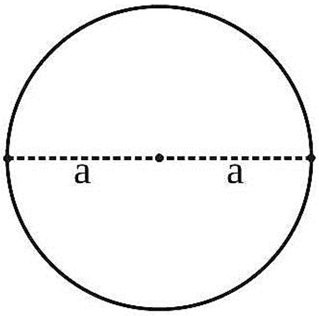
1. **(c)** Electric field due to a dipole at point on its axis $E = \frac{2kp}{r^3}$

2. **(a)**

$F = -kx$	A true
$a = -\omega^2 x$	B true
Velocity is maximum at mean position	C true
Acceleration is maximum at extreme points	D false

3. **(c)**



$F = m\omega^2 r$

$$\Rightarrow \frac{Gmm}{(2a)^2} = m\omega^2 a$$

$$\Rightarrow \omega = \sqrt{\frac{Gm}{4a^3}}$$

4. **(d)** $d_{max} = \sqrt{2Rh_t} + \sqrt{2Rh_r}$

$= \sqrt{2 \times 64 \times 10^5 \times 180} + \sqrt{2 \times 64 \times 10^5 \times 245}$

$= \{(8 \times 6 \times 10^3) + (8 \times 7 \times 10^3)\}m$

$= (48 + 56)km$

$= 104$ km

5. **(c)**

15 year = 3 half lives

Number of active nuclei $= \frac{N_0}{8}$

Number of decay $= \frac{7\,N_0}{8}$

6. **(c)** $= \frac{h}{\sqrt{2mE}}$

$$\lambda' = \frac{h}{\sqrt{2\ m\left(\frac{E}{4}\right)}} = \frac{2\ h}{\sqrt{2mE}} = 2\lambda$$

7. **(c)** For voltmeter

$$R = \frac{V}{I_g} - G$$

$$= \frac{50}{10^{-3}} - 54 \approx 50k\Omega(A)$$

For ammeter

$$S = \frac{I_g G}{I - I_g} = \frac{10^{-3} \times 54}{(10-1) \times 10^{-3}} = 6\Omega(C)$$

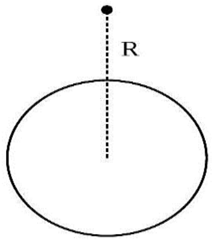
8. **(a)** Average KE per molecule $= \frac{3}{2}kT$

$$\frac{K_{Ar}}{K_H} = \frac{1}{1}$$

9. **(b)** $R_{eq} = R_1 + R_2 + R_3$ So St-1 False

Resistivity depends on temperature. St-2 False

10. **(a)**



By conservation of mechanical energy

$$U_i + K_i = U_f + K_i$$

$$-\frac{GMm}{2R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\frac{GMm}{2R} = \frac{1}{2}mv^2$$

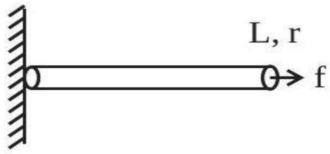
$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

11. **(d)** Induced emf $= -L\frac{dI}{dt}$

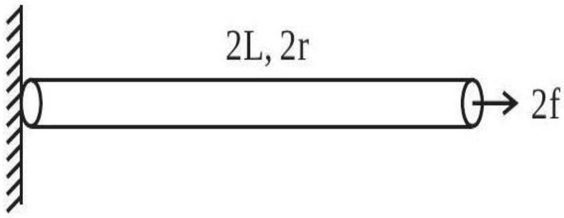
$\Rightarrow 20 = -L\dfrac{(0-2)}{10^{-3}}$

$\Rightarrow L = 10mH$

12. (b)



$\dfrac{f}{\pi r^2} = Y\dfrac{\ell}{L}$



$\dfrac{2f}{\pi(2r)^2} = Y\dfrac{\ell'}{2\,L}$

$\Rightarrow \dfrac{2}{1} = \dfrac{2\ell'}{\ell} \Rightarrow \ell' = \ell$

13. (c) $x = 5t^2 - 4t + 5$

$\mathbf{v} = 10t - 4$

At $t = 2$ s $v = 16$ m/s

14. (a)

$\vec{r} = 10t\hat{\imath} + 15t^2\hat{\jmath} + 7\hat{k}$

$\vec{v} = 10\hat{\imath} + 30t\hat{\jmath}$

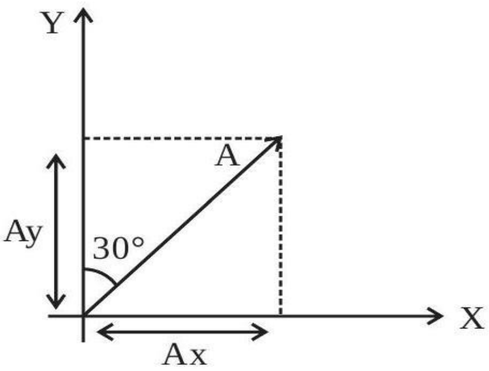
$\vec{a} = 30\hat{\jmath}$

So Net force is along +y direction

15. (b) Increasing order of wave length

X-ray	1 nm to 10^{-3} nm
Ultra Violet	400 nm to 1 nm
Intra red	1 mm to 700 nm
Micro wave	0.1 m to 1 mm

16. (d)



