2 - \tan x}{3 + \tan x} dx = \int \frac{2\cos x - \sin x}{3\cos x + \sin x} dx$

$2\cos x - \sin x = A(3\cos x + \sin x) + B(\cos x - 3\sin x)$

$3A + B = 2$

$A - 3B = -1$
$\Rightarrow A = \frac{1}{2}, B = \frac{1}{2}$

$\therefore \int \frac{2\cos x - \sin x}{3\cos x + \sin x} dx$

$= \frac{x}{2} + \frac{1}{2} \ln|3\cos x + \sin x| + C$

$= \frac{1}{2}(x + \ln|3\cos x + \sin x|) + C$

$= \frac{1}{2}(\alpha x + \ln|\beta \sin x + \gamma \cos x|) + C$

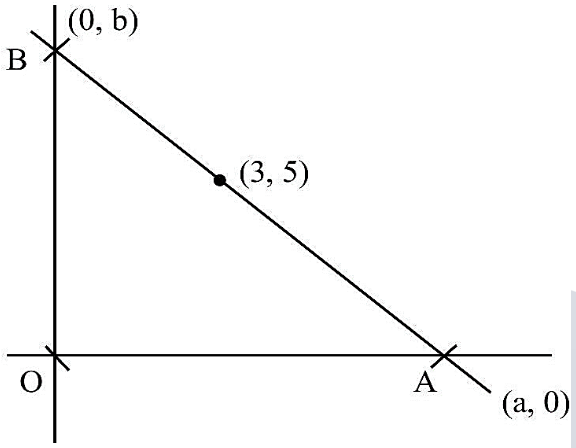
$\alpha = 1, \beta = 1, \gamma = 3$

$\therefore \alpha + \frac{\gamma}{\beta} = 1 + \frac{3}{1} = 4$

68. (a)

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{3}{a} + \frac{5}{b} = 1 \Rightarrow b = \frac{5a}{a-3}, a > 3$



$A = \frac{1}{2}ab = \frac{1}{2}a\frac{5a}{(a-3)} = \frac{5}{2} \cdot \frac{a^2}{a-3}$

$= \frac{5}{2}\left(\frac{a^2 - 9 + 9}{a-3}\right)$

$= \frac{5}{2}\left(a + 3 + \frac{9}{a-3}\right)$

$= \frac{5}{2}\left(a - 3 + \frac{9}{a-3} + 6\right) \geq 30$

69. (c) We know that

$(\cos\theta)\left(\cos(60^\circ - \theta)\right)\left(\cos(60^\circ + \theta)\right) = \frac{1}{4}\cos 3\theta$

So equation reduces to $\left|\frac{1}{4}\cos 3\theta\right| \leq \frac{1}{8}$

$\Rightarrow |\cos 3\theta| \leq \frac{1}{2}$

$\Rightarrow -\frac{1}{2} \leq \cos 3\theta \leq \frac{1}{2}$

$\Rightarrow$ maximum value of $\cos 3\theta = \frac{1}{2}$, here

$\Rightarrow 3\theta = 2n\pi \pm \frac{\pi}{3}$

$\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

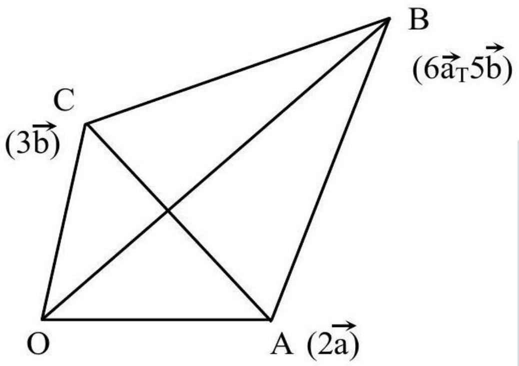
As $\theta \in [0,2\pi]$ possible values are

$\theta = \left\{\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}\right\}$

Whose sum is

$\frac{\pi}{9} + \frac{5\pi}{9} + \frac{7\pi}{9} + \frac{11\pi}{9} + \frac{13\pi}{9} + \frac{17\pi}{9} = \frac{54\pi}{9} = 6\pi$

70. (d)



Area of parallelogram having sides

$\overrightarrow{OA} \& \overrightarrow{OC} = |\overrightarrow{OA} \times \overrightarrow{OC}| = |2\vec{a} \times 3\vec{b}| = 15$

