

PART-I PHYSICS

SECTION-A

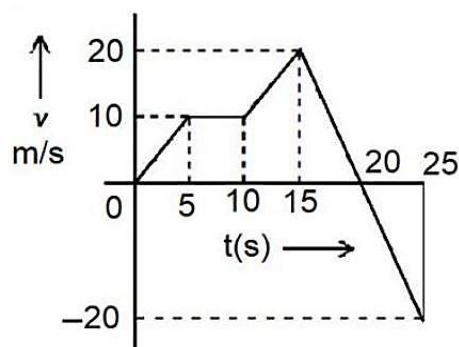
1. The electric field in an electromagnetic wave is given as $\vec{E} = 20\sin \omega \left(t - \frac{x}{c} \right) \hat{j} \text{NC}^{-1}$

Where ω and c are angular frequency and velocity of electromagnetic wave respectively. The energy contained in a volume of $5 \times 10^{-4} \text{ m}^3$ will be

(Given $\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2$)

- (a) $28.5 \times 10^{-13} \text{ J}$ (b) $17.7 \times 10^{-13} \text{ J}$
(c) $8.85 \times 10^{-13} \text{ J}$ (d) $88.5 \times 10^{-13} \text{ J}$

2. From the $v - t$ graph shown, the ratio of distance to displacement in 25 s of motion



- (a) $\frac{3}{5}$ (b) $\frac{1}{2}$
(c) $\frac{5}{3}$ (d) 1

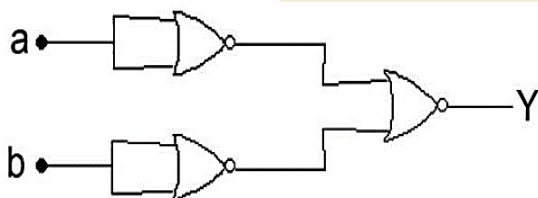
3. The radii of two planets 'A' and 'B' are ' R ' and ' $4R$ ' and their densities are ρ and $\rho/3$ respectively. The ratio of acceleration due to gravity at their surfaces ($g_A : g_B$) will be :

- (a) 1:16 (b) 3:16
(c) 3:4 (d) 4:3

4. A coin placed on a rotating table just slips when it is placed at a distance of 1 cm from the center. If the angular velocity of the table is halved, it will just slip when placed at a distance of from the centre:

- (a) 2 cm (b) 1 cm
(c) 8 cm (d) 4 cm

5. The logic performed by the circuit shown in figure is equivalent to:



- (a) AND (b) NAND
(c) OR (d) NOR

6. A parallel plate capacitor of capacitance 2 F is charged to a potential V . The energy stored in the capacitor is E_1 . The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E_2 . The ratio E_2/E_1 is:

- (a) 2:1 (b) 1:2
(c) 1:4 (d) 2:3

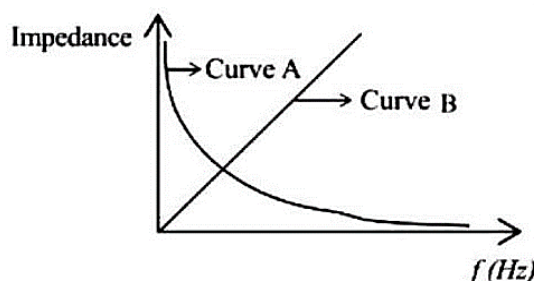
7. Two identical heater filaments are connected first in parallel and then in series. At the same applied voltage, the ratio of heat produced in same time for parallel to series will be:
 (a) 4:1 (b) 2:1
 (c) 1:2 (d) 1:4
8. A transmitting antenna is kept on the surface of the earth. The minimum height of receiving antenna required to receive the signal in line of sight at 4 km distance from it is $x \times 10^{-2}$ m. The value of x is (Let. radius of earth $R = 6400$ km)
 (a) 125 (b) 12.5
 (c) 1.25 (d) 1250
9. As per the given graph choose the correct representation for curve A and curve B.

(Where X_C = reactance of pure capacitive circuit connected with A.C. source

X_L = reactance of pure inductive circuit connected with A.C. source

R = impedance of pure resistive circuit connected with A.C. source

Z = Impedance of the LCR series circuit }



- (a) $A = X_C, B = R$ (b) $A = X_L, B = Z$
 (c) $A = X_C, B = X_L$ (d) $A = X_L, B = R$

10. 1 kg of water at 100°C is converted into steam at 100°C by boiling at atmospheric pressure. The volume of water changes from $1.00 \times 10^{-3} \text{ m}^3$ as a liquid to 1.671 m^3 as steam. The change in internal energy of the system during the process will be

(Given latent heat of vaporisation = 2257 kJ/kg .)

