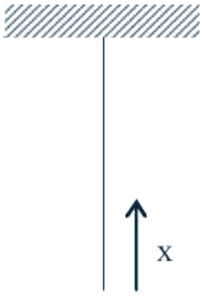
1. (b)

$$T(x)=\frac{Mgx}{L}$$

$$\Rightarrow v(x)=\sqrt{\frac{T}{\mu}}=\sqrt{gx}$$

$$\Rightarrow \frac{dx}{dt}=\sqrt{gx}\Rightarrow \text{time taken}=2\sqrt{2}\text{ s.}$$



2. (c) Let fat used be ' x ' kg

$\Rightarrow$ Mechanical energy available $=x\times 3.8\times 10^{7}\times\frac{20}{100}$

Work done in lifting up $=10\times 9.8\times 1000$

$$\Rightarrow x\times 3.8\times 10^{7}\times\frac{20}{100}=9.8\times 10^{4}$$

$\Rightarrow x\approx 12.89\times 10^{-3}$ kg.

3. (b) Since work done by friction on parts PQ and QR are equal

$$-\mu mg\times\frac{\sqrt{3}}{2}\times 4=-\mu mgx$$

$(QR=x)$

$\Rightarrow x=2\sqrt{3}\text{ m}\approx 3.5$

Applying work energy theorem from P to R

$$mg\sin 30^{0}\times 4-\mu mg\frac{\sqrt{3}}{2}\times 4-\mu mgx=0$$

$$\Rightarrow \mu=\frac{1}{2\sqrt{3}}\approx 0.29.$$

4. (c) $B_A=\frac{\mu_0}{4\pi}\frac{2\pi i}{(\ell/2\pi)}$

$$B_B=\left[\frac{\mu_0}{4\pi}\frac{i}{\ell/8}(\sin 45^{\circ}+\sin 45^{\circ})\right]\times 4$$

$$\frac{B_A}{B_B}=\frac{\pi^2}{8\sqrt{2}}$$

5. **(d)** For full scale deflection

$$100 \times i_g = (i - i_g)S$$

where ' S ' is the required resistance

$$S = \frac{100 \times 1 \times 10^{-3}}{(10 - 10^{-3})}$$

$$S \approx 0.01\Omega$$

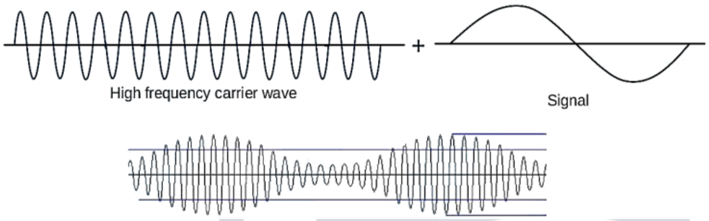
6. **(c)**

7. **(b)** The electric resistance of a typical intrinsic (undoped) semiconductor decreases exponentially with temperature

$$\rho = \rho_0 e^{-aT}$$

where a is a constant.

8. **(d)** In amplitude modulation amplitude of carrier wave varies in proportion to applied signa



Amplitude modulated carrier wave

9. **(c)** for A: $N_0 - \frac{N_0}{2^4} = \frac{15N_0}{16}$

for B: $N_0 - \frac{N_0}{2^2} = \frac{3\ N_0}{4}$

The required ratio is $\frac{5}{4}$

10. **(d)** Equation of line is

$$PV_0 + P_0\ V = 3P_0\ V_0$$

Also, $PV = nRT$

for $T_{max}, \frac{dT}{dV} = 0$

$$\Rightarrow V = \frac{3\ V_0}{2}, P = \frac{3P_0}{2}$$

$$\Rightarrow T_{max} = \frac{9P_0\ V_0}{4nR}$$

11. **(c)** For the lamp with direct current,

$$V = IR$$

$\Rightarrow R = 8\Omega$ and $P = 80 \times 10 = 800$ W

For ac supply

$$P = I_{rms}^2 R = \frac{\varepsilon_{rms}^2}{Z^2} R$$

$$\Rightarrow Z^2 = \frac{(220)^2 \times 8}{800}$$

$$\Rightarrow Z = 22\Omega$$

$$\Rightarrow R^2 + \omega^2 L^2 = (22)^2$$

$$\Rightarrow \omega L = \sqrt{420}$$

$$\Rightarrow L = 0.065H$$

12. (c) For open pipe in air, fundamental frequency:

$$f = \frac{V}{2\ell}$$

For the pipe closed at one end (dipped in water), fundamental frequency:

$$f' = \frac{V}{4\ell'} = \frac{V}{4 \times \frac{\ell}{2}} = \frac{V}{2\ell}$$

$$\therefore f' = f$$

13. (b) We know that

Geometrical spread $= a$

and diffraction spread $= \frac{\lambda L}{a}$

so spot size(b) $= a + \frac{\lambda L}{a}$

for minimum spot size $a = \frac{\lambda L}{a}$

$$\Rightarrow a = \sqrt{\lambda L}$$

and $b_{min} = \sqrt{\lambda L} + \sqrt{\lambda L} = \sqrt{4\lambda L}$

14. (b) Charge on 9μF capacitor $= 18\mu$C

Charge on 4μF capacitor $= 24\mu$C

$$\therefore Q = 24 + 18 = 42\mu C$$

$$\therefore \frac{KQ}{r^2} = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{(30)^2} = 420 \text{ N/C}$$

15. (d) Radiation energy per quantum is

$E = h\nu$

As per EM spectrum, the increasing order of frequency and hence energy is

Radio wave < Yellow light < Blue light < X Ray

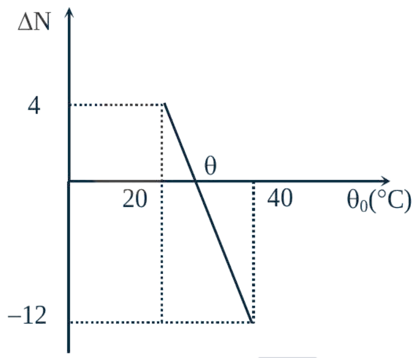
16. (c) For electromagnet and transformer, the coercivity should be low to reduce energy loss
17. (d) First Method

Time period $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell_0}{g}(1+\alpha\Delta\theta)}$

$T = T_0\left[1 + \frac{1}{2}\alpha\Delta\theta\right]$

$N = \frac{2 \times 86400}{T} = \left(\frac{2 \times 86400}{T_0}\right)\left(1 + \frac{1}{2}\alpha\Delta\theta\right) = N\left(1 + \frac{\alpha\Delta\theta}{2}\right)$

$\Delta N = N - N_0 = \frac{1}{2}\alpha\Delta\theta N_0 \Rightarrow \Delta N \propto \Delta\theta$



$\Rightarrow \theta_0 = 25°C$

Putting θ_0, we get $\alpha = 1.85 \times 10^{-5} / °C$

Second Method

According to given conditions

$$86412 = 2\pi\sqrt{\frac{\ell_{40}}{g}}$$

$$86396 = 2\pi\sqrt{\frac{\ell_{20}}{g}}$$

$$86400 = 2\pi\sqrt{\frac{\ell}{g}}$$

From equation (i) and (iii)

$12 = \frac{2\pi}{\sqrt{g}}\left[\sqrt{\ell_{40}} - \sqrt{\ell}\right]$

and $4 = \frac{2\pi}{\sqrt{g}}\left[\sqrt{\ell} - \sqrt{\ell_{20}}\right]$

on dividing (iv) and (v)

$$3 = \frac{\sqrt{1+\alpha(40-\theta)} - 1}{1 - \sqrt{1+\alpha(20-\theta)}}$$

$\Rightarrow 3 = \frac{40-\theta}{\theta-20}$ (by Binomial theorem)

$\Rightarrow \theta = 25°$

on using θ in (i) and (iii)

$$\frac{86412}{86400} = \sqrt{1+\alpha 15} \Rightarrow \alpha = 1.85 \times 10^{-5}/\,°\mathrm{C}$$

18. (d) According Gauss's law, we can write,

$$\mathrm{E} \times 4\pi \mathrm{r}^2 = \frac{1}{\varepsilon_0}\left[\mathrm{Q} + \int_a^r \left(\frac{\mathrm{A}}{\mathrm{r}}\right)(4\pi \mathrm{r}^2 \mathrm{dr})\right] = \frac{1}{\varepsilon_0}[\mathrm{Q} + 2\pi \mathrm{A}(\mathrm{r}^2 - \mathrm{a}^2)]$$