$6|\vec{a} \times \vec{b}| = 15$

$\Rightarrow |\vec{a} \times \vec{b}| = \frac{5}{2}$

Area of quadrilateral

$\mathrm{OABC} = \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

$= \frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{OB}| = \frac{1}{2}|(3\vec{b} - 2\vec{a}) \times (6\vec{a} + 5\vec{b})|$

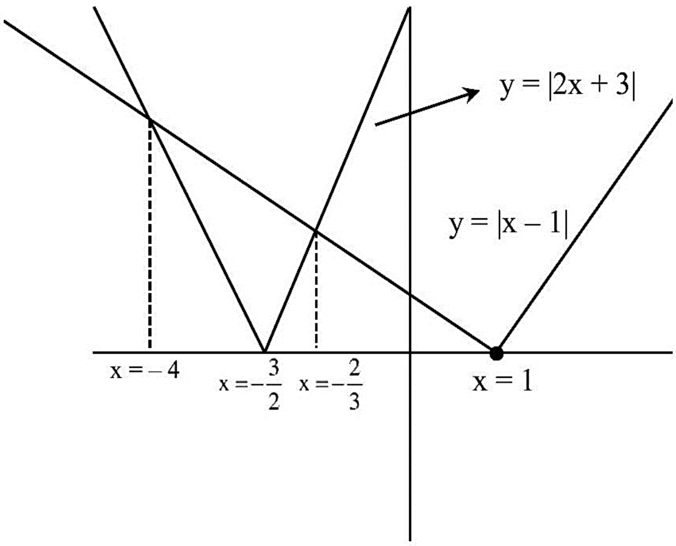
$= \frac{1}{2}|18\vec{b} \times \vec{a} - 10\vec{a} \times \vec{b}| = 14|\vec{a} \times \vec{b}|$

$= 14 \times \frac{5}{2} = 35$

71. (d) Domain of $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is

$2x + 3 \neq 0 \& x \neq \frac{-3}{2}$ and $\left|\frac{(x-1)}{2x+3}\right| \leq 1$

$|x - 1| \leq |2x + 3|$



For $|2x+3| \geq |x-1|$

$x \in (-\infty, -4] \cup \left(-\frac{2}{3}, \infty\right)$

$\alpha = -4 \& \beta = -\frac{2}{3} : 12\alpha\beta = 32$

72. (b) $\frac{1}{1\cdot(1+d)} + \frac{1}{(1+d)(1+2\,d)} + \ldots\ldots\ldots\ldots\ldots$

$\frac{1}{(1+9d)(1+10d)} = 5$

$\frac{1}{d}\left[\frac{(1+d)-1}{1\cdot(1+d)} + \frac{(1+2\,d)-(1-d)}{(1+d)(1+2\,d)}\right] + \ldots\ldots\ldots\ldots\ldots$

$\frac{(1+10\,d)-(1+9\,d)}{(1+9\,d)(1+10\,d)} = 5$

$\frac{1}{d}\left[\left(1-\frac{1}{1+d}\right) + \left(\frac{1}{1+d} - \frac{1}{1+2\,d}\right) + \ldots\ldots\ldots\ldots\ldots\ldots\right.$

$\left.\left(\frac{1}{1+9\,d} - \frac{1}{1+10\,d}\right)\right] = 5$

$\frac{1}{d}\left[1 - \frac{1}{(1+10\,d)}\right] = 5$

$\frac{10\,d}{1+10\,d} = 5\,d$

$50\,d = 5$

73. (d) $f(x) = ax^3 + bx^2 + cx + 41$

$f'(x) = 3ax^2 + 2bx + cx$

$\Rightarrow f'(a) = 3a + 2b + c = 2 \ldots\ldots..(1)$

$f''(n) = 6ax + 2b$

$\Rightarrow f''(a) = 6a + 2b = 4$

$3a + b = 2$

(1) - (2)

$b + c = 0$……… (3)

$f(a) = 40$

$a + b + c + 41 = 40$

use (3)

$a + 41 = 40$

by (2)

