

1. (c) $\vec{E} = 20 \sin \omega \left(t - \frac{x}{c} \right) \hat{j} \text{ N/C}$

Average energy density of an em wave $= \frac{1}{2} \epsilon_0 E_0^2$

Energy stored $= \left(\frac{1}{2} \epsilon_0 E_0^2 \right) (\text{volume})$

$= \frac{1}{2} \times 8.85 \times 10^{-12} \times (20)^2 \times (5 \times 10^{-4}) \text{ J}$

$= 8.85 \times 10^{-13} \text{ J}$

2. (c) Area under the graph from $t = 0$ to $t = 20 \text{ sec} = 200 \text{ m}$

Area under the graph from $t = 20$ to $t = 25 \text{ sec} = 50 \text{ m}$

So distance covered $= (200 + 50) \text{ m} = 250 \text{ m}$

Displacement $= (200 - 50) \text{ m} = 150 \text{ m}$

$\frac{250}{150} = \frac{5}{3}$

3. (c) $g = \frac{GM}{R^2} = \frac{G}{R^2} \times \rho \times \frac{4\pi}{3} R^3 = \left(\frac{4\pi}{3} G \right) \rho R$

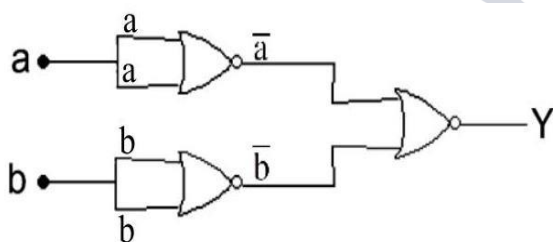
$\frac{g_A}{g_B} = \frac{R \times \rho}{4R \times \frac{\rho}{3}} = \frac{3}{4}$

4. (d) $f_{\text{max}} = \mu mg = m\omega^2 R \Rightarrow R = \frac{\mu g}{\omega^2}$

So, if ω becomes $\frac{\omega}{2}$, R will become $4R$.

So, distance from the center will be 4 cm .

5. (a)



$Y = \overline{\overline{a} + \overline{b}} = a \cdot b$

The truth table for the given circuit will be

a	b	output
0	0	0

0	1	0
1	0	0
1	1	1

Hence it will be equivalent to AND gate.

6. (b) Initially

$$Q_1 = CV = (b)V$$

$$E_1 = \frac{1}{2}CV^2 = \frac{1}{2}(b)V^2 = V^2$$

Finally

$$\text{Charge on each capacitor, } Q_2 = \frac{Q_1}{2} = \frac{2V}{2} = V$$

$$E_2 = 2 \left(\frac{1}{2} \frac{Q_2^2}{C} \right) = \frac{V^2}{2} \therefore \frac{E_2}{E_1} = \frac{1}{2}$$

7. (a) Parallel combination

$$H_p = \left[\frac{V^2}{\left(\frac{R}{2}\right)} \right] t = \frac{2V^2 t}{R}$$

Series combination

$$H_s = \left(\frac{V^2}{2R} \right) t \therefore \frac{H_p}{H_s} = 4$$

$$8. (a) d_r = \sqrt{2 h_r R} \therefore h_r = \frac{d_r^2}{2R}$$

$$= \frac{(4 \text{ km})^2}{2(6400 \text{ km})} = \left(\frac{1}{800} \right) \text{ km} = 1.25 \text{ m}$$

$$9. (c) X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C}$$

$$\therefore X_C \propto \frac{1}{f}$$

$\therefore$ Curve A

$$X_L = \omega L = (2\pi f)L$$

$$\therefore X_L \propto f$$

$\therefore$ Curve B

$$10. (a) \Delta Q = \Delta U + \Delta W$$

$$\therefore \Delta U = \Delta Q - \Delta W$$

$$= mL_V - P\Delta V$$

$$= (1\text{Kg})(2257 \times 10^3 \text{ J/kg})$$

$$-(1 \times 10^5 \text{ Pa})(1.671 \text{ m}^3 - 1 \times 10^{-3} \text{ m}^3)$$

$$= 2257 \times 10^3 \text{ J} - 167 \times 10^3 \text{ J}$$

$$= 2090\text{KJ}$$

11. (b) i_c = Critical angle

$$\frac{v}{c} = \frac{1}{\mu} = \sin i_c = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\Rightarrow v = \frac{c}{\sqrt{2}} = \frac{3 \times 10^8}{\sqrt{2}} \text{ m/s} = 2.12 \times 10^8 \text{ m/s}$$

12. (d) From the equation of photoelectric effect

$$eV_0 = \frac{hc}{\lambda} - \phi_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$\& \frac{eV}{4} = \frac{hc}{2\lambda} = \frac{hc}{\lambda_0}$$

$$\Rightarrow \frac{1}{4} \left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0} \right) = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$$

$$\frac{1}{\lambda_0} - \frac{1}{4\lambda_0} = \frac{1}{2\lambda} - \frac{1}{4\lambda}$$

$$\frac{3}{4\lambda_0} = \frac{1}{4\lambda}$$

$$\Rightarrow \lambda_0 = 3\lambda$$

13. (a)

$$\text{As } X_m = 2 \times 10^{-2}$$

$$\mu_r = 1 + X_m = 1.02$$

$$\Rightarrow B = \mu_r B_0 = 1.02 B_0$$

$$\text{So percentage increase in magnetic field} = \frac{B - B_0}{B_0} \times 100\% = 2\%$$

$$14. (a) I_S = \frac{NBA}{c} \& V_S = \frac{NBA}{CG}$$

$$\Rightarrow V_S = \frac{I_S}{G}, \text{ If } G \text{ (galvanometer resistance) is constant, then } V_S \propto I_S \text{ so percentage change in } V_S \text{ is also } 25\%.$$

15. (d) For a particle executing SHM

$$KE = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

When $x = 0$, KE is maximum & when $x = A$, KE is zero and KEV/Sx graph is parabola

$$16. (d) \frac{X - X_{\text{freez}}}{X_{\text{boil}} - X_{\text{freez}}} = \frac{t - 32}{212 - 32}$$

$$\frac{-95 - (-15)}{65 - (-15)} = \frac{t - 32}{180}$$

$$\frac{-80}{80} = \frac{t - 32}{180}$$

$$t = -180 + 32$$

$$t = -148^\circ\text{F}$$

$$17. (b) 1\text{AU} = 1.496 \times 10^{11} \text{ m}$$

$$1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$$

$$1 \text{ light year} = 9.46 \times 10^{15} \text{ m}$$

So, $\text{Au} < \text{ly} < \text{Per sec}$

18. (b)

$$v_{\text{rms}}(\text{mono}) = \sqrt{\frac{3RT}{4 \times 10^{-3}}}$$

$$v_{\text{rms}}(\text{dia}) = \sqrt{\frac{3RT}{71 \times 10^{-3}}}$$

$$v_{\text{rms}}(\text{ply}) = \sqrt{\frac{3RT}{146 \times 10^{-3}}}$$

So correct relation is

$$v_{\text{rms}}(\text{mono}) > v_{\text{rms}}(\text{dia}) > v_{\text{rms}}(\text{poly})$$

$$19. (b) F = nmv$$

$$\text{where } n = \text{number of bullets fired per second } n = \frac{f}{mv} = \frac{125}{10 \times 10^{-3} \times 250} = 50$$

$$20. (c) T_{1/2}(A) = T_{\text{av}}(B)$$

$$\frac{\ln 2}{\lambda_A} = \frac{1}{\lambda_B}$$

$$\lambda_A = \lambda_B \ell 2$$

$$21. (3) 6 = {}^4C_2 \Rightarrow n_2 = 4$$

$$h\nu = E_4 - E_1$$

$$\therefore v = 13.6 \left(\frac{1}{1^2} - \frac{1}{4^2} \right) \times \frac{1}{4.25 \times 10^{-15}}$$

$$= 3 \times 10^{15} \text{ Hz}$$

22. (4) $P = (1.8 - 1) \left(\frac{1}{20} + \frac{1}{20} \right)$ by lens maker's formula

$$P' = \left(\frac{1.8}{1.5} - 1 \right) \left(\frac{1}{20} + \frac{1}{20} \right)$$