$A_y = A\cos 30^\circ = 2\sqrt{3}$

$\Rightarrow A\dfrac{\sqrt{3}}{2} = 2\sqrt{3}$

$\Rightarrow A = 4$

Now $A_x = A\sin 30^\circ = 4 \times \frac{1}{2} = 2$

17. (a) $v = \lambda^a g^b \rho^c$

using dimension formula

$\Rightarrow [M^0\ L^1\ T^{-1}] = [L^1]^a [L^1\ T^{-2}]^b [M^1\ L^{-3}]^c$

$\Rightarrow [M^0\ L^1\ T^{-1}] = \left[M^c L^{a+b-3c} T^{-2\,b}\right]$

$\therefore c = 0, a + b - 3c = 1, -2\,b = -1 \Rightarrow b = \dfrac{1}{2}$

Now $a + b - 3c = 1$

$\Rightarrow a + \dfrac{1}{2} - 0 = 1$

$\Rightarrow a = \dfrac{1}{2}$

$\therefore a = \dfrac{1}{2}, b = \dfrac{1}{2}, c = 0$

18. (b) On P-V scale area of loop = work done

$\Rightarrow W = +\dfrac{1}{2}(2) \times 300$

$W = 300\ J$

19. (b) As for first minima

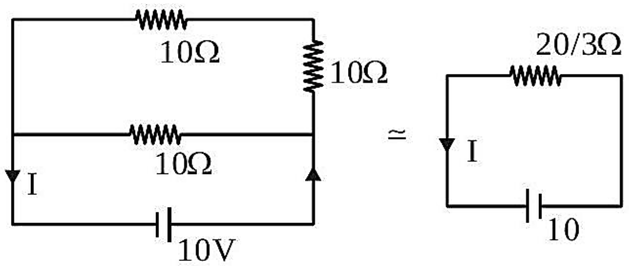
$a\sin\theta = \lambda$

$\Rightarrow a\sin 30^\circ = 600 \times 10^{-9}$

$\Rightarrow a = 1200 \times 10^{-9}$ m

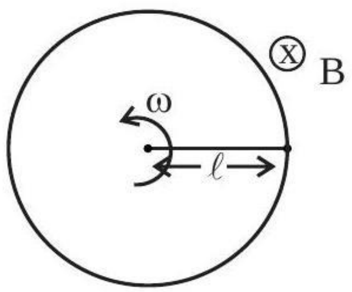
$\Rightarrow a = 1.2\mu m$

20. (a) In the circuit D_1 and D_3 are forward biased and D_2 is reverse biased.



$\therefore I = \frac{10}{20/3} = \frac{3}{2}$ A = 1.5 A

21. (88)



Here $\omega = 210$rpm

$= 210 \times \frac{2\pi}{60}$rad/s

$\Rightarrow \omega = 7\pi$rad/s

$\&\ell = 0.2$ m

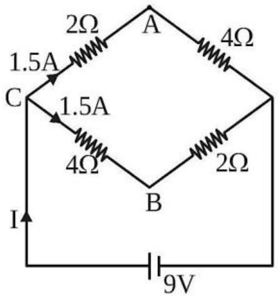
$\&$ B = 0.2 T

emf developed across rod is $= \frac{1}{2}B\omega\ell^2$

$\frac{1}{2} \times 0.2 \times 7\pi \times (0.2)^2$

$= 88$mV

22. (3)



In the circuit $I = \frac{9}{3} = 3\text{ A}$

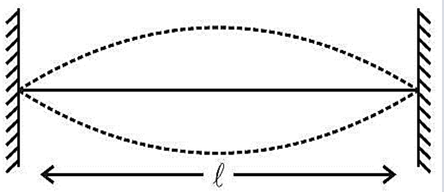
$V_C - V_A = 2 \times 1.5 = 3$

$V_C - V_B = 4 \times 1.5 = 6$

$Eq^n(II) - Eq^n(I)$

$V_A - V_B = 6 - 3 = 3\text{ Volt}$

23. (90)



Fundamental frequency $= 50$ Hz mass / length $= 20$ g/m

mass $= 18$ g

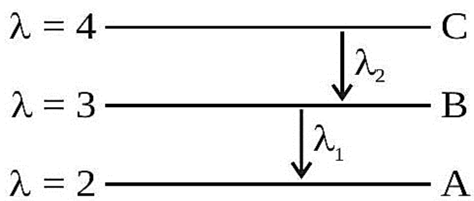
length of string $= \frac{18}{20}\text{ m} = \frac{9}{10}\text{ m}$

from diagram $\frac{\lambda}{2} = \ell$

$\Rightarrow \lambda = 2\ell = \frac{9}{5}\text{ m}$

again speed $v = f\lambda = 50 \times \frac{9}{5} = 90\text{ m/s}$

24. (5)



As $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\frac{1}{\lambda_1} = R(1)^2\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] = R\left(\frac{5}{36}\right)$

$\& \frac{1}{\lambda_2} = R(1)^2\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right] = R\left(\frac{7}{144}\right)$

(ii) ÷ (i) gives

$\frac{\lambda_1}{\lambda_2} = \frac{7/144}{5/36} = \frac{7}{20} = \frac{7}{4 \times 5}$

$\therefore n = 5$

25. (5) For solid sphere $\frac{2}{5}mR^2 = mk_{sph}^2$

$k_{sph} = \sqrt{\frac{2}{5}}R$

For solid cylinder $\frac{mR^2}{2} = mk_{cyl}^2$

$\Rightarrow k_{cyl} = \frac{R}{\sqrt{2}}$

$\frac{k_{sph}}{k_{cyl}} = \frac{\sqrt{\frac{2}{5}}}{\frac{1}{\sqrt{2}}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{x}}$

$\therefore x = 5$

26. (30) As $\mu = \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\sqrt{2} = \frac{\frac{\sin(D_{min}+60)}{2}}{\sin\left(\frac{60}{2}\right)}$

$\Rightarrow \frac{1}{\sqrt{2}} = \sin\left(\frac{D_{min}+60}{2}\right)$

$\Rightarrow \frac{D_{min}+60}{2} = 45$

$\Rightarrow D_{min} = 30$

27. (40) Magnetic field due to moving charge

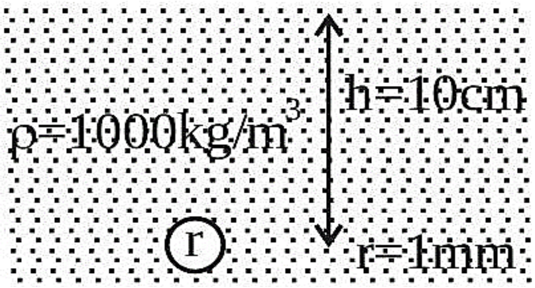
$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$

$B = \frac{\mu_0}{4\pi}\frac{ev\sin(\pi/2)}{r^2}$

$B = \frac{10^{-7} \times 1.6 \times 10^{-19} \times 6.76 \times 10^6}{0.52 \times 0.52 \times 10^{-20}}$

$B = 40\ T$

28. (1150)



Pressure inside the bubble

$$P = P_0 + h\rho g + \frac{2T}{r}$$

$$P - P_0 = h\rho g + \frac{2T}{r}$$

$$= 0.1 \times 1000 \times 10 + \frac{2 \times .075}{10^{-3}}$$

$$= 1000 + (0.15)(1000)$$

$$= 1150\ Pa$$

29. (30) Work done $= \int F dx$

$$\int_2^4 5x dx = 5\left[\frac{x^2}{2}\right]_2^4$$

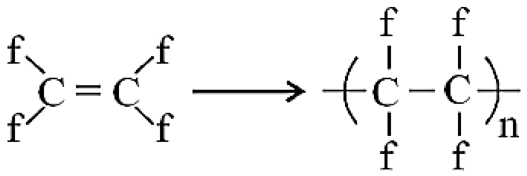
$$= \frac{5}{2}[16 - 4]$$

$$= 30\ J$$

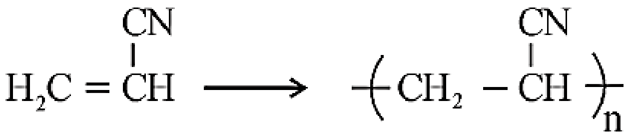
30. (5)