$-3 + b = 2 \Rightarrow b = 5 \& c = -5$

$a^2 + b^2 + c^2 = 1 + 25 + 25 = 51$

74. (a) $(2 + c)x + 5c^2\left(\frac{1-3x}{5}\right) = 1$

$$x = \frac{1 - c^2}{2 + c - 3c^2}, y = \frac{1 - 3x}{5} = \frac{c - 1}{5(2 + c - 3c^2)}$$

$$h = \lim_{c\to 1}\frac{(1 - c)(1 + c)}{(1 - c)(2 + 3c)} = \frac{2}{5}$$

$$K = \lim_{c\to 1}\frac{c - 1}{-5(c - 1)(3c + 2)} = -\frac{1}{25}$$

Centre $\left(\frac{2}{25}, -\frac{1}{25}\right)$,

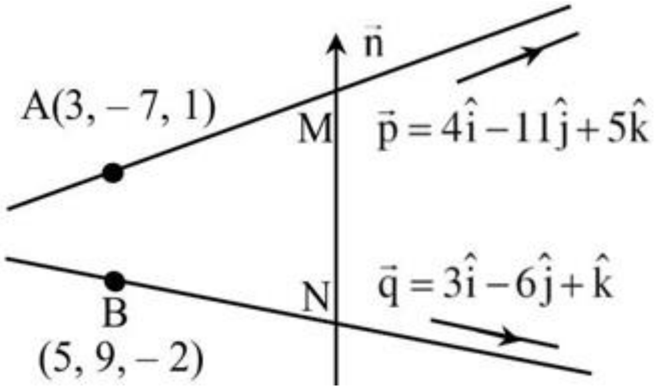
$$r = \sqrt{\left(2 - \frac{2}{5}\right)^2 + \left(0 - \frac{1}{25}\right)^2} = \sqrt{\frac{64}{25} + \frac{1}{625}}$$

$$r = \frac{\sqrt{161}}{25}$$

$$\left(x - \frac{2}{5}\right)^2 + \left(y + \frac{1}{25}\right)^2 = \frac{161}{125}$$

$\Rightarrow 25x^2 + 25y^2 - 20x + 2y - 60 = 0$

75. (a)



$\vec{n}=\vec{p}\times\vec{q}$

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -11 & 5\\ 3 & -6 & 1\end{vmatrix}=19\hat{i}+11\hat{j}+9\hat{k}$$

S.d. = projection of $\overrightarrow{AB}$ on $\vec{n}$

$$=\left|\frac{\overrightarrow{AB}\cdot\vec{n}}{|\vec{n}|}\right|=\left|\frac{(2\hat{i}+16\hat{j}-3\hat{k})\cdot(19\hat{i}+11\hat{j}+9\hat{k})}{\sqrt{361+121+81}}\right|$$

$$=\frac{38+176-27}{\sqrt{563}}$$

S.d. $=\frac{187}{\sqrt{563}}$

76. (b)

$x+y=10\ldots\ldots(1)$

Median $=18=M$

M.D. $=\frac{\Sigma f_i|x_i-M|}{\Sigma f_i}$

$$1.25=\frac{36+x+2y}{40}$$

$x+2y=14\ldots\ldots(2)$

by (1) & (2)

