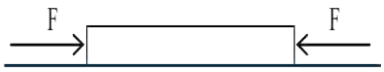
2014

1. (c)

$$0.10 \times 1.1 \times 10^{-5} \times 100 = \frac{\frac{F}{A}}{2 \times 10^{11}} \times 0.10$$

$$\therefore \frac{F}{A} = \text{ Pressure } = 1.1 \times 10^{-5} \times 100 \times 2 \times 10^{11}$$

$$= 2.2 \times 10^{8} \text{ Pa}$$

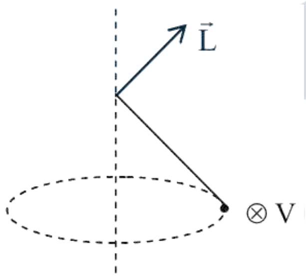


2. (b) $P = \frac{\text{Work Done}}{\text{Time}} = \frac{\int F dx}{t} = \frac{\int I\ell bB \cdot dx}{t}$

$$= \frac{\int_0^2 (10)(c)(3 \times 10^{-4} e^{-0.2x}) dx}{5 \times 10^{-3}}$$

$$= \frac{9 \times 10^{-3}}{5 \times 10^{-3}} \left[\frac{e^{-0.2x}}{-0.2}\right]_0^2 = 9[1 - e^{-0.4}] = 2.97 \text{ W}$$

3. (a) $\vec{L}$ changes in direction not in magnitude



4. (c) $5 = e^{1000\frac{V}{T}} - 1$

$$\Rightarrow e^{1000}\frac{V}{T} = 6$$

Again, $I = e^{1000\frac{V}{T}} - 1$

$$\frac{dI}{dV} = e^{\frac{1000\,V}{T}} \frac{1000}{T}$$

$$dI = \frac{1000}{T} e^{\frac{1000}{T}V} dV$$

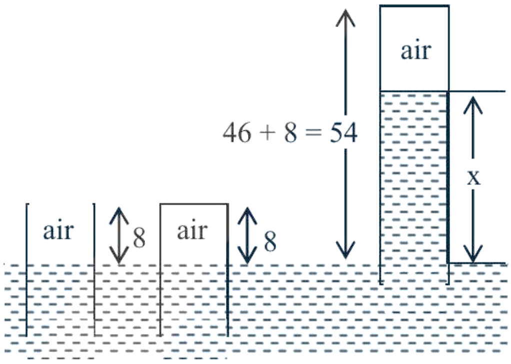
Using (a)

$$\Delta I = \frac{1000}{T} \times 6 \times 0.01 = \frac{60}{T} = \frac{60}{300} = 0.2 \text{ mA}$$

5. (c) $(76)(8) = (54 - x)(76 - x)$

$x = 38$ cm

Length of air column $= 54 - 38 = 16$ cm



6. (b)

Infrared waves → To treat muscular strain

radio waves → for broadcasting

X-rays → To detect fracture of bones

Ultraviolet rays → Absorbed by the ozone layer of the atmosphere;

7. (c) By formula of electric field between the plates of a capacitor $E = \frac{\sigma}{K\varepsilon_0}$

$$\begin{aligned} \Rightarrow \sigma = EK\varepsilon_0 &= 3 \times 10^4 \times 2.2 \times 8.85 \times 10^{-12} \\ &= 6.6 \times 8.85 \times 10^{-8} \\ &= 5.841 \times 10^{-7} \\ &\cong 6 \times 10^{-7} C/m^2 \end{aligned}$$

8. (d) Least count of vernier calliper is $\frac{1}{10}$ mm $= 0.1$ mm $= 0.01$ cm

9. (b) Net force on any one particle

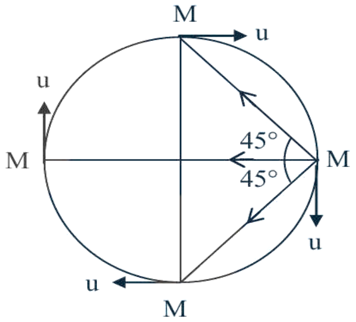
$$= \frac{GM^2}{(2R)^2} + \frac{GM^2}{(R\sqrt{2})^2}\cos 45^\circ + \frac{GM^2}{(R\sqrt{2})^2}\cos 45^\circ$$

$$= \frac{GM^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right]$$

This force will be equal to centripetal force so

$$\frac{Mu^2}{R} = \frac{GM^2}{R^2}\left[\frac{1 + 2\sqrt{2}}{4}\right]$$

$$u = \sqrt{\frac{GM}{4R}[1 + 2\sqrt{2}]} = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$$



10. (a)

Item	No.	Power
40 W bulb	15	600Watt
100 W bulb	5	500Watt
80 W fan	5	400Watt
1000 W heater	1	1000Watt

Total Wattage = 2500 Watt

So current capacity $i = \frac{P}{V} = \frac{2500}{220} = \frac{125}{11} = 11.36 \cong 12$ Amp.