Dividing $\frac{P}{P'} = \frac{0.8}{1.2-1} = 4$

23. (1152)

$$V = \frac{\omega}{k} = \frac{2\pi \times 60}{2\pi \times 0.5} = \frac{160}{0.5} \text{ m/s}$$

$$= \frac{160}{0.5} \times \frac{18}{5} \text{ km/h}$$

$$= 1152 \text{ km}$$

24. (32)

$$W = \int_0^4 (2 + 3x) dx$$

$$= \left[2x + \frac{3x^2}{2} \right]_0^4$$

$$= 8 + 3 \times 8$$

$$= 32 \text{ J}$$

25. (300) For equilibrium along the plane

$$mg \sin \theta = \frac{1}{4\pi\epsilon_0} \times \frac{q_0^2}{(h \csc 30^\circ)^2}$$

$$\therefore h^2 = \frac{1}{4\pi\epsilon_0} \times \frac{q_0^2}{mg \csc 30^\circ}$$

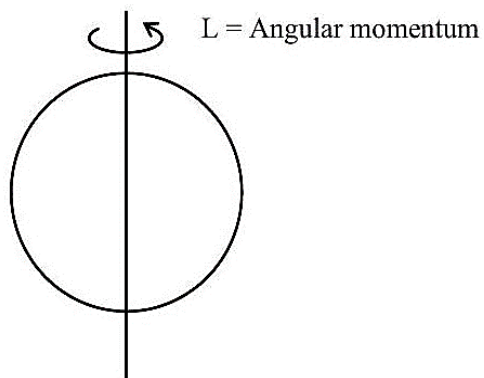
$$= 9 \times 10^9 \times \frac{(2 \times 10^{-6})^2}{0.02 \times 10 \times 2}$$

$$\therefore h = 3 \times 10^4 \times \frac{2 \times 10^{-6}}{0.2}$$

$$= 0.3 \text{ m}$$

$$= 300 \text{ mm}$$

26. (35)

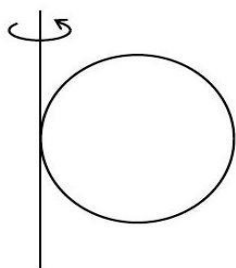


$$m = 500 \text{ g} = 0.5 \text{ kg}$$

$$R = 5 \text{ cm}$$

$$\omega = 10 \text{ rad/sec}$$

moment of inertia about tangent = I_T



$$I_t = x \times 10^{-2} L$$

$$\frac{7}{5} mR^2 = x \times 10^{-2} \frac{2}{5} mR^2 \omega$$

$$\frac{7}{2\omega} = x \times 10^{-2} = \frac{7}{2 \times 10}$$

27. (2) Let the original length be ' ℓ_0 '

When $T_1 = 100 \text{ N}$, Extension = $\ell_1 - \ell_0$

When $T_2 = 120 \text{ N}$, Extension = $\ell_2 - \ell_0$

$$\text{Then } 100 = K(\ell_1 - \ell_0)$$

$$\text{And } 120 = K(\ell_2 - \ell_0)$$

$$\frac{1}{2} \Rightarrow \frac{5}{6} = \frac{\ell_1 - \ell_0}{\ell_2 - \ell_0}$$

$$5\ell_2 - 5\ell_0 = 6\ell_1 - 6\ell_0$$

$$\ell_0 = 6\ell_1 - 5\ell_2$$

$$\ell_0 = 6\ell_1 - 5\left(\frac{11\ell_1}{10}\right)$$

$$\ell_0 = 6\ell_1 - \frac{11\ell_1}{2}$$

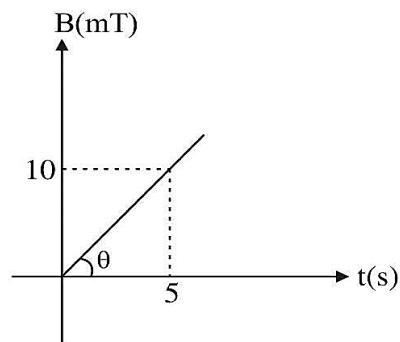
$$\ell_0 = \frac{\ell_1}{2}$$

$$\therefore x = 2$$

$$28. (8) m = \tan \theta = \frac{10}{5} = 2$$

$$B = mt$$

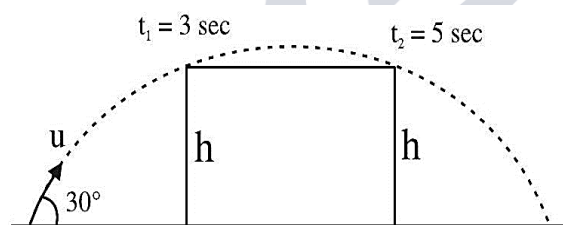
$$B = 2t$$



$$\varepsilon = \left| \frac{d\phi}{dt} \right| = \frac{d(BA)}{dt} = \frac{A dB}{dt}$$

$$\varepsilon = \frac{4 d(2t)}{dt} = 4 \times 2 = 8 \text{ mV}$$

$$29. (80) \text{ Time of flight } t_1 + t_2 = 3 + 5 = 8 \text{ sec}$$

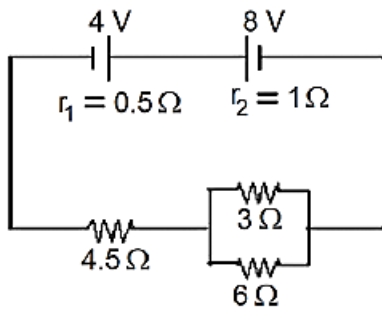


$$T = \frac{2u \sin 30^\circ}{g}$$

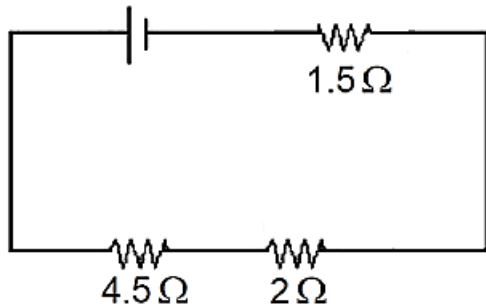
$$8 = \frac{2u \sin(30^\circ)}{10}$$

$$u = 80 \text{ m/s}$$

$$30. (1)$$

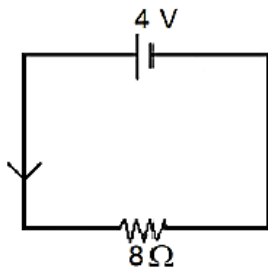


$$E_2 - E_1 = 8 - 4 = 4V$$

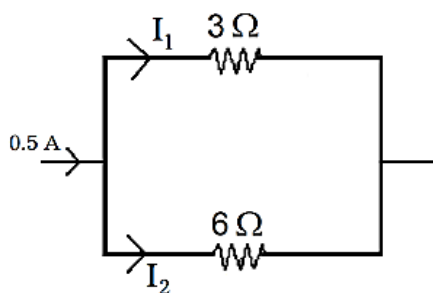


$$\frac{1}{3} + \frac{1}{6} = \frac{1}{2} = \frac{1}{R}$$

$$R = 2\Omega$$



$$I = \frac{4}{8} = 0.5A$$

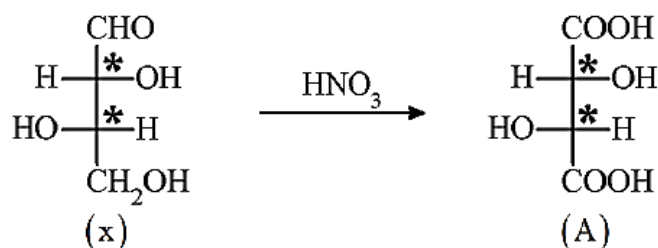


$$I_1 = \left(\frac{6}{3+6} \right) \times 0.5$$

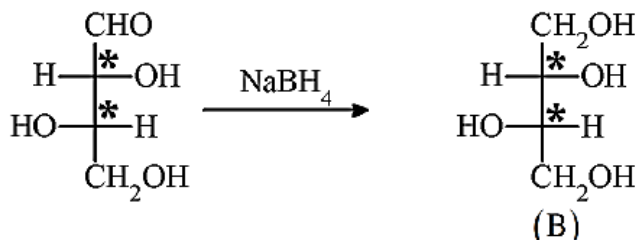
$$I_1 = \frac{2}{3} \times 0.5 = \frac{1}{3} A$$

