**1. (c)** $\frac{N_0 - N}{N} = 0.3$

$$\Rightarrow N = \frac{N_0}{1.3}$$

| | A | $\lambda$ | B |
|---|---|---|---|
| t = 0 | $N_0$ | | 0 |
| t = t | N | | $N_0 - N$ |

$$N = N_0 e^{-\lambda t}$$

$$\Rightarrow \frac{1}{1.3} = e^{-\lambda t}$$

$$t = t\,N$$

$$N_0 - N$$

$$\Rightarrow t = \frac{\ell n(1.3)}{\lambda} = T\frac{\ell n(1.3)}{\ell n(b)}$$

$$\because \lambda = \frac{\ell n 2}{T}.$$

**2. (c)** $T = \frac{rhg}{2} \times 10^3\ N/m$

$$\frac{\Delta T}{T} = \left|\frac{\Delta r}{r}\right| + \left|\frac{\Delta h}{h}\right| = \frac{0.01}{1.25} + \frac{0.01}{1.45}$$

$$\%\ \text{error} = \frac{\Delta T}{T} \times 100 = \frac{1}{1.25} + \frac{1}{1.45} = 0.8 + 0.69 \simeq 1.5\%$$

**3. (b)** $\frac{hc}{\lambda_{min}} = eV$

$$\log\frac{hc}{e} - \log\lambda_{min} = \log V$$

$$\Rightarrow \log\lambda_{min} = k - \log V$$

**4. (b)** $I = \frac{m}{12}[3R^2 + \ell^2]\left(R^2 = \frac{m}{\pi\ell\rho}\right)$

$$= \frac{m}{12}\left[\frac{3\,m}{\pi\rho}\ell^{-1} + \ell^2\right]$$

$$\frac{dI}{d\ell} = \frac{m}{12}\left[-\frac{3\,m}{\pi\rho\ell^2} + 2\ell\right]$$

For minima

$$0 = \frac{-3\,m}{\pi\rho\ell^2} + 2\ell$$

$$\Rightarrow \frac{3\pi R^2\ell\rho}{\pi\rho\ell^2} = 2\ell$$

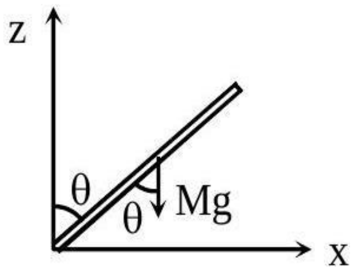
$\Rightarrow \frac{\ell}{R} = \sqrt{\frac{3}{2}}$

**5. (b)**

$\tau = I\alpha$

$\Rightarrow Mg\frac{\ell}{2}\sin\theta = \frac{M\ell^2}{3}\alpha$

$\Rightarrow \alpha = \frac{3}{2}\frac{g}{\ell}\sin\theta$




**6. (d)** For ideal gas

$C_P - C_v = R/M$

If $C_P$ and $C_v$ are specific heats $(J/kg - {}^0C)$

M = molar mass of gas

$\Rightarrow a = R/2$ and $b = R/28$

$\Rightarrow a = 14\, b$

**7. (c)** Final temperature of calorimeter and its contents is given, $T_0 = 75°C$

$\Rightarrow 100 \times 0.1 \times (75 - T) + 100 \times 0.1(75 - 30) + 170 \times 1 \times (75 - 30) = 0$

$\Rightarrow 75 - T + 45 + 765 = 0$

$\Rightarrow T = 885°C$

**8. (b)** Modulated signal can be written as

$C_m(t) = (A_c + A_m \sin\omega_m t)\sin\omega_c t$

$\Rightarrow C_m(t) = A_c \sin\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t$

where $\mu = \frac{A_m}{A_c}$

**9. (a)**

Using, $n = \left(\frac{PV}{RT}\right)$

$$n_f - n_i = \frac{PV}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right) \text{ moles}$$
$$= \frac{1 \times 10^5 \times 30}{8.32}\left(\frac{1}{300} - \frac{1}{290}\right) \times 6.023 \times 10^{23} \text{ molecules}$$
$$= -2.5 \times 10^{25} \text{ molecules}$$

**10. (c)** $y = \frac{m \times 650 \times 10^{-9} \times D}{d} = \frac{n \times 520 \times 10^{-9} \times D}{d}$

$\Rightarrow \frac{m}{n} = \frac{4}{5} \Rightarrow$ minimum values of m and n will be 4 and 5 respectively.

