1. (b) $v \propto r^2$

$\frac{v_1}{v_2} = \left(\frac{r}{R}\right)^2$

$8 . \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = 2r$

$\frac{10}{v_2} = \left(\frac{1}{2}\right)^2$

$v_2 = 40 \text{ cm/s}$

2. (b) $f = f_0\left(\frac{c+v_0}{c+v_s}\right)$

$f = 400\left(\frac{360+40}{360+20}\right)$

$f = 421 \text{ Hz}$

3. (b) $\vec{E} = 6.6\hat{j}$

$v = 20\text{MHz}$

$\vec{c} = 3 \times 10^8\hat{i}$

$|\vec{B}| = \frac{|\vec{E}|}{c} = 2.2 \times 10^{-8} \text{ T}$

$\hat{E} \times \hat{B} = \hat{c}$

$\vec{B} = 2.2 \times 10^{-8}\hat{k}T$

4. (c) $\phi = \frac{q_{in}}{\epsilon_0}$

$= \frac{Q}{\epsilon_0}$

$= \frac{CV}{\epsilon_0}$

5. (b) $[ML^{-3}] = [MLT^{-2}]^a[LT^{-1}]^b[T]^c$

$= \left[M^a L^{a+b} T^{-2a-b+c}\right]$

$a = 1,$

$a + b = -3,$

$\Rightarrow b = -4,$

also $-2a - b + c = 0$

$c = -2$

6. **(b)** Conceptual

7. **(a)** $V=\frac{GM}{2R^3}(3R^2-r^2)$ at $r=R \Rightarrow V=\left(\frac{GM}{R}\right)$

at $r=0,\ V_0=\frac{3GM}{2R}=\left(\frac{3\,V}{2}\right)$

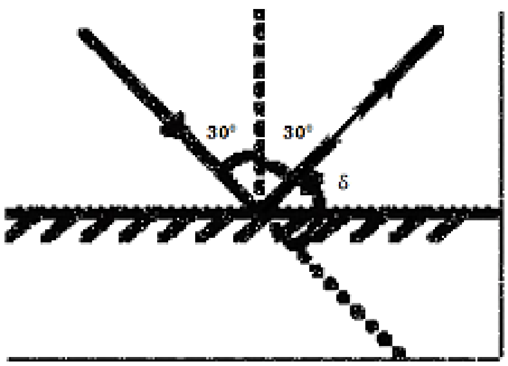
8. **(b)** $v=10\sqrt{x} \Rightarrow v^2=100x$

$$2v\frac{dv}{dx}=100 \Rightarrow a=50\ m/s^2$$

$F=25\ N$

9. **(a)** At $t=2$ particle is at maximum height moving with velocity $V=40\cos 30^\circ=20\sqrt{3}\ ms^{-1}$.

10. **(b)**



$\delta=180^\circ-60^\circ=120^\circ$

11. **(c)** $v_{orbit}=\sqrt{\frac{GM}{R}}=\sqrt{gR};$

$$v_{escape}=\sqrt{\frac{2GM}{R}}=\sqrt{2gR}$$

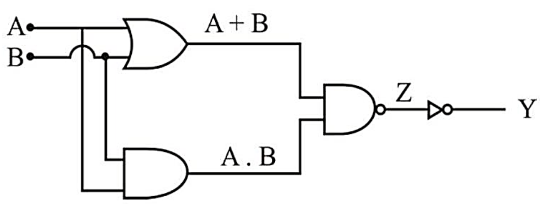
$\Delta v=(\sqrt{2}-1)\sqrt{gR}=8(\sqrt{2}-1)km/s$

12. **(a)** $\lambda \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{\lambda_p}{\lambda_e}=\sqrt{\frac{m_e}{m_p}}=1:43$

13. **(b)** $T=$ constant $\Rightarrow U=$ constant

14. **(c)** $E_n=\frac{-13.6Z^2}{n^2}=\frac{-13.6\times 4}{4}=-13.6eV$

15. **(a)**



$Z=\overline{(A+B)\cdot(A\cdot B)}$

$Y=\overline{Z}=(A+B)\cdot(A\cdot B)$

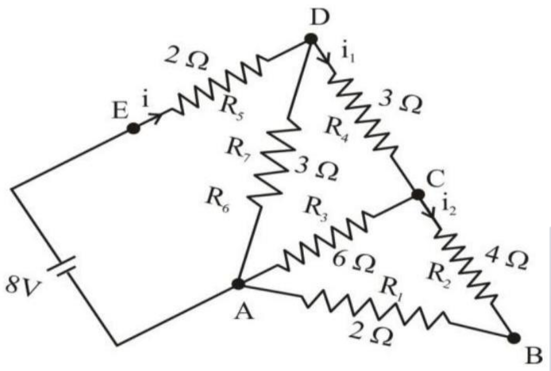
$Y = A.B$

$\therefore$ It is an AND Gate

16. (a)

$$V_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3 \times 1.4 \times 10^{-23} \times 300}{4.6 \times 10^{-26}}} = 523\ m/s$$

17. (d)