For E to be independent of r,

$$\mathrm{Q} - 2\pi \mathrm{Aa}^2 = 0 \Rightarrow \mathrm{a} = \frac{\mathrm{Q}}{2\pi \mathrm{a}^2}$$

19. (d) $\delta = \mathrm{i} + \mathrm{e} - \mathrm{A} \Rightarrow \mathrm{A} = 74°$

$$\mu = \frac{\sin\left(\frac{\mathrm{A}+\delta_{\min}}{2}\right)}{\sin\left(\frac{\mathrm{A}}{2}\right)} = \frac{5}{3}\sin\left(37° + \frac{\delta_{\min}}{2}\right)$$

$\mu_{\max}$ can be $\frac{5}{3}$, so μ will be less than $\frac{5}{3}$

Since $\delta_{\min}$ will be less than 40°, so

$$\mu < \frac{5}{3}\sin 57° < \frac{5}{3}\sin 60° \Rightarrow \mu < 1.446$$

So, the nearest possible value of μ should be 1.5

20. (d) $\mathrm{t} = \frac{\mathrm{t}_1+\mathrm{t}_2+\mathrm{t}_3+\mathrm{t}_4}{4} = \frac{90+91+95+92}{4} = 92$

Now mean deviation is equal to $\left(\frac{2+1+3+0}{4}\right) = 1.5$

Since least count of clock is one second, so $\Delta \mathrm{t} = 2\mathrm{sec}$

21. (d)

22. (d) $\frac{\mathrm{hc}}{\lambda} - \phi = \frac{1}{2}\mathrm{mv}^2$

$$\frac{4\mathrm{hc}}{3\lambda} - \phi = \frac{1}{2}\mathrm{mv}_1^2$$

So, $\frac{hc}{3\lambda} = \frac{1}{2}m(v_1^2 - v^2)$

$$\frac{1}{3}\left(\frac{1}{2}mv^2 + \phi\right) = \frac{1}{2}m(v_1^2 - v^2)$$

$$\therefore\ v_1 > v\left(\frac{4}{3}\right)^{\frac{1}{2}}$$

23. (c)

$$3\omega\sqrt{A^2 - \left(\frac{2\ A}{3}\right)^2} = \omega\sqrt{A_1^2 - \left(\frac{2\ A}{3}\right)^2}$$

$$\therefore\ A_1 = \frac{7\ A}{3}$$

24. (a, c)

$\vec{L}_O = mv\frac{R}{\sqrt{2}}(-\hat{k})$ [D to A]

$\vec{L}_O = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ [C to D]

25. (a)

$$C = C_V - \frac{R}{n-1}$$

$$\frac{R}{n-1} = C_V - C$$

$$n = \frac{C - C_P}{C - C_V}$$

26. (a) Reading $= 0.5 + 25\left(\frac{0.5}{50}\right) + 5\left(\frac{0.5}{50}\right) = 0.8$ mm

27. (d) From normal reactions of roller, we can conclude it moves towards left.

28. (b) From truth Table, it is OR gate.

29. (a, c)

$\beta = \frac{\alpha}{1-\alpha}$, is not satisfied by option (a, c)

30. (c) At height h escape velocity

$$V_e = \sqrt{\frac{2GM}{h+h}}$$

Orbital velocity $V_0 = \sqrt{\frac{GM}{R+h}}$

$\therefore$ Increase in orbital velocity required to escape gravitational field

$\Rightarrow V_e - V_0$

$\Rightarrow \sqrt{gR}(\sqrt{2} - 1)$

31. (c)

$$\lambda = \frac{h}{mv}$$
$$= \frac{h}{\sqrt{2\,m(KE)}}$$
$$\therefore\ \frac{h}{\lambda} = \sqrt{2meV}$$

32. (d) via S_N1 and E_2 mechanisms
33. (a) CrO_2 is metallic and ferromagnetic
34. (c) HDPE is used in making buckets and dustbins
35. (b)

$$\frac{x}{m} = kp^{\frac{1}{n}}$$
$$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$$

