

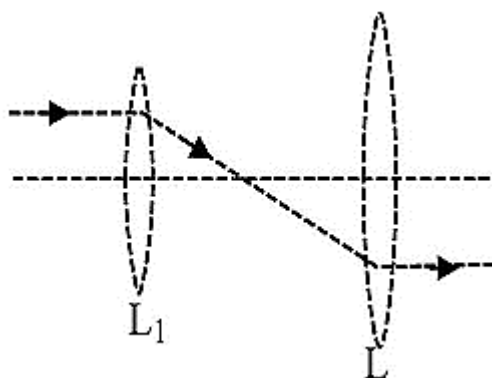
PART- I PHYSICS

SECTION- A

1. A nucleus at rest disintegrates into two smaller nuclei with their masses in the ratio of 2: 1. After disintegration they will move: -

- (a) In opposite directions with speed in the ratio of 1: 2 respectively
- (b) In opposite directions with speed in the ratio of 2: 1 respectively
- (c) In the same direction with same speed.
- (d) In opposite directions with the same speed.

2. The following figure represents two biconvex lenses L_1 and L_2 having focal length 10 cm and 15 cm respectively. The distance between L_1 & L_2 is:



- (a) 10 cm
- (b) 15 cm
- (c) 25 cm
- (d) 35 cm

3. The temperature of a gas is -78°C and the average translational kinetic energy of its molecules is K . The temperature at which the average translational kinetic energy of the molecules of the same gas becomes $2K$ is:

- (a) -39°C
- (b) 117°C
- (c) 127°C
- (d) -78°C

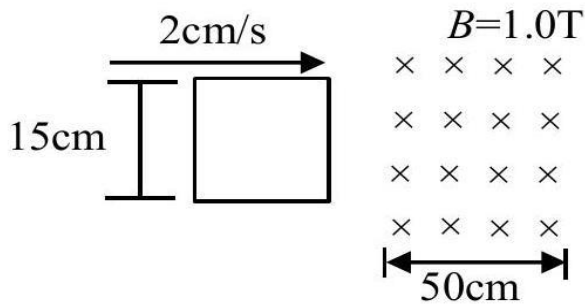
4. A hydrogen atom in ground state is given an energy of 10.2 eV. How many spectral lines will be emitted due to transition of electrons?

- (a) 6
- (b) 3
- (c) 10
- (d) 1

5. The magnetic field in a plane electromagnetic wave is $B_y = (3.5 \times 10^{-7})\sin(1.5 \times 10^3x + 0.5 \times 10^{11}t)\text{T}$. The corresponding electric field will be

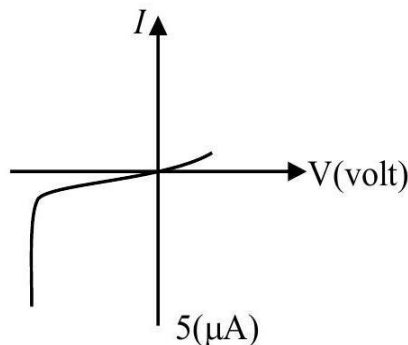
- (a) $E_y = 1.17\sin(1.5 \times 10^3x + 0.5 \times 10^{11}t)\text{Vm}^{-1}$
- (b) $E_z = 105\sin(1.5 \times 10^3x + 0.5 \times 10^{11}t)\text{Vm}^{-1}$
- (c) $E_z = 1.17\sin(1.5 \times 10^3x + 0.5 \times 10^{11}t)\text{Vm}^{-1}$
- (d) $E_y = 10.5\sin(1.5 \times 10^3x + 0.5 \times 10^{11}t)\text{Vm}^{-1}$

6. A square loop of side 15 cm being moved towards right at a constant speed of 2 cm/s as shown in figure. The front edge enters the 50 cm wide magnetic field at $t = 0$. The value of induced emf in the loop at $t = 10$ s will be:

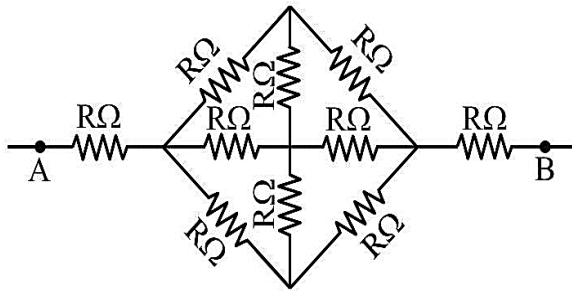


- (a) 0.3mV (b) 4.5mV
(c) zero (d) 3mV

7. Two cars are travelling towards each other at speed of 20 m s^{-1} each. When the cars are 300 m apart, both the drivers apply brakes and the cars retard at the rate of 2 m s^{-2} . The distance between them when they come to rest is:
(a) 200 m (b) 50 m
(c) 100 m (d) 25 m
8. The $I - V$ characteristics of an electronic device shown in the figure. The device is :