$R_{eq} = 4\Omega$

$i = \frac{8}{4} = 2\ A$

$i_1 = \frac{2 \times 3}{3 + 6} = \frac{2}{3}\ A$

$i_2 = \frac{2/3}{2} = \frac{1}{3}\ A$

18. (d) $\vec{B} - \vec{A} = 2j$

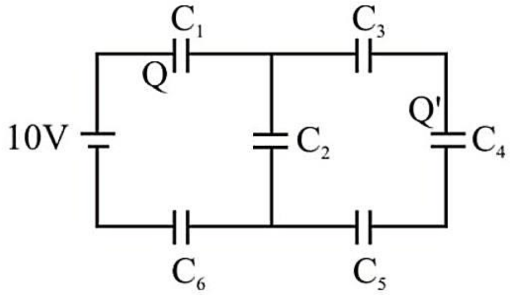
$\vec{B} = 2\hat{i} + 5\hat{j} + 2\hat{k}$

$|\vec{B}| = \sqrt{33}$

19. (c)

20. (b) $\vec{F} = q(\vec{v} \times \vec{B})$ as angle between $\vec{v}$ and $\vec{B}$ is 0° $\vec{F} = 0$

21. (4)



$C_{eq} = 0.5\mu F$

$Q = 0.5 \times 10 = 5\mu C$

$Q' = \dfrac{5\mu C \times 0.8}{0.8 + 0.2} = 4\mu C$

22. (3) $KE = \frac{L^2}{2I} \Rightarrow \frac{KE_{final}}{KE_{initial}} = \frac{I_{initial}}{I_{final}} \Rightarrow \frac{KE_{final}}{E} = \frac{1}{3}$

$\Rightarrow KE_{final} = \dfrac{E}{3}$

23. (2) $\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{A_2}{A_1} = \frac{2}{1}$

24. (5) $V = I \times 10$

$1.5 = \left(\dfrac{3}{10 + 2r}\right) \times 10$

$r = 5\Omega$

25. (300)

$F - 5\,g\sin 30^\circ = 5a \Rightarrow F = 5 + 25 = 30\text{ N}$

$V_{10} = u + at \Rightarrow v_{10} = 0 + 1(10) = 10\text{ m/s}$

$P_{10} = Fv = 300\text{ W}$

26. (625) $I = \frac{V}{R} = \frac{5}{2}A$

$E = \dfrac{1}{2}LI^2 = \dfrac{1}{2} \times 2 \times \left(\dfrac{5}{2}\right)^2$

$E = 625 \times 10^{-2}\text{ J}$

27. (150) $\Delta V = (v \times B)d$

$\Delta V = (2 \times 1/2)0.15$

$\Delta V = 150\text{mV}$

28. (80) $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{YA\Delta L}{\rho AL}}$

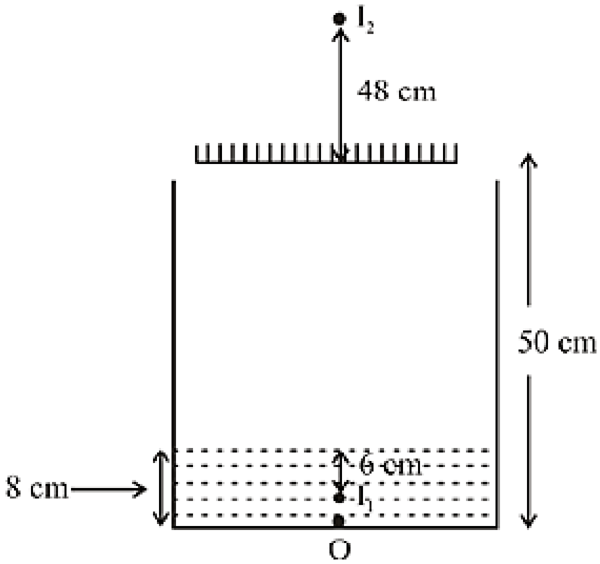
$f = 80\text{ Hz}$

29. (264) $W = T.(\Delta A)$

$W = T\left(8\pi(r_2^2 - r_1^2)\right)$

$W = 264 \times 10^{-4}\text{ J}$

30. (98)

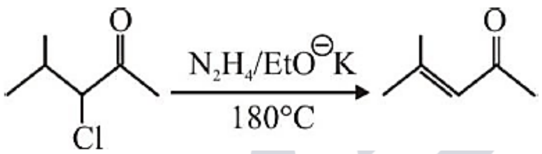


Apparent depth of O $= \frac{d}{\mu} = 6$

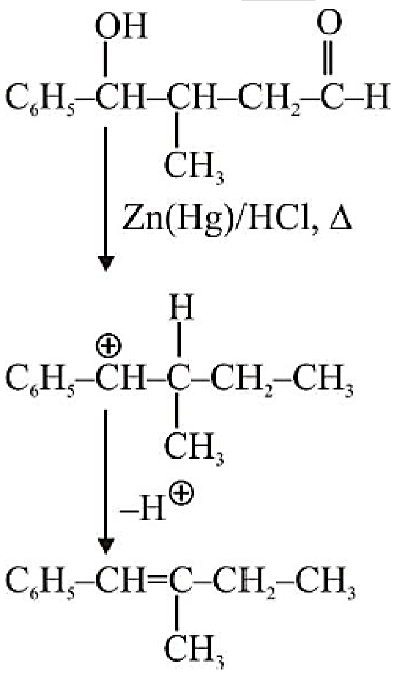
Distance between O and $I_2 = 48 + 50 = 98$ cm

31. (a) BeH_2 is hypovalent

32. (c) Wolff-Kishner reduction is not suitable for base sensitive group.