36. (c) $C + O_2 \longrightarrow CO_2$

$$CO + \frac{1}{2}O_2 \longrightarrow CO_2$$

$\Delta H_1 \Rightarrow -393.5$ kJ/ mole

$\Delta H_2 \Rightarrow -283.5$ kJ/ mole

$$C + \frac{1}{2}O_2 \longrightarrow CO$$

$\Delta_f H_{(CO,g)} = \Delta H_1 - \Delta H_2$

$\Rightarrow -393.5 - (-283.5)$

$= -393.5 + 283.5$

$\Rightarrow -110$ kJ/mole

37. (a) region 2
38. (a) Sodium $^+$(lauryl sulphate) $^-$is an anionic detergent.
39. (b) $n_{C_6H_{12}O_6} = 0.1$

$$n_{H_2O} = \frac{178.2}{18} = 9.9$$

$$\frac{p^0 - p}{p^0} \Rightarrow \frac{0.1}{10}$$

$$p^0 - p \Rightarrow \frac{0.1}{10} \times 760 = 7.6$$

$p = p^0 - 7.6$

$= 760 - 7.6$

$= 752.4$ torr

40. **(c)** Glycerol decomposes before its boiling point.

41. **(d)** $O = \overset{\oplus}{N} = O$

42. **(a)** $H_2O_2 \longrightarrow H_2O + \frac{1}{2}O_2$

$2t_{1/2} = 50\text{mins}$

$t_{1/2} = 25\text{mins}$

$\therefore k = \frac{0.693}{25}$

$-\frac{d[H_2O_2]}{dt} = \frac{d[O_2]}{dt} \times 2$

$\therefore \frac{d[O_2]}{dt} = \frac{1}{2} \times \frac{0.693}{25} \times 0.05$

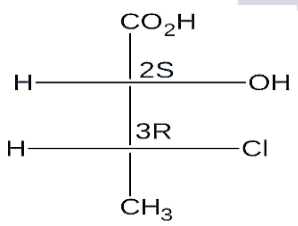
$= 6.93 \times 10^{-4}$

43. **(a)**

$Cr^{2+} \longrightarrow d^4$
No of unpaired electron 4
$Fe^{2+} \longrightarrow d^6$
No of unpaired electron 4

Both configurations have 4 unpaired electrons due to weak field ligands.

44. **(a)**



45. **(b)**

$A + B \rightleftharpoons C + D$

$1 \quad 1 \quad 1 \quad 1$

$1 - X \; 1 - X \; 1 + x \; 1 + x$

Given $K_{eq} = 100$

$\frac{(1+x)^2}{(1-x)^2} = 100$

$\frac{1+x}{1-x} = 10$

$1 + x = 10 - 10x$

$11x = 9$

$x = \frac{9}{11}$

$[D] = 1 + \frac{9}{11}$

$= 1 + 0.818$

$= 1.818$

46. (b) Galena (PbS) is a sulphide ore.

47. None

Volume of $O_2 = 375 \times \frac{20}{100} = 75ml$

Volume of gaseous hydrocarbon $= 15ml$

$C_xH_y + \left(x + \frac{y}{4}\right) O_2 \longrightarrow xCO_2 + \frac{y}{2} H_2O(\ell)$

$15\left(x + \frac{y}{4}\right) = 75$

$x + \frac{y}{4} = 5$

Excess air $= 375 - 75 = 300$

$300 +$ volume of $CO_2 = 330$

Volume of $CO_2 = 30$

$15x = 30$

$x = 2$

(Correct option is not given)

$2 + \frac{y}{4} = 5$

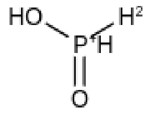
$\frac{y}{4} = 3$

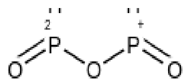
$y = 12$

C_2H_{12} (not possible)

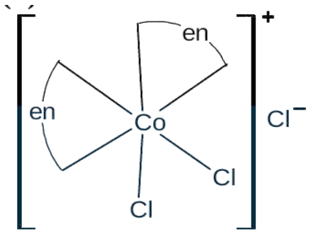
48. (d) $H_4P_2O_5 \Rightarrow$ pyrophosphorous (+3 state)

$H_3PO_3 \Rightarrow$ orthophosphorous (+3 state)





49. (a)