11. (b)

$A(1 - \cos\omega\tau) = a$

$A(1 - \cos 2\omega\tau) = 3a$

$\cos\omega\tau = \left(1 - \frac{a}{A}\right)$

$\cos 2\omega\tau = \left(1 - \frac{3a}{A}\right)$

$2\left(1 - \frac{a}{A}\right)^2 - 1 = 1 - \frac{3a}{A}$

Solving the equation

$\frac{a}{A} = \frac{1}{2}$

$A = 2a$

$\cos\omega\tau = \frac{1}{2}$

$T = 6\tau$

12. (a) $\mu_0 H = \mu_0 ni$

$3 \times 10^3 = \frac{100}{0.1} \times i \Rightarrow i = 3\text{ A}$

13. Sol (3) By diagram

14. (a)

15. (d) Since resistance and inductor are in parallel, so ratio will be 1.

16. (d) For the mass m,

$$\mathrm{mg} - \mathrm{T} = \mathrm{ma}$$

for the cylinder,

$$\mathrm{TR} = \mathrm{mR}^2 \frac{\mathrm{a}}{\mathrm{R}}$$

$$\Rightarrow \mathrm{T} = \mathrm{ma}$$

$$\Rightarrow \mathrm{mg} = 2\mathrm{ma}$$

$$\Rightarrow \mathrm{a} = \mathrm{g}/2$$

17. (b)

$$\Delta\mathrm{U}_{\mathrm{AB}} = \mathrm{nC}_{\mathrm{V}}(\mathrm{T}_{\mathrm{B}} - \mathrm{T}_{\mathrm{A}}) = 1 \times \frac{5\mathrm{R}}{2}(800 - 400) = 1000\mathrm{R}$$

$$\Delta\mathrm{U}_{\mathrm{BC}} = \mathrm{nC}_{\mathrm{V}}(\mathrm{T}_{\mathrm{C}} - \mathrm{T}_{\mathrm{B}}) = 1 \times \frac{5\mathrm{R}}{2}(600 - 800) = -500\mathrm{R}$$

$$\Delta\mathrm{U}_{\mathrm{total}} = 0$$

$$\Delta\mathrm{U}_{\mathrm{CA}} = \mathrm{nC}_{\mathrm{v}}(\mathrm{T}_{\mathrm{A}} - \mathrm{T}_{\mathrm{C}}) = 1 \times \frac{5\mathrm{R}}{2}(400 - 600) = -500\mathrm{R}$$

18. (a) Time to reach the maximum height

$$\mathrm{t}_1 = \frac{\mathrm{u}}{\mathrm{g}}$$

If t_2 be the time taken to hit the ground

$$-\mathrm{H} = \mathrm{ut}_2 - \frac{1}{2}\mathrm{gt}_2^2$$

But $\mathrm{t}_2 = \mathrm{nt}_1$ (given)

$$\Rightarrow -\mathrm{H} = \mathrm{u}\frac{\mathrm{nu}}{\mathrm{g}} - \frac{1}{2}\mathrm{g}\frac{\mathrm{n}^2\mathrm{u}^2}{\mathrm{g}^2}$$

$$\Rightarrow 2\mathrm{gH} = \mathrm{nu}^2(\mathrm{n} - 2)$$

19. (d) $\frac{f_m}{f} = \frac{(\mu - 1)}{\left(\frac{\mu}{\mu_m} - 1\right)}$

$$\Rightarrow \frac{\mathrm{f}_1}{\mathrm{f}} = \frac{\left(\frac{3}{2} - 1\right)}{\left(\frac{3/2}{4/3} - 1\right)} = 4$$

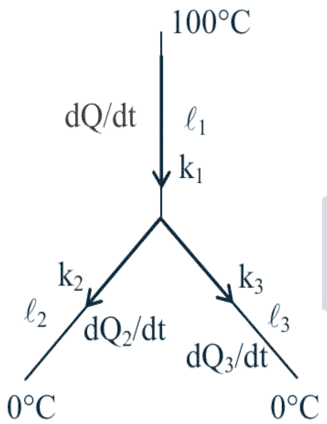
$$\Rightarrow \mathrm{f}_1 = 4\mathrm{f}$$

$$\frac{f_2}{f}=\frac{\left(\frac{3}{2}-1\right)}{\left(\frac{3/2}{5/3}-1\right)}=-5$$