Atmospheric pressure = $1 \times 10^5 \text{ Pa}$)

- (a) $+2090 \text{ kJ}$ (b) -2090 kJ
 (c) -2426 kJ (d) $+2476 \text{ kJ}$

11. The critical angle for a denser-rarer interface is 45° . The speed of light in rarer medium is $3 \times 10^8 \text{ ms}$. The speed of light in the denser medium is:

- (a) $5 \times 10^7 \text{ m/s}$ (b) $2.12 \times 10^8 \text{ m/s}$
 (c) $3.12 \times 10^7 \text{ m/s}$ (d) $\sqrt{2} \times 10^8 \text{ m/s}$

12. A metallic surface is illuminated with radiation of wavelength λ , the stopping potential is V_0 . If the same surface is illuminated with radiation of wavelength 2λ , the stopping potential becomes $\frac{V_0}{4}$. The threshold wavelength for this metallic surface will be -

- (a) $\frac{\lambda}{4}$ (b) 4λ
 (c) $\frac{3}{2}\lambda$ (d) 3λ

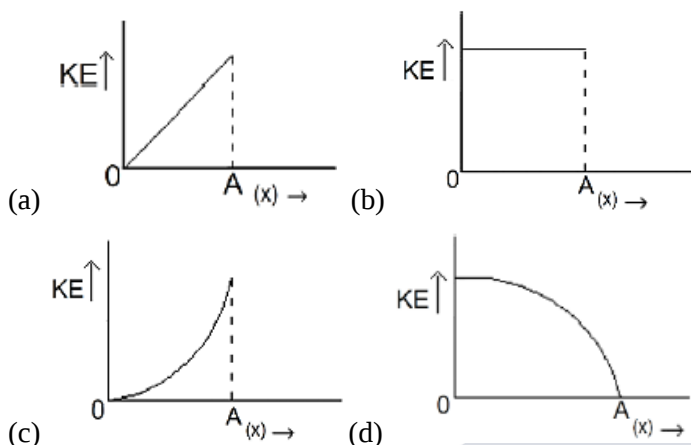
13. The free space inside a current carrying toroid is filled with a material of susceptibility 2×10^{-2} . The percentage increase in the value of magnetic field inside the toroid will be

- (a) 2% (b) 0.2%
 (c) 0.1% (d) 1%

14. The current sensitivity of moving coil galvanometer is increased by 25%. This increase is achieved only by changing in the number of turns of coils and area of cross section of the wire while keeping the resistance of galvanometer coil constant. The percentage change in the voltage sensitivity will be:

- (a) +25% (b) -50%
(c) Zero (d) -25%

15. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by



16. On a temperature scale 'X'. The boiling point of water is 65°X and the freezing point is -15°X . Assume that the X scale is linear. The equivalent temperature corresponding to -95°X on the Fahrenheit scale would be:

- (a) -63°F (b) -112°F
(c) -48°F (d) -148°F

17. Given below are two statements:

Statements I: Astronomical unit (Au), Parsec (Pc) and Light year (ly) are units for measuring astronomical distances.

Statements II: $\text{Au} < \text{Parsec (Pc)} < \text{ly}$

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Both Statements I and Statements II are correct.
(b) Statements I is correct but Statements II is incorrect.
(c) Both Statements I and Statements II are incorrect.
(d) Statements I is incorrect but statements II is correct.

18. Three vessels of equal volume contain gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and third contains uranium hexafluoride (polyatomic). Arrange these on the basis of their root mean square speed (v_{rms}) and choose the correct answer from the options given below:

- (a) $v_{\text{rms}}(\text{mono}) = v_{\text{rms}}(\text{dia}) = v_{\text{rms}}(\text{poly})$
(b) $v_{\text{rms}}(\text{mono}) > v_{\text{rms}}(\text{dia}) > v_{\text{rms}}(\text{poly})$
(c) $v_{\text{rms}}(\text{dia}) < v_{\text{rms}}(\text{poly}) < v_{\text{rms}}(\text{mono})$
(d) $v_{\text{rms}}(\text{mono}) < v_{\text{rms}}(\text{dia}) < v_{\text{rms}}(\text{poly})$

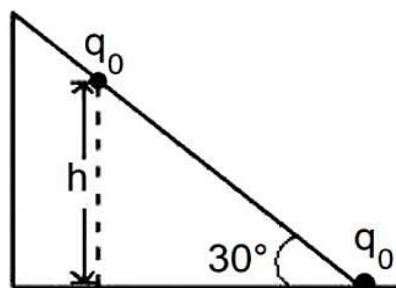
19. An average force of 125 N is applied on a machine gun firing bullets each of mass 10 g at the speed of 250 m/s to keep it in position. The number of bullets fired per second by the machine gun is:
 (a) 5 (b) 50
 (c) 100 (d) 25
20. Two radioactive elements A and B initially have same number of atoms. The half life of A is same as the average life of B. If λ_A and λ_B are decay constants of A and B respectively, then choose the correct relation from the given options.
 (a) $\lambda_A = \lambda_B$ (b) $\lambda_A = 2\lambda_B$
 (c) $\lambda_A = \lambda_B \ln 2$ (d) $\lambda_A \ln 2 = \lambda_B$

