2015

1. (d) As electron goes to ground state, total energy decreases.

$TE = -KE$

$PE = 2TE$

So, kinetic energy increases but potential energy and total energy decreases.

2. (a)

$$g = \frac{4\pi^2 L}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\left(\frac{\Delta T}{T}\right)$$

$$\frac{\Delta L}{L} = \frac{0.1}{20}, \frac{\Delta T}{T} = \frac{0.01}{0.9}$$

$$100\left(\frac{\Delta g}{g}\right) = 100\left(\frac{\Delta L}{L}\right) + 2 \times 100 \times \left(\frac{\Delta T}{T}\right) \simeq 3\%$$

3. (d) It originates from + Ve charge and terminates at - Ve charge. It can not form close loop.

4. (a) $f_R = f_C + f_m = 2000\text{kHz} + 5\text{kHz} = 2005\text{kHz}$

$f_R = f_C - f_m = 2000\text{kHz} - 5\text{kHz} = 1995\text{kHz}$

So, frequency content of resultant wave will have frequencies $1995\text{kHz}, 2000\text{kHz}$ and 2005kHz

5. (b)

$dQ = dU + dW$

$dU = -pdV$

$$\frac{dU}{dV} = -p = -\frac{1}{3}\frac{U}{V}$$

$$\frac{dU}{U} = -\frac{1}{3}\frac{dV}{V}$$

$$\ell nU = -\frac{1}{3}\ell nV + \ell nC$$

$U \cdot V^{1/3} = C$

$VT^4 \cdot V^{1/3} = C'$

$$T \propto \frac{1}{R}$$

6. (c) When K_1 is closed and K_2 is open,

$$I_0 = \frac{E}{R}$$

when K_1 is open and K_2 is closed, current as a function of time ' t ' in L.R. circuit.

$$I = I_0 e^{-R\frac{t}{L}}$$

$= \frac{1}{10} e^{-5} = \frac{1}{1500} = 0.67$ mA

7. **(d)** Time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

When additional mass M is added to its bob

$$T_M = 2\pi\sqrt{\frac{\ell + \Delta\ell}{g}}$$

$$\Delta\ell = \frac{Mg\ell}{AY}$$

$$\Rightarrow T_M = 2\pi\sqrt{\frac{\ell + \frac{Mg\ell}{AY}}{g}}$$

$$\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

$$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

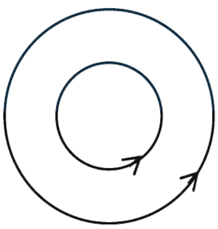
8. **(a)** Intensity, $I = \frac{1}{2}\varepsilon_0 E_0^2 C$, where E_0 is amplitude of the electric field of the light.

$$\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 C$$

$$E_0 = \sqrt{\frac{2P}{4\pi r^2 C\varepsilon_0}} = 2.45 \text{ V/m}$$

9. **(d)** Both solenoid are in equilibrium so, net Force on both solenoids due to other is zero.

So, $\vec{F}_1 = \vec{F}_2 = 0$



10. **(b)** Average time between collision $= \frac{\text{Mean free Path}}{V_{rms}}$

$t = \frac{\frac{1}{\pi d^2\, N/V}}{\sqrt{\frac{3RT}{M}}}$; $t = \frac{CV}{\sqrt{T}}$ (where $C = \frac{\sqrt{M}}{\pi d^2\, N\sqrt{3R}}$ = constant)

$$\Rightarrow T \propto \frac{V^2}{t^2}$$

For adiabatic

$$TV^{\gamma-1} = k$$

$\frac{V^2}{t^2} V^{\gamma-1} = k$

$\frac{V^{\gamma+1}}{t^2} = k$

$t \propto V^{\frac{\gamma+1}{2}}$

so, $q = \frac{\gamma+1}{2}$

11. (d) As $L_1 > L_2$, therefore $\frac{1}{2} L_1 i^2 > \frac{1}{2} L_2 i^2$,

$\therefore$ Rate of energy dissipated through R from L_1 will be slower as compared to L_2.

12. (a) $V_{\text{required}} = V_M - V_{M/8}$

$= -\frac{GM}{2R^3}\left[3R^2 - \frac{R^2}{4}\right] + \frac{GM/8}{2(R/2)^3}[3(R/2)^2]$

$= -\frac{11GM}{8R} + \frac{3GM}{8R} = -\frac{GM}{R}$.