Charge on C_1 is $Q_1 = 2 \times 10 = 20\mu C$
Charge on C_2 is $Q_2 = x \times 10 = 10x\mu C$
Charge on C_3 is $Q_3 = 3 \times 10 = 30\mu C$
Total charge $20 + 10x + 30 = 100 \Rightarrow x = 5$

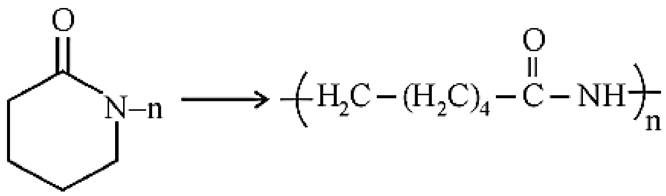
31. (a)



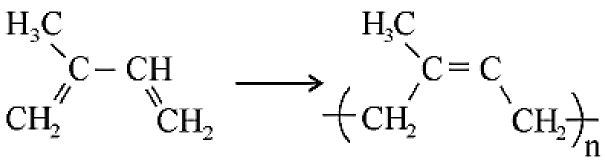
Tetra Fluoroethene Teflon



Acrylonitrile Orlon

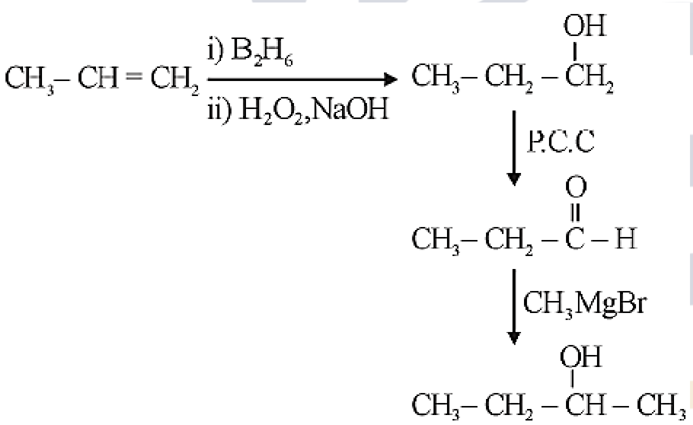


Caprolactum Nylon - 6



Isoprene Natural rubber

32. (a)



33. (d) Photochemical smog occurs in warm, dry and sunny climate.

34. (d) $2PbS + 3O_2(g) \xrightarrow{\Delta} 2PbO + 2SO_2(g)$

It is a roasting reaction.

35. (d)

(A) NF_3 has trigonal pyramidal shape.

(B) Bond order $\Rightarrow N_2 > O_2$

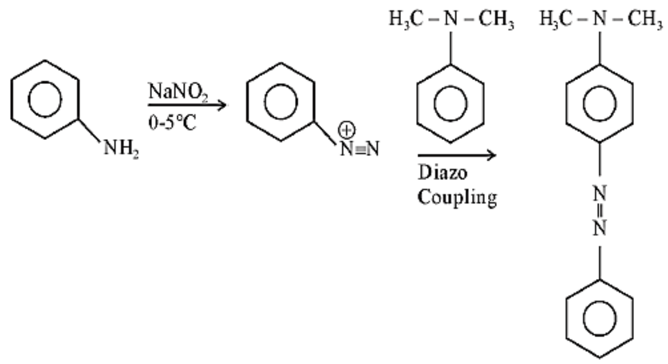
Bond length $\Rightarrow N_2 < O_2$

⇒ (C)

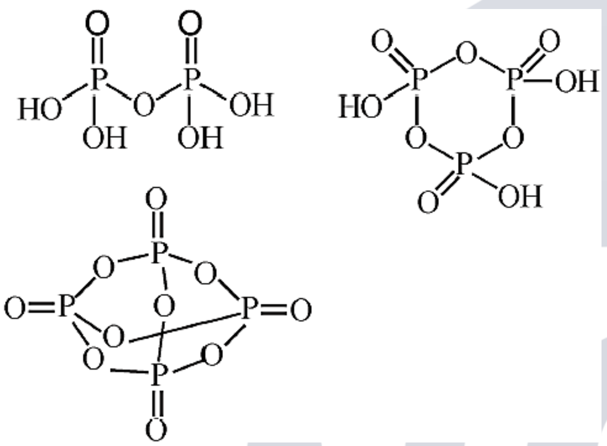
(D) Dipole moment $H_2O > H_2S$

Due to Electronegativity difference.

36. (b)



37. (a)

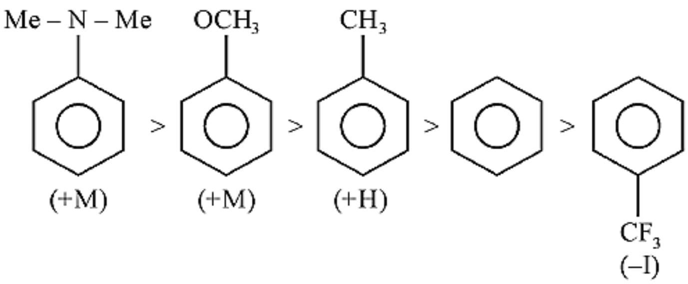


Molecule	Number of P-O-P Bond
$H_4P_2O_7$	1
$(HPO_3)_3$	3
P_4O_{10}	6

38. (a) According to Bohr's model the angular momentum is quantised and equal to $\frac{nh}{2\pi}$.

Heisenberg uncertainty principle explains orbital concept, which is based on probability of finding electron.

39. (b) Higher the electron density on Benzene Ring, Higher its Reactivity towards electrophilic substitution Reaction



40. (d)

(A) pH of 10^{-8} MHCl is in acidic range (6.98).