SECTION-B

21. A monochromatic light is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. The frequency of incident light is $x \times 10^{15}$ Hz. The value of x is. ____+(Given $h = 4.25 \times 10^{-15}$ eVs)
22. The radius of curvature of each surface of a convex lens having refractive index 1.8 is 20 cm. The lens is now immersed in a liquid of refractive index 1.5. The ratio of power of lens in air to its power in the liquid will be x: 1. The value of x is ____
23. The equation of wave is given by

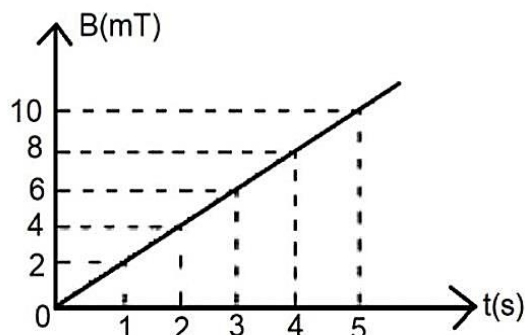
$$Y = 10^{-2} \sin 2\pi \left(160t - 0.5x + \frac{\pi}{4} \right)$$

 Where x and Y are in m and t in s. The speed of the wave is ____ kmh^{-1}
24. A force $\vec{F} = (2 + 3x)\hat{i}$ acts on a particle in the x direction where F is in newton and x is in meter. The work done by this force during a displacement from $x = 0$ to $x = 4$ m, is J.
25. As shown in the figure. a configuration of two equal point charges ($q_0 = +2\mu\text{C}$) is placed on an inclined plane. Mass of each point charge is 20 g. Assume that there is no friction between charge and plane. For the system of two-point charges to be in equilibrium (at rest) the height $h = x \times 10^{-3}$ m The value of x is ____
 (Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$, $g = 10 \text{ ms}^{-1}$)

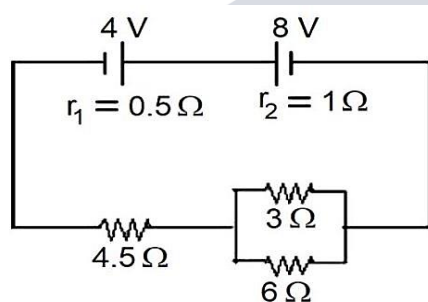


26. A solid sphere of mass 500 g and radius 5 cm is rotated about one of its diameter with angular speed of 10 rad s^{-1} . If the moment of inertia of the sphere about its tangent is $x \times 10^{-2}$ times its angular momentum about the diameter. Then the value of x will be

27. The length of wire becomes l_1 and l_2 when 100 N and 120 N tensions are applied respectively. If $10l_2 = 11l_1$, the natural length of wire will be $\frac{1}{x}l_1$. Here the value of x is ____
28. The magnetic field B crossing normally a square metallic plate of area 4 m^2 is changing with time as shown in figure. The magnitude of induced emf in the plate during $t = 2 \text{ s}$ to $t = 4 \text{ s}$, is ____ mV



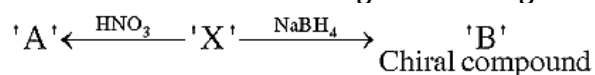
29. A projectile fired at 30° to the ground is observed to be at same height at time 3 s and 5 s after projection, during its flight. The speed of projection of the projectile is ____ ms^{-1} (Given $g = 10 \text{ m s}^{-2}$)
30. In the circuit diagram shown in figure given below, the current flowing through resistance 3Ω is $\frac{x}{3} \text{ A}$. The value of x is



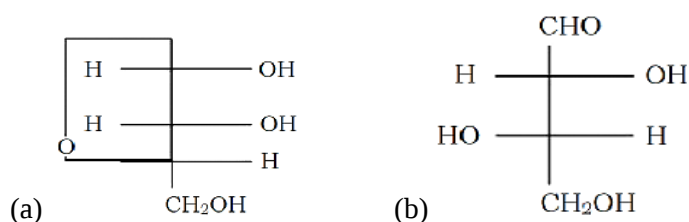
PART-II CHEMISTRY

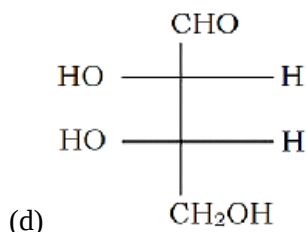
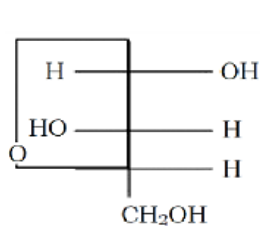
SECTION-A