13. (a)

f_1 (train approaches) $= 1000\left(\frac{320}{320-20}\right) = 1000\left(\frac{320}{300}\right)$ Hz.

f_2 (train recedes) $= 1000\left(\frac{320}{320+20}\right) = 1000\left(\frac{320}{340}\right)$ Hz.

$\Delta f = \left(\frac{f_1 - f_2}{f_1}\right) \times 100\% = \left(1 - \frac{300}{340}\right) \times 100\% = \frac{40}{340} \times 100\%$

$= 11.7\% \approx 12\%$.

14. (b) Normal force on block A due to B and between B and wall will be F.

Friction on A due to $B = 20$ N

$\therefore$ Friction on B due to wall $= 100 + 20 = 120$ N

15. (a) $Z_0 = h - \frac{h}{4} = \frac{3h}{4}$

16. (b) Since $\vec{B}$ is uniform, only torque acts on a current carrying loop. $\vec{\tau} = (I\vec{A}) \times \vec{B}$

$\vec{A} = A\hat{k}$ for (b) and $\vec{A} = -A\hat{k}$ for (d).

$\therefore\ \vec{\tau} = 0$ for both these cases.

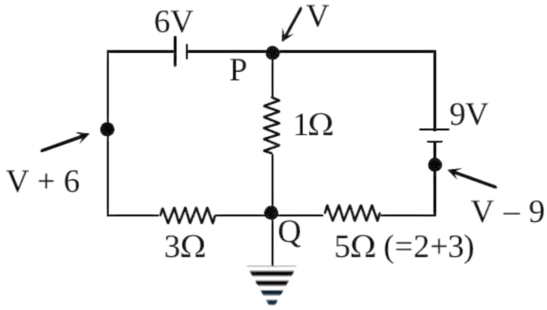
The energy of the loop in the $\vec{B}$ field is : $U = -I\vec{A} \cdot \vec{B}$, which is minimum for (b).

17. (b) Taking the potential at Q to be 0 and at P to be V, we apply Kirchoff's current law at Q:

$\frac{V+6}{3} + \frac{V}{1} + \frac{V-9}{5} = 0$

$V = -\frac{3}{23} = -0.13$ volt

The current will flow from Q to P.



18. (a) &(c) The potential at the centre $= k\frac{Q}{\frac{4}{3}\pi R^3}\int_0^R \frac{4\pi r^2 dr}{r} = \frac{3}{2}\frac{kQ}{R} = \frac{3}{2}V_0; k = \frac{1}{4\pi\varepsilon_0}$

$\therefore R_1 = 0$

Potential at surface, $V_0 = \frac{kQ}{R}$

Potential at $R_2 = \frac{5V_0}{4} \Rightarrow R_2 = \frac{R}{\sqrt{2}}$

Potential at $R_3, \frac{kQ}{R_3} = \frac{3}{4}\frac{kQ}{R} \Rightarrow R_3 = \frac{4R}{3}$

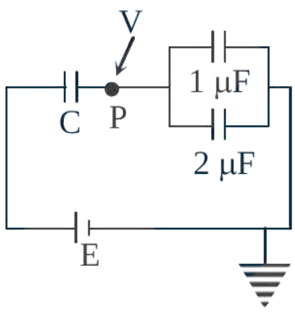
Similarly, at $R_4, \frac{kQ}{R_4} = \frac{kQ}{4R} \Rightarrow R_4 = 4R$

19. (b) Let the potential at P be V,

Then, $C(E - V) = 1 \times V + 2 \times V$ (we take C in μF)

Or, $V = \frac{CE}{3+C}$

$\therefore Q_2 = \frac{2CE}{3+C}$



20. (b)

$E_{\text{initial}} = \frac{1}{2}m(2v)^2 + \frac{1}{2}2m(v)^2 = 3mv^2$

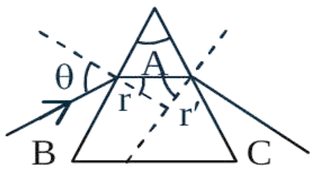
$E_{\text{final}} = \frac{1}{2}3m\left(\frac{4}{9}v^2 + \frac{4}{9}v^2\right) = \frac{4}{3}mv^2$