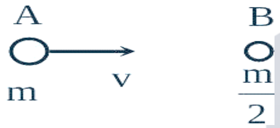
$y = \frac{4 \times 650 \times 10^{-9} \times 1.5}{5 \times 10^{-4}}$ meter

$= 7.8$ mm

**11. (c)**

$mv = mv_A + \frac{m}{2}v_B$ (conservation of linear momentum)

$\therefore v = v_A + \frac{v_B}{2} = v_B - v_A$ (elastic collision)




Before collision


After collision

$\therefore \frac{v_B}{v_A} = 4$

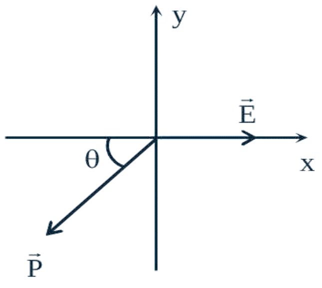
$\therefore \frac{\lambda_A}{\lambda_B} = \frac{m_B v_B}{m_A v_A} = 2$

**12. (b)**

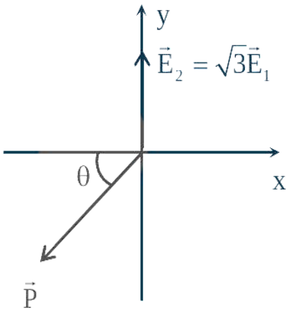
$$T = 2\pi\sqrt{\frac{I}{MB}}$$
$$= 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665\text{sec}$$

So, time of 10 oscillations $= 6.65$sec

**13. (d)** From the given information




Case - I




Case - II

$\therefore\ PE_1 \sin\theta = \sqrt{3} PE_1 \sin\left(\frac{\pi}{2} + \theta\right)$

$\therefore\ \theta = 60^\circ$

**14. (d)** Change in flux $= R\int idt = 250$ Wb
**15. (b)** From impulse momentum theorem

$\int_0^1 6tdt = mv$

$\therefore v = 3$ m/s

So, work done by the force $= \Delta K \cdot E. = \frac{1}{2}(a)(3)^2 = 4.5$ J

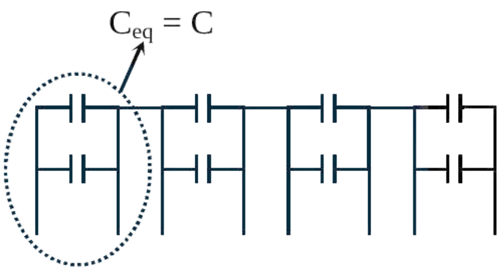
**16. (a)** $\Delta E \propto \frac{1}{\lambda}$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\Delta E_2}{\Delta E_1} = \frac{1}{3}$

**17. (a)** Potential difference across each resistor is zero.
**18. (d)** $v = u - gt$
**19. (a)** $\frac{C}{4} = 2 \Rightarrow C = 8\mu F$

Which requires eight $1\mu F$ capacitors in parallel.

$\Rightarrow$ Minimum number of capacitors required is 32.

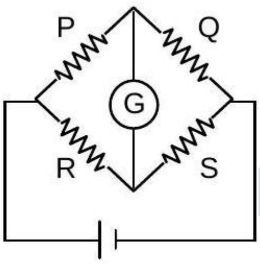



**20. (d)** $q = CV$

$$= \frac{CEr_2}{r + r_2}$$

**21. (a)** In common emitter amplifier circuit, the output voltage is out of phase w.r.t. input voltage.

**22. (c)** The balanced condition is given by $\frac{P}{Q} = \frac{R}{S}$; When battery and Galvanometer are exchanged, it become $\frac{P}{R} = \frac{Q}{S}$; which is same as previous




**23. (a)** For given SHM $x = A \sin \omega t$

$$v = \frac{dx}{dt} = A\omega \cos \omega t$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} mA^2\omega^2 \cos^2 \omega t = KE_{max}\left(\frac{1 + \cos 2\omega t}{2}\right)$$

**24. (d)**

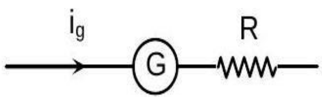
$$f' = \left(\sqrt{\frac{1+\beta}{1-\beta}}\right) f, \text{ where } \beta = \frac{v}{c}$$