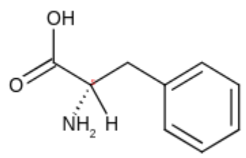
cis $[Co(en)_2Cl_2]Cl$

(optically active)

50. (d)

$$Zn + 4HNO_3 \longrightarrow Zn(NO_3)_2 + 2H_2O + +2NO_2 \uparrow$$
$$4Zn + 10HNO_3 \longrightarrow 4Zn(NO_3)_2 + N_2O \uparrow + + 5H_2O$$

51. (b) Ice shows intermolecular H - bonding.
52. (b) Maximum limit of nitrate in potable water is 50ppm.
53. (c)
54. (b)



55. (c)
56. (c)
57. (c)

$$R-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2 + 4NaOH + Br_2 \longrightarrow RNH_2 + 2NaBr + Na_2CO_3 + 2H_2O$$

58. (b) Initial total moles:

$$n = \frac{p_iV}{RT_1} + \frac{p_iV}{RT_1} = \frac{2p_iV}{RT_1}$$

Final total moles:

$$n=\frac{p_f V}{RT_1}+\frac{p_f V}{RT_2}$$

Equating the two:

$$\frac{2p_i}{T_1}=\frac{p_f}{T_1}+\frac{p_f}{T_2}$$
$$p_f=\frac{2p_i}{T_1}\times\frac{T_1\ T_2}{(T_1+T_2)}$$
$$p_f=2p_i\times\frac{T_2}{T_1\times T_2}$$

59. (a) $CH_3-CH=CH_2+Cl^{\oplus}\longrightarrow CH_3-\overset{\oplus}{C}H-CH_2Cl$ (Markovnikov's addition)

60. (a) Allylic bromination followed by hydrolysis.

61. (b) Remaining two sides of rhombus are $x-y-3=0$ and $7x-y+15=0$.

So, on solving, we get vertices as $\left(\frac{1}{3},-\frac{8}{3}\right),(1,2),\left(-\frac{7}{3},-\frac{4}{3}\right)$ and $(-3,-6)$.

62. (a) Let the G.P. be a, ar, ar^2 and terms of A.P. are A + d, A +4d, A + 8d

then $\frac{ar^2-ar}{ar-a}=\frac{(A+8\ d)-(A+4\ d)}{(A+4\ d)-(A+d)}=\frac{4}{3}$

$\Rightarrow r=\frac{4}{3}$.

Alternate Solution:

Let AP is $a, a+d, a+2\ d$....

$2^{\text{nd}}, 5^{\text{th}}$ and 9^{th} terms $a+d, a+4d, a+8d$ are in GP

$\Rightarrow (a+4\ d)^2=(a+d)(a+8\ d)$

$\Rightarrow d(8\ d-a)=0\Rightarrow 8\ d=a$ as $d\neq 0$

Hence common ratio of GP $\frac{a+4d}{a+d}=\frac{8d+4d}{8d+d}=\frac{12d}{9d}=\frac{4}{3}$.

63. (d)

Equation of normal at P is

$y=-tx+4t+2t^3$

It passes through $C(0,-6)$

$\therefore\ -6=4t+2t^3$

$\Rightarrow t^3+2t+3=0$

$\Rightarrow t=-1$

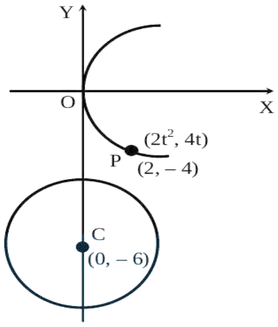
Hence P is $(2,-4)$

$r = \sqrt{4+4} = 2\sqrt{2}$

Equation of required circle

$(x-2)^2 + (y+4)^2 = 8$

$\Rightarrow\ x^2 + y^2 - 4x + 8y + 12 = 0$



64. **(c)** For non-trivial solution

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$
$$1(\lambda+1) - \lambda(-\lambda^2+1) - 1(\lambda+1) = 0$$
$$\lambda + 1 + \lambda^3 - \lambda - \lambda - 1 = 0$$
$$\lambda(\lambda^2 - 1) = 0 \Rightarrow \lambda = 0, \lambda = \pm 1.$$