(B) Conjugate Base of $H_2PO_4^-$ is HPO_4^{2-}

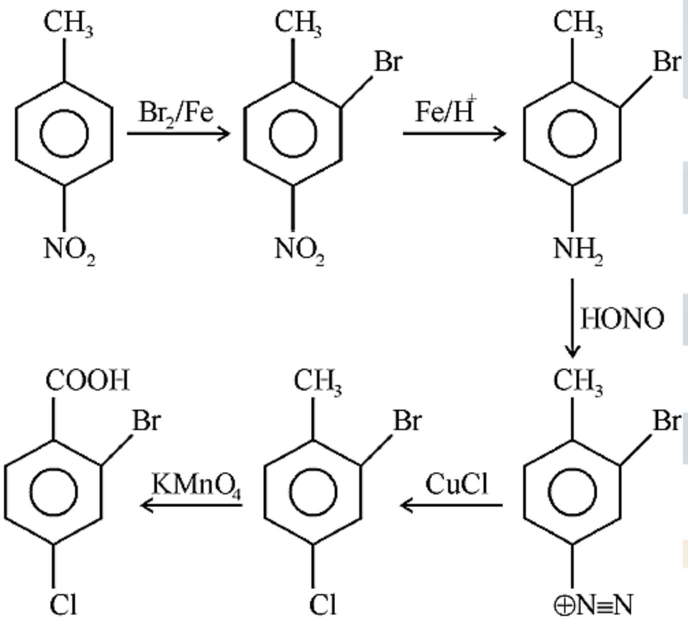
(C) K_w increases with increasing Temperature, as the temperature increases, the dissociation of water increases.

(D) At half neutralisation point, half of the acid is present in the form of salt.

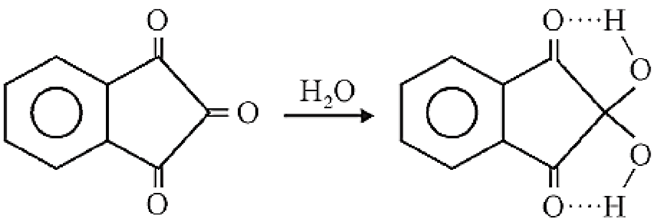
$$pH = Pk_a + \log\frac{1}{1} = Pk_a$$

41. (b) Be, Mg do not give colour to flame due to high excitation energy.

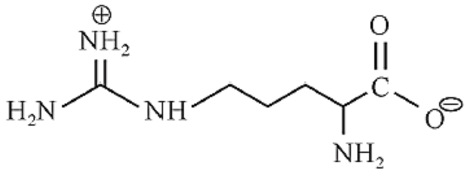
42. (c)



43. (d)



44. (d)



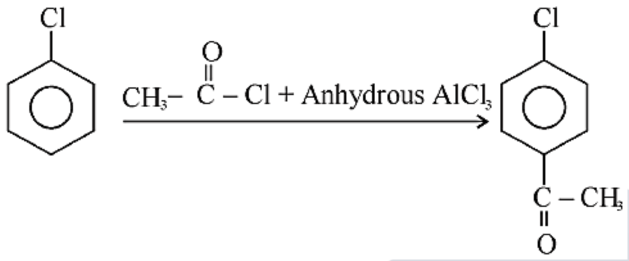
Arginine exist is zwitterion, so solid in nature and soluble in polar solvent.

45. (a)

46. (a) For good quality cement, the ratio of silica (SlO_2) to Alumina (Al_2O_3) should be between 2.5 to 4.

47. (b) In paper chromatography, a special quality paper known as chromatography paper is used. Paper contains water trapped in it, which acts as the stationary phase.

48. (a)



Chlorine is ortho/para directing, para is major.

49. (a) $Ti^{+3} = 67$pm radius

$Cr^{3+} = 62$pm radius

$Mn^{+3} = 65$pm radius

$Fe^{+3} = 65$ pm radius

So, Cr^{3+} has highest tendency to attract ligand.

50. (c) For CsCl, $Cs^{\oplus}$ is present at Body centre and

$Cl^{\ominus}$ at all corner. $\frac{\sqrt{3}a}{2} = r_{cs}{}^{\oplus} + r_{Cl}$

51. (1)

Co^{2+}: 3 d^7 configuration

$t_{2g}^{221} e_g^{11}$

52. (4)

$[Cr(H_2O)_5Cl$	$2AgNO_3 \rightarrow$
0.01M, 20 mL	0.1M
For 0.2 milimole	$AgNO_3$ required
	= 0.4 milimole

$0.4 = 0.1 \times V(\text{ml})$	
$J = 4\ \text{mL}$	

53. (5)

$KMnO_4 \rightarrow Mn^{2+}$

$\downarrow \qquad \downarrow$

+7 **+2**

Change in oxidation state of Mn $= 5$

54. (6)

$CrO_2Cl_2 x - 4 - 2 = 0$

Oxidation State $= +6$

55. (5) Isoelectronic species $O^{2\ominus}, F^{\ominus}, Mg^{2+}, Na^{\oplus}, Al^{3+}$

56. (23) $\frac{24-P_s}{P_s} = \frac{m\times 18}{1000}$

wt of solute $= 30$gm

Volume of solution $= 100$ mL

wt. of solution $= 1.2 \times 100 = 120$gm

wt. of solvent $= 120 - 30 = 90$gm

$$m = \frac{30 \times 1000}{180 \times 90} = 1.85$$

$$\frac{24 - P_s}{P_s} = \frac{1.85 \times 18}{1000}$$

$24 - P_s = 0.0333P_s$

$P_s(1.033) = 24$

$P_s = 23.22$

57. (50) Coagulating value is required milimole of electrolyte needed to coagulate 1 L sol in 2 hours.

Coagulating value $= \frac{20\times 0.5}{200} \times 1000 = 50$

58. (130) $E_a = 300$ kJ mol^{-1}

$$\frac{E_a}{T} = \frac{E'_a}{T'}$$

(Since rate of catalysed and uncatalysed reaction is same)

$$\frac{300}{600}=\frac{E'_{a,f}}{300}$$
$$E'_{a,f}=150$$
$$20=150-E'_{a,b}$$
$$E'_{a,b}=130$$

59. (3)

(A) Conductivity decreases with dilution for strong electrolyte as well as weak electrolyte.

(B) On dilution, The number of ions per unit volume that carry current in a solution decreases.

(C) Molar conductivity increases with dilution.

(D) Molar conductivity of strong electrolyte follows DHO equation but it is not applicable for weak electrolyte.

(E) On dilution degree of dissociation of weak electrolyte increases.

So answer is (A), (C) & (D).

60. (1070)

$$\Delta S=\frac{\Delta H}{T_{mp}}$$

$$28.4=\frac{30.4\times 1000}{T_{mp}}$$

$T_{mp}=1070.422$ K.