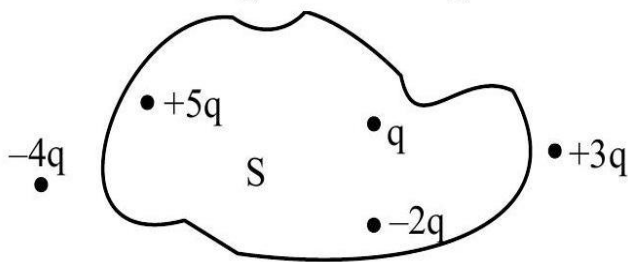


- (a) a solar cell
(b) a transistor which can be used as an amplifier
(c) a zener diode which can be used as voltage regulator
(d) a diode which can be used as a rectifier
9. The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. The ratio between the volume of the first and the second bubble is :
(a) 1:9 (b) 1:3
(c) 1:81 (d) 1:27
10. The de-Broglie wavelength associated with a particle of mass m and energy E is $h/\sqrt{2mE}$. The dimensional formula for Planck's constant is :
(a) $[ML^{-1} T^{-2}]$ (b) $[ML^2 T^{-1}]$
(c) $[MLT^{-2}]$ (d) $[M^2 L^2 T^{-2}]$
11. A satellite of 10^3 kg mass is revolving in circular orbit of radius $2R$. If $\frac{10^4 R}{6} \text{ J}$ energy is supplied to the satellite, it would revolve in a new circular orbit of radius:
(use $g = 10 \text{ m/s}^2$, $R = \text{radius of earth}$)
(a) $2.5R$ (b) $3R$
(c) $4R$ (d) $6R$
12. The effective resistance between A and B, if resistance of each resistor is R , will be



- (a) $\frac{2}{3}R$ (b) $\frac{8R}{3}$
(c) $\frac{5R}{3}$ (d) $\frac{4R}{3}$

13. Five charges $+q$, $+5q$, $-2q$, $+3q$ and $-4q$ are situated as shown in the figure. The electric flux due to this configuration through the surface S is :



- (a) $\frac{5q}{\epsilon_0}$ (b) $\frac{4q}{\epsilon_0}$
(c) $\frac{3q}{\epsilon_0}$ (d) $\frac{q}{\epsilon_0}$

14. A proton and a deuteron ($q = +e$, $m = 2.0u$) having same kinetic energies enter a region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. The ratio of the radius r_d of deuteron path to the radius r_p of the proton path is:

- (a) 1:1 (b) $1:\sqrt{2}$
(c) $\sqrt{2}:1$ (d) 1:2

15. UV light of 4.13 eV is incident on a photosensitive metal surface having work function 3.13 eV. The maximum kinetic energy of ejected photoelectrons will be:

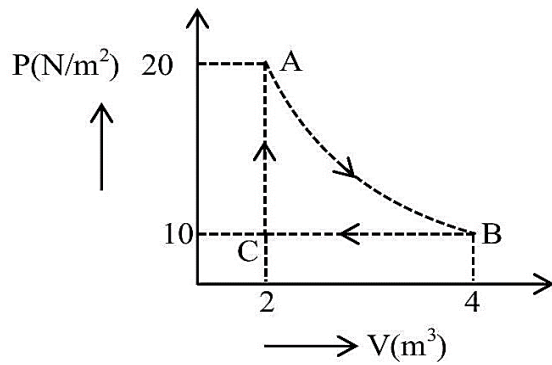
- (a) 4.13 eV (b) 1 eV
(c) 3.13 eV (d) 7.26 eV

16. The energy released in the fusion of 2 kg of hydrogen deep in the sun is E_H and the energy released in the fission of 2 kg of ^{235}U is E_U . The ratio $\frac{E_H}{E_U}$ is approximately:

(Consider the fusion reaction as $4^1_1\text{H} + 2e^- \rightarrow ^4_2\text{He} + 2\nu + 6\gamma + 26.7\text{MeV}$, energy released in the fission reaction of ^{235}U is 200 MeV per fission nucleus and $N_A = 6.023 \times 10^{23}$)