$\therefore$ Fractional loss $= \frac{3-\frac{4}{3}}{3} = \frac{5}{9} = 56\%$

21. (d) At face AB, $\sin\theta = \mu\sin r$

At face AC $r' < \theta_C$

$A\text{-}r < \sin^{-1}\frac{1}{\mu}$



$\therefore\ r > A - \sin^{-1}\frac{1}{\mu}$

$\therefore\ \sin r > \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)$

$\frac{\sin\theta}{\mu} > \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)$

$\theta > \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\frac{1}{\mu}\right)\right]$

22. (b) For maximum possible volume of cube

$2R = \sqrt{3}a$, a is side of the cube.

Moment of inertia about the required axis $= I = \rho a^3\frac{a^2}{6}$, where $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

$$I = \frac{3M}{4\pi R^3}\frac{1}{6}\left(\frac{2R}{\sqrt{3}}\right)^5 = \frac{3M}{4\pi R^3}\frac{1}{6}\frac{32R^5}{9\sqrt{3}} = \frac{4MR^2}{9\sqrt{3}\pi} = \frac{4MR^2}{9\sqrt{3}\pi}$$

23. (2)

24. (c)

$J = ne\,v_d$

$\frac{A\Delta V}{\rho\ell A} = nev_d$

$\therefore\ \rho = \frac{\Delta v}{\ell nev_d} = \frac{5}{0.1 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}}$

$= 1.56 \times 10^{-5}$

$\approx 1.6 \times 10^{-5}\Omega m$

25. (a) At mean position, K.E. is maximum whereas P.E. is minimum

26. (b) $y_1 = 10t - \frac{1}{2}gt^2$

$y_2 = 40t - \frac{1}{2}gt^2$

$y_2 - y_1 = 30t$ (straight line)

but stone with $10\ m/s$ speed will fall first and the other stone is still in air. Therefore path will become parabolic till other stone reaches ground.

27. (a)

Case (i) $\int dS = C\left[\int_{373}^{423}\frac{dT}{T} + \int_{423}^{473}\frac{dT}{T}\right] = \ln(473/373)$

Case (ii) $= \int dS = C\left[\int_{373}^{385.5}\frac{dT}{T} + \int_{385.5}^{398}\frac{dT}{T} + \int_{398}^{410.5}\frac{dT}{T} + \int_{410.5}^{423}\frac{dT}{T} + \int_{423}^{435.5}\frac{dT}{T} + \int_{435.5}^{448}\frac{dT}{T} + \int_{448}^{460.5}\frac{dT}{T} + \int_{460.5}^{473}\frac{dT}{T}\right] = \ln(473/373)$

Note: If given temperatures are in Kelvin then answer will be option (a).

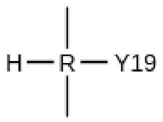
28. (a)

$\theta = 1.22\frac{\lambda}{D}$

Minimum separation $= (25 \times 10^{-2})\theta = 30\mu m$

29. (a) $\tan\theta = \frac{F\ell}{\lambda\ell g} = \frac{\left(\frac{\mu_0 I^2}{4\pi L\sin\theta}\right)\ell}{\lambda\ell g}$

$\Rightarrow I = 2\sin\theta\sqrt{\frac{\pi\lambda Lg}{\mu_0\cos\theta}}$



30. (c) According to Huygens' principle, each point on wave front behaves as a point source of light.

31. (b) $E_n = -13.6 \times \frac{Z^2}{n^2}\,eV$

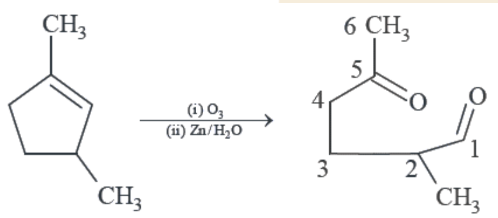
For hydrogen $Z = 1$

$E_n = -13.6 \times \frac{1}{n^2}\,eV$

$E_2 = \frac{-13.6}{4} = -3.4eV$

32. (c)

33.