61. (a)

(1,1,1) (3,3,3) (5,5,5) (8,8,8)
(5,5,8) (8,8,5) (1,3,5) (1,3,8)

$$\text{Total number }=1+1+1+1+\frac{3!}{2!}+\frac{3!}{2!}+3!+3!=22$$

62. (c)

$$\text{Shor test distance}=\frac{\left|\begin{vmatrix}0&4&1\\3&-4&0\\2\lambda&3&-12\end{vmatrix}\right|}{\left|\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&4&1\\3&-4&0\end{vmatrix}\right|}$$

$$13=\frac{|153+8\lambda|}{|4\hat{i}+3\hat{j}-12\hat{k}|}$$

$$=\frac{|153+8\lambda|}{13}$$

$$|153+8\lambda|=169$$

$$153+8\lambda=169,-169$$

$\lambda = \frac{16}{8}, \frac{-322}{8}$

$8\left|\sum_{\lambda \in S} \lambda\right| = 306$

63. (b) $\mu = 20, \sigma = 8$

$$\mu_{\text{Corrected}} = \frac{200 - 50 + 40}{10} = 19$$

$$\sigma^2 = \frac{1}{10}\sum x_i^2 - 20^2$$

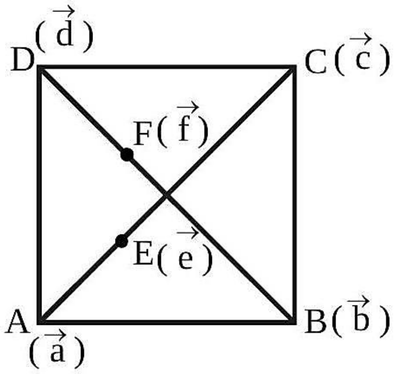
$$(64 + 400)10 = \sum x_i^2$$

$$\sigma^2_{\text{Corrected}} = \frac{1}{10}[(64 + 400)10 - 2500 + 1600] - 19^2$$

$= 374 - 361$

$= 13$

64. (d)



$\overrightarrow{AB} - \overrightarrow{BC} + \overrightarrow{AB} - \overrightarrow{DC} = k\overrightarrow{FE}$

$(\vec{b} - \vec{a}) - (\vec{c} - \vec{b}) + (\vec{d} - \vec{a}) - (\vec{c} - \vec{d}) = k\overrightarrow{FE}$

$2(\vec{b} + \vec{d}) - 2(\vec{a} - \vec{c}) = k\overrightarrow{FE}$

$2(2f - 2(2e) = kFE$

$4(\vec{f} - \vec{e}) = k\overrightarrow{FE}$

$-4\overrightarrow{FE} = k\overrightarrow{FE}$

$k = -4$

65. (c) $2(y + 2)\ln(y + 2)dx + (x + 4 - 2\ln(y + 2))dy = 0$ $2\ln(y + 2) + (x + 4 - 2\ln(y + 2))\frac{1}{y+2} \cdot \frac{dy}{dx} = 0$

let, $\ln(y + 2) = t$

$$\frac{1}{y + 2} \cdot \frac{dy}{dx} = \frac{dt}{dx}$$

$$2t+(x+4-2t)\cdot\frac{dt}{dx}=0$$

$$(x+4-2t)\frac{dt}{dx}=-2t$$

$$\frac{dx}{dt}=\frac{2t-4-x}{2t}$$

$$\frac{dx}{dt}+\frac{x}{2t}=\frac{2t-4}{2t}$$

$$x\cdot t^{1/2}=\int\frac{2t-4}{2t}\cdot t^{1/2}\cdot dt$$

$$x.t^{1/2}=\int\left(t^{1/2}-\frac{2}{t^{1/2}}\right)\cdot dt$$

$$=\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-2\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$x\cdot t^{\frac{1}{2}}=\frac{2t^{\frac{3}{2}}}{3}-4t^{\frac{1}{2}}+C$$

$$x=\tfrac{2}{3}\cdot t-4+C.\,t^{\frac{-1}{2}}$$

$$x=\frac{2}{3}\ln(y+2)-4+C\cdot(\ln(y+2))^{\frac{-1}{2}}$$

Put $y=e^4-2, x=1$ $1=\frac{2}{3}\times 4-4+C\times\frac{1}{2}$

$$\frac{C}{2}=5-\frac{8}{3}=\frac{7}{3}$$

$$C=\frac{14}{3}$$

$$x=\frac{2}{3}\times 9-4+\frac{14}{3}\times\frac{1}{3}$$

$$=2+\frac{14}{9}$$

$$=\frac{32}{9}$$

66. (d)

$$\text{Let } g(x)=1+x+[x]=\begin{cases}1+x; & x\in[0,1)\\ 2+x; & x\in[1,2)\\ 5; & x=2\end{cases}$$

$$\lambda(x)=x+2[x]=\begin{cases}x; & x\in[0,1)\\ x+2; & x\in[1,2)\\ 6; & x=2\end{cases}$$

$r(x) = 2 + x$

$$f(x) = \begin{cases} 2 + x; & x \in [0,2) \\ 6; & x = 2 \end{cases}$$

$f(x)$ is discontinuous only at $x = 2 \Rightarrow m = 1$

$f(x)$ is differentiable in $(0,2) \Rightarrow n = 0$

$(m + n)^2 + 2 = 3$

67. (b) $x|x| - 5|x + 2| + 6 = 0$

$C - 1: -x \in [0, \infty]$

$x^2 - 5x - 4 = 0$

$$x = \frac{5 \pm \sqrt{25 + 16}}{2} = \frac{5 + \sqrt{41}}{2}$$

$$x = \frac{5 \pm \sqrt{41}}{2}$$

$C - 2$:- :- $x \in [-2,0)$

$-x^2 - 5x - 4 = 0$

$x^2 + 5x + 4 = 0$

$x = -1, -4$

$x = -1$

$C - 3: x \in [-\infty, -2)$

$-x^2 + 5x + 16 = 0$

$x^2 - 5x - 16 = 0$

$$x = \frac{5 \pm \sqrt{25 + 64}}{2}$$

$$\frac{5 \pm \sqrt{89}}{2}$$

$$x = \frac{5 - \sqrt{89}}{2}$$

68. (b) $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$

Coefficient of $x^1 = 20$

$$20 = \frac{10!}{9!\,1!} \times a^9 \times b^1$$

$a^9 \cdot b = 2$

$a = 1, b = 2$

Coefficient of $x^2 = 210$

$210 = \frac{10!}{9!\,1!} \times a^9 \times c^1 + \frac{10!}{8!\,2!} \times a^8 b^2$

$210 = 10.c + 45 \times 4$

$10c = 30$

$c = 3$

$2(a + b = c) = 12$

69. (a)

$|A| = m - n$

$4\,m + n = 22$

$17\,m + 4n = 93$

$m = 5, n = 2$

$|A| = 3$

$|\,2\text{adj}(\text{adj}\,5\,A)\big| = 2^5\big|5\,A\big|^{16}$

$$\begin{aligned} &= 2^5 \cdot 5^{80} |\,A|^{16} \\ &= 2^5 \cdot 5^{80} \cdot 3^{16} \\ &= 3^{11} \cdot 5^{80} \cdot 6^5 \end{aligned}$$

$a + b + c = 96$

70. (b) $a, A_1,\ A_2, b$ are in A.P.