$$I_1 = \frac{x}{3} = \frac{1}{3} \therefore x = 1$$

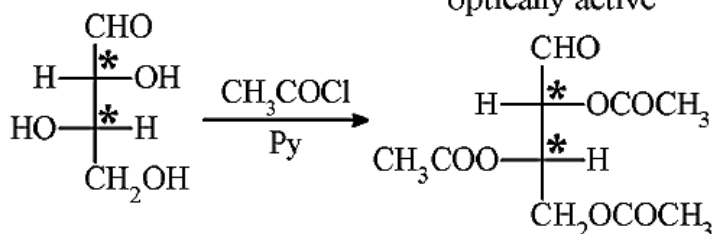
31. (b)



L-tetrose with two chiral centre



optically active

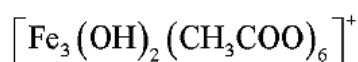
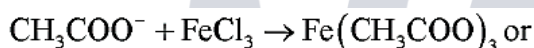


(x) gives positive schiff's test due $-\text{CHO}$ group

(x) is L-tetrose

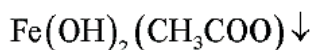
32. (d) Ethene undergoes addition polymerisation to high density polythene in the presence of catalyst such as AlEt_3 and TiCl_4 (Ziegler - Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6-7 atmosphere.

33. (a)

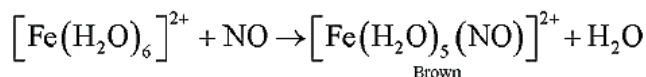
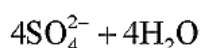
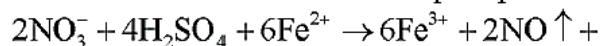


Blood red colour

$\downarrow \Delta$



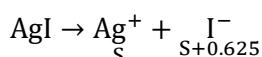
Red-brown precipitate



Brown

34. (b) There is a characteristic minimum frequency, or "threshold frequency," for each metal below which the photoelectric effect is not seen. The ejected electrons leave with a specific amount of kinetic energy at a frequency $\nu > \nu_0$ with an increase in light frequency of these electron kinetic energies also rise.

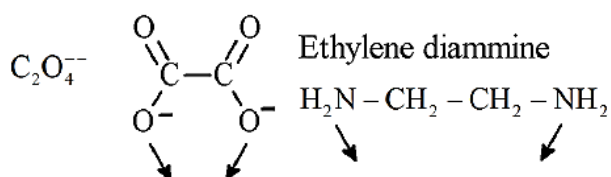
35. (c) $\text{AgNO}_3 + \text{KI} \rightarrow \text{AgI} \downarrow + \text{KNO}_3$



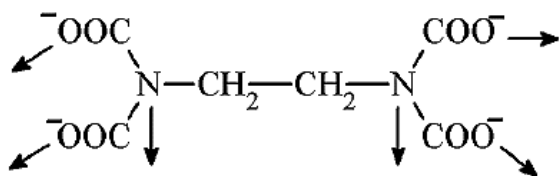
AgI is an insoluble salt so concentration Ag^+ and I^- will be negligible.

36. (d)

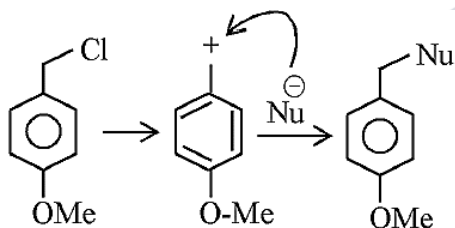
37. (a) NO_2^- , NCS^- are ambidentate ligands



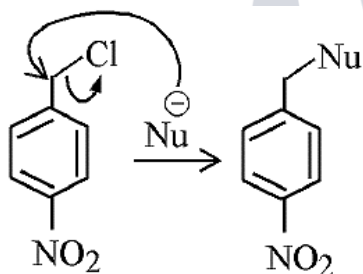
EDTA Ethylenediamine tetra acetate



38. (c)



Electron Donating group
 S_N^1 Mech.: 1st order



Electron withdrawing group
 S_N^2 Mech: 2nd order

39. (c) First I.E.

$\text{F} > \text{N} > \text{O} > \text{C} > \text{Be} > \text{B} > \text{Li}$

Li – 520 kJ/mol

Be – 899 kJ/mol

B – 801 kJ/mol

C – 1086 kJ/mol

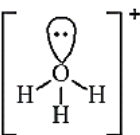
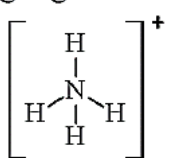
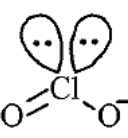
N – 1402 kJ/mol

O – 1314 kJ/mol

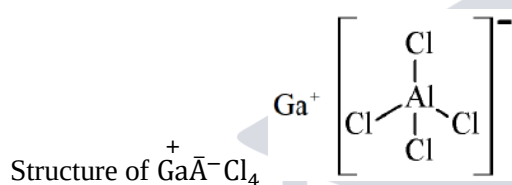
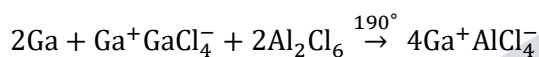
F – 1681 kJ/mol

40. (a)

Molecule/Ion Hybridisation Shape

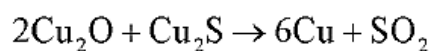
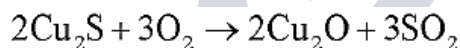
H_3O^+	sp^3	Pyramidal	
Acetylide	sp	linear	$\text{C} \equiv \text{C}$
NH_4^+	sp^3	tetrahedral	
ClO_2^-	sp^3	Bent	

41. (a) Gallous tetrachloro aluminate $\text{Ga}^+\text{AlCl}_4^-$



Ga is cationic part of salt GaAlCl_4 .

42. (a)



Blister copper

Due to evolution of SO_2 , the solidified copper formed has a blistered look and is referred to as blister copper.

43. (a)

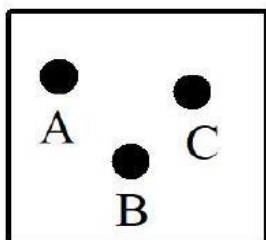
K^+ - Sodium - Potassium Pump

KCl - Fertiliser

KOH - absorber of CO_2

Li - used in thermonuclear reactions

44. (a)



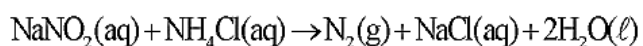
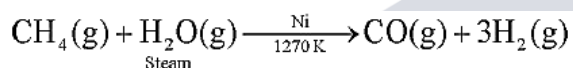
According to the observation, A is more mobile and interacts with the mobile phase more than C, and C is more drawn to the mobile phase than B.

Hence, the correct order of elution in the silico gel column chromatography is $-B < C < A$

45. (a) $[MA_3 B_3]$ type of compound exists as facial and meridional isomer.



46. (a)



47. (c) Clean water would have BOD value of less than 5ppm.