5 - keto- 2 - methyl hexanal

34. (b) keto- 2 - methyl hexanal

35. (b) L $\rightarrow$ M charge transfer transition

36. (a)

37. (b)

38. (c) Na_3AlF_6 serves as the electrolyte

39. (a)

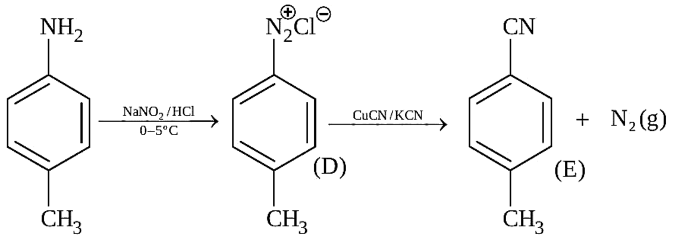
$TiCl_4 + (C_2H_5)_3Al \rightarrow$ Ziegler Natta catalyst, used for coordination polymerization.

$$CH_2 = CH_2 \xrightarrow[H_2O + PCCl_2]{Warker} CH_3 - CHO + 2HCl$$

$CuCl_2 \rightarrow$ used as catalyst in Deacon's process of production of Cl_2.

$V_2O_5 \rightarrow$ used as catalyst in Contact Process of manufacturing of H_2SO_4.

40.



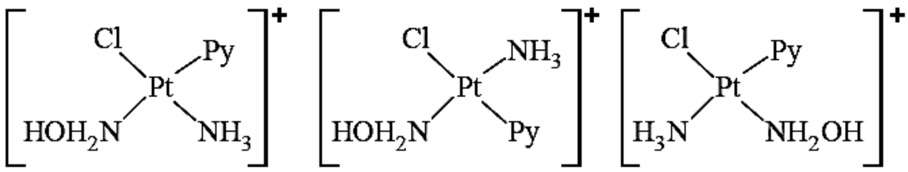
41. (a) Glyptal $\rightarrow$ Manufacture of paints and lacquers

Polypropene $\rightarrow$ Manufacture of ropes, toys

Poly vinyl chloride $\rightarrow$ Manufacture of raincoat, handbags

Bakelite $\rightarrow$ Making combs and electrical switch

42. (a)



43. (d) Higher order (>3) reactions are less probable due to low probability of simultaneous collision of all the reacting species.

44. (c) Inter halogen compounds are more reactive than corresponding halogen molecules due to polarity of bond.

45. (a) $Cu^{2+} + 2e^- \longrightarrow Cu$

$2\ F$ of electricity will give 1 mole of Cu.

46. (d) Amount adsorbed

$= (0.060 - 0.042) \times 50 \times 10^{-3} \times 60$

$= 0.018 \times 50 \times 60 \times 10^{-3}$

$= .018 \times 3$

$= .054gm = 54mg$

Amount adsorbed per gram of activated charcoal $= \frac{54}{3} = 18mg$

47. (c) $2R - Cl + Hg_2\ F_2 \xrightarrow{\Delta} 2R - F + Hg_2Cl_2$

48. (c)

$2C_8H_7SO_3Na + Ca^{+2} \longrightarrow (C_8H_7SO_3)_2Ca$

2 mole 1 mole

412gm 1 mole

Maximum uptake of Ca^{+2} ions by the resin $= 1/412$ (mole per gm resin)

49. (d) Vitamin C is water soluble

50. (c)

51. (c)

$\Delta G^0_{Rex^n} = 2\Delta G^0_f(NO_2) - 2\Delta G^0_f(NO) - \Delta G^0_f(O_2)$

$2\Delta G^o_f(NO_2) = \Delta G^o_{(Rex^n)} + 2\Delta G^o_f(NO) + \Delta G^o_f(O_2)$

$\Delta G = \Delta G^0 + RT\ln\ k_p$

At equilibrium,

$\Delta G = 0, Q = k_p$

$\Delta G^0 = -RT\ln\ k_p$

$\Delta G^0(O_2) = 0$

$\Delta G^0_f(NO_2) = 0.5[2 \times 86{,}600 - R(298)\ln(1.6 \times 10^{12})]$

52. (d) $Zn_2[Fe(CN)_6]$ is bluish white and rest are yellow colored compounds.