31. L-isomer of tetrose $X(\text{C}_4\text{H}_8\text{O}_4)$ gives positive Schiff's test and has two chiral carbons. On acetylation, 'X' yields triacetate. 'X' also undergoes following reactions



X is





32. The polymer X - consists of linear molecules and is closely packed. It is prepared in the presence of triethylaluminium and titanium tetrachloride under low pressure. The polymer X is -

- (a) Polyacrylonitrile (b) Low density polythene
(c) Polytetrafluoroethane (d) High density polythene

33. When a solution of mixture having two inorganic salts was treated with freshly prepared ferrous sulphate in acidic medium, a dark brown ring was formed whereas on treatment with neutral FeCl_3 , it gave deep red colour which disappeared on boiling and a brown red ppt was formed. The mixture contains

- (a) CH_3COO^- & NO_3^- (b) $\text{C}_2\text{O}_4^{2-}$ & NO_3^-
(c) SO_3^{2-} & CH_3COO^- (d) SO_3^{2-} & $\text{C}_2\text{O}_4^{2-}$

34. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: In the photoelectric effect, the electrons are ejected from the metal surface as soon as the beam of light of frequency greater than threshold frequency strikes the surface.

Reason R: When the photon of any energy strikes an electron in the atom, transfer of energy from the photon to the electron takes place.

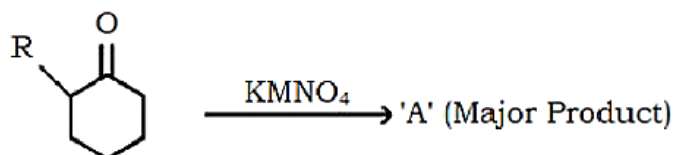
In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Both A and R are correct but R is NOT the correct explanation of A
(b) A is correct but R is not correct
(c) Both A and R are correct and R is the correct explanation of A
(d) A is not correct but R is correct

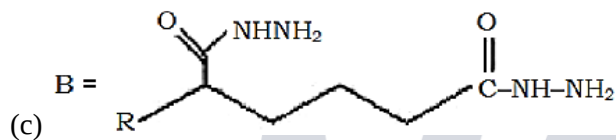
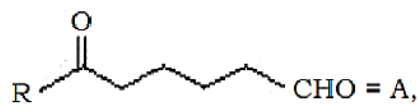
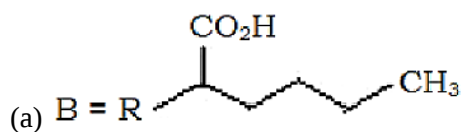
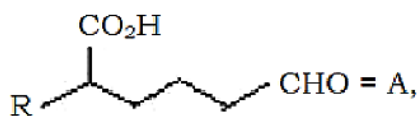
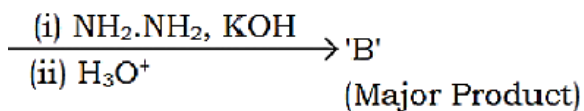
35. 25 mL of silver nitrate solution (1 M) is added dropwise to 25 mL of potassium iodide (1.05M) solution. The ion(s) present in very small quantity in the solution is/are

- (a) NO_3^- only (b) K^+ only
(c) Ag^+ and I^- both (d) I^- only

36. 'A' and 'B' in the below reactions are:



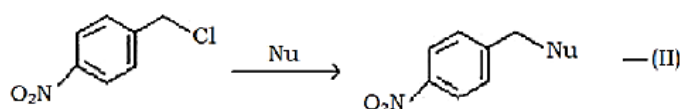
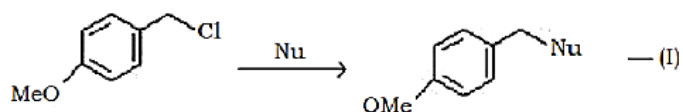
(R = alkyl)



37. The set which does not have ambidentate ligand(s) is

- $\text{C}_2\text{O}_4^{2-}$, ethylene diammine, H_2O
- EDTA^{4-} , NCS^- , $\text{C}_2\text{O}_4^{2-}$
- NO_2^- , $\text{C}_2\text{O}_4^{2-}$, EDTA^{4-}
- $\text{C}_2\text{O}_4^{2-}$, NO_2^- , NCS^-

38.



Where Nu = Nucleophile

Find out the correct statement from the options given below for the above 2 reactions.