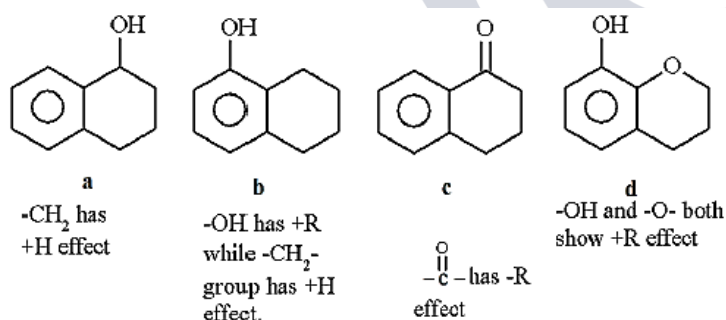
Maximum limit of Zn in clean water

$$= 5.0\text{ppm or mgdm}^{-3}$$

Maximum limit of NO_3^- in clean water

$$= 50\text{ppm or mgdm}^{-3}$$

48. (c) Benzene becomes more reactive towards EAS when any substituent raises the electron density.



Correct order

$$c < a < b < d$$

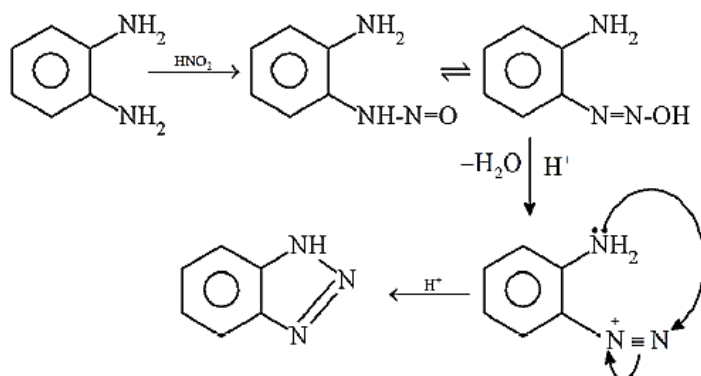
49. (b) $Fe_4[Fe(CN)_6]_3$ Prussian Blue-water insoluble

$K_3[Co(NO_2)_6]$ very poorly water soluble

$(NH_4)_3[As(MO_3O_{10})_4]$ water insoluble ammonium arseno molybdate

$[\text{Fe}_3(\text{OH})_2(\text{OAc})_6]\text{Cl}$ is water soluble.

50. (a) Orthophenyl amine



51. (44)

$\text{CO}_{(\text{g})} + \text{H}_2\text{O}_{(\text{g})} \rightleftharpoons \text{CO}_{2(\text{g})} + \text{H}_{2(\text{g})}$
 $t = 0$ 1 mol 1 mol 0 0
 at equ. $1 - x$ $1 - x$ x x
 at equilibrium 40% by mass water reacts with CO

$$x = 0.4 \quad 1 - x = 0.6$$

$$K_c = \frac{[\text{CO}_2][\text{H}_2]}{[\text{CO}][\text{H}_2\text{O}]} = \frac{0.4 \times 0.4}{0.6 \times 0.6} = 0.44$$

$$K_c \times 10^2 = 44$$

52. (1)

Spin magnetic moment of $[\text{Cr}(\text{CN})_6]^{3-} (t_2^3 e_g^0) \mu_1 = \sqrt{3(3+2)} = \sqrt{15} \text{ BM}$

Spin magnetic moment of $[\text{Cr}(\text{H}_2\text{O})_6]^{3+} (t_2^3 e_g^0)$

$$\mu_2 = \sqrt{3(3+2)} = \sqrt{15} \text{ BM}$$

$$\frac{\mu_1}{\mu_2} = \frac{\sqrt{15}}{\sqrt{15}} = 1$$

53. (4)

$$d = 3 \text{ g/cc}$$

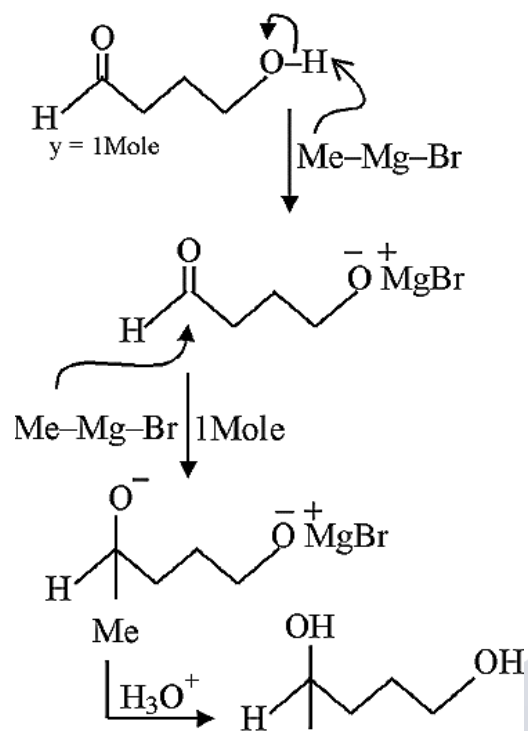
$$M = 12 \text{ g/mol}$$

$$a = 300 \text{ pm} = 3 \times 10^{-8} \text{ cm}$$

$$Z = \frac{d \times N_A \times a^3}{M} = \frac{3 \times 6.02 \times 10^{23} \times (3 \times 10^{-8})^3}{12}$$

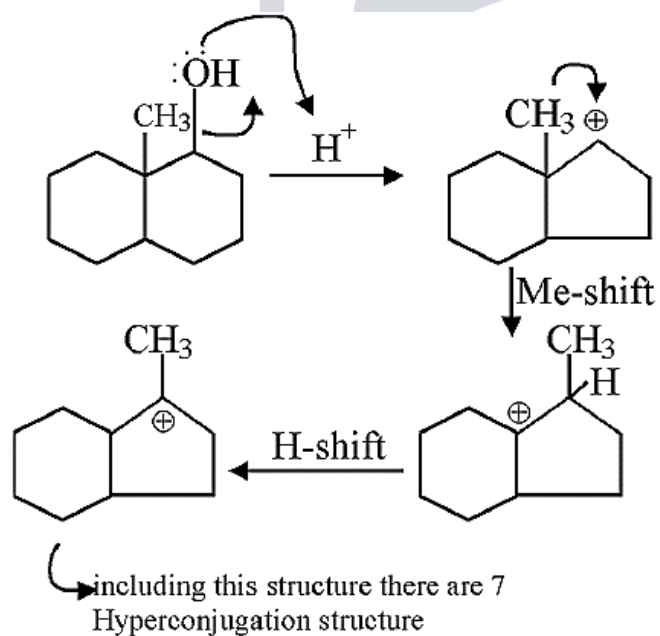
$$= 4.06 \approx 4$$

54. (2)

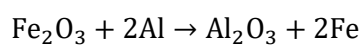

 $\therefore x = 2 \text{ mole}$

$$\frac{x}{y} = \frac{2}{1} = 2$$

55. (7)



56. (4)



Molar mass 160 g 27 g

$$\begin{aligned}
 (\Delta H_f^0)_{\text{reaction}} &= [(\Delta H_f^0)_{\text{Al}_2\text{O}_3} + 2(\Delta H_f^0)_{\text{Fe}}] - \\
 &\quad [(\Delta H_f^0)_{\text{Fe}_2\text{O}_3} + 2(\Delta H_f^0)_{\text{Al}}] \\
 &= [-1700 + 0] - [-840 + 0] \\
 &= -860 \text{ kJ/mol}
 \end{aligned}$$

Total mass of mixture = $\text{Fe}_2\text{O}_3 + \text{Al}$ (1: 2 molar ratio)

$$\begin{aligned}
 &= 160 + 2 \times 27 \\
 &= 214 \text{ g/mol}
 \end{aligned}$$

$$\text{Heat evolved per gram} = \frac{860}{214} = 4 \text{ kJ/g}$$

57. (36)