53. (d) Let the organic compound is $R - Br$

$R - Br + AgNO_3 \longrightarrow AgBr \downarrow$

So number of moles of AgBr $\equiv$ Number of moles of $R - Br \equiv$ Number of moles of Br

$\frac{141 \times 10^{-3}}{188}$ = number of moles of Br $= 0.75 \times 10^{-3}$

Mass of bromine $= 0.75 \times 80 \times 10^{-3}\ g = 60mg$

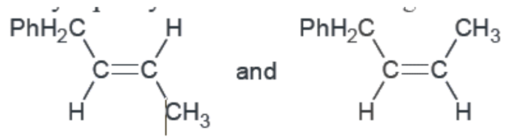
Percentage of bromine $= \frac{60}{250} \times 100 = 24\%$

54. (d) For body center unit cell.

$4r = a\sqrt{3}$

So, radius of Na $= \frac{\sqrt{3} \times 4.29}{4} = 1.857 \approx 1.86Å$

55. (a) phenyl -2-butene can show geometrical isomerism



trans cis

56. (a) Vapour pressure of pure acetone $(p^0) = 185$ torr

Vapour pressure of solution $(p) = 183$ torr

Then from Raoult's law,

$$\frac{p^0 - p}{p} = \frac{n_{solute}}{n_{solvent}}$$

$$\frac{185 - 183}{183} = \frac{1.2/MW}{100/58}$$

$$\frac{2}{183} = \frac{1.2 \times 58}{100 \times MW}$$

$$MW = \frac{1.2 \times 58 \times 183}{200} = 63.68 \approx 64$$

57. (d) H_2O_2 can act both as oxidizing agent as well as reducing agent and H_2O_2 decomposes on exposures to light and dust; so as to kept in plastic or wax lined glass bottles in dark.

58. (a) Because of smallest size of Be^{2+}, its hydration energy is maximum and is greater than the lattice energy of $BeSO_4$

59. (a) $\Delta G^0 = -RT\ln K$

$2494.2 = -8.314 \times 300\ln K$

$\ln K = -1$

$\log_{2.718} K = -1$

$K = (2.718)^{-1} = \frac{1}{2.718} = 0.3679$

$$Q_c = \frac{[B][C]}{[A]^2} = \frac{2 \times \frac{1}{2}}{\left(\frac{1}{2}\right)^2} = 4$$

$Q_c > K_c \Rightarrow$ Reverse direction.

60. (c) The boiling point of Noble gases in increasing order:

$He < Ne < Ar < Kr < Xe < Rn$

Boiling point $(K): 4.2 < 27.1 < 87.2 < 119.7 < 165 < 211$

61. (d)

$t_{r+1} = {}^{50}C_r \cdot (1)^{50-r} \cdot \left(-2x^{1/2}\right)^r$

$= {}^{50}C_r \cdot 2^r \cdot x^{r/2}(-1)^r \Rightarrow r =$ even integer.

$\Rightarrow$ Sum of coefficient $= \sum_{r=0}^{25} {}^{50}C_{2r} \cdot 2^{2r} = \frac{1}{2}\left((1+2)^{50} + (1-2)^{50}\right) = \frac{1}{2}(3^{50} + 1)$

62. (b)

$\lim_{x\to 0}\left(\frac{x^2+f(x)}{x^2}\right) = 3$, since limit exits hence $x^2 + f(x) = ax^4 + bx^3 + 3x^2$

$\Rightarrow f(x) = ax^4 + bx^3 + 2x^2$

$\Rightarrow f'(x) = 4ax^3 + 3bx^2 + 4x$

also $f'(x) = 0$ at $x = 1,2$

$\Rightarrow\ a = \frac{1}{2}, b = -2$

$\Rightarrow f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

$\Rightarrow f(x) = 8 - 16 + 8 = 0.$

63. (c) New sum $\sum y_i = (16 \times 16 - 16) + (3 + 4 + 5) = 252$

Number of observation $= 18$

$\Rightarrow$ New mean

$\Rightarrow \bar{y} = \frac{252}{18} = 14.$

64. (a) $t_r = \frac{\sum r^3}{\sum(2r-1)} = \frac{r^2(r+1)^2}{4r^2} = \frac{1}{4}(r+1)^2$