- (a) Reaction (I) is of 2nd order and reaction (II) is of 1st order
- (b) Reaction (I) and (II) both are of 2nd order
- (c) Reaction (I) is of 1st order and reaction (II) is of 2nd order
- (d) Reactions (I) and (II) both are of 1st order

39. For elements B, C, N, Li, Be, O and F the correct order of first ionization enthalpy is

- (a) $\text{Li} < \text{Be} < \text{B} < \text{C} < \text{N} < \text{O} < \text{F}$
- (b) $\text{B} > \text{Li} > \text{Be} > \text{C} > \text{N} > \text{O} > \text{F}$
- (c) $\text{Li} < \text{B} < \text{Be} < \text{C} < \text{O} < \text{N} < \text{F}$
- (d) $\text{Li} < \text{Be} < \text{B} < \text{C} < \text{O} < \text{N} < \text{F}$

40. Match List-I with List-II :

List-I Species	List-II Geometry/Shape
(A) H_3O^+	(I) Tetrahedral
(B) Acetylide anion	(II) Linear
(C) NH_4^+	(III) Pyramidal
(D) ClO_2^-	(IV) Bent

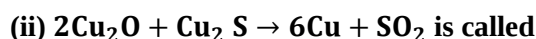
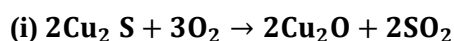
Choose the correct answer from the options given below:

- (a) A-III, B-II, C-I, D-IV
- (b) A-III, B-I, C-II, D-IV
- (c) A-III, B-IV, C-I, D-II
- (d) A-III, B-IV, C-II, D-I

41. For compound having the formula GaAlCl_4 , the correct option from the following is

- (a) Ga is more electronegative than Al and is present as a cationic part of the salt GaAlCl_4
- (b) Oxidation state of Ga in the salt GaAlCl_4 is +3 .
- (c) Cl forms bond with both Al and Ga in GaAlCl_4
- (d) Ga is coordinated with Cl in GaAlCl_4

42. In the extraction process of copper, the product obtained after carrying out the reactions



- (a) Blister copper
- (b) Copper scrap
- (c) Reduced copper
- (d) Copper matte

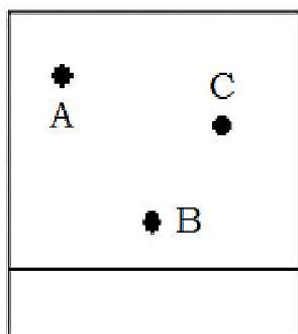
43. Match List-I with List-II :

List-I	List-II
(A) K	(I) Thermonuclear reactions
(B) KCl	(II) Fertilizer
(C) KOH	(III) Sodium potassium pump
(D) Li	(IV) Absorbent of CO ₂

Choose the correct answer from the options given below:

- (a) A-III, B-II, C-IV, D-I (b) A-IV, B-I, C-III, D-II
(c) A-IV, B-III, C-I, D-II (d) A-III, B-IV, C-II, D-I

44. Thin layer chromatography of a mixture shows the following observation:



The correct order of elution in the silica gel column chromatography is

- (a) A, C, B (b) B, C, A
(c) C, A, B (d) B, A, C

45. Which of the following complex has a possibility to exist as meridional isomer?

- (a) $[\text{Co}(\text{NH}_3)_3(\text{NO}_2)_3]$ (b) $[\text{Co}(\text{en})_3]$
(c) $[\text{Co}(\text{en})_2\text{Cl}_2]$ (d) $[\text{Pt}(\text{NH}_3)_2\text{Cl}_2]$

46. Given below are two statements:

Statement-I: Methane and steam passed over a heated Ni catalyst produces hydrogen gas.

Statement-II: Sodium nitrite reacts with NH_4Cl to give H_2O , N_2 and NaCl .

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Both the statements I and II are correct
(b) Both the statements I and II are incorrect
(c) Statement I is incorrect but Statement II is correct
(d) Statement I is correct but Statement II is incorrect

47. Given below are two statements:

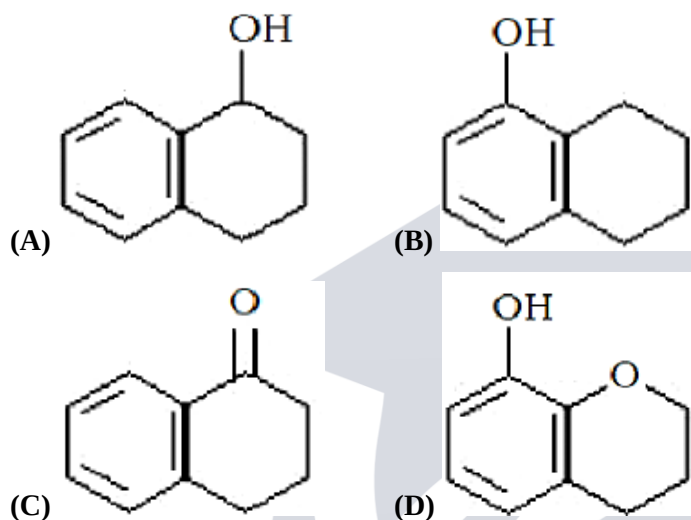
Statement I: If BOD is 4ppm and dissolved oxygen is 8ppm, then it is a good quality water.