So, $f' = 17.3 GHz$

**25. (b)** Stress $= \frac{F}{A} = \frac{mg}{A} = \frac{\rho \ell A g}{A} = \rho \ell g$

So, $\frac{\text{Stress}_f}{\text{Stress}_i} = 9$

**26. (b)** $i_g(R + R_g) = V$




$$R = \frac{V}{i_g} - R_g$$

$R = \frac{10}{5 \times 10^{-3}} - 15 = 1.985 \times 10^3 \Omega$

**27. (a)** The variation of magnitude of acceleration due to gravity is given by

$g = \left(\frac{GM}{R^3}\right) d$, where $0 \leq d \leq R$

$= \frac{GM}{d^2}$, where $d > R$

**28. (b)** By applying pressure, $\Delta P = -\frac{B\Delta V}{V}$

$\Rightarrow -\frac{\Delta V}{V} = \frac{\Delta P}{B} = \frac{P}{K}$ ( given $B = K$)

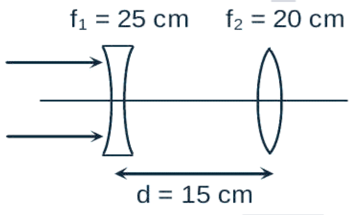
By increasing temperature, fractional increase in volume

$$-\frac{\Delta V}{V} = 3\alpha\Delta\theta$$

$$\frac{P}{K} = 3\alpha\Delta\theta$$

$$\Delta\theta = \frac{P}{3\alpha K}$$

**29. (b)** For diverging lens,




$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{\infty} = \frac{1}{-25}$$

$\Rightarrow v = -25$ cm

First image is formed at a distance 25 cm left to the diverging lens.

For the converging lens.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-40} = \frac{1}{20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

$\Rightarrow v = +40$ cm

**30. (d)** $\frac{1}{2}mv_f^2 = \frac{1}{8}mv_0^2 \Rightarrow v_f = \frac{v_0}{2}$

Now, $\frac{mdv}{dt} = -kv^2$

$$\Rightarrow m\int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2} = -k\int_0^{10} dt$$

$$\Rightarrow m\left[-\frac{1}{v}\right]_{v_0}^{\frac{v_0}{2}} = -k[t]_0^{10}$$

$$\Rightarrow m\left(\frac{2}{v_0} - \frac{1}{v_0}\right) = 10k$$

$$\Rightarrow \frac{m}{v_0} = 10k$$

$$\Rightarrow k = \frac{m}{10v_0} = \frac{10^{-2}}{10 \times 10} = 10^{-4}\ \text{kg m}^{-1}$$

**31. (a)** $M_2CO_3 + 2HCl \longrightarrow CO_2 + 2MCl + H_2O$

Moles of $M_2CO_3$ = Moles of $CO_2$ produced.

moles of $M_2CO_3 = \frac{w}{\text{molar mass}} = 0.01186$

$\therefore$ Molar mass $= 84.3\ \text{g mol}^{-1}$

**32. (b)**

$$C_{(\text{graphite})} + O_2(g) \longrightarrow CO_2(g);\ \Delta_r H^o = -393.5\ \text{kJ/mol}^{-1}$$
$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\ell);\ \Delta_r H^\circ = -285.8\ \text{kJ/mol}^{-1}$$
$$CO_2(g) + 2H_2O(\ell) \longrightarrow CH_4(g) + 2O_2(g);\ \Delta H_r^o = +890.3\ \text{kJ/mol}^{-1}$$

$C_{(\text{graphite})} + 2H_2(g) \longrightarrow CH_4(g); \Delta H = ?$

[Eq. (a) + Eq. (c) ] + [2 × Eq. (b) ] = Eq. (d)

$\therefore [\Delta H_1 + \Delta H_3] + [2 \times \Delta H_2] = \Delta H_4$

$[(-393.5) + (890.3)] + [2(-285.8)] = -74.8$ kJ/mol

$= -74.8\ \text{kJ/mol}^{-1}$

**33. (c)**

$$\Delta T_f = i \times K_f \times m$$
$$\Rightarrow 0.45 = i \times 5.12 \times \frac{0.2 \times 1000}{60 \times 20}$$