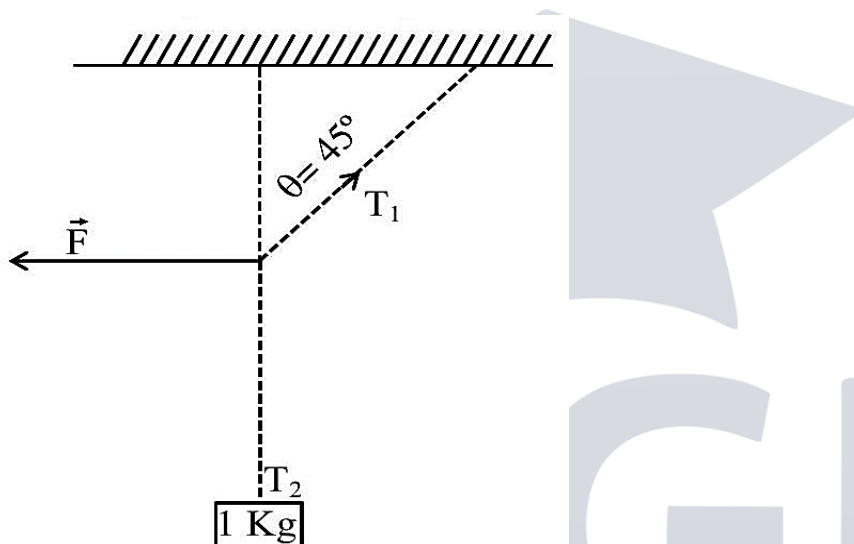
- (a) 9.13 (b) 15.04
(c) 7.62 (d) 25.6

17. A real gas within a closed chamber at 27°C undergoes the cyclic process as shown in figure. The gas obeys $PV^3 = RT$ equation for the path A to B. The net work done in the complete cycle is (assuming $R = 8 \text{ J/molK}$):



- (a) 225J (b) 205J
(c) 20J (d) -20J

18. A 1 kg mass is suspended from the ceiling by a rope of length 4 m. A horizontal force 'F' is applied at the mid point of the rope so that the rope makes an angle of 45° with respect to the vertical axis as shown in figure. The magnitude of F is:



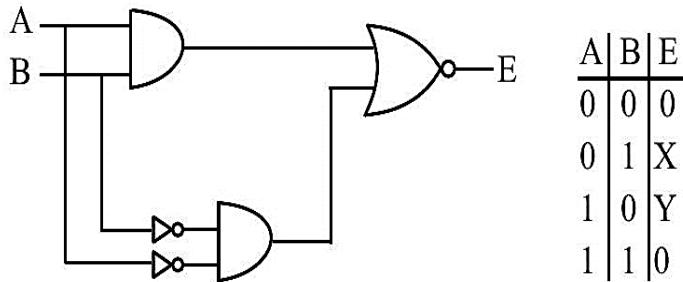
- (a) $\frac{10}{\sqrt{2}}$ N (b) 1 N
(c) $\frac{1}{10\sqrt{2}}$ N (d) 10 N

19. A spherical ball of radius 1×10^{-4} m and density 10^5 kg/m³ falls freely under gravity through a distance h before entering a tank of water, If after entering in water the velocity of the ball does not change, then the value of h is approximately :

(The coefficient of viscosity of water is 9.8×10^{-6} Ns/m²)

- (a) 2296 m (b) 2249 m
(c) 2518 m (d) 2396 m

20.



In the truth table of the above circuit the value of X and Y are:

- (a) 1,1 (b) 1,0
(c) 0,1 (d) 0,0

SECTION- B

21. A straight magnetic strip has a magnetic moment of 44Am^2 . If the strip is bent in a semicircular shape, its magnetic moment will be _____ Am^2

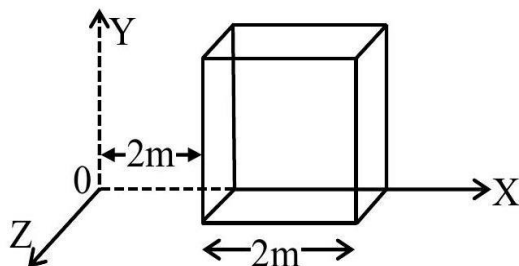
(Given $\pi = \frac{22}{7}$)

22. A particle of mass 0.50 kg executes simple harmonic motion under force $F = -50(\text{Nm}^{-1})x$. The time period of oscillation is $\frac{x}{35}$ s. The value of x is _____

(Given $\pi = \frac{22}{7}$)

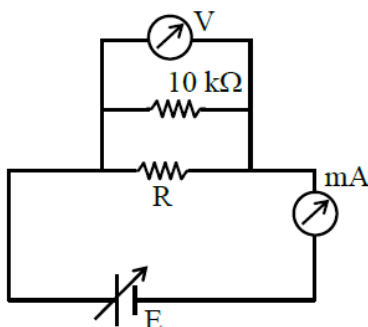
23. A capacitor of reactance $4\sqrt{3}\Omega$ and a resistor of resistance 4Ω are connected in series with an ac source of peak value $8\sqrt{2}$ V. The power dissipation in the circuit is _____ W.

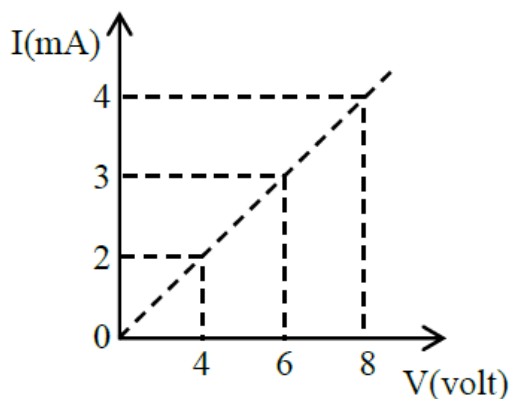
24. An electric field $\vec{E} = (2 \times \hat{i})\text{NC}^{-1}$ exists in space. A cube of side 2 m is placed in the space as per figure given below. The electric flux through the cube is _____ Nm^2/C .