33. (b)



34. (a) Freons are chlorofluoro carbon.

35. (a) $r = K[A]^1[B]^1$

$0.1 = K(20)^1(0.5)^1$

$0.40 = K(x)^1(0.5)^1$

$0.80 = K(40)^1(y)^1$

From (i) and (ii)

$x = 80$

From (i) and (iii)

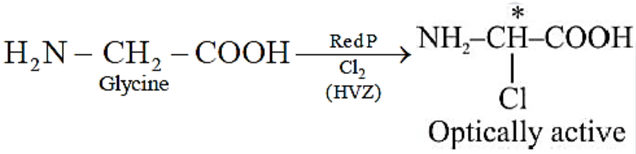
$y = 2$

36. (a) Since H_2O is strong field ligand compared to chloride and Co^{3+} ion is present.

$\therefore$ CFSE is higher for $[Co(NH_3)_5H_2O]^{+3}$, hence it will absorb at lower wavelength

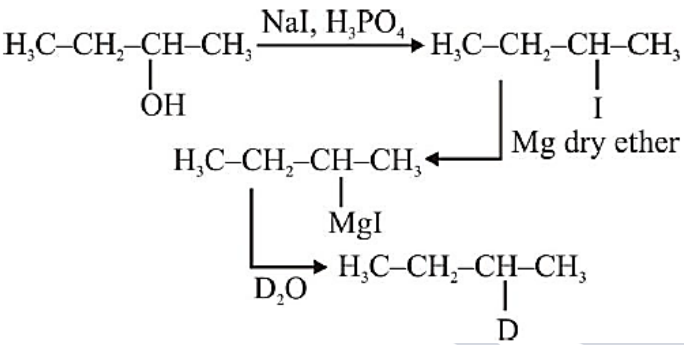
37. (b)

(1)



(2) Meso compound are optically inactive.

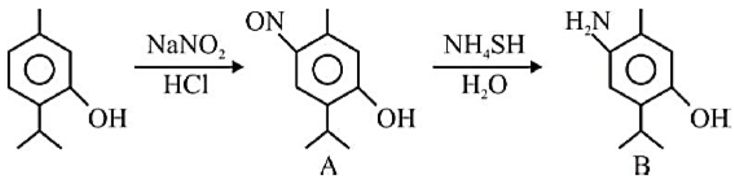
38. (a)



39. (b)

$S_1 \Rightarrow$ HDPE is formed by $TiCl_4$ & $Al(Et)_3$.
$S_2 \Rightarrow$ Nylon-6 is formed by caprolactam.

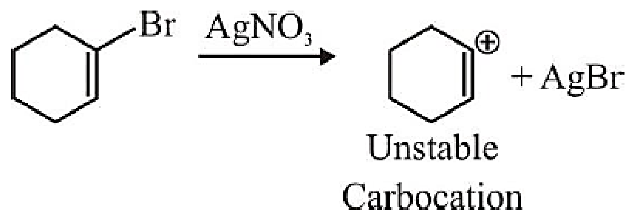
40. (b)



41. (a) SO_2 and NH_3 are polar molecules. They are constituent particles of polar molecular solids.

42. (a) $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 2\ S_2Cl_2 + 4SO_2$

43. (a)



44. (d) Solute (X) = 2 g

Solvent $(H_2O) = 1$ mole = 18 g

Total mass = 2 + 18 = 20 g

% mass of X $= \frac{2}{20} \times 100 = 10\%$

45. (a) $2ZnS + 3O_2 \rightarrow 2ZnO + 2SO_2$

Oxides on carbon reduction forms CO_2 while sulphide on carbon reduction gives CS_2.

CO_2 is more volatile compared to CS_2 therefore oxides are easy to reduce.

46. (d) On moving down the group in alkali metals melting point decreases.

47. (d) $\frac{P^0 - P_s}{P^0} = \frac{n}{N}$ (for dilute solution)

$$\frac{0.2}{54.2} = \frac{n \times 18}{100}$$

$$n = \frac{100}{271 \times 18}$$

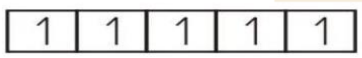
$$w = \frac{100 \times 180}{271 \times 18};\ w = 3.69\ g$$

48. (d)

$[Fe(H_2O)_6]^{3+}$

$Fe^{3+} \Rightarrow [Ar]3\,d^5 4\,s^0$

No pairing



$\therefore$ Unpaired $e^- = 5$

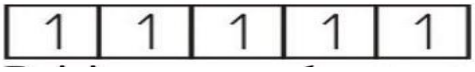
$\mu = \sqrt{n(n+2)}$

$= \sqrt{5(5+2)}$

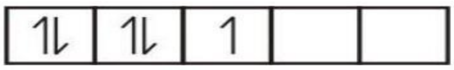
$\mu = \sqrt{35} = 5.92$ B.M.