Total mass of sugar in mixture of 25% of 200 and 40% of 500 g

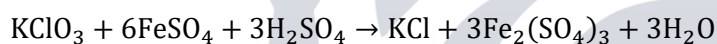
$$\text{Sugar solution} = 0.25 \times 200 + 0.40 \times 500$$

$$= 50 + 200 = 250 \text{ g}$$

$$\text{Total mass of solution} = 200 + 500 = 700 \text{ g}$$

$$\begin{aligned}
 \text{Mass of sugar in solution} &= \frac{250}{700} \times 100 = 35.7\% \\
 &\approx 36\%
 \end{aligned}$$

58. (333)



$$\begin{aligned}
 \text{ROR} &= -\frac{\Delta[\text{KClO}_3]}{\Delta t} = \frac{-1 \Delta[\text{FeSO}_4]}{6 \Delta t} \\
 &= \frac{+1 \Delta[\text{Fe}_2(\text{SO}_4)_3]}{3 \Delta t}
 \end{aligned}$$

$$\frac{\Delta[\text{Fe}_2(\text{SO}_4)_3]}{\Delta t} = \frac{1 - \Delta[\text{FeSO}_4]}{2 \Delta t}$$

$$= \frac{1(10 - 8.8)}{2 \times 30 \times 60}$$

$$= 0.333 \times 10^{-3}$$

$$= 333 \times 10^{-6} \text{ mol litre}^{-1} \text{ sec}^{-1}$$

59. (75) Isotonic solutions,

$$\pi_{\text{K}_2\text{SO}_4} = \pi_{\text{Glucose}}$$

$$i \times 0.004 \times RT = 0.01 \times RT$$

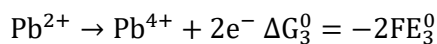
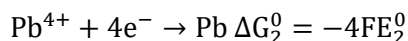
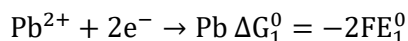
$$i = 2.5$$

For K_2SO_4 for dissociation $i = 1 + (n - 1)\alpha$

$$\text{DOD}(\alpha) = \frac{i - 1}{n - 1} = \frac{2.5 - 1}{3 - 1} = 0.75$$

% dissociation = 75

60. (2)



$$\Delta G_3^0 = \Delta G_1^0 - \Delta G_2^0$$

$$-2FE_3^0 = 2F(2n - m)$$

$$E_3^0 = m - 2n = m - xn$$

Hence $x = 2$

61. (d) $I = \int_{-\ln 2}^{\ln 2} e^x \left(\ln(e^x + \sqrt{1 + e^{2x}}) \right) dx$

Put $e^x = t \Rightarrow e^x dx = dt$

$$I = \int_{1/2}^2 \ln(t + \sqrt{1 + t^2}) dt$$

Applying integration by parts.

$$= \left[t \ln(t + \sqrt{1 + t^2}) \right]_{1/2}^2 - \int_{1/2}^2 \frac{t}{t + \sqrt{1 + t^2}} \left(1 + \frac{2t}{2\sqrt{1 + t^2}} \right) dt$$

$$= 2 \ln(2 + \sqrt{5}) - \frac{1}{2} \ln\left(\frac{1 + \sqrt{5}}{2}\right) - \int_{1/2}^2 \frac{t}{\sqrt{1 + t^2}} dt$$

$$= 2 \ln(2 + \sqrt{5}) - \frac{1}{2} \ln\left(\frac{1 + \sqrt{5}}{2}\right) - \frac{\sqrt{5}}{2}$$

$$= \ln\left(\frac{(2 + \sqrt{5})^2}{\left(\frac{\sqrt{5} + 1}{2}\right)^{\frac{1}{2}}}\right) - \frac{\sqrt{5}}{2}$$

62. (d) The equation of plane through $(-2, 3, 5)$ is

$$a(x + 2) + b(y - 3) + c(z - 5) = 0$$

it is perpendicular to $2x + 4y + 5z = 8$ & $3x - 2y + 3z = 5$

$$\therefore 2a + 4b + 5c = 0$$

$$3a - 2b + 3c = 0$$

$$\therefore \frac{a}{\begin{vmatrix} 4 & 5 \\ -2 & 3 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & 5 \\ 3 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix}}$$

$$\Rightarrow \frac{a}{22} = \frac{b}{9} = \frac{c}{-16}$$

∴ Equation of Plane is

$$22(x + 2) + 9(y - 3) - 16(z - 5) = 0$$

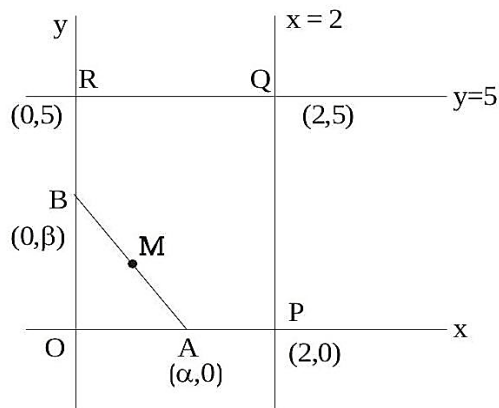
$$\Rightarrow 22x + 9y - 16z + 97 = 0$$

$$\text{Comparing with } \alpha x + \beta y + \gamma z + 97 = 0$$

$$\text{We get } \alpha + \beta + \gamma = 22 + 9 - 16 = 15$$

$$63. (b) \frac{\text{ar(OPQR)}}{\text{ar(OAB)}} = \frac{4}{1}$$

Let M be the mid-point of AB.



$$M(h, k) \equiv \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$$

$$\Rightarrow \frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta} = 4$$

$$\Rightarrow \frac{5}{2}\alpha\beta = 10 \Rightarrow \alpha\beta = 4$$

$$\Rightarrow (2h)(2k) = 4$$

∴ Locus of M is $xy = 1$

Which is a hyperbola.

$$64. (b) \omega A = \{a_1, a_2, a_3, a_4, a_5\}$$

$$B = \{b_1, b_2, b_3, b_4, b_5\}$$

$$\text{Given, } \sum_{i=1}^5 a_i = 25, \sum_{i=1}^5 b_i = 40$$

$$\frac{\sum_{i=1}^5 a_i^2}{5} - \left(\frac{\sum_{i=1}^5 a_i}{5}\right)^2 = 12, \frac{\sum_{i=1}^5 b_i^2}{5} - \left(\frac{\sum_{i=1}^5 b_i}{5}\right)^2 = 20$$

$$\Rightarrow \sum_{i=1}^5 a_i^2 = 185, \sum_{i=1}^5 b_i^2 = 420$$

$$\text{Now, } C = \{C_1, C_2, \dots, C_{10}\}$$

$$\text{s.f. } C_i = a_i = 3 \quad \text{or} \quad b_i + 2$$

First five elements
Last five elements

$$\therefore \text{Mean of } C, \bar{C} = \frac{(\sum a_i - 15) + (\sum b_i + 10)}{10}$$

$$\bar{C} = \frac{10 + 50}{10} = 6$$

$$\therefore \sigma^2 = \frac{\sum_{i=1}^{10} C_i^2}{10} = (\bar{C})^2$$

$$= \frac{\sum (a_i - 3)^2 + \sum (b_i + 2)^2}{10} - (6)^2$$

$$= \frac{\sum a_i^2 + \sum b_i^2 - 6\sum a_i + 4\sum b_i + 65}{10} - 36$$

$$= \frac{185 + 420 - 150 + 160 + 65}{10} - 36$$

$$= 32$$

$$\therefore \text{Mean} + \text{Variance} = \bar{C} + \sigma^2 = 6 + 32 = 38$$

65. (d) Here $f(x) = [x(x-1)] + \{x\}$

$f(0^+) = -1 + 0 = -1$	$f(1^+) = 0 + 0 = 0$
$f(0) = 0$	$f(a) = 0$
	$f(1^-) = -1 + 1 = 0$

$$\therefore f(x) \text{ is continuous at } x = 1, \text{ discontinuous at } x = 0$$