$x=6, y=4$

$\Rightarrow 4x+5y=24+20=44$

Age (x_i)	f	$\lvert x_i-M\rvert$	$f_i\lvert x_i-M\rvert$
15	5	3	15
16	8	2	16

17	5	1	5
18	12	0	0
19	x	1	x
20	y	2	2y

77. (a) $(x^2+y^2)dx = 5xydy$

$\Rightarrow \frac{dy}{dx} = \frac{x^2+y^2}{5xy}$

Put $y = Vx$

$\Rightarrow V + x\frac{dv}{dx} = \frac{1+V^2}{5\text{ V}}$

$\Rightarrow \frac{xdv}{dx} = \frac{1-4\text{ V}^2}{5\text{ V}}$

$\Rightarrow \int \frac{V}{1-4V^2} dV = \int \frac{dx}{5x}$

Let $1 - 4\text{ V}^2 = t$

$\Rightarrow -8VdV = dt$

$\Rightarrow \int \frac{dt}{(-8)(t)} = \int \frac{dx}{5x}$

$\Rightarrow \frac{-1}{8} \ln|t| = \frac{1}{5}\ln|x| + \ln C$

$\Rightarrow -5\ln|t| = 8\ln|x| + \ln K$

$\Rightarrow \ln x^8 + \ln|t^5| + \ln K = 0$

$\Rightarrow x^8|t^5| = C$

$\Rightarrow x^8|1-4\text{ V}^2|^5 = C$

$\Rightarrow x^8\left|\frac{x^2-4y^2}{x^2}\right|^5 = C$

$\Rightarrow |x^2-4y^2|^5 = Cx^2$

given $y(1) = 0$

$\Rightarrow |1|^5 = C \Rightarrow C = 1$

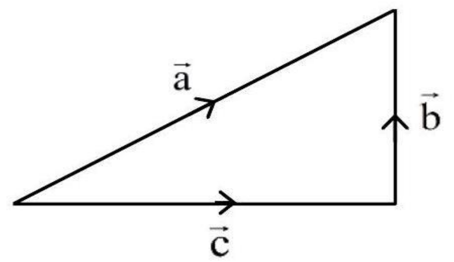
$\Rightarrow |x^2-4y^2|^5 = x^2$

78. (b)

$\vec{c} = \vec{a} - \vec{b}$

$\Rightarrow (x, y, z) = (\alpha - 5, 1, -2)$

$\Rightarrow x = \alpha - 5, y = 1, z = -2$............. (1)



Area of $\Delta = 5\sqrt{6}$ (given) $\frac{1}{2}|\vec{a} \times \vec{c}| = 5\sqrt{6}$

$$\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 4 & 2 \\ x & 1 & -2 \end{matrix}\right\| = 10\sqrt{6}$$

$\Rightarrow |-10\hat{i} - \hat{j}(-2\alpha - 2x) + \hat{k}(\alpha - 4x)| = 10\sqrt{6}$

$\Rightarrow (2\alpha + 2\alpha - 10)^2 + (\alpha - 4\alpha + 20)^2 = 500$

$\Rightarrow (4\alpha - 10)^2 + (20 - 3\alpha)^2 = 500$

$\Rightarrow 25\alpha^2 - 80\alpha - 120\alpha = 0$

$\Rightarrow \alpha(25\alpha - 200) = 0$

$\Rightarrow \alpha = 8$ (given α is + ve number)

$\Rightarrow x = \alpha - 5 = 3$

$|\vec{c}|^2 = x^2 + y^2 + z^2$

$= 9 + 1 + 4$

$= 14$

79. (c) $x^2 + 2\sqrt{2}x - 1 = 0$

$\alpha + \beta = -2\sqrt{2}$
$\alpha\beta = -1$
$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$
$= ((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2$
$= (8 + 2)^2 - 2(-1)^2$
$= 100 - 2 = 98$
$\alpha^6 + \beta^6 = (\alpha^3 + \beta^3)^2 - 2\alpha^3\beta^3$
$= ((\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)^2 - 2(\alpha\beta)^3$

$= (-2\sqrt{2}(8 + 3))^2 + 2$

$= (8)(121) + 2 = 970$

$\frac{1}{10}(\alpha^6 + \beta^6) = 97$

$x^2 - (98 + 97)x + (98)(97) = 0$

$\Rightarrow x^2 - 195x + 9506 = 0$

80. (b) $f(x) = x^2 + 9$ $g(x) = \frac{x}{x-9}$

$a = f(g(10)) = f\left(\frac{10}{10-9}\right)$

$= f(10) = 109$

$b = g(f(c)) = g(9 + 9)$

$= g(18) = \frac{18}{9} = 2$

$E: \frac{x^2}{109} + \frac{y^2}{2} = 1$

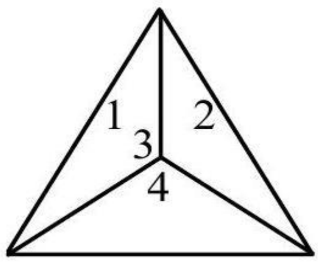
$e^2 = 1 - \frac{2}{109} = \frac{107}{109}$

$\ell = \frac{2(b)}{\sqrt{109}} = \frac{4}{\sqrt{109}}$

$8e^2 + \ell^2 = \frac{8(107)}{109} + \frac{16}{109}$

$= 8$

81. (19) $a, b, c \in \{1,2,3,4\}$



Tetrahedral dice

$ax^2 + bx + c = 0$

has all real roots

$\Rightarrow D \geq 0$

$\Rightarrow b^2 - 4ac \geq 0$

Let $b = 1 \Rightarrow 1 - 4ac \geq 0$ (Not feasible)