$[Fe(CN)_6]^{-3}$

$Fe^{3+} \Rightarrow [Ar]3\,d^5 4\,s^\circ$



Pairing occur due to strong field ligand CN^-



$\therefore$ Unpaired $e^- \Rightarrow 1$

$\mu = \sqrt{n(n+2)}$

$= \sqrt{1(1+2)} = \sqrt{3} = 1.732$ B.M.

49. (a)

$Mg(NH_4)PO_4 \Rightarrow$ White

$K_3[Co(NO_2)_6] \Rightarrow$ Yellow

$MnO(OH)_2 \Rightarrow$ Brown

$Fe_4[Fe(CN)_6]_3 \Rightarrow$ Blue

50. (b) $[NiBr_2Cl_2]^{2-} \rightarrow$ This complex species is tetrahedral as $Br^{\ominus}$&$Cl^{\ominus}$ are weak field ligands.

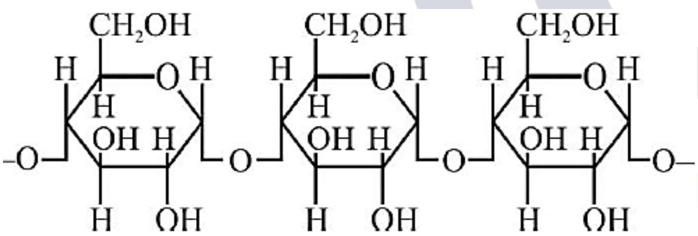
$[PtBr_2Cl_2]^{2-} \rightarrow$ As Pt belongs to 5 d series. This complex species is square planar.

Both the complex species are optically inactive.

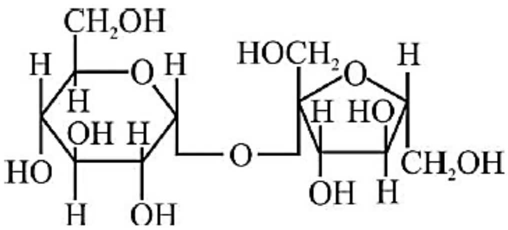
$[NiBr_2Cl_2]^{2-}$, being tetrahedral does not show Geometrical Isomerism.

$[PtBr_2Cl_2]^{2-}$ shows two Geometrical Isomers.

51. (3) Amylose



Sucrose :



Both Amylose and Sucrose does not give Benedict's test.

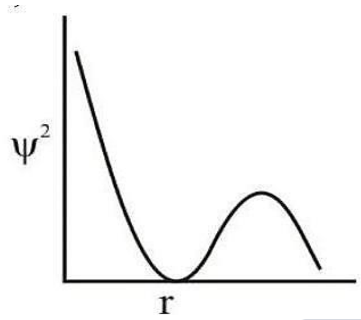
52. (1)

	$H_{2(g)}$ +	$I_{2(g)}$ ⇌	$2HI_{(g)}$
$t = 0$	4.5	4.5	–
t_{eq}	3	3	3

$$K_c = \frac{[HI]^2}{[H_2][I_2]} = \frac{(3)^2}{3\times3} = \frac{9}{9} = 1$$

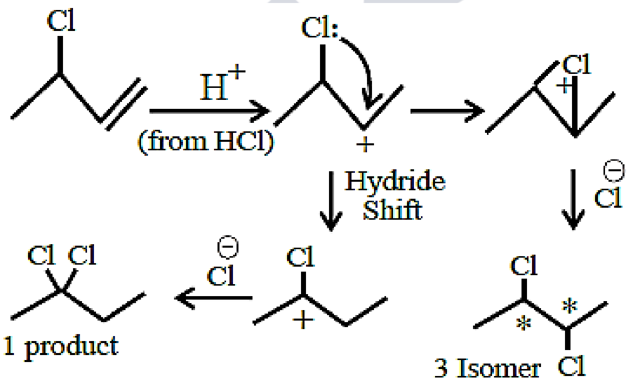
53. (3) B, C and D are correct.

54. (3) A, D and E statements are correct.



For 2 s orbital, the probability density first decreases and then increases. At any distance from nucleus the probability density of finding electron is never zero and it always have some finite value.

55. (4)



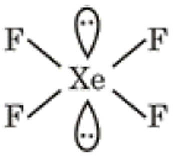
Total Possible Isomeric product = 1 + 3 = 4

56. (6) $Mg(NO_3)_2 \cdot 6H_2O$ is a hydrated salt whereas $Ba(NO_3)_2$ is a anhydrous salt.

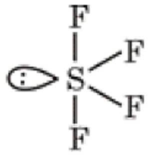
$\therefore x + y = 6$

57. (4) Extensive ⇒ Mole, Volume, Gibbs free energy.
Intensive ⇒ Molar mass, Molar heat capacity, Molarity, E^θ cell.