25. A circular disc reaches from top to bottom of an inclined plane of length l . When it slips down the plane, it takes t_s . When it rolls down the plane then it takes $\left(\frac{\alpha}{2}\right)^{1/2} t_s$, where α is

26. To determine the resistance (R) of a wire, a circuit is designed below, The V-I characteristic curve for this circuit is plotted for the voltmeter and the ammeter readings as shown in figure. The value of R is _____ Ω .



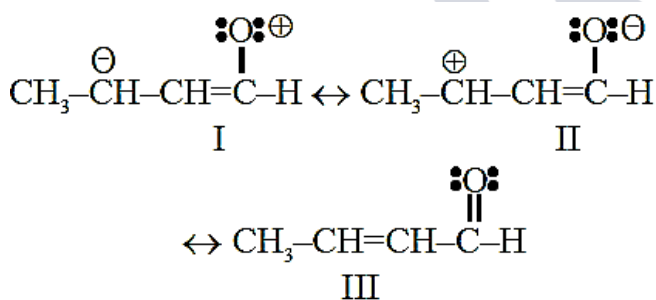


27. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$ and its magnitude is half that of $\vec{B}$. The angle between vectors $\vec{A}$ and $\vec{B}$ is.....
28. Monochromatic light of wavelength 500 nm is used in Young's double slit experiment. An interference pattern is obtained on a screen. When one of the slits is covered with a very thin glass plate (refractive index = 1.5), the central maximum is shifted to a position previously occupied by the 4th bright fringe. The thickness of the glass-plate is _____ μm .
29. A force $(3x^2 + 2x - 5)\text{N}$ displaces a body from $x = 2\text{ m}$ to $x = 4\text{ m}$. Work done by this force is J.
30. At room temperature (27°C), the resistance of a heating element is 50Ω . The temperature coefficient of the material is $2.4 \times 10^{-4} ^\circ\text{C}^{-1}$. The temperature of the element, when its resistance is 62Ω , is _____ $^\circ\text{C}$.

PART- II CHEMISTRY

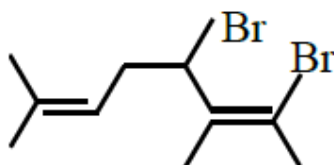
SECTION- A

31. The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 'A' $\times 10^{12}$ hertz and that has a radiant intensity in that direction of $\frac{1}{B'}$ watt per steradian. 'A' and 'B' are respectively
- (a) 540 and $\frac{1}{683}$ (b) 540 and 683
- (c) 450 and $\frac{1}{683}$ (d) 450 and 683
32. The correct stability order of the following resonance structures of $\text{CH}_3 - \text{CH} = \text{CH} - \text{CHO}$ is



- (a) $\text{II} > \text{III} > \text{I}$ (b) $\text{III} > \text{II} > \text{I}$
- (c) $\text{I} > \text{II} > \text{III}$ (d) $\text{II} > \text{I} > \text{III}$

33. Total number of stereo isomers possible for the given structure:



- (a) 8 (b) 2
(c) 4 (d) 3

34. The correct increasing order for bond angles among BF_3 , PF_3 and ClF_3 is :

- (a) $\text{PF}_3 < \text{BF}_3 < \text{ClF}_3$ (b) $\text{BF}_3 < \text{PF}_3 < \text{ClF}_3$
(c) $\text{ClF}_3 < \text{PF}_3 < \text{BF}_3$ (d) $\text{BF}_3 = \text{PF}_3 < \text{ClF}_3$

35. Match List I with List II

LIST-I (Test)		LIST-II (Observation)	
A.	Br 2 water test	I.	Yellow orange or orange red precipitate formed
B.	Ceric ammonium nitrate test	II.	Reddish orange colour disappears
C.	Ferric chloride test	III.	Red colour appears
D.	2, 4-DNP test	IV.	Blue, Green, Violet or Red colour appear

Choose the correct answer from the options given below:

- (a) A-I, B-II, C-III, D-IV (b) A-II, B-III, C-IV, D-I
(c) A-III, B-IV, C-I, D-II (d) A-IV, B-I, C-II, D-III

36. Match List I with List II

LIST-I (Cell)		LIST-II (Use/Property/Reaction)	
A.	Leclanche cell	I.	Converts energy of combustion into electrical energy

B.	Ni-Cd cell	II.	Does not involve any ion in solution and is used in hearing aids
C.	Fuel cell	III.	Rechargeable
D.	Mercury cell	IV.	Reaction at anode $\text{Zn} \rightarrow \text{Zn}^{2+} + 2\text{e}^-$

Choose the correct answer from the options given below:

- (a) A-I, B-II, C-III, D-IV (b) A-III, B-I, C-IV, D-II
(c) A-IV, B-III, C-I, D-II (d) A-II, B-III, C-IV, D-I

37. Match List I with List II

LIST-I		LIST-II	
A.	$\text{K}_2[\text{Ni}(\text{CN})_4]$	I.	sp^3
B.	$[\text{Ni}(\text{CO})_4]$	II.	$\text{sp}^3 \text{d}^2$
C.	$[\text{Co}(\text{NH}_3)_6]\text{Cl}_3$	III.	dsp^2
D.	$\text{Na}_3[\text{CoF}_6]$	IV.	d^2sp^3

Choose the correct answer from the options given below:

- (a) A-III, B-I, C-II, D-IV (b) A-III, B-II, C-IV, D-I
(c) A-I, B-III, C-II, D-IV (d) A-III, B-I, C-IV, D-II

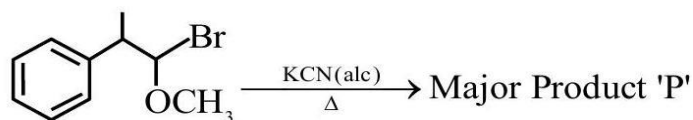
38. The coordination environment of Ca^{2+} ion in its complex with EDTA^{4-} is:

- (a) tetrahedral (b) octahedral
(c) square planar (d) trigonal prismatic

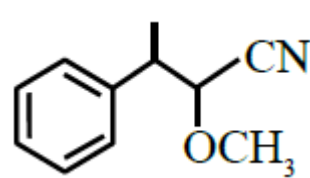
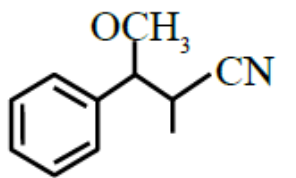
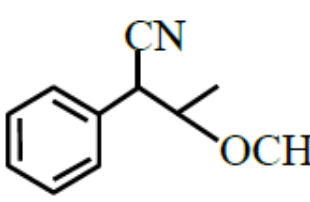
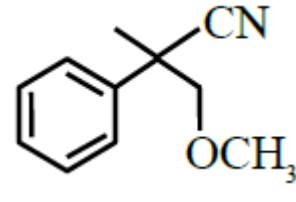
39. The incorrect statement about Glucose is:

- (a) Glucose is soluble in water because of having aldehyde functional group
(b) Glucose remains in multiple isomeric form in its aqueous solution
(c) Glucose is an aldohexose
(d) Glucose is one of the monomer unit in sucrose

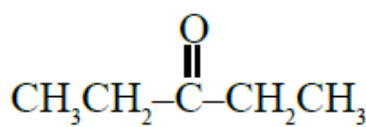
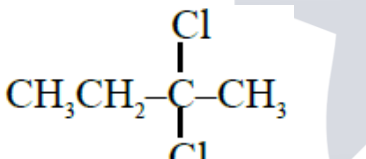
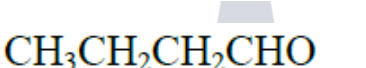
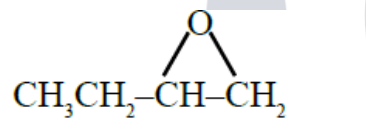
40.



In the above reaction product 'P' is

- (a)  (b) 
- (c)  (d) 

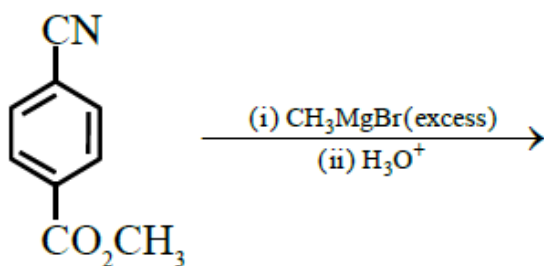
41. Which of the following compound can give positive iodoform test when treated with aqueous KOH solution followed by potassium hypoiodite?

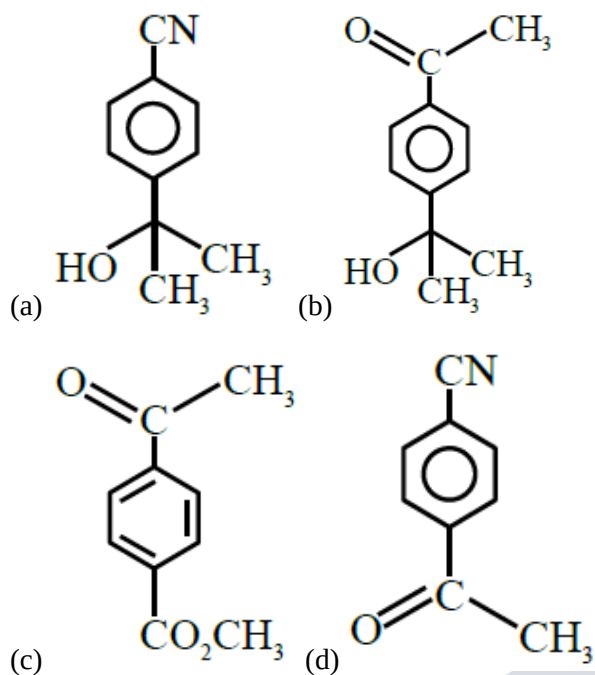
- (a) 
- (b) 
- (c) 
- (d) 