$S_9 = \frac{1}{4}\sum_{r=1}^{9}(r+1)^2$, let $t = r + 1$

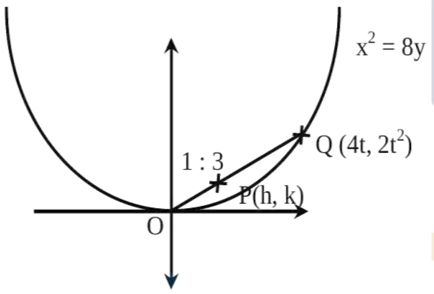
$= \frac{1}{4}\left(\sum_{t=1}^{10} t^2 - 1\right) = 96.$

65. (c)

$h = \frac{4t}{4} = t$

$k = \frac{2t^2}{4} = \frac{t^2}{2}$

$\Rightarrow\ x^2 = 2y$



66. (b)

$x^2 = 6x + 2 \Rightarrow \alpha^2 = 6\alpha + 2$

$\Rightarrow \alpha^{10} = 6\alpha^9 + 2\alpha^8$

and $\beta^{10} = 6\beta^9 + 2\beta^8$

$\Rightarrow$ Subtract (2) from (1)

$a_{10} = 6a_9 + 2a_8$

$\Rightarrow \frac{a_{10} - 2a_8}{2a_9} = 3.$

67. Correct option is not available

Required probability $=\left(\frac{{}^{3}C_{1}\,{}^{12}C_{3}2^{9}-{}^{3}C_{2}\,{}^{12}C_{3}\,{}^{9}C_{3}}{3^{12}}\right)$

68. (b)

$\left|\frac{z_1-2z_2}{z-z_1\bar{z}_2}\right|=1$

$(z_1-2z_2)(\bar{z}_1-2\bar{z}_2)$

$=(2-z_1\bar{z}_2)(2-\bar{z}_1\mathbf{z}_2)$

$\Rightarrow |z_1|^2-2z_2\bar{z}_1-2\bar{z}_2z_1+4|z_2|^2$

$=4-2z_1\bar{z}_2-2\bar{z}_1z_2+|z_1|^2|z_2|^2$

$\Rightarrow |z_1|^2+4|z_2|^2-4-|z_1|^2|z_2|^2=0$

$|z_1|^2(1-|z_2|^2)-4(1-|z_2|^2)=0$

$\Rightarrow |z_1|=2(\quad \text{as } |z_2|\neq 1)$

69. (c) $\int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

$1+\frac{1}{x^4}=t$

$\frac{-4}{x^5}dx=dt$

$\Rightarrow -\frac{1}{4}\int \frac{1}{t^{3/4}}dt$

$=-\frac{1}{4}\times 4t^{1/4}+c=-\left(1+\frac{1}{x^4}\right)^{1/4}+c.$

70. (c) $x+y<41, x>0, y>0$ is bounded region.

Number of positive integral solutions of the equation $x+y+k=41$ will be number of integral co-ordinates in the bounded region.

$\Rightarrow {}^{41-1}C_{3-1}={}^{40}C_2=780.$

71. (c) Let the point of intersection be $(2+3\lambda, 4\lambda-1, 12\lambda+2)$

$(2+3\lambda)-(4\lambda-1)+12\lambda+2=16$

$11\lambda=11$

$\lambda=1$

$\Rightarrow$ point of intersection is $(5,3,14)$

$\Rightarrow$ distance $=\sqrt{(5-1)^2+9+12^2}$

$=\sqrt{16+9+144}=13.$

72. (b)

73. (c) The required region

$$= \int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy$$

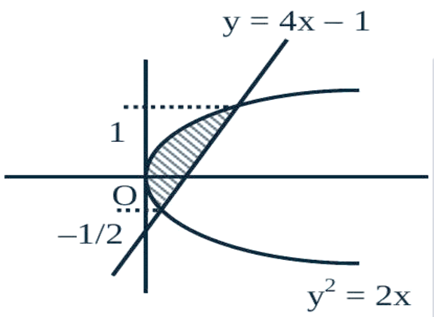
$$= \frac{1}{4}\left(\frac{y^2}{2}+y\right)_{-\frac{1}{2}}^{1}-\frac{1}{2}\left(\frac{y^3}{3}\right)_{-\frac{1}{2}}^{1}$$

$$= \frac{1}{4}\left(\frac{1}{2}+1-\left(\frac{1}{8}-\frac{1}{2}\right)\right)-\frac{1}{2}\frac{1}{3}\left(1+\frac{1}{8}\right)$$

$$= \frac{1}{4}\times\frac{15}{8}-\frac{3}{16}=\frac{9}{32}.$$

74. (b) Given

$l + n = 2m$



$1, G_1, G_2, G_3, n$ are in G.P.