65. **(b)** $f(x) + 2f\left(\frac{1}{x}\right) = 3x$

replace x by $\frac{1}{x}$

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$

$\Rightarrow f(x) = \frac{2}{x} - x$ as $f(x) = f(-x) \Rightarrow x = \pm\sqrt{2}$

66. **(b)**

$$p = \lim_{x\to 0^+}\left(1 + \tan^2\sqrt{x}\right)^{\frac{1}{2x}}$$
$$= e^{\lim_{x\to 0^+}\frac{\tan^2\sqrt{x}}{2x}}$$
$$= e^{\lim_{x\to 0^+}\frac{1}{2}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)^2}$$

$p = e^{\frac{1}{2}}$

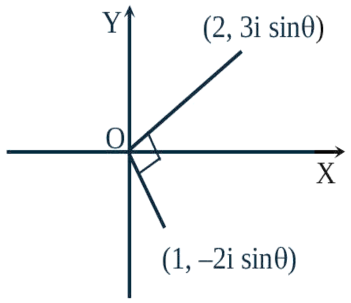
67. **(c)**

$\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary $\in$ Arg $\left(\frac{2+3i\sin\theta}{1-2i\sin\theta}\right) = \frac{\pi}{2}, -\frac{\pi}{2}$

$\Rightarrow$ product of slopes taken as in xy plane is -1

$$\Rightarrow \frac{3\sin\theta}{2}\cdot\frac{-2\sin\theta}{1}=-1$$

$$\Rightarrow \sin^2\theta=\frac{1}{3}, \theta=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$



Alternate Solution:

$\frac{2+3i\sin\theta}{1-2i\sin\theta}=\frac{2-6\sin^2\theta+7i\sin\theta}{1+4\sin^2\theta}$ is purely imaginary

$\Rightarrow \frac{2-6\sin^2\theta}{1+4\sin^2\theta}=0 \Rightarrow 6\sin^2\theta=2.$

68. (b) Given, $\frac{2\,b^2}{a}=8$ and $2\,b=\frac{1}{2}(2ae)$

$2\,b=ae$

$4b^2=a^2\cdot e^2$

$3e^2=4\Rightarrow e=\frac{2}{\sqrt{3}}$

69. (a)

$$\bar{x}=\frac{2+3+a+11}{4}=\frac{a}{4}+4$$

$$\sigma=\sqrt{\sum\frac{x_i^2}{n}-(\bar{x})^2}$$

$$3.5=\sqrt{\frac{4+9+a^2+121}{4}-\left(\frac{a}{4}+4\right)^2}$$

$$\Rightarrow \frac{49}{4}=\frac{4(134+a^2)-(a^2+256+32a)}{16}$$

$$\Rightarrow 3a^2-32a+84=0$$

70. (a)

$$I=\int\frac{\left(\frac{2}{x^3}+\frac{5}{x^6}\right)}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx,\ \text{let } 1+\frac{1}{x^2}+\frac{1}{x^5}=t$$

Hence $I=\frac{x^{10}}{2(x^5+x^3+1)^2}+c.$

71. (c) As line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in plane $\ell x+my-z=9$

So $2\ell-m-3=0$ (as line is perpendicular to normal of the plane)

Also point $(3,-2,-4)$ lies in plane

So $3\ell-2\text{ m}-5=0$

From equation (a) and (b), we get $\ell=1, \text{m}=-1$

So $\ell^2+\text{m}^2=2$

72. (b) $2\cos\frac{5x}{2}\cdot\cos\frac{3x}{2}+2\cos\frac{5x}{2}\cdot\cos\frac{x}{2}=0$

$$\Rightarrow \cos\frac{5x}{2}\cdot\left(2\cdot\cos x\cdot\cos\frac{x}{2}\right)=0$$

$\Rightarrow \frac{x}{2}=(2n+1)\frac{\pi}{2},\ x=(2\text{ m}+1)\frac{\pi}{2},\frac{5x}{2}=(2k+1)\frac{\pi}{2}$, (where $n, m, k \in Z$)

$$\Rightarrow x=(2n+1)\pi,\ x=(2\text{ m}+1)\frac{\pi}{2},\ x=(2k+1)\frac{\pi}{5}$$

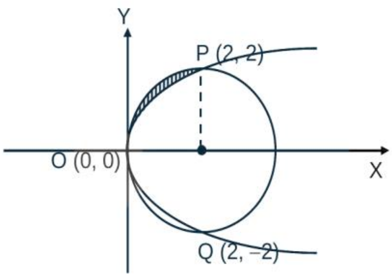
$\Rightarrow x=\pi,\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{5},\frac{3\pi}{5},\frac{7\pi}{5},\frac{9\pi}{5}$.