$d = \frac{b-a}{3}; A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$

$A_2 = \frac{a+2\,b}{3}$

$A_1 + A_2 = a + b$

a, G_1, G_2, G_3, b are in G.P.

$r = \left(\frac{b}{a}\right)^{\frac{1}{4}}$

$G_1 = (a^3\,b)^{\frac{1}{4}}$

$G_2 = (a^2\,b^2)^{\frac{1}{4}}$

$G_3 = (ab^3)^{\frac{1}{4}}$

$G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2 =$

$a^3b + a^2b^2 + ab^3 + (a^3b)^{\frac{1}{2}} \cdot (ab^3)^{\frac{1}{2}}$

$= a^3b + a^2b^2 + ab^3 + a^2 \cdot b^2$

$= ab(a^2 + 2ab + b^2)$

$= ab(a+b)^2$

$= G_1 \cdot G_3 \cdot (A_1 + A_2)^2$

71. (d) Let $z_1 = \left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right)$

Let $z = 3 + iy$

$\bar{z} = 3 - iy$

$$z_1 = \frac{2iy + (9+y^2)}{2 - 3(3+iy) + 5(3-iy)}$$

$$= \frac{9 + y^2 + i(2y)}{8 - 8iy}$$

$$= \frac{(9+y^2) + i(2y)}{8(1-iy)}$$

$$\text{Re}(z_1) = \frac{(9+y^2) - 2y^2}{8(1+y^2)}$$

$$= \frac{9-y^2}{8(1+y^2)}$$

$$= \frac{1}{8}\left[\frac{10-(1+y^2)}{(1+y^2)}\right]$$

$$= \frac{1}{8}\left[\frac{10}{1+y^2} - 1\right]$$

$1 + y^2 \in [1, \infty]$

$$\frac{1}{1+y^2} \in (0,1]$$

$$\frac{10}{1+y^2} \in (0,10]$$

$$\frac{10}{1+y^2} - 1 \in (-1,9]$$

$$\text{Re}(z_1) \in \left(\frac{-1}{8}, \frac{9}{8}\right]$$

$\alpha = \frac{-1}{8}, \beta = \frac{9}{8}$

$24(\beta - \alpha) = 24\left(\frac{9}{8} + \frac{1}{8}\right) = 30$

72. (a)

$C_1\left(9, \frac{15}{2}\right) r_1 = \sqrt{81 + \frac{225}{4} - 131} = \frac{5}{2}$

$C_2(3,3) r_2 = 5$

$C_1C_2 = \sqrt{6^2 + \frac{81}{4}} = \frac{15}{2}$

$r_1 + r_2 = \frac{15}{2}$

$C_1C_2 = r_1 + r_2$

Number of common tangents = 3

73. (d)

$\sim [p \wedge (q \wedge \sim (p \wedge q))]$

$\sim p \vee (\sim q \vee (p \wedge q))$

$\sim p \vee ((\sim q \vee p) \wedge (\sim q \vee q))$

$\sim p \vee (\sim q \vee p)$

$\sim (p \wedge q) \vee p$

74. (d) $-x + 2y - 9z = 7$

$-x + 3y - 7z = 9$

$-2x + y + 5z = 8$

(b) $-$(a)

$y + 16z = 2$

(c) $-2 \times$ (a)

$-3y + 23z = -6 - (5)$

$3 \times$ (d) + (5)

$71z = 0 \Rightarrow z = 0$

$y = 2$
$x = -3$

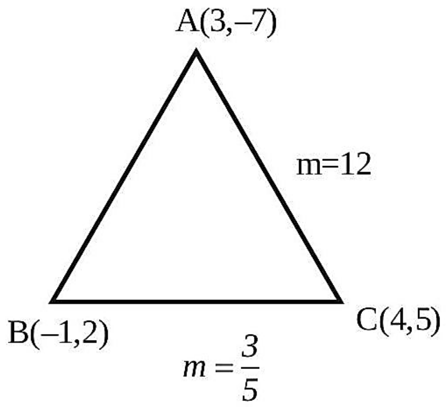
$(-3,2,0) \rightarrow (\alpha, \beta, \gamma)$

Put in $-3x + y + 13z = \lambda$

$\lambda = 9 + 2 = 11$

$$d = \left|\frac{-6-4-11}{3}\right| = 7$$

75. (b)



Altitude of BC: $y + 7 = \frac{-5}{3}(x - 3)$

$3y + 21 = -5x + 15$

$5x + 3y + 6 = 0$

Altitude of AC: $y - 2 = \frac{-1}{12}(x + 1)$

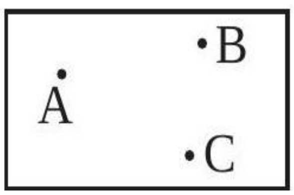
$12y - 24 = -x - 1$

$x + 12y = 23$

$$\alpha = \frac{-47}{19}, \beta = \frac{121}{57}$$

$9\alpha - 6\beta + 60 = 25$

76. (c) $P(3, -2, -9)$



Equation of plane through A,B,C.

$$\begin{vmatrix} x+1 & y+2 & z+3 \\ 10 & 5 & 7 \\ 10 & 0 & 4 \end{vmatrix} = 0$$

$2x + 3y - 5z - 7 = 0$

Foot of I^r of $P(3,-2,-9)$ is

$$\frac{x-3}{2}=\frac{y+2}{3}=\frac{z+9}{-5}=-\frac{(\cancel{6}-\cancel{6}+45-7)}{4+9+25}$$

$$\frac{x-3}{2}=\frac{y+2}{3}=\frac{z+9}{-5}=-1$$

$Q(1,-5,-4)\equiv(\alpha,\beta,\gamma)$

$OQ=\sqrt{\alpha^2+\beta^2+\gamma^2}=\sqrt{42}$

77. (c)