$\Rightarrow\ G_1 = lr$ (let r be the common ratio)

$G_2 = lr^2$

$G_3 = lr^3$

$n = 1r^4$

$$r = \left(\frac{n}{l}\right)^{1/4}$$

$$\Rightarrow\ G_1^4 + 2G_2^4 + G_3^4 = (lr)^4 + 2(lr^2)^4 + (lr^3)^4$$
$$= l^4 \times r^4[1 + 2r^4 + r^8]$$
$$= l^4 \times r^4[r^4 + 1]^2$$
$$= l^4 \times \frac{n}{l}\left[\frac{n+l}{l}\right]^2$$
$$= \ln \times 4m^2$$
$$= 4lnm^2.$$

75. (a) Let M is mid point of BB′ and AM is ⊥ bisector of BB′ (where A is the point of intersection given lines)

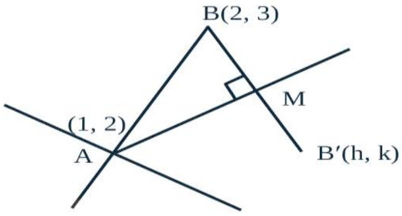
$(x-2)(x-1)+(y-2)(y-3)=0$

$$\Rightarrow\ \left(\frac{h+2}{2}-2\right)\left(\frac{h+2}{2}-1\right)+\left(\frac{k+3}{2}-2\right)\left(\frac{k+3}{2}-3\right)=0$$

$\Rightarrow (h-2)(h)+(k-1)(k-3)=0$

$\Rightarrow x^2-2x+y^2-4y+3=0$

$\Rightarrow (x-1)^2+(y-2)^2=2.$



76. (c)

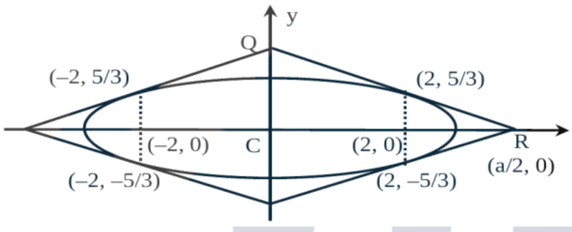
$a=3, b=\sqrt{5}$

$$e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$$

foci $=(\pm 2,0)$

tangent at P $\Rightarrow \frac{2x}{9}+\frac{5y}{3.5}=1$

$$\frac{2x}{9}+\frac{y}{3}=1$$



$2x+3y=9$

Area of quadrilateral

$=4\times$ (area of triangle QCR)

$$=\left(\frac{1}{2}\times\frac{9}{2}\times 3\right)\times 4=27$$

77. (a) Four-digit numbers will start from 6,7,8

$3\times 4\times 3\times 2=72$

Five-digit numbers $=5!=120$

Total number of integers $=192$.

78. $(\boldsymbol{d})$ $n(A)=4, n(B)=2$

$n(A\times B)=8$

number of subsets having atleast 3 elements

$= 2^8 - (1 + {}^8C_1 + {}^8C_2) = 219$

79. (d)

$$\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$$

$$= \tan^{-1}\left(\frac{x + \frac{2x}{1-x^2}}{1 - x\left(\frac{2x}{1-x^2}\right)}\right)$$

$$= \tan^{-1}\left(\frac{x - x^3 + 2x}{1 - x^2 - 2x^2}\right)$$

$$\tan^{-1} y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

$y = \frac{3x-x^3}{1-3x^2}$.

80. (b) Apply the property

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

And then add

$2I = \int_2^4 1dx$

$2I = 2$

$I = 1$

81. (c)

$\sim (\sim s \vee (\sim r \wedge s))$

$\equiv s \wedge (\sim (\sim r \wedge s))$

$\equiv s \wedge (r \vee \sim s) \equiv (s \wedge r) \vee (s \wedge \sim s)$

$\equiv (s \wedge r) \vee F$

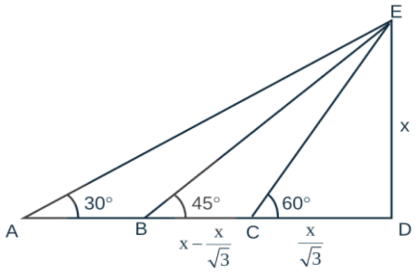
$\equiv s \wedge r$.