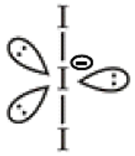
58. (3)



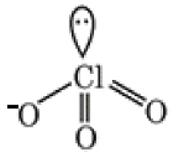
[2 lone pair]



[1 lone pair]



[3 lone pair]



[1 lone pair]

59. (224)

$$Mg + 2HCl \rightarrow MgCl_2 + H_2 \uparrow$$
$$w = 2.4 \text{ g}$$
$$N = \frac{2.4}{24} = 0.1 \text{ mole}$$
$$1 \text{ mole of gas at STP } \Rightarrow 22.4 \text{ lit.}$$
$$\therefore \ 0.1 \text{ mole of gas } = 0.1 \times 22.4$$
$$= 2.24 \text{ lit. } = 224 \times 10^{-2} \text{ litre}$$

60. (4) Given statements A, B, C and D are correct.

61. (d)

Put $x = 0$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$$

$$\lambda^3 = \frac{9^3}{8} \therefore \lambda = \frac{9}{2}$$

$$\therefore \frac{\lambda}{3} = \frac{3}{2}$$

$\therefore$ Required equation is : $x^2 - x\left(\frac{9}{2} + \frac{3}{2}\right)x + \frac{27}{4} = 0 \ 4x^2 - 24x + 27 = 0$

62. (d) Line: $\frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$

$R(3\lambda + 5, 4\lambda + 3, 2\lambda + 4)$

$\therefore 3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$

$\lambda + 6 = 4 \therefore \lambda = -2$

$\therefore R \equiv (-1, -5, 0)$

Line: $\frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$

Point $T = (2\mu - 1, 2\mu - 5, \mu)$

It lies on plane

$2\mu - 1 + 2(2\mu - 5) + 3\mu + 2 = 0$

$\mu = 1$

$\therefore T = (1, -3, 1)$

$\therefore RT = 3$

63. (c) T_{1011} from beginning $= T_{1010+1}$

$= {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$

T_{1011} from end

$= {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

Given: ${}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

$= 2^{10} \cdot {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$

$\left(\frac{-5}{2x}\right)^{2} = 2^{10}\left(\frac{4x}{5}\right)^{2}$

$x^4 = \frac{5^4}{2^{16}}$

$|x| = \frac{5}{16}$

64. (c) Minimum $\{x^2, \{x\}\} = x^2; x \in [0,1)$

$[x - \log_e x] = 1; x \in [1,2)$

$\therefore f(x) = \begin{cases} e^{x^2}; x \in [0,1) \\ e; x \in [1,2) \end{cases}$

$\int_0^2 xf(x)dx = \int_0^1 xe^{x^2}dx + \int_1^2 exdx$

$= \frac{1}{2}(e-1) + \frac{1}{2}(4-1)e$

$= 2e - \frac{1}{2}$

65. (c) I.F $= e^{\int \frac{5dx}{x(x^5+1)}} = e^{\int \frac{5x^{-6}dx}{(x^{-5}+1)}}$

Put, $1 + x^{-5} = t \Rightarrow -5x^{-6}dx = dt$

$\Rightarrow e^{\int \frac{-dt}{t}} = \frac{1}{t} = \frac{x^5}{1+x^5}$

$$y \cdot \frac{x^5}{1+x^5} = \int \frac{x^5}{(1+x^5)} \times \frac{(1+x^5)^2}{x^7} dx$$

$$= \int x^3 dx + \int x^{-2} dx$$

$$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + c$$

Given that : $x = 1 \Rightarrow y = 2\ 2 \cdot \frac{1}{2} = \frac{1}{4} - 1 + c$

$$c = \frac{7}{4}$$

$$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + \frac{7}{4}$$

Now put, $x = 2$

$$y \cdot \left(\frac{32}{33}\right) = \frac{21}{4}$$

$$y = \frac{693}{128}$$

66. (a) a,b,c,d are coplanar points.

$\vec{b} - \vec{a}, \vec{c} - \vec{a}, \vec{d} - \vec{a}$ are coplanar vectors.

So, $[\vec{b} - \vec{a}\vec{c} - \vec{a}\vec{d} - \vec{a}] = 0$

$(\vec{b} - \vec{a}) \cdot ((\vec{c} - \vec{a}) \times (\vec{d} - \vec{a})) = 0$

$[\vec{b}\vec{c}\vec{d}] - [\vec{b}\vec{c}\vec{a}] - [\vec{b}\vec{a}\vec{d}] - [\vec{a}\vec{c}\vec{d}] = 0$

$\Rightarrow [\vec{a}\vec{b}\vec{c}] = [\vec{c}\vec{d}\vec{b}] + [\vec{b}\vec{d}\vec{a}] + [\vec{d}\vec{c}\vec{a}]$

67. (d) $I = \int_0^{\frac{\pi}{4}} f(\sin 2x) \sin x dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(\sin 2x) \sin x dx$