66. (b) $x + y + z = 15$

$$\text{Total no. of solution} = {}^{15+3-1}C_{3-1} = 136$$

$$\text{Let } x = y \neq z$$

$$2x + z = 15 \Rightarrow z = 15 - 2x$$

$$\Rightarrow x \in \{0, 1, 2, \dots, 7\} - \{5\}$$

$$\therefore 7 \text{ solutions}$$

$$\therefore \text{there are 21 solutions in which exactly}$$

$$\text{Two of } x_1 y_1 z \text{ are equal}$$

$$\text{There is one solution in which } x = y = z$$

Required answer = $136 - 21 - 1 = 114$

67. (d) Without loss of generality

$$\text{Let } |a_1| \leq |a_2| \leq |a_3|$$

$$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \geq |a_3|^2$$

$$\Rightarrow |\vec{a}| \geq |a_3| = \max\{|a_1|, |a_2|, |a_3|\}$$

A is true

$$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \leq |a_3|^2 + |a_3|^2 + |a_3|^2$$

$$\Rightarrow |\vec{a}|^2 \leq 3|a_3|^2$$

$$\Rightarrow |\vec{a}| \leq \sqrt{3}|a_3| = \sqrt{3}\max\{|a_1|, |a_2|, |a_3|\}$$

$$\leq 3\max\{|a_1|, |a_2|, |a_3|\}$$

(B) is true

68. (d) $W_1 = z_1 i = (5 + 4i)i = -4 + 5i$

$$W_2 = z_2(-i) = (3 + 5i)(-i) = 5 - 3i$$

$$W_1 - W_2 = -9 + 8i$$

$$\text{Principal argument} = \pi - \tan^{-1}\left(\frac{8}{9}\right)$$

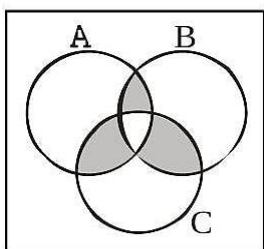
69. (c) $|A| = 48$

$$|B| = 25$$

$$|C| = 18$$

$$|A \cup B \cup C| = 60 \text{ [Total]}$$

$$|A \cap B \cap C| = 5$$



$$|A \cup B \cup C| = \sum |A| - \sum |A \cap B| + |A \cap B \cap C|$$

$$\Rightarrow \sum |A \cap B| = 48 + 25 + 18 + 5 - 60$$

$$= 36$$

No. of men who received exactly 2 medals

$$= \sum |A \cap B| - 3 |A \cap B \cap C|$$

$$= 36 - 15$$

$$= 21$$

70. (a) $M \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a, b, c, d, \in \{0,1,2\}$

$$n(s) = 3^4 = 81$$

we first bound $p(\bar{A})$

$$|m| = 0 \Rightarrow ad = bc$$

$$ad = bc = 0 \Rightarrow \text{no. of } (a, b, c, d) = (3^2 - 2^2)^2 = 25$$

$$ad = bc = 1 \Rightarrow \text{no. of } (a, b, c, d) = 1^2 = 1$$

$$ad = bc = 2 \Rightarrow \text{no. of } (a, b, c, d) = 2^2 = 4$$

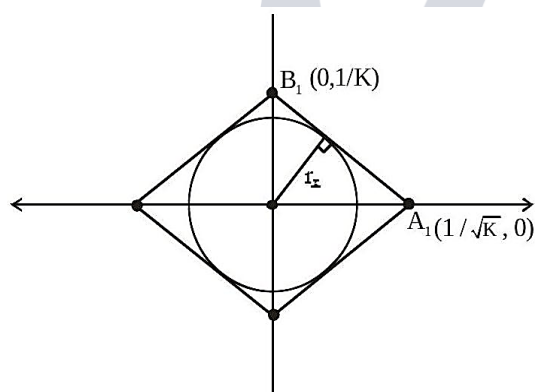
$$ad = bc = 4 \Rightarrow \text{no. of } (a, b, c, d) = 1^2 = 1$$

$$: P(\bar{A}) = \frac{31}{81} \Rightarrow p(A) = \frac{50}{81}$$

71. (a) $Kx^2 + K^2y^2 = 1$

$$\frac{x^2}{1/K} + \frac{y^2}{1/K^2} = 1$$

Now



Equation of

$$A_1 B_2; \frac{x}{1/\sqrt{K}} + \frac{y}{1/K} = 1 \Rightarrow \sqrt{K}x + Ky = 1$$

$$r_k = \perp r \text{ distance of } (0,0) \text{ from line } A_1 B_1$$

$$r_k = \left| \frac{(0 + 0 - 1)}{\sqrt{K + K^2}} \right| = \frac{1}{\sqrt{K + K^2}}$$

$$\frac{1}{r_k^2} = K + K^2 \Rightarrow \sum_{k=1}^{20} \frac{1}{r_k^2} = \sum_{k=1}^{20} (K + K^2)$$

$$\begin{aligned}
 &= \sum_{K=1}^{20} K + \sum_{K=1}^{20} K^2 \\
 &= \frac{20 \times 21}{2} + \frac{20 \cdot 21 \cdot 41}{6} \\
 &= 210 + 10 \times 7 \times 41 \\
 &= 210 + 2870 \\
 &= 3080
 \end{aligned}$$

72. (a) $\log_{x+\frac{7}{2}} \left(\frac{x-7}{2x-3} \right)^2 \geq 0$

Feasible region: $x + \frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$ And $x + \frac{7}{2} \neq 1 \Rightarrow x \neq -\frac{5}{2}$
 And $\frac{x-7}{2x-3} \neq 0$ and $2x - 3 \neq 0$

$$x \neq 7 \quad x \neq \frac{3}{2}$$

Taking intersection: $x \in \left(-\frac{7}{2}, \infty \right) - \left\{ -\frac{5}{2}, \frac{3}{2}, 7 \right\}$

Now $\log_2 b \geq 0$ if $a > 1$ and $b \geq 1$

Or

$$a \in (0,1) \text{ and } b \in (0,1)$$

$$\text{C-I; } x + \frac{7}{2} > 1 \text{ and } \left(\frac{x-7}{2x-3} \right)^2 \geq 1$$

$$x > -\frac{5}{2} \quad (2x-3)^2 - (x-7)^2 \leq 0$$

$$(2x-3+x-7)(2x-3-x+7) \leq 0$$

$$(3x-10)(x+4) \leq 0$$

$$x \in \left[-4, \frac{10}{3} \right]$$

Intersection: $x \in \left(-\frac{5}{2}, \frac{10}{3} \right]$

$$\text{C-II } x + \frac{7}{2} \in (0,1) \text{ and } \left(\frac{x-7}{2x-3} \right)^2 \in (0,1)$$

$$0 < x + \frac{7}{2} < 1$$

$$\left(\frac{x-7}{2x-3} \right)^2 < 1$$

$$-\frac{7}{2} < x < -\frac{5}{2} \quad (x-7)^2 < (2x-3)^2$$

$$x \in (-\infty, -4) \cup \left(\frac{10}{3}, \infty \right)$$

No common values of x .