6	W
4	R

$$\frac{1}{6}\times\left[\frac{{}^6C_1}{{}^{10}C_1}+\frac{{}^6C_2}{{}^{10}C_2}+\frac{{}^6C_3}{{}^{10}C_3}+\frac{{}^6C_4}{{}^{10}C_4}+\frac{{}^6C_5}{{}^{10}C_5}+\frac{{}^6C_6}{{}^{10}C_6}\right]$$
$$=\frac{1}{6}\left(\frac{126+70+35+15+5+1}{210}\right)=\frac{42}{210}=\frac{1}{5}$$

78. (a)

$$I=\int_0^1\frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$$

$x\to 1-x$

$$I=\int_0^1\frac{e^{2-4x}dx}{(5+2x-2x^2)(1+e^{2-4x})}$$

Add (i) and (ii)

$$2I=\int_0^1\frac{dx}{5+2x-2x^2}=\int_0^1\frac{dx}{2\left(\frac{11}{4}-\left(x-\frac{1}{2}\right)^2\right)}$$

$$I=\frac{1}{\sqrt{11}}\ln\left(\frac{\sqrt{11}+1}{\sqrt{10}}\right)\qquad \alpha=\sqrt{11}\quad \beta=\sqrt{10}$$

$\alpha^4-\beta^4=121-100=21$

79. (c)

$$\begin{vmatrix}\lambda & -1 & 1\\ 1 & 2 & \mu\\ 3 & -4 & 5\end{vmatrix}=0\ \&\lambda-\mu=5$$

$\lambda(10+4\mu)+(5-3\mu)+(-10)=0$

$(\mu+5)(4\mu+10)+5-3\mu-10=0$

$\mu = -15; \lambda = 5/4$

$\mu = -3; \lambda = 2$

Hence $\sum_{(\lambda,\mu)\in S} 80(\lambda^2 + \mu^2)$

$= 80\left(\frac{250}{16} + 13\right)$

$= 1250 + 1040$

$= 2290$

80. (b)

$f(x) = \ln(4x^2 + 11x + 6) + \sin^{-1}(4x + 3)$

$+\cos^{-1}\left(\frac{10x + 6}{3}\right)$

(i) $4x^2 + 11x + 6 > 0$

$4x^2 + 8x + 3x + 6 > 0$

$(4x + 3)(x + 2) > 0$

$x \in (-\infty, -2) \cup \left(-\frac{3}{4}, \infty\right)$

(ii) $4x + 3 \in [-1,1]$

$x \in [-1, -1/2]$

(iii) $\frac{10x+6}{3} \in [-1,1]$

$x \in \left[-\frac{9}{10}, -\frac{3}{10}\right]$

$x \in \left(-\frac{3}{4}, -\frac{1}{2}\right]$ $\alpha = -\frac{3}{4}, \beta = -\frac{1}{2}$

$\alpha + \beta = -\frac{5}{4}$

$36|\alpha + \beta| = 45$

81. (7)

$P = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2.3} + \frac{1}{3^2}\right) +$

$\left(\frac{1}{2^3} - \frac{1}{2^2 \cdot 3} + \frac{1}{2 \cdot 3^2} - \frac{1}{3^3}\right) + \cdots \ldots$

$P\left(\frac{1}{2} + \frac{1}{3}\right) = \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{2^3} + \frac{1}{3^3}\right) + \left(\frac{1}{2^4} - \frac{1}{3^4}\right) + \cdots$

$$\frac{5P}{6}=\frac{\frac{1}{4}}{1-\frac{1}{2}}-\frac{\frac{1}{9}}{1+\frac{1}{3}}$$

$$\frac{5P}{6}=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$$

$$\therefore P=\frac{1}{2}=\frac{\alpha}{\beta}\quad \therefore \alpha=1,\beta=2$$

$$\alpha+3\beta=7$$

82. (72)
83. (26)

Plane $P\equiv P_1+\lambda P_2=0$

$$(2x+y-z-3)+\lambda(5x-3y)+4z+9)=0$$

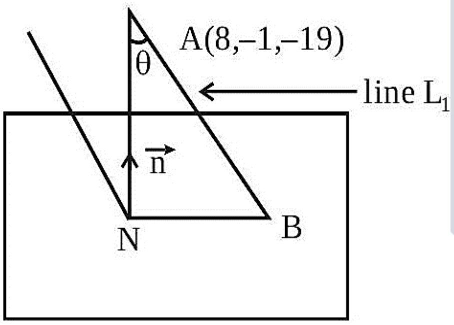
$$(5\lambda+2)x+(1-3\lambda)y+(4\lambda-1)z+9\lambda-3=0$$

$\vec{n}.\vec{b}=0$ where $\vec{b}(2,4,5)$

$$2(5\lambda+2)+4(1-3\lambda)+5(4\lambda-1)=0$$

$$\lambda=-\frac{1}{6}$$

Plane $7x+9y-10z-27=0$



Equation of line AB is

$$\frac{x-8}{-3}=\frac{y+1}{4}=\frac{z+19}{12}=\lambda$$

Let $B=(8-3\lambda,-1+4\lambda,-19+12\lambda)$ lies on plane P

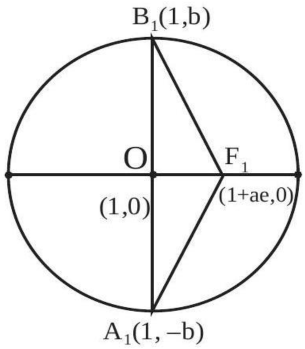
$$\therefore 7(8-3\lambda)+9(4\lambda-1)-10(12\lambda-19)=27$$

$$\lambda=2$$

$\therefore$ Point $B=(2,7,5)$

$$AB=\sqrt{6^2+8^2+24^2}=26$$

84. (9)



L.R. $=\frac{2b^2}{a}=\frac{1}{2}$

$4\,b^2=a$

Ellipse $\frac{(x-1)^2}{a^2}+\frac{y^2}{b^2}=1$

$m_{B_2\,F_1}=\frac{1}{\sqrt{3}}$

$\frac{b}{ae}=\frac{1}{\sqrt{3}}$

$3b^2=a^2e^2=a^2-b^2$

$4b^2=a^2$

From (i) and (ii)

$a=a^2$

$\therefore a=1$

$b^2=\frac{1}{4}$

$((2a)+(2\,b))^2=9$

85. (6)