73. (a) The point of intersection of the curve $x^2+y^2=4x\ y^2=2x$ are $(0,0)$ and $(2,2)$ for $x\geq 0$ and $y\geq 0$

So required area $=\frac{1}{4}\pi\times 4-\int_0^2\sqrt{2x}dx$

$$=\pi-\sqrt{2}\cdot\frac{2}{3}\left[x^{3/2}\right]_0^2$$

$$=\pi-\frac{8}{3}$$



74. (c) $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

$$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$$

$\Rightarrow \vec{a}\cdot\vec{c}=\frac{\sqrt{3}}{2}$ and $\vec{a}\cdot\vec{b}=-\frac{\sqrt{3}}{2}$

$\Rightarrow \cos\theta=-\frac{\sqrt{3}}{2}$ where θ is angle between $\vec{a}\&\vec{b}$

$\therefore \theta=\frac{5\pi}{6}$.

75. (b)

$f(x)=x^2+\frac{(1-2x)^2}{\pi}\left(\quad \text{As } r=\frac{1-2x}{\pi}\right)$

$f'(x)=2x-\frac{4(1-2x)}{\pi}$

$f''(x)=2+\frac{8}{\pi}>0$

For minimum value of sum of area $f'(x)=0$

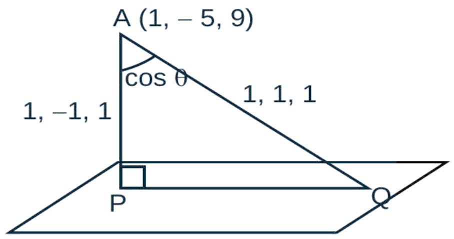
$x=\frac{2}{\pi+4}\Rightarrow r=\frac{1}{\pi+4}$

$\Rightarrow x=2r$

76. (a) $\cos\theta=\frac{1-1+1}{3}=\frac{1}{3}$

$\cos\theta=\frac{AP}{AQ}$

$AQ=\frac{AP}{\cos\theta}=10\sqrt{3}$



77. (c) $y(1+xy)dx=xdy$

$\frac{xdy-ydx}{y^2}=xdx$

$\int -d\left(\frac{x}{y}\right)=\int xdx$

$-\frac{x}{y}=\frac{x^2}{2}+c$ as $y(a)=-1\Rightarrow c=\frac{1}{2}$

Hence $y=\frac{-2x}{x^2+1}\Rightarrow f\left(-\frac{1}{2}\right)=\frac{4}{5}$.

78. (c) Total number of terms $= {}^{n+2}C_2=28$

$(n+2)(n+1)=56$
$n=6$

Sum of coefficients $=(1-2+4)^n$

$=3^6=729$

79. (a) $f(x)=\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$, where $x\in\left(0,\frac{\pi}{2}\right)$

$=\tan^{-1}\left(\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|\right)$

$\Rightarrow f(x)=\frac{\pi}{4}+\frac{x}{2},\ f\left(\frac{\pi}{6}\right)=\frac{\pi}{3}$

$f'(x)=\frac{1}{2}$

Equation of normal is

$y-\frac{\pi}{3}=-2\left(x-\frac{\pi}{6}\right)$

It passes through $\left(0,\frac{2\pi}{3}\right)$.