$\Rightarrow f_2<0$

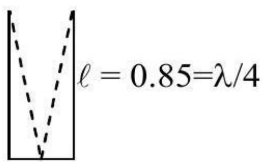
20. (a)

$$\frac{dQ_1}{dt}=\frac{dQ_2}{dt}+\frac{dQ_3}{dt}$$
$$\Rightarrow \frac{0.92(100-T)}{46}=\frac{0.26(T-0)}{13}+\frac{0.12(T-0)}{12}$$
$$\Rightarrow T=40^\circ C$$
$$\frac{dQ_1}{dt}=\frac{0.92\times 4(100-40)}{40}=4.8 cal/s$$



21. (a) In fundamental mode

$$\frac{\lambda}{4}=0.85$$
$$\lambda=4\times 0.85$$
$$f=v/\lambda=\frac{340}{4\times 0.85}$$
$$=100\text{ Hz.}$$



$\therefore$ Possible frequencies $=100$ Hz, 300 Hz, 500hz, 700 Hz, 900 Hz1100 Hz below 1250 Hz

22. (a) $P_A=P_B$

$$P_0+d_1gR(\cos\alpha-\sin\alpha)=P_0+d_2gR(\cos\alpha+\sin\alpha)$$

$$\Rightarrow \frac{d_1}{d_2}=\frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}=\frac{1+\tan\alpha}{1-\tan\alpha}$$

23. (d) As frequency of visible light increases refractive index increases. With the increase of refractive index critical angle decreases. So that light having frequency greater than green will get total internal reflection and the light having frequency less than green will pass to air.

24. (a) $\frac{1}{\lambda}=\mathrm{Rz}^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$

$\therefore \lambda=\frac{4}{3\mathrm{Rz}^2}$

$\lambda_1=\frac{4}{3\mathrm{R}}$

$\lambda_2=\frac{4}{3\mathrm{R}}$

$\lambda_3=\frac{4}{12\mathrm{R}}$

$\lambda_4=\frac{4}{27\mathrm{R}}$

$\Rightarrow \lambda_1=\lambda_2=4\lambda_3=9\lambda_4$

25. (d) $\mathrm{mv}=\mathrm{qBR}$

$\mathrm{KE}_{(\max)}=\frac{(\mathrm{mv})^2}{2\ \mathrm{m}}=0.8\mathrm{eV}$

$\mathrm{h}\nu=13.6\left[\frac{1}{4}-\frac{1}{9}\right]$

$\therefore\ \mathrm{W}=\mathrm{h}\nu-\mathrm{KE}.(\max)$

$=13.6\frac{5}{36}-0.8=1.1\mathrm{eV}$

26. (c) $\mathrm{mg}\sin\theta=\mu\mathrm{mg}\cos\theta$

$\tan\theta=\mu$

$\frac{\mathrm{dy}}{\mathrm{dx}}=\tan\theta=\mu=\frac{1}{2}$

$\frac{x^2}{2}=\frac{1}{2}, x=\pm 1$

$\mathrm{y}=\frac{1}{6}\ \mathrm{m}.$

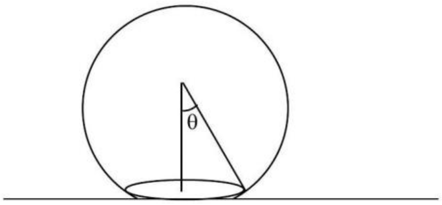
27. (a)
28. None

$$(2\pi r\ \mathrm{T})\sin\theta = \frac{4}{3}\pi \mathrm{R}^3\rho_\omega \cdot g$$

$$T \times \frac{r}{R} \times 2\pi r = \frac{4}{3}\pi R^3 \rho_w g$$

$$r^2 = \frac{2}{3}\frac{R^4\rho_w g}{T}$$

$$r = R^2\sqrt{\frac{2\rho_\omega g}{3T}}$$



29. (b)

$$\mathrm{I_A}\cos^2 30 = \mathrm{I_B}\cos^2 60$$

$$\frac{\mathrm{I_A}}{\mathrm{I_B}} = \frac{1}{3}$$

30. None

$$\vec{\mathrm{E}} = 30\mathrm{x}^2\hat{\imath}$$

$$\mathrm{dV} = --\int \mathrm{E}\cdot \mathrm{dx}$$

$$\int_{\mathrm{V_0}}^{\mathrm{V_A}} \mathrm{dV} = -\int_0^2 30\mathrm{x}^2\mathrm{dx}$$

$$\mathrm{V_A} - \mathrm{V_0} = -80 \text{ Volt}$$

31. (c) NO is paramagnetic in gaseous state due to the presence of unpaired electron in its structure.