Statement II: If the concentration of zinc and nitrate salts are 5ppm each, then it can be a good quality water.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Both the statements I and II are incorrect
- (b) Statement I is incorrect but Statement II is correct
- (c) Both the statements I and II are correct
- (d) Statement I is correct but Statement II is incorrect

48. Arrange the following compounds in increasing order of rate of aromatic electrophilic substitution reaction

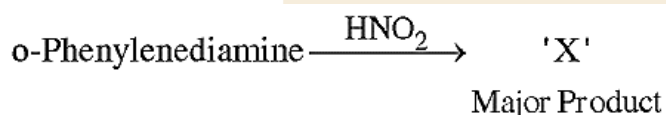


- (a) D, B, C, A
- (b) B, C, A, D
- (c) C, A, B, D
- (d) D, B, A, C

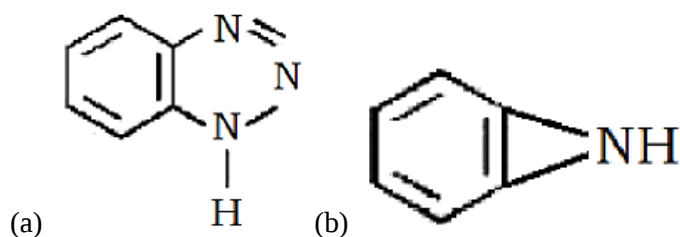
49. The complex that dissolves in water is

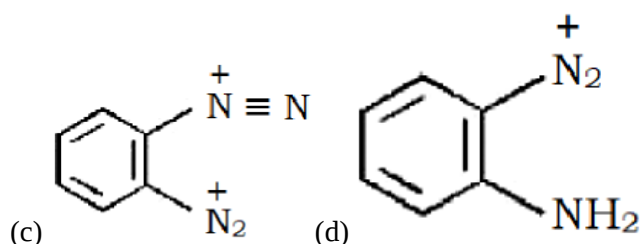
- (a) $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$
- (b) $[\text{Fe}_3(\text{OH})_2(\text{OAc})_6]\text{Cl}$
- (c) $\text{K}_3[\text{Co}(\text{NO}_2)_6]$
- (d) $(\text{NH}_4)_3[\text{As}(\text{Mo}_3\text{O}_{10})_4]$

50.



X is





SECTION-B

51. A mixture of 1 mole of H_2O and 1 mole of CO is taken in a 10 litre container and heated to 725 K. At equilibrium 40% of water by mass reacts with carbon monoxide according to the equation: $\text{CO(g)} + \text{H}_2\text{O(g)} \rightleftharpoons \text{CO}_2(\text{g}) + \text{H}_2(\text{g})$.

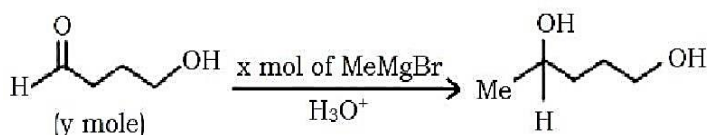
The equilibrium constant $K_c \times 10^2$ for the reaction is___ (Nearest integer)

52. The ratio of spin-only magnetic moment values $\mu_{\text{eff}} [\text{Cr}(\text{CN})_6]^{3-} / \mu_{\text{eff}} [\text{Cr}(\text{H}_2\text{O})_6]^{3+}$ is___

53. An atomic substance A of molar mass 12 g mol^{-1} has a cubic crystal structure with edge length of 300pm. The no. of atoms present in one unit cell of A is___ (Nearest integer)

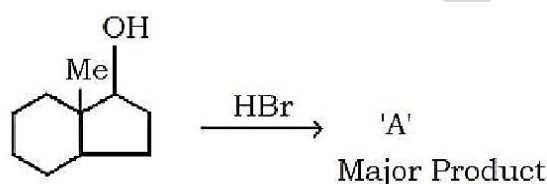
Given the density of A is 3.0 g mL^{-1} and $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$

54.



The ratio x/y on completion of the above reaction is___

55.