$+\alpha \int_0^{\frac{\pi}{4}} f(\cos 2x) \cos x dx = 0$

Apply king in first part and put $x - \frac{\pi}{4} = t$ in second part.

$$I=\int_0^{\frac{\pi}{4}} f(\cos 2x)\sin\left(\frac{\pi}{4}-x\right)dx+\int_0^{\frac{\pi}{4}} f(\cos 2t)\sin\left(\frac{\pi}{4}+t\right)dt$$

$$+\alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x dx=0$$

$$I=\int_0^{\frac{\pi}{4}} f(\cos 2x)\left[2\sin\frac{\pi}{4}\cdot\cos x+\alpha\cos x\right]dx=0$$

$$I=(\alpha+\sqrt{2})\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x dx=0$$

$\therefore \alpha=-\sqrt{2}$

68. (a) $7x+11y+\alpha z=13$

$5x+4y+7z=\beta$

$175x+194y+57z=361$

(i) $\times 10+$ (ii) $\times 21-$ (iii)

$z(10\alpha+147-57)=130+21\beta-361$

$\therefore 10\alpha+90=0$

$\alpha=-9$

$130-361+21\beta=0$

$\beta=11$

$\alpha+\beta+2=4$

69. (c) $[x]^2-3[x]-10>0$

$[x]<-2$ or $[x]>5$

70. (b) $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 8 & 0 & -2\\ 4 & -3 & -2\end{vmatrix}=\hat{i}(-6)+8\hat{j}-24\hat{k}$

Normal of the plane $=3\hat{i}-4\hat{j}+12\hat{k}$

Plane: $3x-4y+12z=3$

Distance from A $(3,4,\alpha)$

$$\left|\frac{9-16+12\alpha-3}{13}\right|=2$$

$\alpha=3$

$\alpha=-8(\text{ rejected })$

Distance from B $(2,3,a)$

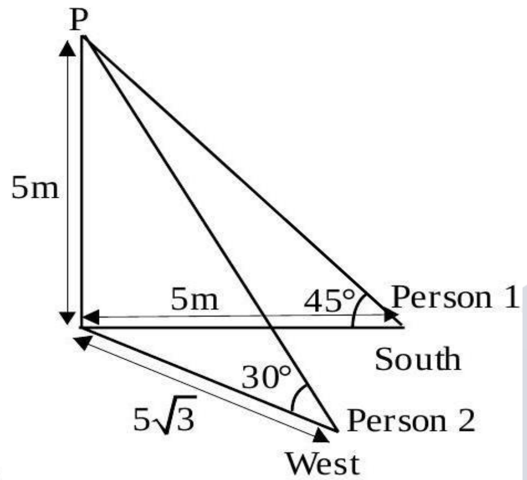
$$\left|\frac{6-12+12a-3}{13}\right|=3$$

$a = 4$

71. (d) Converse of $(\sim p) \wedge q) \Rightarrow r$

$\equiv r \Rightarrow (\sim p \wedge q)$
$\equiv \sim r \vee (\sim p \wedge q)$
$\equiv \sim r \vee (p \vee \sim q) \equiv (p \vee \sim q) \Rightarrow \sim r$

72. (10)



Distance = 10 (By Pythagoras theorem)

73. (a) $\frac{5\left(\frac{a}{5}\right)+3\left(\frac{b}{3}\right)+2\left(\frac{c}{2}\right)+d}{11} \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2}\right)^{1/11}$

$$1 \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2}\right)^{1/11}$$

$\beta = 90$

74. (c) $(x-2)^2 + y^2 = r^2$

Solving with ellipse, we get

$$(x-2)^2 + \frac{36 - x^2}{4} = r^2$$

$$3x^2 - 16x + 52 - 4r^2 = 0$$

$$D = 0 \Rightarrow 4r^2 = \frac{92}{3}$$

75. (d) $x + y = 18 \ \because$ mean $= 5$

$$10 = \frac{1 + 4 + 16 + 25 + x^2 + y^2}{6} - 25$$

$$x^2 + y^2 = 164$$

By solving (i) and (ii)

$x = 8, y = 10$

M.D. $(\bar{x}) = \frac{\sum|x_i - \bar{x}|}{6} = \frac{8}{3}$

76. (b) ${}^{n+2}C_{r-1}: {}^{n+2}C_r: {}^{n+2}C_{r+1} = 1:3:5$

$\frac{{}^{n+2}C_{r-1}}{{}^{n+2}C_r} = \frac{1}{3}$

$n = 4r - 3$

$\frac{{}^{n+2}C_r}{{}^{n+2}C_{r+1}} = \frac{3}{5}$

$8r - 1 = 3n$

From, (i) and (ii)

$r = 2$ and $n = 5$

Required sum $= 63$

77. (a) $4 \times 4! + 1 \times 3! + 1 = 103$
78. (b) Let $a = x_1 + iy_1 z = x + iy$

Now $\mathrm{Re}(a + \bar{z}) > \mathrm{Im}(\bar{a} + z)$

$\therefore x_1 + x > -y_1 + y$

$x_1 = 2, y_1 = 10, x = -12, y = 0$

Given inequality is not valid for these values.