32.(d)

$$\left(\mathrm{P} + \frac{\mathrm{n^2a}}{\mathrm{V^2}}\right)(\mathrm{V} - \mathrm{nb}) = \mathrm{nRT}$$

For 1 mole, $\left(\mathrm{P} + \frac{\mathrm{a}}{\mathrm{V^2}}\right)(\mathrm{V} - \mathrm{b}) = \mathrm{RT}$

$$\mathrm{PV} = \mathrm{RT} + \mathrm{Pb} - \frac{\mathrm{a}}{\mathrm{V}} + \frac{\mathrm{ab}}{\mathrm{V^2}}$$

at low pressure, terms Pb& $\frac{\mathrm{ab}}{\mathrm{V^2}}$ will be negligible as compared to RT.

So, $\mathrm{PV} = \mathrm{RT} - \frac{\mathrm{a}}{\mathrm{V}}$

$$\mathrm{Z} = 1 - \frac{\mathrm{a}}{\mathrm{RTV}}$$

33. (d) During the electrolysis of aqueous solution of s-block elements, H_2 gas is obtained at cathode.

34. (c) $50 = \frac{1}{K} \times \frac{\ell}{A}$

$50 = \frac{1}{1.4} \times \frac{\ell}{A}$

$\frac{\ell}{A} = 70\ m^{-1}$

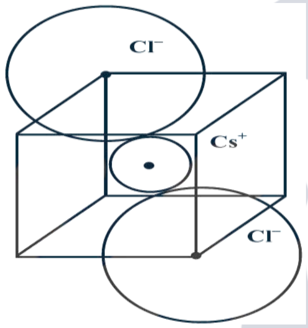
$280 = \frac{1}{K} \times 70$

$K = \frac{1}{4} Sm^{-1}$

$\Lambda_m = \frac{1}{4} \times \left(\frac{1000}{M}\right)(10^{-2}\ m)^3$
$= \frac{1}{4} \times \frac{1000}{0.5} \times 10^{-6}$

$= 500 \times 10^{-6} = 5 \times 10^{-4} Sm^2\ mol^{-1}$

35. (a) In CsCl structure, Cs^+ion is in contact with Cl^-ion at the nearest distance which is equal to $\sqrt{3}\frac{a}{2}$



36. (c) For $C_2H_5OH, \pi = 1 \times 0.5 \times RT$

For $KBr, \pi = 2 \times 0.25 \times RT$

For $Mg_3(PO_4)_2, \pi = 5 \times 0.1 \times RT$

For $Na_3PO_4, \pi = 4 \times 0.125 \times RT$

So, all are isotonic solutions.

37. (b) A reducing agent loses electrons during redox reaction.

Hence (b, d) is correct.

38. (d) Rate of S_N2 reaction $\propto \frac{1}{\text{steric over crowding in transition state}}$

39. (d) Strong field ligands cause higher magnitude of crystal field splitting which is accompanied by the absorption of higher energy radiation.

□V□I□B□G□Y□O□R□□decreasing energy

40. (d)

$\%$ of N $= \frac{1.4\times \text{ milliequivalents of acid consumed}}{\text{mass of organic compound}}$

Meq of acid consumed $= \left(60 \times \frac{1}{10} \times 2\right) - \left(20 \times \frac{1}{10} \times 1\right)$

$= 10$

$\therefore$ % of N $= \frac{1.4\times10}{1.4} = 10\%$

41. (a) According to Debye Huckel's Theory for a strong electrolyte,

$\lambda_C = \lambda_\infty - B\sqrt{C}$

42. (d) For reaction:

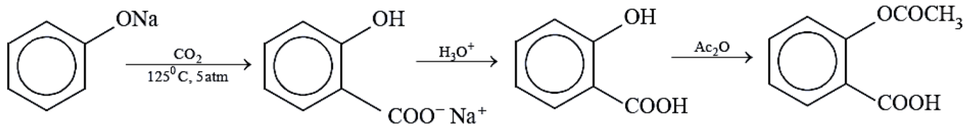
$$SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$$

$\Delta n_g = -\frac{1}{2} = x$

43. (a)

$$CH_3COOH \xrightarrow{LiAlH_4} CH_3CH_2OH \xrightarrow{PCl_5} CH_3CH_2Cl \xrightarrow{\text{alc. KOH}} \underset{\text{(ethylene)}}{H_2C{=}CH_2}$$

44. (c)



(Aspirin)

45. (b)

$$RNH_2 + CHCl_3 + \underset{\text{(alc.)}}{3KOH} \longrightarrow \underset{\text{(alkyl isocyanide)}}{RNC\uparrow} + 3KCl + 3H_2O$$

46. (d) $CsI_3 \longrightarrow Cs^+ + I_3^-$

$\Rightarrow$ Cs cannot show +3 oxidation state.