The number of hyperconjugation structures involved to stabilize carbocation formed in the above reaction is

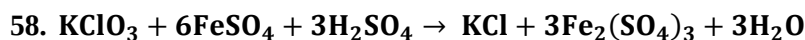
56. Solid fuel used in rocket is a mixture of Fe_2O_3 and Al (in ratio 1: 2). The heat evolved (kJ) per gram of the mixture is___ (Neatest integer)

Given: $\Delta H_f^\ominus (\text{Al}_2\text{O}_3) = -1700 \text{ kJ mol}^{-1}$

$\Delta H_f^\ominus (\text{Fe}_2\text{O}_3) = -840 \text{ kJ mol}^{-1}$

Molar mass of Fe, Al and O are 56, 27 and 16 g mol^{-1} respectively.

57. A solution of sugar is obtained by mixing 200 g of its 25% solution and 500 g of its 40% solution (both by mass). The mass percentage of the resulting sugar solution is _____ (Nearest integer)



The above reaction was studied at 300 K by monitoring the concentration of FeSO_4 in which initial concentration was 10M and after half an hour became 8.8M. The rate of production of $\text{Fe}_2(\text{SO}_4)_3$ is $___ \times 10^{-6} \text{ mol L}^{-1} \text{ s}^{-1}$. (Nearest integer)

59. 0.004M K_2SO_4 solution is isotonic with 0.01M glucose solution. Percentage dissociation of K_2SO_4 is _____ (Nearest integer)

60. In an electrochemical reaction of lead, at standard temperature, if $E_{(\text{Pb}^{2+}/\text{Pb})}^0 = m$ Volt and $E_{(\text{Pb}^{4+}/\text{Pb})}^0 = n$ Volt, then the value of $E_{(\text{Pb}^{2+}/\text{Pb}^{4+})}^0$ is given by $m - xn$. The value of x is _____ (Nearest integer)

PART-III MATHEMATICS

SECTION-A

61. The value of the integral

$$\int_{-\log_e 2}^{\log_e 2} e^x \left(\log_e \left(e^x + \sqrt{1 + e^{2x}} \right) \right) dx \text{ is equal to}$$

$$(a) \log_e \left(\frac{2(2+\sqrt{5})}{\sqrt{1+\sqrt{5}}} \right) - \frac{\sqrt{5}}{2} \quad (b) \log_e \left(\frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}} \right) + \frac{\sqrt{5}}{2}$$

$$(c) \log_e \left(\frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}} \right) + \frac{\sqrt{5}}{2} \quad (d) \log_e \left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}} \right) - \frac{\sqrt{5}}{2}$$

62. If equation of the plane that contains the point $(-2, 3, 5)$ and is perpendicular to each of the planes

$$2x + 4y + 5z = 8 \text{ and } 3x - 2y + 3z = 5 \text{ is}$$

$$\alpha x + \beta y + \gamma z + 97 = 0 \text{ then } \alpha + \beta + \gamma =$$

$$(a) 18 \quad (b) 17$$

$$(c) 16 \quad (d) 15$$

63. Let R be a rectangle given by the lines $x = 0, x = 2, y = 0$ and $y = 5$. Let $A(\alpha, 0)$ and $B(0, \beta)$, $\alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4:1. Then, the mid-point of AB lies on a

$$(a) \text{ parabola} \quad (b) \text{ hyperbola}$$

$$(c) \text{ straight line} \quad (d) \text{ circle}$$

64. Let sets A and B have 5 elements each. Let the mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is

formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is____

- (a) 32 (b) 38
(c) 40 (d) 36

65. Let $f(x) = x^2 - x + | -x + [x] |$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t . Then, f is

- (a) continuous at $x = 0$, but not continuous at $x = 1$
(b) continuous at $x = 0$ and $x = 1$
(c) not continuous at $x = 0$ and $x = 1$
(d) continuous at $x = 1$, but not continuous at $x = 0$

66. The number of triplets (x, y, z) , where x, y, z are distinct non negative integers satisfying $x + y + z = 15$, is

- (a) 80 (b) 114
(c) 92 (d) 136

67. For any vector $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10|a_i| < 1, i = 1, 2, 3$, consider the following statements:

(A): $\max\{|a_1|, |a_2|, |a_3|\} \leq |\vec{a}|$

(B) : $|\vec{a}| \leq 3\max\{|a_1|, |a_2|, |a_3|\}$

- (a) Only (B) is true (b) Only (A) is true
(c) Neither (A) nor (B) is true (d) Both (A) and (B) are true

68. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to

- (a) $-\pi + \tan^{-1} \frac{33}{5}$ (b) $-\pi - \tan^{-1} \frac{33}{5}$
(c) $-\pi + \tan^{-1} \frac{8}{9}$ (d) $\pi - \tan^{-1} \frac{8}{9}$

69. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events?