Hence intersection with feasible region

We get $x \in \left(-\frac{5}{2}, \frac{10}{3} \right] - \left\{ \frac{3}{2} \right\}$

Integral value of x are $\{-2, -1, 0, 1, 2, 3\}$

No. of integral values = 6

73. (a) $x^2 + (y - 2)^2 \leq 2^2$ and $x^2 \geq 2y$

Solving circle and parabola simultaneously:

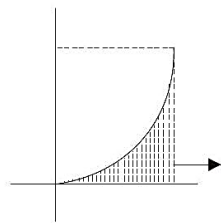
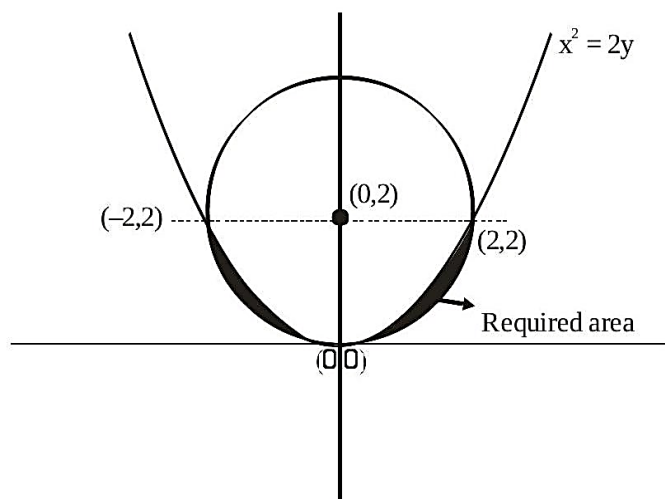
$$2y + y^2 - 4y + 4 = 4$$

$$y^2 - 2y = 0$$

$$y = 0, 2$$

$$\text{Put } y = 2 \text{ in } x^2 = 2y \rightarrow x = \pm 2$$

$$\Rightarrow (2, 2) \text{ and } (-2, 2)$$



$$= 2 \times 2 - \frac{1}{4} \cdot \pi \cdot 2^2 = 4 - \pi$$

$$\text{Required area} = 2 \left[\int_0^2 \frac{x^2}{2} dx - (4 - \pi) \right]$$

$$= 2 \left[\frac{x^3}{6} \Big|_0^2 - 4 + \pi \right]$$

$$= 2 \left[\frac{4}{3} + \pi - 4 \right]$$

$$= 2 \left[\pi - \frac{8}{3} \right]$$

$$= 2\pi - \frac{16}{3}$$

74. (c)

75. (d) $(1 - x^2y^2)dx = ydx + xdy, y(a) = 2$

$$y(b) = \alpha = ?$$

$$dx = \frac{d(xy)}{1-(xy)^2} \int dx = \int \frac{dxy}{1-(xy)^2}$$

$$x = \frac{1}{2} \ln \left| \frac{1+xy}{1-xy} \right| + C$$

$$\text{Put } x = 1 \text{ and } y = 2 :$$

$$1 = \frac{1}{2} \ln \left| \frac{1+2}{1-2} \right| + C$$

$$C = 1 - \frac{1}{2} \ln 3$$

$$\text{Now put } x = 2 :$$

$$2 = \frac{1}{2} \ln \left| \frac{1+2\alpha}{1-2\alpha} \right| + 1 - \frac{1}{2} \ln 3$$

$$1 + \frac{1}{2} \ln 3 = \frac{1}{2} \ln \left| \frac{1+2\alpha}{1-2\alpha} \right|$$

$$2 + \ln 3 = \ln \left(\frac{1+2\alpha}{1-2\alpha} \right)$$

$$\left| \frac{1+2\alpha}{1-2\alpha} \right| = 3e^2$$

$$\frac{1+2\alpha}{1-2\alpha} = 3e^2, -3e^2$$

$$\frac{1+2\alpha}{1-2\alpha} = 3e^2 \Rightarrow \alpha = \frac{3e^2 - 1}{2(3e^2 + 1)}$$

$$\text{And } \frac{1+2\alpha}{1-2\alpha} = -3e^2 \Rightarrow \alpha = \frac{3e^2 + 1}{2(3e^2 - 1)}$$

$$76. (c) A^T = \alpha A + I$$

$$A = \alpha A^T + I$$

$$A = \alpha(\alpha A + I) + I$$

$$A = \alpha^2 A + (\alpha + 1)I$$

$$A(1 - \alpha^2) = (\alpha + 1)I$$

$$A = \frac{I}{1 - \alpha}$$

$$|A| = \frac{1}{(1 - \alpha)^2}$$

$$|A^2 - A| = |A||A - I|$$

$$A - I = \frac{I}{1 - \alpha} - I = \frac{\alpha}{1 - \alpha} I$$

$$|A - I| = \left(\frac{\alpha}{1 - \alpha} \right)^2$$

$$\text{Now } |A^2 - A| = 4$$

$$|A| |A - I| = 4$$

$$\Rightarrow \frac{1}{(1 - \alpha)^2} \frac{\alpha^2}{(1 - \alpha)^2} = 4$$

$$\Rightarrow \frac{\alpha}{(1 - \alpha)^2} = \pm 2$$

$$\Rightarrow 2(1 - \alpha)^2 = \pm \alpha$$

$$(C_1) 2(1 - \alpha)^2 = \alpha \quad \left| \quad (C_2) 2(1 - \alpha)^3 = -\alpha \right.$$

$$2\alpha^2 - 5\alpha + 2 = 0 \quad \alpha_1 \quad 2\alpha^2 - 3\alpha + 2 = 0 \quad \alpha_2$$

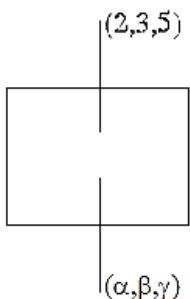
$$\alpha_1 + \alpha_2 = \frac{5}{2} \alpha \notin \mathbb{R}$$

$$\text{Sum of value of } \alpha = \frac{5}{2}$$

77. (a)

$$\frac{\alpha - 2}{2} = \frac{\beta - 3}{1} = \frac{\gamma - 5}{-3} = -2 \left(\frac{2 \times 2 + 3 - 3 \times 5 - 6}{2^2 + 1^2 + 1 - 3^2} \right) = 2$$

$\frac{\alpha - 2}{2} = 2$	$\beta - 3 = 2$	$\gamma - 5 = -6$
$\alpha = 6$	$\beta = 5$	$\gamma = -1$



$$\alpha + \beta + \gamma = 10$$

78. (d) n_1 and n_2 are normal vector to the plane $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}; \hat{j} - \hat{k}$ respectively

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} = \lambda |\vec{n}_2 \times \vec{n}_2|$$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \lambda(-2\hat{j} + 2\hat{k})$$

$$\vec{a} \cdot \vec{b} = \lambda|0 + 4 + 2| = 6$$

$$\Rightarrow \lambda = 1$$

$$\vec{a} = -2\hat{j} + 2\hat{k}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\cos \theta = \frac{6}{2\sqrt{2} \times 3} = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

$$\text{Now } |\vec{a} \cdot \vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$36 + |\vec{a} \times \vec{b}|^2 = 8 \times 9 = 72$$

$$|\vec{a} \times \vec{b}|^2 = 36$$

$$|\vec{a} \times \vec{b}| = 6$$

79. (c)

$$3\cos^4 \theta - 5\cos^2 \theta - 2\sin^6 \theta + 2 = 0$$

$$\Rightarrow 3\cos^4 \theta - 3\cos^2 \theta - 2\cos^2 \theta - 2\sin^6 \theta + 2 = 0$$

$$\Rightarrow 3\cos^4 \theta - 3\cos^2 \theta + 2\sin^2 \theta - 2\sin^6 \theta = 0$$

$$\Rightarrow 3\cos^2 \theta (\cos^2 \theta - 1) + 2\sin^2 \theta (\sin^4 \theta - 1) = 0$$

$$\Rightarrow -3\cos^2 \theta \sin^2 \theta + 2\sin^2 \theta (1 + \sin^2 \theta) \cos^2 \theta - 1$$

$$\Rightarrow \sin^2 \theta \cos^2 \theta (2 + 2\sin^2 \theta - 3) = 0$$

$$\Rightarrow \sin^2 \theta \cos^2 \theta (2\sin^2 \theta - 1) = 0$$

$$(C1) \sin^2 \theta = 0 \rightarrow 3 \text{ solution; } \theta = \{0, \pi, 2\pi\}$$

$$(C2) \cos^2 \theta = 0 \rightarrow 2 \text{ solution; } \theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

$$(C3) \sin^2 \theta = \frac{1}{2} \rightarrow 4 \text{ solution; } \theta = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