$\Rightarrow$ I_2 molecules are too large to be accommodated in lattice.

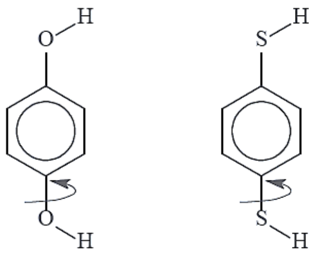
47. (d) Equation -1 is not balanced w.r.t. charge.

Equation -2 gives $K_3[Cu(CN)_4]$ as product.

Equation -3 reaction is unfavourable in the forward direction (K_2O is unstable, while Li_2O is stable).

Equation -4 is correct & balanced.

48. (b)



Due to infinite possible conformations in the above cases (of which only one has zero μ); a weighted μ will finally exist.

49. (b) $R = k[A]^x[B]^y$

$1.2 \times 10^{-3} = k[0.1]^x[0.1]^y$

$1.2 \times 10^{-3} = k[0.1]^x[0.2]^y$

$2.4 \times 10^{-3} = k[0.2]^x[0.1]^y$

Solving $x = 1, y = 0$

$R = k[A]$

50. (b) In Eq. (a) $FeCl_3$ cannot be reduced when heated in air

In Eq. (c) $Fe_2(SO_4)_3$ cannot convert to Fe on heating; instead oxide(s) will be formed.

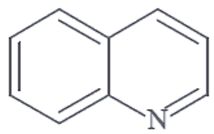
In Eq. (d) $FeSO_4$ cannot be converted to Fe on heating; instead oxide(s) will be formed.

51. (c) Aliphatic amines are more basic than aromatic amines.

$(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N$ (among aliphatic amines in water).

52. (c) Adenine, Thymine, Cytosine, Guanine are bases present in DNA.

Quinoline an aromatic compound is NOT present in DNA.

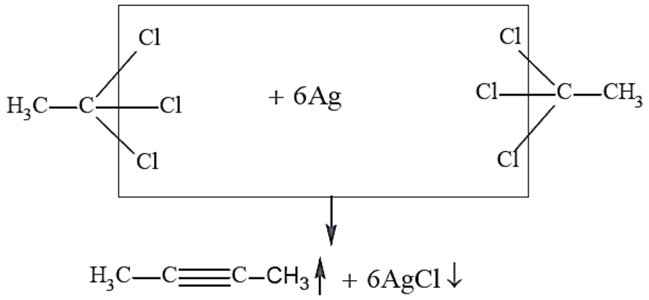


(Quinoline)

53. (c) $_{37}Rb = [Kr]5s^1$

$n = 5, l = 0, m = 0, s = +\frac{1}{2}$

54. (a)



2 – butyne

55. (c)

$$
\begin{array}{ll}
Mn^{+2} + 2e^- \longrightarrow Mn & E^\circ = -1.18\ V \\
2Mn^{+2} \longrightarrow 2Mn^{+3} + 2e^- & E^\circ = -1.51\ V \\
\hline
3Mn^{+2} \longrightarrow Mn + 2Mn^{+3} & E^\circ = SOP + SRP \\
 & = -1.18 + (-1.51) \\
 & = -2.69\ V
\end{array}
$$

Negative EMF reflects non-spontaneous cell reaction.

56. (d) Moles of $O_2 = \frac{w}{32}$

Moles of $N_2 = \frac{4w}{28}$

$$\frac{n_{O_2}}{n_{N_2}} = \frac{w}{32} \times \frac{28}{4w} = \frac{7}{32}$$

57. (c) Teflon, Acrylonitrile and Neoprene are addition polymers while Dacron is a condensation polymer.

58. (a) $HClO < HClO_2 < HClO_3 < HClO_4$

Increasing acid strength due to increase in oxidation state of central atom.

59. (c)

$C_2H_5OH(\ell) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(\ell)$

$\Delta E = -1364.47$ kJ/ mole

$\Delta H = ?$

$T = 298$ K

$\Delta n_g = -1$

$\because \Delta H = \Delta E + \Delta n_g RT$

So, $\Delta H = -1366.95$ kJ/ mole

60. (b) $RCH_2OH \overset{PCC}{\rightarrow} RCHO$

61. (a) Line is parallel to plane

Image of $(1,3,4)$ is $(-3,5,2)$.