- (a) 10 (b) 9
(c) 21 (d) 15

70. Let $S = \{M = [a_{ij}], a_{ij} \in \{0, 1, 2\}, 1 \leq i, j \leq 2\}$ be a sample space and $A = \{M \in S: M \text{ is invertible}\}$ be an event. Then $P(A)$ is equal to

- (a) $\frac{50}{81}$ (b) $\frac{47}{81}$
(c) $\frac{49}{81}$ (d) $\frac{16}{27}$

71. Consider ellipses $E_k: kx^2 + ky^2 = 1, k = 1, 2, \dots$

20. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k . If r_k is the radius of the circle C_k , then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is

- (a) 3080 (b) 3210
(c) 3320 (d) 2870

72. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)} \left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is

- (a) 6 (b) 8
(c) 5 (d) 7

73. Area of the region $\{(x, y): x^2 + (y - 2)^2 \leq 4, x^2 \geq 2y\}$ is

- (a) $2\pi - \frac{16}{3}$ (b) $\pi - \frac{8}{3}$
(c) $\pi + \frac{8}{3}$ (d) $2\pi + \frac{16}{3}$

74. Let $f: [2, 4] \rightarrow \mathbb{R}$ be a differentiable function such that $(x \log_e x) f'(x) + (\log_e x) f(x) + f(x) \geq 1$,

$x \in [2, 4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$.

Consider the following two statements:

(A): $f(x) \leq 1$, for all $x \in [2, 4]$

(B): $f(x) \geq \frac{1}{8}$, for all $x \in [2, 4]$

Then,

- (a) Only statement (B) is true
(b) Neither statement (A) nor statement (B) is true
(c) Both the statement (A) and (B) are true
(d) Only statement (A) is true

75. Let $y = y(x)$ be a solution curve of the differential equation, $(1 - x^2 y^2) dx = y dx + x dy$.

If the line $x = 1$ intersects the curve $y = y(x)$ at $y = 2$ and the line $x = 2$ intersects the curve $y = y(x)$ at $y = \alpha$, then a value of α is

- (a) $\frac{3e^2}{2(3e^2-1)}$ (b) $\frac{3e^2}{2(3e^2+1)}$
(c) $\frac{1-3e^2}{2(3e^2+1)}$ (d) $\frac{1+3e^2}{2(3e^2-1)}$

76. Let A be a 2×2 matrix with real entries such that $A' = \alpha A + I$, where $\alpha \in \mathbb{R} - \{-1, 1\}$. If $\det(A^2 - A) = 4$, then the sum of all possible values of α is equal to

- (a) 0 (b) $\frac{3}{2}$
(c) $\frac{5}{2}$ (d) 2

77. Let (α, β, γ) be the image of the point $P(2, 3, 5)$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to

- (a) 10 (b) 5
(c) 12 (d) 9

78. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}, \hat{j} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a} \cdot \vec{b} = 6$ then the ordered pair $(\theta, |\vec{a} \times \vec{b}|)$ is equal to

- (a) $(\frac{\pi}{4}, 3\sqrt{6})$ (b) $(\frac{\pi}{3}, 3\sqrt{6})$
(c) $(\frac{\pi}{3}, 6)$ (d) $(\frac{\pi}{4}, 6)$

79. The number of elements in the set $S = \{\theta \in [0, 2\pi]: 3\cos^4 \theta - 5\cos^2 \theta - 2\sin^2 \theta + 2 = 0\}$ is

- (a) 10 (b) 8
(c) 9 (d) 12

80. Let $x_1, x_2, \dots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i(x_i - i)$, $1 \leq i \leq 100$, then the mean of y_1, y_2, y_{100} is.

- (a) 10101.50 (b) 10051.50
(c) 10049.50 (d) 10100

SECTION-B

81. The mean of the coefficients of $x, x^2, \dots, x^7$ in the binomial expansion of $(2 + x)^9$ is__

82. Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \dots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to__

83. For $m, n > 0$, let $\alpha(m, n) = \int_0^2 t^m (1 + 3t) dt$. If $11\alpha(10, 6) + 18\alpha(11, 5) = p(14)^6$, then p is equal to__

84. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat, is__

85. Let a line l pass through the origin and be perpendicular to the lines

$$l_1: \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R}$$

$$\text{and } l_2: \vec{r} = (-\hat{i} + \hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$$

If P is the point of intersection of l and l_1 , and $Q(\alpha, \beta, \gamma)$ is the foot of perpendicular from P on l_2 , then $9(\alpha + \beta + \gamma)$ is equal to__

86. The number of integral terms in the expansion of $(3^{\frac{1}{2}} + 5^{\frac{1}{4}})^{680}$ is equal to

87. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is True, is equal to__

88. Let $H_n = \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1, n \in \mathbb{N}$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is length of the latus rectum of H_k , then $21l$ is equal to__

89. If a and b are the roots of equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to__

90. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$, where $a, c \in \mathbb{R}$. If $A^3 = A$ and the positive value of a belongs to the interval $(n - 1, n]$, where $n \in \mathbb{N}$, then n is equal to ____