No. of solution = 9

80. (c) Mean = 200

$$\Rightarrow \frac{\frac{100}{2}(2 \times 2 + 99d)}{100} = 200$$

$$\Rightarrow 4 + 99d = 400$$

$$\Rightarrow d = 4$$

$$y_i = i(x_i - i)$$

$$= i(2 + (i - 1)4 - i) = 3i^2 - 2i$$

$$\text{Mean} = \frac{\sum y_i}{100}$$

$$= \frac{1}{100} \sum_{i=1}^{100} 3i^2 - 2i$$

$$= \frac{1}{100} \left\{ \frac{3 \times 100 \times 101 \times 201}{6} - \frac{2 \times 100 \times 101}{2} \right\}$$

$$= 101 \left\{ \frac{201}{2} - 1 \right\} = 101 \times 99.5$$

$$= 10049.50$$

81. (2736) Coefficient of $x = {}^9C_1 2^8$

$$\text{Of } x^2 = {}^9C_2 2^7$$

$$\text{Of } x^7 = {}^9C_7 \cdot 2^2$$

$$\text{Mean} = \frac{{}^9C_1 \cdot 2^8 + {}^9C_2 \cdot 2^7 + \dots + {}^9C_7 \cdot 2^2}{7}$$

$$= \frac{(1+2)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2^1 - {}^9C_9}{7}$$

$$= \frac{3^9 - 2^9 - 18 - 1}{7}$$

$$= \frac{19152}{7} = 2736$$

82. (2175)

$$S = 109 + \frac{108}{5} + \frac{107}{5^2} + \dots + \frac{1}{5^{108}}$$

$$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} + \dots + \frac{2}{5^{108}} + \frac{1}{5^{109}}$$

$$\frac{4S}{5} = 109 - \frac{1}{5} - \frac{1}{5^2} - \dots - \frac{1}{5^{108}} - \frac{1}{5^{109}}$$

$$= 109 - \left(\frac{1 \left(1 - \frac{1}{5^{109}} \right)}{\frac{1}{5} \left(1 - \frac{1}{5} \right)} \right)$$

$$= 109 - \frac{1}{4} \left(1 - \frac{1}{5^{109}} \right)$$

$$= 109 - \frac{1}{4} + \frac{1}{4} \times \frac{1}{5^{109}}$$

$$s = \frac{5}{4} \left(109 - \frac{1}{4} + \frac{1}{4 \cdot 5^{109}} \right)$$

$$16S = 20 \times 109 - 5 + \frac{1}{5^{108}}$$

$$16S - (25)^{-54} = 2180 - 5 = 2175$$

83. (32)

$$\alpha(m, n) = \int_0^2 t^m (1 + 3t)^n dt$$

If $11\alpha(10, 6) + 18\alpha(11, 5) = p(14)^6$ then P

$$= 11 \int_0^2 \frac{t^{10} (1 + 3t)^6}{11} + 10 \int_0^2 t^{11} (1 + 3t)^5 dt$$

$$= 11 \left[(1 + 3t)^6 \cdot \frac{t^{11}}{11} - \int 6(1 + 3t)^5 \cdot 3 \frac{t^{11}}{11} dt \right]_0^2 + 18 \int_0^2 t^{11} (1 + 3t)^5 dt$$

$$= (t^{11} (1 + 3t)^6)_0^2$$

$$= 2^{11} (7)^6$$

$$= 2^5 (14)^6$$

$$= 32 (14)^6$$

84. (44)

Derangement of 5 students

$$D_5 = 5! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right)$$

$$= 120 \left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} \right)$$

$$= 60 - 20 + 5 - 1$$

$$= 40 + 4$$

$$= 44$$

85. (5) Let $\ell = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \gamma(a\hat{i} + b\hat{j} + c\hat{k})$

$$= \gamma(a\hat{i} + b\hat{j} + c\hat{k})$$

$$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(2 - 6) - \hat{j}(1 - 6) + \hat{k}(2 - 4)$$

$$= -4\hat{i} - 5\hat{j} - 2\hat{k}$$

$$\ell = \gamma(-4\hat{i} + 5\hat{j} - 2\hat{k})$$

P is intersection of ℓ and ℓ_1

$$-4\gamma = 1 + \lambda, 5\gamma = -11 + 2\lambda, -2\gamma = -7 + 3\lambda$$

By solving there equation $\gamma = -1, P(4, -5, 2)$

Let $Q(-1 + 2\mu, 2\mu, 1 + \mu)$

$$\overrightarrow{PQ} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0$$

$$-2 + 4\mu + 4\mu + 1 + \mu = 0$$

$$9\mu = 1$$

$$\mu = \frac{1}{9}$$

$$Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$$

$$9(\alpha + \beta + \gamma) = 9\left(\frac{-7}{9} + \frac{2}{9} + \frac{10}{9}\right)$$

$$= 5$$

86. (171) The number of integral term in the expression of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to

$$\text{General term} = {}^{680}C_r \left(3^{\frac{1}{2}}\right)^{680-r} \left(5^{\frac{1}{4}}\right)^r$$

$$= {}^{680}C_r 3^{\frac{680-r}{2}} 5^{\frac{r}{4}}$$

Values of r , where $\frac{r}{4}$ goes to integer

$$r = 0, 4, 8, 12, \dots \dots \dots 680$$

All value of r are accepted for $\frac{680-r}{2}$ as well so No of integral terms = 171.

87. (7)

p	q	r	Pvq	Pvr	(pvq) $\wedge$ (pvr)	qvr	(pvq) $\wedge$ (pvr) $\rightarrow$ qvr
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

Hence total no of ordered triplets are 7

88. (306) $H_n \Rightarrow \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{3+n}{1+n}} = \sqrt{\frac{2n+4}{n+1}}$$

$$e = \sqrt{\frac{2n+4}{n+1}}$$

$n = 48$ (smallest even value for which $e \in \mathbb{Q}$)

$$e = \frac{10}{7}$$

$$a^2 = n+1 \quad b^2 = n+3$$

$$= 49, = 51$$

$$1 = \text{length of LR} = \frac{2b^2}{a}$$

$$L = 2 \cdot \frac{51}{7}$$

$$1 = \frac{102}{7}$$

$$21\ell = 306$$

89. (51) $x^2 - 7x - 1 = 0 < \leq_a^b$

By newton's theorem

$$S_{n+2} - 7 S_{n+1} - S_n = 0$$

$$S_{21} - 7 S_{20} - S_{19} = 0$$

$$S_{20} - 7 S_{19} - S_{18} = 0$$

$$S_{19} - 7 S_{18} - S_{17} = 0$$

$$\frac{S_{21} + S_{17}}{S_{19}} = \frac{S_{21} + (S_{19} - 7 S_{18})}{S_{19}}$$

$$= \frac{S_{21} + S_{19} - 7(S_{20} - 7 S_{19})}{S_{19}}$$

$$= \frac{50 S_{19} + (S_{21} - 7 S_{20})}{S_{19}}$$

$$= 51 \cdot \frac{S_{19}}{S_{19}} = 51$$

$$90. (2) A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^3 = A$$

$$A^2 = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & a & 2+3c \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2ac+3 & a+2+3c & 2a+4+6c \\ a(a+3c)+2a & 3+2ac & 6+3a+9c \\ a+2+3c & ac+c(2+3c) & 2ac+3 \end{bmatrix}$$

$$\text{Given } A^3 = A$$

$$2ac+3=0 \dots (a) \text{ and } a+2+3c=1$$

$$a+1+3c=0$$

$$a+1-\frac{9}{2a}=0$$

$$2a^2+2a-9=0$$

$$f(a) < 0, f(b) > 0$$

$$a \in (1,2]$$

$$n=2$$