62. (d)

$1(1-2x)^{18} + ax(1-2x)^{18} + bx^2(1-2x)^{18}$

Coefficient of x^3 : $(-2)^3\,{}^{18}C_3 + a(-2)^2\,{}^{18}C_2 + b(-2)\,{}^{18}C_1 = 0$

$\frac{4\times(17\times16)}{(3\times2)} - 2a\cdot\frac{17}{2} + b = 0$

Coefficient of x^4: $(-2)^4\,{}^{18}C_4 + a(-2)^3\,{}^{18}C_3 + b(-2)^2\,{}^{18}C_2 = 0$

$(4\times20) - 2a\cdot\frac{16}{3} + b = 0$

From equation (i) and (ii), we get

$4\left(\frac{17\times8}{3} - 20\right) + 2a\left(\frac{16}{3} - \frac{17}{2}\right) = 0$

$4\left(\frac{17\times8-60}{3}\right) + \frac{2a(-19)}{6} = 0$

$a = \frac{4\times76\times6}{3\times2\times19}$

$\Rightarrow a = 16$

$\Rightarrow b = \frac{2\times16\times16}{3} - 80 = \frac{272}{3}$

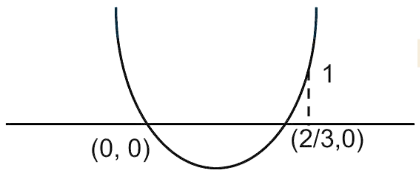
63. (a) $a^2 = 3t^2 - 2t$

For non-integral solution

$0 < a^2 < 1$

$a \in (-1,0) \cup (0,1)$.

[Note: It is assumed that a real solution of given equation exists.]



64. (d) $[\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}] = [\vec{a}\vec{b}\quad \vec{c}]^2$

$\lambda = 1.$

65. (b)

$\sigma^2 = \left(\frac{\sum x_i^2}{n}\right) - \bar{x}^2$

$\bar{x} = \frac{\sum_{r=1}^{50} 2r}{50} = 51$

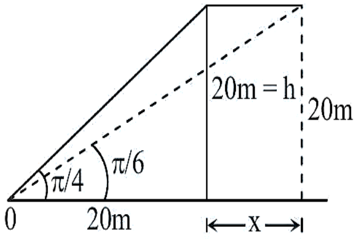
$$\sigma^2=\frac{\sum_{r=1}^{50}4r^2}{50}-(51)^2=833$$

66. (d) $\tan 30^\circ=\frac{20}{20+x}=\frac{1}{\sqrt{3}}$

$20+x=20\sqrt{3}$

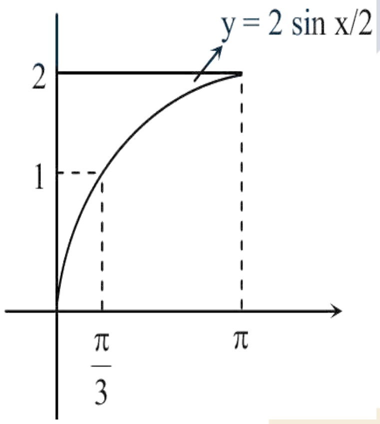
$x=20(\sqrt{3}-1)$

$\Rightarrow$ Speed is $20(\sqrt{3}-1)\mathrm{m/sec}$.



67. (d)

$$I=\int_0^\pi\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$$
$$=\int_0^\pi\left|1-2\sin\frac{x}{2}\right|dx$$
$$=\int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$$



$$=\left(x+4\cos\frac{x}{2}\right)\Big|_0^{\pi/3}+\left(-4\cos\frac{x}{2}-x\right)\Big|_{\pi/3}^{\pi}$$
$$=-\frac{\pi}{3}+8\cdot\frac{\sqrt{3}}{2}-4$$
$$=4\sqrt{3}-4-\frac{\pi}{3}$$

68.

P	q	~ q	p ↔~ q	~ (p ↔~ q)	p ↔ q
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

69. (b)

$$B = A^{-1} A' \Rightarrow AB = A'$$
$$ABB' = A'B' = (BA)' = (A^{-1} A'A)' = (A^{-1}AA')' = A.$$
$$\Rightarrow BB' = I.$$

70. (b)

$$\int \left(1 + x - \frac{1}{x}\right) e^{\left(x+\frac{1}{x}\right)} dx$$

$$= \int e^{\left(x+\frac{1}{x}\right)} dx + \int x \left(1 - \frac{1}{x^2}\right) e^{\left(x+\frac{1}{x}\right)} dx$$

$$= \int e^{\left(x+\frac{1}{x}\right)} dx + xe^{\left(x+\frac{1}{x}\right)} - \int e^{\left(x+\frac{1}{x}\right)} dx$$

$$= xe^{\left(x+\frac{1}{x}\right)} + c$$

71. (b)

$$|z| \geq 2$$
$$\left|z + \frac{1}{2}\right| \geq \left||z| - |\frac{1}{2}|\right| \geq \left|2 - \frac{1}{2}\right| \geq \frac{3}{2}.$$