$b = 2 \Rightarrow 4 - 4ac \geq 0$

$1 \geq ac \Rightarrow a = 1, c = 1,$

$b = 3 \Rightarrow 9 - 4ac \geq 0$

$\frac{9}{4} \geq ac$

$\Rightarrow a = 1, c = 1$

$\Rightarrow a = 1, c = 2$

$\Rightarrow a = 2, c = 1$

$b = 4 \Rightarrow 16 - 4ac \geq 0$

$4 \geq ac$

$\Rightarrow a = 1, c = 1$

$\Rightarrow a = 1, c = 2 \;\; \Rightarrow a = 2, c = 1$

$\Rightarrow a = 1, c = 3 \;\; \Rightarrow a = 3, c = 1$

$\Rightarrow a = 1, c = 4 \;\; \Rightarrow a = 4, c = 1$

$\Rightarrow a = 2, c = 2$

$\text{Probability} = \frac{12}{(d)(d)(d)} = \frac{3}{16} = \frac{m}{m}$

$m + n = 19$

82. (9)

$|z - 1| \leq 1$

$\Rightarrow |(x - 1) + iy| \leq 1$

$\Rightarrow \sqrt{(x-1)^2 + y^2} \leq 1$

$\Rightarrow (x - 1)^2 + y^2 \leq 1 \ldots\ldots\ldots (1)$

Also $|z - 5| \leq |z - 5i|$

$(x - 5)^2 + y^2 \leq x^2 + (y - 5)^2$

$-10x \leq -10y$

$\Rightarrow x \geq y \ldots\ldots\ldots\ldots\ldots (2)$

Solving (1) and (2)

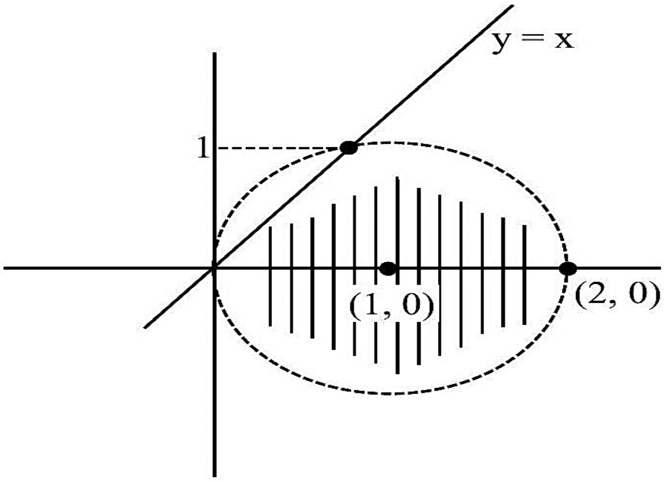
$\Rightarrow (x - 1)^2 + x^2 = 1$

$\Rightarrow 2x^2 - 2x = 0$

$\Rightarrow x(x - 1) = 0$

$\Rightarrow x = 0$ or $x = 1$

$y = 0$ or $y = 1$



Given $x, y \in I$
Points $(0,0), (1,0), (2,0), (1,1), (1,-1)$

to find

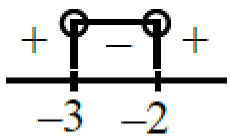
$|z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 + |z_5|^2$

$= 0 + 1 + 4 + 1 + 1 + 1 + 1 = 9$

83. (39)

$$\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$$

$$\Rightarrow \frac{1}{(x+2)(x+3)} < 0$$



$x \in (-3, -2)$…………… (1)

$f(x) = 1 + x(\lambda^2 - x^2)$

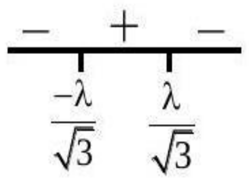
Finding local minima

$f'(x) = (\lambda^2 - x^2) + (-2x) \cdot x$

Put $f'(x) = 0$

$\Rightarrow \lambda^2 = 3x^2$

$$\Rightarrow x = \pm\frac{\lambda}{\sqrt{3}}$$



Local min Local max

We want local min

$\Rightarrow x = \frac{-\lambda}{\sqrt{3}}$

from (1)