$A=\{1,2,3,4\}$

$R=\{(a,b),(c,d)\}$

$2a+3\,b=4c+5\,d=\alpha$ let

$2a=\{2,4,6,8\}$ $4c=\{4,8,12,16\}$

$3b=\{3,6,9,12\}$ $5d=\{5,10,15,20\}$

$2a+3b=\begin{Bmatrix}5,8,11,14\\7,10,13,16\\9,12,15,18\\11,14,17,20\end{Bmatrix}$ 4c+5d $\begin{Bmatrix}9,14,19,24\\13,18\ldots.\\17,22\ldots.\\21,26\ldots.\end{Bmatrix}$

Possible value of $\alpha=9,13,14,14,17,18$

Pairs of $\{(a,b),(c,d)\} = 6$

86. (15)

$n \in [10,100]$

$3^n - 3$ is multiple of 7

$3^n = 7\lambda + 3$

$n = 1,7,13,20, \ldots \ldots 97$

Number of possible values of $n = 15$

87. (5) $\sin A + \sin B + \sin C = \frac{18}{x}$

$\sin 2A + \sin 2B + \sin 2C = \frac{9}{x}$

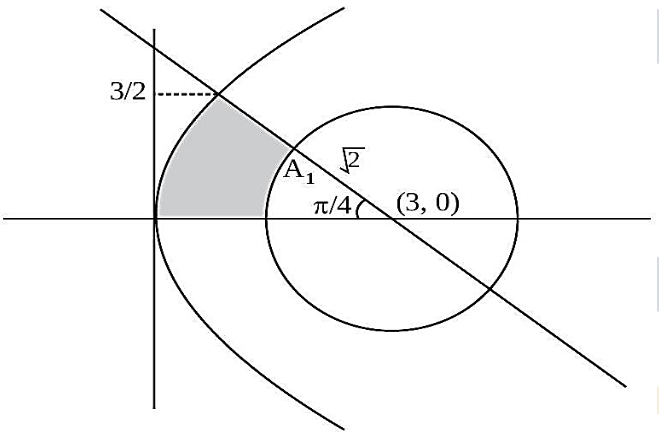
$\therefore \sin A + \sin B + \sin C = 2(\sin 2A + \sin 2B + \sin 2C)$

$4\cos A/2 \cos B/2 \cos C/2 = 2(4\sin A \sin B \sin C)$

$16\sin A/2 \sin B/2 \sin C/2 = 1$

Hence Ans. $= 5$.

88. (42)



$y^2 = \frac{3x}{2}, x + y = 3, y = 0$

$2y^2 = 3(3 - y)$

$2y^2 + 3y - 9 = 0$

$2y^2 - 3y + 6y - 9 = 0$

$(2y - 3)(y + 2) = 0; y = 3/2$

Area $\left(\int_0^{\frac{3}{2}} (x_R - x_2) dy\right) - A_1$

$= \int_0^{\frac{3}{2}} \left((3-y) - \frac{2y^2}{3}\right) dy - \frac{\pi}{8}(2)$

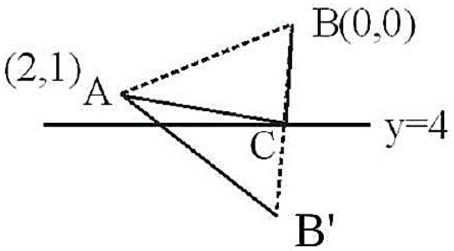
$A = \left(3y - \frac{y^2}{2} - \frac{2y^3}{9}\right)_0^{\frac{3}{2}} - \frac{\pi}{4}$

$4\text{ A} + \pi = 4\left[\frac{9}{2} - \frac{9}{8} - \frac{3}{4}\right] = \frac{21}{2} = 10.50$

$\therefore 4(4\text{ A} + \pi) = 42$

89. (48)

$A(2,1), B(0,0), C(t,4): t \in [0,4]$



$\mathrm{B_1}(0,8) \equiv$ image of B w.r.t. y = 4

for AC + BC + AB to be minimum.

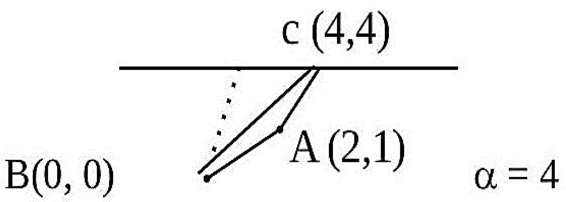
$\mathrm{m_{AB}} = \frac{-7}{2}$

line $\mathrm{AB_1} \equiv 7\mathrm{x} + 2\mathrm{y} = 16$

$C\left(\frac{8}{7}, 4\right)$

$\beta = \frac{8}{7}$

For max. perimeter



$\mathrm{AB} = \sqrt{5}: \mathrm{BC} = 4\sqrt{2}, \mathrm{AC} = \sqrt{13}$

$6\alpha + 21\beta = 24 + 24 = 48$

90. (28) $\mathrm{f(x)} = \int \frac{\mathrm{dx}}{(3+4\mathrm{x}^2)\sqrt{4-3\mathrm{x}^2}}$

$\mathrm{x} = \frac{1}{\mathrm{t}}$

$$= \int \frac{\frac{-1}{t^2}dt}{\frac{(3t^2+4)}{t^2}\frac{\sqrt{4t^2-3}}{t}}$$

$$= \int \frac{-dt.t}{(3t^2+4)\sqrt{4t^2-3}}: \text{Put } 4t^2-3=z^2$$

$$= -\frac{1}{4}\int \frac{zdz}{\left(3\left(\frac{z^2+3}{4}\right)+4\right)z}$$

$$= \int \frac{-dz}{3z^2+25} = -\frac{1}{3}\int \frac{dz}{z^2+\left(\frac{5}{\sqrt{3}}\right)^2}$$

$$= -\frac{1}{3}\frac{\sqrt{3}}{5}\tan^{-1}\left(\frac{\sqrt{3}z}{5}\right)+C$$

$$= -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{4t^2-3}\right)+C$$

$$f(x) = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{\frac{4-3x^2}{x^2}}\right)+C$$

$$\because f(0)=0 \because c = \frac{\pi}{10\sqrt{3}}$$

$$f(1) = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\right)+\frac{\pi}{10\sqrt{3}}$$

$$f(1) = \frac{1}{5\sqrt{3}}\cot^{-1}\left(\frac{\sqrt{3}}{5}\right) = \frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$$

$$\alpha = 5: \beta = \sqrt{3} \therefore \alpha^2+\beta^2 = 28$$