Hence, minimum distance between z and $\left(-\frac{1}{2}, 0\right)$ is $\frac{3}{2}$

72. (d)

$$f(g(x)) = x$$

$$f'(g(x))g'(x) = 1$$

$$g'(x) = 1 + (g(x))^5$$

73. (c)

$$\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha-1 & \beta-1 \\ 1 & \alpha^2-1 & \beta^2-1 \end{vmatrix}$$

$= \left((\alpha-1)(\beta^2-1)-(\beta-1)(\alpha^2-1)\right)^2$

$= (\alpha-1)^2(\beta-1)^2(\alpha-\beta)^2 \Rightarrow \mathrm{k}=1$

74. (d) $\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$

$= \frac{3(\sin^4 x+\cos^4 x)-2(\sin^6 x+\cos^6 x)}{12}$

$= \frac{3(1-2\sin^2 x\cos^2 x)-2(1-3\sin^2 x\cos^2 x)}{12}$

$= \frac{1}{12}.$

75. (d) $\frac{1}{\alpha}+\frac{1}{\beta}=4$

$2\mathrm{q}=\mathrm{p}+\mathrm{r}$

$\Rightarrow -2(\alpha+\beta)=1+\alpha\beta$

$\Rightarrow -2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=\frac{1}{\alpha\beta}+1$

$\Rightarrow \frac{1}{\alpha\beta}=-9$

Equation having roots α,β is $9\mathrm{x}^2+4\mathrm{x}-1=0$

$\alpha,\beta=\frac{-4\pm\sqrt{16+36}}{2\times 9}$

$|\alpha-\beta|=\frac{2\sqrt{13}}{9}.$

76. (c) $\mathrm{P}(\overline{\mathrm{A}\cup\mathrm{B}})=\frac{1}{6}$

$\mathrm{P}(\mathrm{A}\cup\mathrm{B})=\frac{5}{6},\ \mathrm{P}(\mathrm{A})=\frac{3}{4}$

$\mathrm{P}(\mathrm{A}\cup\mathrm{B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A}\cap\mathrm{B})=\frac{5}{6}$

$\mathrm{P}(\mathrm{B})=\frac{5}{6}-\frac{3}{4}+\frac{1}{4}=\frac{1}{3}.$

$\mathrm{P}(\mathrm{A}\cap\mathrm{B})=\mathrm{P}(\mathrm{A})\cdot\mathrm{P}(\mathrm{B})$

$\frac{1}{4}=\frac{3}{4}\times\frac{1}{3}.$

77. (d) Let $h(f)=f(x)-2g(x)$

as $\mathrm{h}(0)=\mathrm{h}(\mathrm{a})=2$

Hence, using Rolle's theorem

$\mathrm{h}'(\mathrm{c})=0$

$\Rightarrow \mathrm{f}'(\mathrm{c})=2\,\mathrm{g}'(\mathrm{c})$

78. (a) $\frac{dp}{dt}=\frac{p-400}{2}$

$$\frac{dp}{p-400}=\frac{1}{2}dt$$

$$\ln|p-400|=\frac{1}{2}t+c$$

at $t=0, p=100$

$\ln 300=c$

$$\ln\left|\frac{p-400}{300}\right|=\frac{t}{2}$$

$\Rightarrow |p-400|=300e^{t/2}$

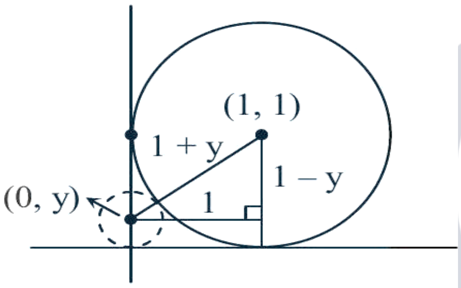
$\Rightarrow 400-p=300e^{t/2}$ (as $p<400$)