82. (d)

$AB = \sqrt{3}x - x$

$$BC = x - \frac{x}{\sqrt{3}}$$

$\frac{AB}{BC} = \frac{\sqrt{3}x - x}{x - \frac{x}{\sqrt{3}}} = \frac{\sqrt{3}}{1}$.



83. (b) $\lim_{x\to 0} \frac{2\sin^2 x\times(3+\cos x)}{x\times\left(\frac{\tan 4x}{4x}\right)\times 4x} = \frac{2\times 4}{4} = 2.$

84. (d) $(\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow -\vec{b}\cdot\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow -|\vec{b}||\vec{c}|\cos\theta = \frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow \cos\theta = -\frac{1}{3}$

$\Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}.$

85. (c)

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}, A^T = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$

$AA^T = [b_{ij}]_{3\times 3}$

$b_{13} = 0 \Rightarrow 0 = a + 4 + 2b$

$b_{23} = 0 \Rightarrow 0 = 2a + 2 - 2b$

$\Rightarrow 3a + 6 = 0 \Rightarrow a = -2, b = -1.$

86. (d) for f(x) to be continuous

$2k = 3m + 2$

$2k - 3m = 2$

for f(x) to be differentiable

$\frac{k}{4} = m$

$k = 4m.$

from (i), $8m - 3m = 2$

$5m = 2$

$m = \frac{2}{5}$

$k = 4 \times \frac{2}{5} = \frac{8}{5}$

$k + m = \frac{2}{5} + \frac{8}{5} = \frac{10}{5} = 2$

87. (b)

$(2-\lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

$\Rightarrow \begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$(2-\lambda)(3\lambda + \lambda^2 - 4) + 2(-2\lambda + 2) + 1(4 - 3 - \lambda) = 0$

$(2-\lambda)(\lambda^2 + 3\lambda - 4) + 4(1-\lambda) + (1-\lambda) = 0$

$(2-\lambda)((\lambda+4)(\lambda-1)) + 5(1-\lambda) = 0$

$(1-\lambda)((\lambda+4)(\lambda-2) + 5) = 0 \Rightarrow \lambda = 1, 1, -3.$

88. (c) $x^2 + 3xy - xy - 3y^2 = 0$

$x(x+3y) - y(x+3y) = 0$

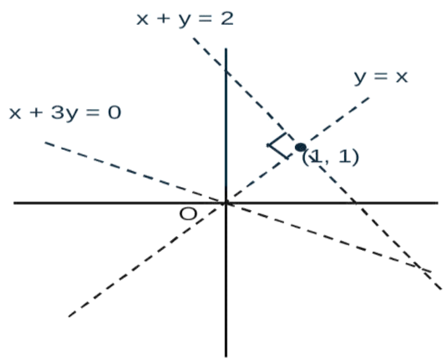
$(x+3y)(x-y) = 0$

Equation of normal is $(y-1) = -1(x-1)$

$\Rightarrow x + y = 2$

It intersects $x + 3y = 0$ at $(3, -1)$

And hence meets the curve again in the 4th quadrant.



89. (b)

$C_1\,(2,3)$;

$r_1 = \sqrt{4+9+12} = 5$

and $C_2(-3,-9)$;

$r_2 = \sqrt{9+81-26} = 8$

$C_1 C_2 = \sqrt{25+144} = 13$

$C_1 C_2 == r_1 + r_2$ touching externally.

$\Rightarrow 3$ common tangents.

90. (b) $\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{2x\ln x}{x\ln x}$

I.F. $= e^{\int \frac{1}{x\ln x}dx} = e^{\ln(\ln x)} = \ln x$

$y\ln x = \int 2\ln x dx$

$y\ln x = 2(x\ln x - x) + c$

For $x = 1, c = 2$

$y\ln x = 2(x\ln x - x + 1)$

put $x = e \Rightarrow y(e) = 2$.