42. For a sparingly soluble salt AB_2 , the equilibrium concentrations of A^{2+} ions and B^- ions are $1.2 \times 10^{-4} M$ and $0.24 \times 10^{-3} M$, respectively. The solubility product of AB_2 is:

- (a) 0.069×10^{-12} (b) 6.91×10^{-12}
- (c) 0.276×10^{-12} (d) 27.65×10^{-12}

43. Major product of the following reaction is





44. Given below are two statements:

Statement I: The higher oxidation states are more stable down the group among transition elements unlike p-block elements.

Statement II: Copper cannot liberate hydrogen from weak acids.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement I and Statement II are false
- (b) Statement I is false but Statement II is true
- (c) Both Statement I and Statement II are true
- (d) Statement I is true but Statement II is false

45. The incorrect statement regarding ethyne is

- (a) The C – C bonds in ethyne is shorter than that in ethene
- (b) Both carbons are sp hybridised
- (c) Ethyne is linear
- (d) The carbon-carbon bonds in ethyne is weaker than that in ethene

46. Match List I with List II

List-I (Element)		List-II (Electronic Configuration)	
A.	N	I.	[Ar]3 d ¹⁰ 4 s ² 4p ⁵
B.	S	II.	[Ne]3 s ² 3p ⁴

C.	Br	III.	$[\text{He}]2s^22p^3$
D	Kr	IV.	$[\text{Ar}]3d^{10}4s^24p^6$

Choose the correct answer from the options given below:

- (a) A-IV, B-III, C-II, D-I (b) A-III, B-II, C-I, D-IV
(c) A-I, B-IV, C-III, D-II (d) A-II, B-I, C-IV, D-III

47. Match List I with List II

List-I		List-II	
A.	Melting point [K]	I.	$\text{Tl} > \text{In} > \text{Ga} > \text{Al} > \text{B}$
B.	Ionic Radius $[\text{M}^{+3}/\text{pm}]$	II.	$\text{B} > \text{Tl} > \text{Al} \approx \text{Ga} > \text{In}$
C.	$\Delta_f H_1$ $[\text{kJ mol}^{-1}]$	III.	$\text{Tl} > \text{In} > \text{Al} > \text{Ga} > \text{B}$
	Atomic Radius [pm]	IV.	$\text{B} > \text{Al} > \text{Tl} > \text{In} > \text{Ga}$

Choose the correct answer from the options given below:

- (a) A-III, B-IV, C-I, D-II (b) A-II, B-III, C-IV, D-I
(c) A-IV, B-I, C-II, D-III (d) A-I, B-II, C-III, D-IV

48. Which of the following compounds will give silver mirror with ammoniacal silver nitrate?

- (A) Formic acid (B) Formaldehyde
(C) Benzaldehyde (D) Acetone

Choose the correct answer from the options given below:

- (a) C and D only (b) A, B and C only
(c) A only (d) B and C only

49. Which out of the following is a correct equation to show change in molar conductivity with respect to concentration for a weak electrolyte, if the symbols carry their usual meaning:

- (a) $\Lambda_m^2 - K_a \Lambda_m^{\circ 2} + K_a \Lambda_m \Lambda_m^{\circ} = 0$

$$(b) \Lambda_m - \Lambda_m^\circ + AC^{\frac{1}{2}} = 0$$

$$(c) \Lambda_m - \Lambda_m^\circ - AC^{\frac{1}{2}} = 0$$

$$(d) \Lambda_m^2 C + K_a \Lambda_m^2 - K_a \Lambda_m \Lambda_m^\circ = 0$$

50. The electronic configuration of Einsteinium is : (Given atomic number of Einsteinium = 99)

- (a) $[Rn]5f^{12}6d^07s^2$ (b) $[Rn]5f^{11}6d^07s^2$
(c) $[Rn]5f^{13}6d^07s^2$ (d) $[Rn]5f^{10}6d^07s^2$

SECTION- B

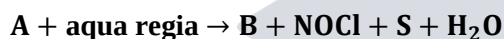
51. Number of oxygen atoms present in chemical formula of fuming sulphuric acid is ____

52. A transition metal 'M' among Sc, Ti, V, Cr, Mn and Fe has the highest second ionisation enthalpy. The spin only magnetic moment value of M^+ ion is ____ BM (Near integer)

(Given atomic number Sc: 21, Ti: 22, V: 23, Cr : 24, Mn: 25, Fe: 26)

53. The vapour pressure of pure benzene and methyl benzene at 27°C is given as 80 Torr and 24 Torr, respectively. The mole fraction of methyl benzene in vapour phase, in equilibrium with an equimolar mixture of those two liquids (ideal solution) at the same temperature is $\times 10^{-2}$ (nearest integer)

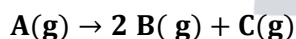
54. Consider the following test for a group-IV cation.