$x \in (-3, -2)$

$-3 < \frac{-\lambda}{\sqrt{3}} < -2$

$3\sqrt{3} > \lambda > 2\sqrt{3}$

$\alpha = 2\sqrt{3}, \beta = 3\sqrt{3}$

$\alpha^2 + \beta^2 = 12 + 27 = 39$

84. (32) $\sum_{r=1}^{\infty} \frac{n}{\sqrt{n^4+r^4}} - \frac{2nr^2}{(n^2+r^2)\sqrt{n^4+r^4}}$

$$\sum_{r=1}^{\infty} \frac{\frac{1}{n}}{\sqrt{1+\left(\frac{r}{n}\right)^4}} - \frac{2\left(\frac{1}{n}\right)\left(\frac{r}{n}\right)^2}{\left(1+\left(\frac{r}{n}\right)^2\right)\sqrt{1+\left(\frac{r}{n}\right)^4}}$$

$$\Rightarrow \int_0^1 \frac{dx}{\sqrt{1+x^4}} - \frac{2x^2 dx}{(1+x^2)\sqrt{1+x^4}}$$

$$\Rightarrow \int_0^1 \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx$$

$$\Rightarrow \int_0^1 \frac{\frac{1}{x^2}-1}{\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}} dx$$

$$\Rightarrow -\int_0^1 \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}} dx$$

$x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2} dx = dt \Rightarrow -\int_{\infty}^{2} \frac{dt}{t\sqrt{t^2-2}}$

$$\Rightarrow -\int_{\infty}^{2} \frac{tdt}{t^2\sqrt{t^2-2}}$$

take $t^2 - 2 = \alpha^2$

$tdt = \alpha d\alpha$

$\Rightarrow -\int_{\infty}^{\sqrt{2}} \frac{\alpha d\alpha}{(\alpha^2+2)\alpha}$

$\Rightarrow -\int_{\infty}^{\sqrt{2}} \frac{d\alpha}{\alpha^2+2}$

$\Rightarrow \frac{-1}{\sqrt{2}} \tan^{-1} \frac{\alpha}{\sqrt{2}}\Big]_{\infty}^{\sqrt{2}}$

$\Rightarrow \frac{-1}{\sqrt{2}}\{\tan^{-1} 1\} + \frac{1}{\sqrt{2}} \tan^{-1} \infty$

$\Rightarrow \frac{1}{\sqrt{2}}\left\{\frac{\pi}{2} - \frac{\pi}{4}\right\}$

$\Rightarrow \frac{\pi}{4\sqrt{2}} = \frac{\pi}{K}$

So $K = 4\sqrt{2}$

$K^2 = 32$

85. (1)

$(428)^{2024} = (420+8)^{2024}$

$= (21 \times 20 + 8)^{2024}$
$= 21\text{ m} + 8^{2024}$

Now $8^{2024} = (8^2)^{1012}$

$= (64)^{1012}$

$= (63+1)^{1012}$

$= (21 \times 3 + 1)^{1012}$

$= 2\ln + 1$

$\Rightarrow$ Remainder is 1

86. (81)

LHL at $x = \frac{\pi}{2}$

$\lim_{x \to \frac{\pi}{2}} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} = \left(\frac{8}{7}\right)^0 = 1$

RHL at $x = \frac{\pi}{2}$

$\lim_{x \to \frac{\pi}{2}} (1 + |\cot x|)^{\frac{b}{1}\tan x|}$

$$= e^{\lim_{x \to \frac{\pi}{2}} |\cot x| \frac{b}{a} |\tan x|} = e^{\frac{b}{a}}$$

$\Rightarrow 1 = a - 8 = e^{\frac{b}{a}}$

$\Rightarrow a = 9, b = 0$

$\Rightarrow a^2 + b^2 = 81$

87. (14)

$|3\operatorname{adj}(2\operatorname{adj}(|A|A))| = |3\operatorname{adj}(2|A|^2\operatorname{adj}(A)|$