$\Rightarrow p=400-300e^{t/2}$

79. (d) According to the figure

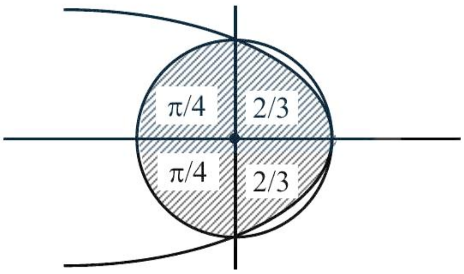
$(1+y)^2=(1-y)^2+1\ (y>0)$

$$\Rightarrow y=\frac{1}{4}$$



80. (a) $A=\frac{1}{2}\times\pi+2\int_0^1\sqrt{1-x}dx$

$=\frac{\pi}{2}+\frac{4}{3}.$



81. (c) Let point of intersection is $(h,-h)$

$$\Rightarrow\begin{cases}4ah-2ah+c=0\\5bh-2bh+d=0\end{cases}$$

So, $-\frac{c}{2a}=-\frac{d}{3b}$

$3bc-2ad=0$

82. **(b)** $S\left(\frac{13}{2},1\right), P(2,2)$

Slope $=-\frac{2}{9}$

Equation will be $\frac{y+1}{x-1}=-\frac{2}{9}$

$9y+9+2x-2=0$

$2x+9y+7=0$

83. **(d)** $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$

$$=\lim_{x\to 0}\frac{\sin(\pi-\pi\sin^2 x)}{x^2}$$

$=\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\times\frac{\pi\sin^2 x}{x^2}=\pi.$

84. **(d)** Set X contains elements of the form

$4^n-3n-1=(1+3)^n-3n-1$

$=3^n+{}^nC_{n-1}3^{n-1}\ldots\ldots{}^nC_2 3^2$

$=9(3^{n-2}+{}^nC_{n-1}3^{n-1}\ldots+{}^nC_2)$

Set X has natural numbers which are multiples of 9 (not all)

Set Y has all multiples of 9

$X\cup Y=Y$

85. **(c)** Let the foot of perpendicular be $P(h,k)$

Equation of tangent with slope m passing $P(h,k)$ is
$y=mx\pm\sqrt{6m^2+2}$, where $m=-\frac{h}{k}$

$$\Rightarrow\sqrt{\frac{6h^2}{k^2}+2}=\frac{h^2+k^2}{k}$$

$6h^2+2k^2=(h^2+k^2)^2$

So required locus is $6x^2+2y^2=(x^2+y^2)^2$.

86. **(d)** Let numbers be a, ar, a^2

Now, $2(2ar)=a+ar^2$ $[a\neq 0]$

$\Rightarrow 4r = 1 + r^2$

$\Rightarrow r^2 - 4r + 1 = 0$

$\Rightarrow r = 2 \pm \sqrt{3}$

$r = 2 + \sqrt{3}$ (Positive value)

87. (c)

$$S = 10^9 + 2 \cdot 11^1 \cdot 10^8 + \cdots + 10 \cdot 11^9$$
$$\frac{11}{10} \cdot S = 11^1 \cdot 10^8 + \cdots + 9 \cdot 11^9 + 11^{10}$$
$$\Rightarrow -\frac{1}{10} S = 10^9 + 11^1 \cdot 10^8 + 11^2 \cdot 10^7 + \cdots + 11^9 - 11^{10}$$
$$\Rightarrow -\frac{1}{10} S = 10^9 \left(\frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1} \right) - 11^{10} \Rightarrow -\frac{1}{10} S = 11^{10} - 10^{10} - 11^{10}$$
$$S = 10^{11}$$
$$S = 100 \cdot 10^9$$
$$\Rightarrow k = 100.$$

88. (a)

$$1 = -m - n$$
$$m^2 + n^2 = (m + n)^2$$
$$\Rightarrow mn = 0$$

So possibilities are $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ or $\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$.

89. (a) Equation of tangent at $A(t^2, 2t)$

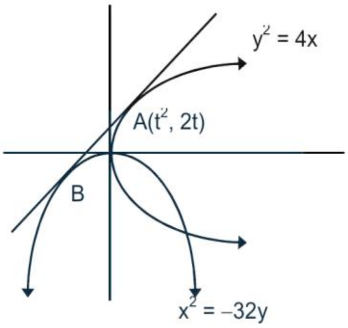
$yt = x + t^2$ is tangent to $x^2 + 32y = 0$ at B

$\Rightarrow x^2 + 32\left(\frac{x}{t} + t\right) = 0$

$\Rightarrow x^2 + \frac{32}{t} x + 32t = 0$

$\Rightarrow \left(\frac{32}{t}\right)^2 - 4(32t) = 0$

$\Rightarrow 32\left(\frac{32}{t^2} - 4t\right) = 0$



$\Rightarrow t^3 = 8 \Rightarrow t = 2$

$\Rightarrow$ Slope of tangent is $\frac{1}{t} = \frac{1}{2}$.

90. (c) $f'(x) = \frac{\alpha}{x} + 2\beta x + 1$

$2\beta x^2 + x + \alpha = 0$ has roots -1 and 2