80. (a) At $x=0, f$ is differential and $f'(0)=-\cos 0=-1$ $g'(0)=f'(f(0))\cdot f'(0)$

$=-\cos(\log 2)\times -1$ (at $x=0, f(0)=\log 2$)

$=\cos(\log 2)$

81. (c) $P(E_1)=\frac{1}{6}, P(E_2)=\frac{1}{6}, P(E_3)=\frac{1}{2}$

Also $P(E_1\cap E_2)=\frac{1}{36}, P(E_2\cap E_3)=\frac{1}{12}, P(E_1\cap E_3)=\frac{1}{12}$

And $P(E_1\cap E_2\cap E_3)=0\neq P(E_1)\cdot P(E_2)\cdot P(E_3)$

82. (a) $A\operatorname{adj}A=|A|I_n=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}\begin{bmatrix}5a & 3\\ -b & 2\end{bmatrix}$

$\Rightarrow(10a+3b)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}25a^2+b^2 & 15a-2b\\ 15a-2b & 13\end{bmatrix}$

$\Rightarrow 15a-2b=0$ and $10a+3b=13$

$\Rightarrow 5a+b=5\times\frac{2}{5}+3=5$

83.(b) $(p\wedge\sim q)\vee q\vee(\sim p\wedge q)$

$\equiv\{(p\vee q)\wedge(\sim q\vee q)\}\vee(\sim p\wedge q)$

$\equiv\{(p\vee q)\wedge T\}\vee(\sim p\wedge q)$

$\equiv(p\vee q)\vee(\sim p\wedge q)$

$\equiv\{(p\vee q)\vee\sim p\}\wedge(p\vee q\vee q)$

$\equiv T\wedge(p\vee q)$

$\equiv p\vee q$

84. (d) Either $x^2-5x+5=1$ or $x^2+4x-60=0$

$x=1,4$ or $x=-10,6$

Also $x^2-5x+5=-1$ and $x^2+4x-60 \in$ even number

$x=2,3$

For $x=3x^2+4x-60$ is odd

Total solutions are $x=1,4,-10,6,2$

$\Rightarrow$ Sum $=3$

85. (c) Let (h,k) be the centre of the circle which touch $x-axis$ and $x^2+y^2-8x-8y-4=0$ externally.

$\Rightarrow$ Radius of that circle is | k

$\Rightarrow (h-4)^2+(k-4)^2=(|k|+6)^2$

$\Rightarrow x^2-8x-20y-4=0$ if $y \geq 0$

and $x^2-8x+4y-4=0$ if $y<0$

$\Rightarrow$ The curve is parabola.

86. (c) Words starting with A, L, M $=\frac{4!}{2!}+4!+\frac{4!}{2!}=48$

Words starting with SA, SL $=\frac{3!}{2!}+3!=9$

$\Rightarrow$ Rank of the word SMALL $=58$.

87. (a)

$$\ln y=\lim_{n\to\infty}\frac{1}{n}\cdot\log\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{2n}{n}\right)$$
$$=\frac{1}{n}\cdot\sum_{r=1}^{2n}\ln\left(1+\frac{r}{n}\right)$$

$\ln y=\int_0^2\ln(1+x)dx$, let $t=1+x$

$$=\int_1^3\ln t dt$$
$$=\ln\frac{27}{e^2}$$

88. (a)

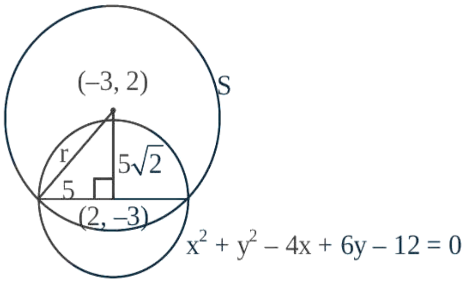
Let $S=\left(\frac{8}{5}\right)^2+\left(\frac{12}{5}\right)^2+\left(\frac{16}{5}\right)^2+\left(\frac{20}{5}\right)^2+\cdots..+\left(\frac{44}{5}\right)^2$

$\Rightarrow S=\frac{16}{25}[2^2+3^2+4^2+5^2 \ldots..+11^2]$

$\Rightarrow S = \frac{16}{25}[1^2 + 2^2 ... + 11^2 - 1] = \frac{16}{5} \times 101$

$\Rightarrow m = 101$

89. (a) Let 'r' be the radius of circle S $\Rightarrow r = 5\sqrt{3}$



90. (c) $\tan 60^\circ = \frac{h}{x} \Rightarrow h = \sqrt{3x}$

$\tan 30^\circ = \frac{h}{x+y} \Rightarrow \sqrt{3}h = x + y$

$3x = x + y$

$\Rightarrow 2x = y$

Time taken from A to B is 10 min

So time taken from B to pillar is 5 min