The spin only magnetic moment value of the metal complex C is ____ BM.

(Nearest integer)

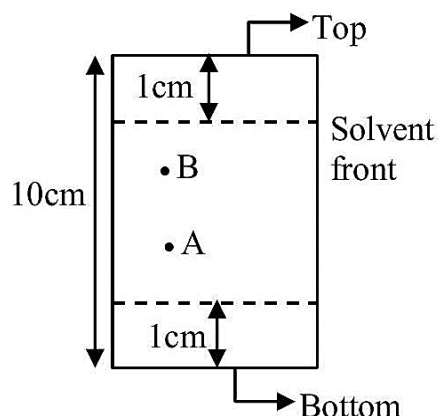
55. Consider the following first order gas phase reaction at constant temperature



If the total pressure of the gases is found to be 200 torr after 23sec. and 300 torr upon the complete decomposition of A after a very long time, then the rate constant of the given reaction is $\times 10^{-2} \text{ s}^{-1}$ (nearest integer)

[Given: $\log_{10}(b) = 0.301$]

56.



In the given TLC, the distance of spot A & B are 5 cm & 7 cm, from the bottom of TLC plate, respectively. R_f value of B is $x \times 10^{-1}$ times more than A. The value of x is _____.

57. Based on Heisenberg's uncertainty principle, the uncertainty in the velocity of the electron to be found within an atomic nucleus of diameter 10^{-15} m is _____ $\times 10^9$ ms^{-1} (nearest integer)

[Given: mass of electron = 9.1×10^{-31} kg, Planck's constant (h) = 6.626×10^{-34} Js]

(Value of $\pi = 3.14$)

58. Number of compounds from the following which cannot undergo Friedel-Crafts reactions is : _____ toluene, nitrobenzene, xylene, cumene, aniline, chlorobenzene, m-nitroaniline, m-dinitrobenzene

59. Total number of electron present in (π^*) molecular orbitals of O_2 , O_2^+ and O_2^- is _____

60. When $\Delta H_{\text{vap}} = 30$ kJ/mol and $\Delta S_{\text{vap}} = 75$ J mol^{-1} K^{-1} , then the temperature of vapour, at one atmosphere is _____ K.

PART-III MATHEMATICS

SECTION- A

61. $\lim_{x \rightarrow 0} \frac{e^{-(1+2x)^{\frac{1}{2x}}}}{x}$ is equal to:

- (a) e (b) $\frac{-2}{e}$
(c) 0 (d) $e - e^2$

62. Consider the line L passing through the points (1, 2, 3) and (2, 3, 5). The distance of the point $(\frac{11}{3}, \frac{11}{3}, \frac{19}{3})$ from the line L along the line

$\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to:

- (a) 3 (b) 5
(c) 4 (d) 6

63. Let $\int_0^x \sqrt{1 - (y'(t))^2} dt = \int_0^x y(t) dt$, $0 \leq x \leq 3$, $y \geq 0$, $y(0) = 0$. Then at $x = 2$, $y'' + y + 1$ is equal to:

- (a) 1 (b) 2
(c) $\sqrt{2}$ (d) $1/2$

64. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z - (6 + 8i)|$ is equal to:

- (a) 12 (b) ∞
(c) 10 (d) 8

65. The area (in square units) of the region enclosed by the ellipse $x^2 + 3y^2 = 18$ in the first quadrant below the line $y = x$ is :

- (a) $\sqrt{3}\pi + \frac{3}{4}$ (b) $\sqrt{3}\pi$
(c) $\sqrt{3}\pi - \frac{3}{4}$ (d) $\sqrt{3}\pi + 1$

66. Two vertices of a triangle ABC are A(3, -1) and B(-2, 3), and its orthocentre is P(1, 1). If the coordinates of the point C are (α , β) and the centre of the circle circumscribing the triangle PAB is (h, k), then the value of $(\alpha + \beta) + 2(h + k)$ equals :

- (a) 51 (b) 81
(c) 5 (d) 15

67. If the variance of the frequency distribution is 160, then the value of $c \in \mathbb{N}$ is

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

- (a) 5 (b) 8
(c) 7 (d) 6

68. Let the range of the function

$$f(x) = \frac{1}{2 + \sin 3x + \cos 3x}, x \in \mathbb{R} \text{ be } [a, b].$$

If α and β are respectively the A.M. and the G.M.

of a and b , then $\frac{\alpha}{\beta}$ is equal to:

- (a) $\sqrt{2}$ (b) 2
(c) $\sqrt{\pi}$ (d) π

69. Between the following two statements:

Statement-I: Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a} \times \vec{r} = \vec{a} \times \vec{b}$ and $\vec{a} \cdot \vec{r} = 0$ is of magnitude $\sqrt{10}$.