S1 is false.

Now $\mathrm{Re}(a + \bar{z}) < \mathrm{Im}(\bar{a} + z)$

$x_1 + x < -y_1 + y$

$x_1 = -2, y_1 = -10, x = 12, y = 0$

Given inequality is not valid for these values.

S2 is false.

79. (b) Let $a_1 = 1 \Rightarrow 5$ choices of b_2

$a_1 = 3 \Rightarrow 4$ choices of b_2

$a_1 = 4 \Rightarrow 4$ choices of b_2

$a_1 = 6 \Rightarrow 2$ choices of b_2

$a_1 = 9 \Rightarrow 1$ choices of b_2

For (a_1, b_2)16 ways.

Similarly, $b_1 = 2 \Rightarrow 4$ choices of a_2

$b_1 = 4 \Rightarrow 3$ choices of a_2

$b_1 = 5 \Rightarrow 2$ choices of a_2

$b_1 = 8 \Rightarrow 1$ choices of a_2

Required elements in $R = 160$

80. (b) $f(x) = \begin{cases} x+1, \ x<0 \\ 1-x, \ 0 \leq x < 1 \\ x-1, 1 \leq x \end{cases}$

$g(x) = \begin{cases} x+1, x<0 \\ 1, x \geq 0 \end{cases}$

$g(f(x)) = \begin{cases} x+2, x<-1 \\ 1, x \geq -1 \end{cases}$

$\therefore$ $g(f(x))$ is continuous everywhere

$g(f(x))$ is not differentiable at $x = -1$

Differentiable everywhere else

81. (2) Let $e^{2x} = t$

$\Rightarrow t^4 - t^3 - 3t^2 - t + 1 = 0$

$\Rightarrow t^2 + \frac{1}{t^2} - \left(t + \frac{1}{t}\right) - 3 = 0$

$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 5 = 0$

$\Rightarrow t + \frac{1}{t} = \frac{1 + \sqrt{21}}{2}$

Two real values of t.

82. (27) $64x^2 + 5Nx + 1 = 0$

$D = 25\ N^2 - 256 < 0$

$\Rightarrow N^2 < \frac{256}{25} \Rightarrow N < \frac{16}{5}$

$\therefore N = 1,2,3$

$\therefore$ Probability $= \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{37}{64}$

$\therefore q - p = 27$

83. (285) $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

$\vec{b}\cdot(\vec{a}\times\vec{c}) = 27, \vec{a}\cdot\vec{b} = 0$

$\vec{b}\times(\vec{a}\times\vec{c}) = -3\vec{a}$

Let θ be angle between $\vec{b}, \vec{a}\times\vec{c}$

Then $|\vec{b}|\cdot|\vec{a}\times\vec{c}|\sin\theta = 3\sqrt{14}$

$|\vec{b}|\cdot|\vec{a}\times\vec{c}|\cos\theta = 27$

$\Rightarrow \sin\theta = \frac{\sqrt{14}}{\sqrt{95}}$

$\therefore |\vec{b}|\times|\vec{a}\times\vec{c}| = 3\sqrt{95}$

$\Rightarrow |\vec{a}\times\vec{c}| = \sqrt{3}\times\sqrt{95}$

84. (1680)

$\left(\frac{z^2+8iz-15}{z^2-3iz-2}\right)\in R$

$\Rightarrow 1+\frac{(1liz-13)}{(z^2-3iz-2)}\in R$

Put $z=\alpha-\frac{13}{11}i$
$\Rightarrow (z^2-3iz-2)$ is imaginary

Put $z = x+iy$

$\Rightarrow (x^2-y^2+2xyi-3x+3y-2)\in$ Imaginary
$\Rightarrow \operatorname{Re}(x^2-y^2+3y-2+(2xy-3x)i)=0$
$\Rightarrow x^2-y^2+3y-2=0$
$x^2=y^2-3y+2$
$x^2=(y-1)(y-2) \therefore z=\alpha-\frac{13}{11}i$

Put $x=\alpha, y=\frac{-13}{11}$

$\alpha^2=\left(\frac{-13}{11}-1\right)\left(\frac{-13}{11}-2\right)$

$\alpha^2=\frac{(24\times35)}{121}$

$242\alpha^2=48\times35=1680$

85. (2) $10=1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+\cdots$.. upto ∞

$9=\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+\cdots$.. upto ∞

$\frac{9}{k}=\frac{4}{k^2}+\frac{8}{k^3}+\frac{13}{k^4}+\cdots$.. upto ∞

$S = 9\left(1-\frac{1}{k}\right) = \frac{4}{k}+\frac{4}{k^2}+\frac{5}{k^3}+\frac{6}{k^4}+\cdots$.. upto ∞

$\frac{S}{k} = \frac{4}{k^2}+\frac{4}{k^3}+\frac{5}{k^4}+\cdots$.. upto ∞

$$\left(1-\frac{1}{k}\right)S = \frac{4}{k}+\frac{1}{k^3}+\frac{1}{k^4}+\frac{1}{k^5}+\cdots..\infty$$

$$9\left(1-\frac{1}{k}\right)^2 = \frac{4}{k}+\frac{\frac{1}{k^3}}{\left(1-\frac{1}{k}\right)}$$