$= |3.2^2|A|^4\operatorname{adj}(\operatorname{adj}(A)| = 2^6 3^3 |A|^{12}|A|^4$

$= 2^6 3^3 |A|^{16} = 2^{-10} 3^{-13}$

$\Rightarrow |A|^{16} = 2^{-16} 3^{-16} \Rightarrow |A| = 2^{-1} 3^{-1}$

Now $|3\operatorname{adj}(2A)| = |3.2^2\operatorname{adj}(A)|$

$= 2^6 3^3 |A|^2 = 2^{-m} 3^{-n}$

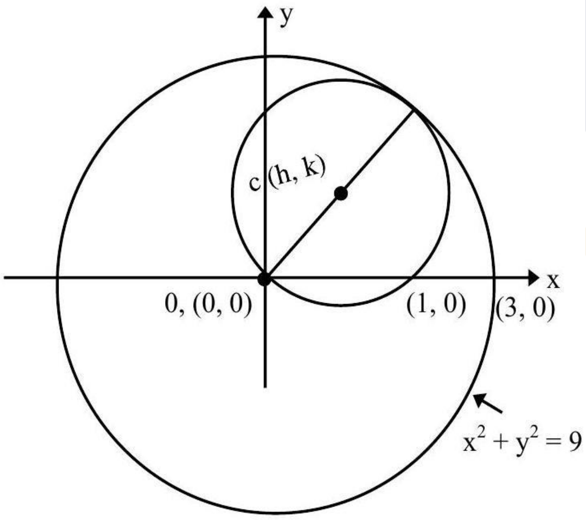
$\Rightarrow 2^6 3^3 2^{-2} 3^{-2} = 2^{-m} 3^{-n}$

$\Rightarrow 2^{-m} 3^{-n} = 2^4 3^1$

$\Rightarrow m = -4, n = -1$

$\Rightarrow |3m + 2n| = |-12 - 2| = 14$

88. (9)



$(x - h)^2 + (y - k)^2 = h^2 + k^2$

$x^2 + y^2 - 2hx - 2ky = 0$

$\because$ passes through (1,0)

$\Rightarrow 1+0-2\,h=0$

$\Rightarrow h=1/2$

$\because OC=\frac{OP}{2}$

$\sqrt{\left(\frac{1}{2}\right)^2+k^2}=\frac{3}{2}$

$\frac{1}{4}+k^2=\frac{9}{4}$

$k^2=2$

$k=\pm\sqrt{2}$

$\therefore$ Possible coordinate of

$c(h,k)\left(\frac{1}{2},\sqrt{2}\right)\left(\frac{1}{2},-\sqrt{2}\right)$

$4(h^2+k^2)=4\left(\frac{1}{4}+2\right)=4\left(\frac{9}{4}\right)=9$

89. (1010) $f(m+n)=f(m)+f(n)$

$\Rightarrow f(x)=kx$

$\Rightarrow f(a)=1$

$\Rightarrow k=1$

$f(x)=x$

Now

$$\sum_{k=1}^{2022} f(\lambda+k)\le(2022)^2$$

$\Rightarrow \sum_{k=1}^{2022}(\lambda+k)\le(2022)^2$

$\Rightarrow 2022\lambda+\frac{2022\times2023}{2}\le(2022)^2\Rightarrow\lambda\le2022-\frac{2023}{2}$

$\Rightarrow \lambda\le1010.5$

$\therefore$ largest natural no. λ is 1010.

90. (25)

$A=\{2,3,6,7\}$

$B=\{2,5,6,8\}$

$(a_1,b_1)R(a_2,b_2)$

$a_1 + a_2 = b_1 + b_2$

$$\left.\begin{array}{ll}
1.\ (2, 4)\ R\ (6, 4) & 2.\ (2, 4)\ R\ (7, 5) \\
3.\ (2,5)\ R\ (7, 4) & 4.\ (3, 4)\ R\ (6, 5) \\
5.\ (3, 5)\ R\ (6, 4) & 6.\ (3, 5)\ R\ (7, 5) \\
7.\ (3,6)\ R\ (7, 4) & 8.\ (3, 4)\ R\ (7, 6) \\
9.\ (6, 5)\ R\ (7, 8) & 10.\ (6, 8)\ R\ (7, 5) \\
11.\ (7,8)\ R\ (7, 6) & 12.\ (6, 8)\ R\ (6, 4)
\end{array}\right] \times 2$$

$13.\ (6,6)\ R\ (6, 6)$

Total $24 + 1 = 25$