Statement-II: In a triangle ABC, $\cos 2A + \cos 2B$

$$+ \cos 2C \geq -\frac{3}{2}.$$

- (a) Both Statement-I and Statement-II are incorrect
(b) Statement-I is incorrect but Statement-II is correct
(c) Both Statement-I and Statement-II are correct
(d) Statement-I is correct but Statement-II is incorrect

70. $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\int_{\frac{\pi}{2}}^x \frac{\sin(2t^{1/3}) + \cos(t^{1/3})}{(x - \frac{\pi}{2})^2} dt \right)$ is equal

to:

- (a) $\frac{9\pi^2}{8}$ (b) $\frac{11\pi^2}{10}$
(c) $\frac{3\pi^2}{2}$ (d) $\frac{5\pi^2}{9}$

71. Let the foci of a hyperbola H coincide with the foci of the ellipse $E: \frac{(x-1)^2}{100} + \frac{(y-1)^2}{75} = 1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β , then $3\alpha^2 + 2\beta^2$ is equal to:

- (a) 242 (b) 225
(c) 237 (d) 205

72. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $(x^{2/3} + \frac{1}{2}x^{-2/5})^9$ is :

- (a) 21/4 (b) 69/16
(c) 63/16 (d) 19/4

73. Let $B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$ and A be a 2×2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4 + \alpha C^2 + \beta I = O$, then $2\beta - \alpha$ is equal to:

- (a) 16 (b) 2
(c) 8 (d) 10

74. If $\log_e y = 3\sin^{-1} x$, then $(1-x)^2 y'' - xy'$ at $x = \frac{1}{2}$ is equal to :

- (a) $9e^{\pi/6}$ (b) $3e^{\pi/6}$
(c) $3e^{\pi/2}$ (d) $9e^{\pi/2}$

75. The integral $\int_{1/4}^{3/4} \cos \left(2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right) dx$ is equal to:
- (a) $-1/2$ (b) $1/4$
(c) $1/2$ (d) $-1/4$
76. Let $a, ar, ar^2, \dots$ be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$, then $a + 18r$ is equal to :
- (a) 27 (b) 46
(c) 38 (d) 31
77. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{\text{th}}$ roll, $i = 2, 3$, is equal to :
- (a) $3/54$ (b) $2/54$
(c) $5/54$ (d) $1/54$
78. The value of the integral $\int_{-1}^2 \log_e (x + \sqrt{x^2 + 1}) dx$ is:
- (a) $\sqrt{5} - \sqrt{2} + \log_e \left(\frac{9+4\sqrt{5}}{1+\sqrt{2}} \right)$ (b) $\sqrt{2} - \sqrt{5} + \log_e \left(\frac{9+4\sqrt{5}}{1+\sqrt{2}} \right)$
(c) $\sqrt{5} - \sqrt{2} + \log_e \left(\frac{7+4\sqrt{5}}{1+\sqrt{2}} \right)$ (d) $\sqrt{2} - \sqrt{5} + \log_e \left(\frac{7+4\sqrt{5}}{1+\sqrt{2}} \right)$
79. Let $\alpha, \beta; \alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $P_n = \alpha^n - \beta^n, n \in \mathbb{N}$. Then $(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$ is equal to:
- (a) $10\sqrt{2}P_9$ (b) $10\sqrt{3}P_9$
(c) $11\sqrt{2}P_9$ (d) $11\sqrt{3}P_9$
80. Let $\vec{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = \beta\hat{j} - \hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let the values of the ordered pair (α, β) for which the area of the parallelogram of diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2) .
- Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$ is equal to
- (a) 17 (b) 24
(c) 21 (d) 19

SECTION- B

81. Consider the circle $C: x^2 + y^2 = 4$ and the parabola $P: y^2 = 8x$. If the set of all values of α , for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q) , then $(2q - p)^2$ is equal to _____
82. Let the set of all values of p , for which $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b) . Then $16ab$ is equal to _____
83. For a differentiable function $f: \mathbb{R} \rightarrow \mathbb{R}$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in \mathbb{R}, f(0) = 1$ and $\lim_{x \rightarrow -\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to _____
84. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is _____.
85. Let $A = \{(x, y): 2x + 3y = 23, x, y \in \mathbb{N}\}$ and $B = \{x: (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to _____
86. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x -axis at the point D . Let M and N respectively be the feet of the perpendiculars from A and B on L .

Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to _____

87. The square of the distance of the image of the point $(6, 1, 5)$ in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is

88. If $\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+1012}\right)$

$-\left(\frac{1}{2 \cdot 1} + \frac{1}{4 \cdot 3} + \frac{1}{6 \cdot 5} + \dots + \frac{1}{2024 \cdot 2023}\right)$

$= \frac{1}{2024}$, then α is equal to-

89. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1} x + 3\cos^{-1} x = \frac{2\pi}{5}$, is ____

90. Consider the matrices: $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ m \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m , for which the system of equations $AX = B$ has a negative solution (i.e., $x < 0$ and $y < 0$), be the interval (a, b) . Then $8 \int_a^b |A| dm$ is equal to. _____