$9(k-1)^3 = 4k(k-1)+1$

$k = 2$

86. (360) $f(a)+f(b)+1 = f(d) \leq 6$

$f(a)+f(b) \leq 5$

Case (i) $f(a) = 1 \Rightarrow f(b) = 1,2,3,4 \Rightarrow 4$ mappings

Case (ii) $f(a) = 2 \Rightarrow f(b) = 1,2,3 \Rightarrow 3$ mappings

Case (iii) $f(a) = 3 \Rightarrow f(b) = 1,2 \Rightarrow 2$ mappings

Case (iv) $f(a) = 4 \Rightarrow f(b) = 1 \Rightarrow 1$ mapping

$f(5)\&f(6)$ both have 6 mappings each

Number of functions $= (4+3+2+1)\times 6\times 6 = 360$

87. (116) $P(3,\alpha)$ lies on $y^2 = 12x$

$\Rightarrow \alpha = \pm 6$

But, $\left.\frac{dy}{dx}\right|_{(3,\alpha)} = \frac{6}{\alpha} = 1 \Rightarrow \alpha = 6(\alpha = -6$ reject $)$

Now, hyperbola $\frac{x^2}{9}-\frac{y^2}{36} = 1$, normal at

$Q(\alpha-1,\alpha+2)$ is $\frac{9x}{5}+\frac{36y}{8} = 45$

$\Rightarrow 2x+5y-50 = 0$

Now, distance of $(6,-4)$ from $2x+5y-50 = 0$ is equal to

$$\left|\frac{2(6)-5(d)-50}{\sqrt{2^2+5^2}}\right| = \frac{58}{\sqrt{29}}$$

$\Rightarrow$ Square of distance $= 116$

88. (11) $\ell: x = \frac{y-1}{2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$

DR's of line $\ell(1,2,\lambda)$

DR's of normal vector of plane $P: x+2y+3z = 4$ are $(1,2,3)$

Now, angle between line ℓ and plane P is given by

$$\sin\theta = \left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}\right| = \frac{3}{\sqrt{14}}\left(\text{given } \cos\theta = \sqrt{\frac{5}{14}}\right)$$

$$\Rightarrow \lambda = \frac{2}{3}$$

Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$ lies on plane P.

$$\Rightarrow t = -1$$

$$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$$

89. (348) Point of intersection of $\ell_1: 3y - 2x = 3$

$\ell_2: x - y + 1 = 0$ is $P \equiv (0,1)$

Which lies on $\ell_3: \alpha x + \beta y + 17 = 0$,

$\Rightarrow \beta = -17$

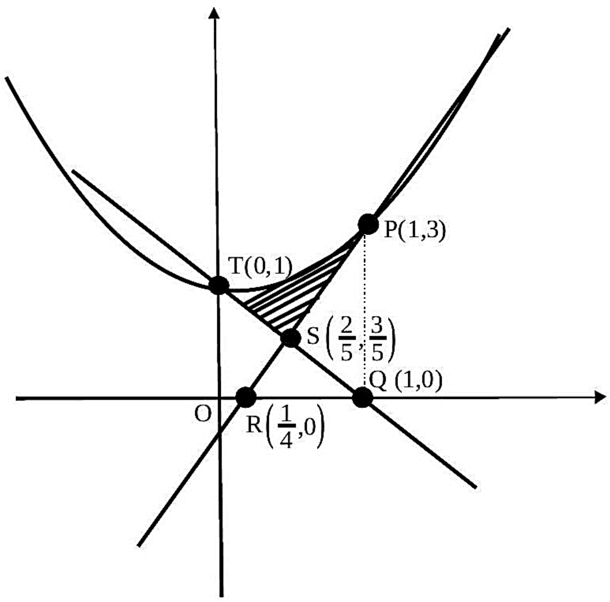
Consider a random point $Q \equiv (-1,0)$ on $\ell_2: x - y + 1 = 0$, image of Q about $\ell_2: x - y + 1 = 0$ is $Q' \equiv \left(\frac{-17}{13}, \frac{6}{13}\right)$ which is calculated by formulae $\frac{x-(-1)}{2} = \frac{y-0}{-3} = -2\left(\frac{-2+3}{13}\right)$

Now, Q' lies on $\ell_3: \alpha x + \beta y + 17 = 0$

$\Rightarrow \alpha = 7$

Now, $\alpha^2 + \beta^2 - \alpha - \beta = 348$

90. (16)



$y = 2x^2 + 1$

Tangent at (1, 3)

$y = 4x - 1$

$A = \int_0^1 (2x^2 + 1)dx$ – area of (ΔQOT) – area of

(ΔPQR) + area of (ΔQRS)

$$A = \left(\frac{2}{3} + 1\right) - \frac{1}{2} - \frac{9}{8} + \frac{9}{40} = \frac{16}{60}$$