$i = 0.527$

$2CH_3COOH \rightleftharpoons (CH_3COOH)_2$

$1 - \alpha \quad \frac{\alpha}{2}$

$$i = 1 - \alpha + \frac{\alpha}{2}$$

$\alpha = 0.946$

$\therefore$ % dissociation is 94.6%

**34. (b)** Total hydrogen $\left({}_1H^1\right) = \frac{10}{100} \times 75 = 7.5$ kg

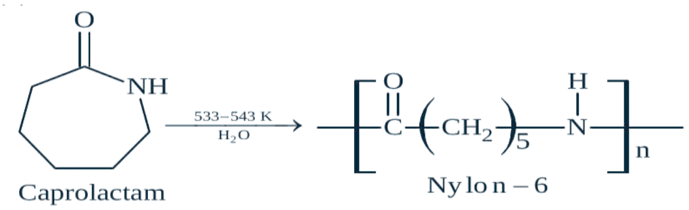
If it is replaced by ${}_1H^2$ then mass will be doubled so now hydrogen mass $= 15$ kg So, mass of person will be increased by 7.5 kg.

**35. (b)** $\Delta U = q + w$

$q = 0$ in adiabatic process.

So, $\Delta U = W$

**36. (d)**




**37. (d)** Reduction potential of

$E^0_{Cr^{3+}/Cr} = -0.74$ V

So, $E_{Cr/Cr^{3+}} = +0.74$ V

$\therefore$ Cr would be strongest reducing agent.

**38. (a)** Tyndall effect is observed only when

(i) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

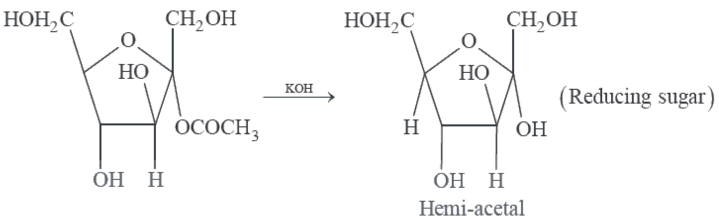
(ii) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude. So, (b) and (d) are correct.

**39. (c)**

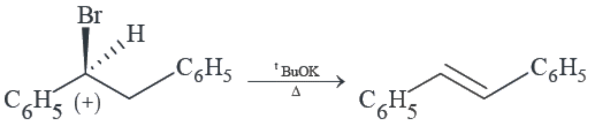
$ZnO + Na_2O \longrightarrow Na_2ZnO_2$; ZnO behaving as an acid.

$ZnO + CO_2 \longrightarrow ZnCO_3$; ZnO behaving as a base.

**40. (d)**




**41. (a)**



Elimination major product

**42. (a)** (14) CO - diamagnetic

(16) $O_2$ - paramagnetic

(10) $B_2$ - paramagnetic

(15) NO - paramagnetic

**43. (d)** Moles of $CoCl_3 \cdot 6H_2O \longrightarrow 100 \text{ mL} \times 0.1\text{M} = 10 \times 10^{-3}$ moles

Ions $\rightarrow 6.023 \times 10^{23} \times 0.01$

$= 6.023 \times 10^{21}$ ions

Precipitated ions $= 1.2 \times 10^{22}$

$\therefore$ $1Ag^+$ion and $1Cl^-$ion.

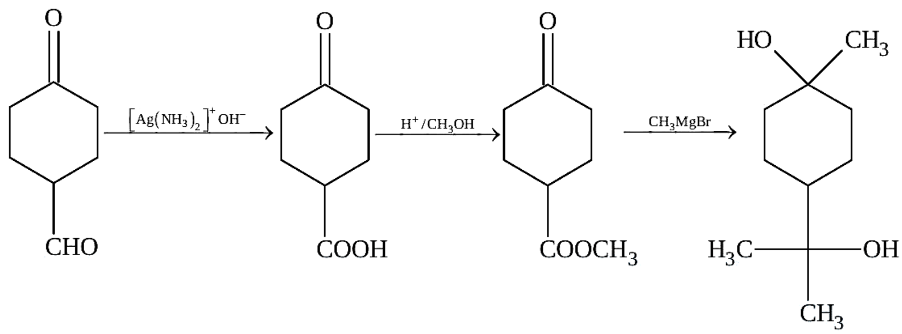
**44. (a)**

$$\begin{aligned} pH &= 7 + \frac{pK_a}{2} - \frac{pK_b}{2} \\ &= 7 + \frac{3.2}{2} - \frac{3.4}{2} \\ &= 6.9 \end{aligned}$$

**45. (a)** Rate of $S_N1 \propto$ carbocation stability
$\therefore II < I < III$

**46. (a)**

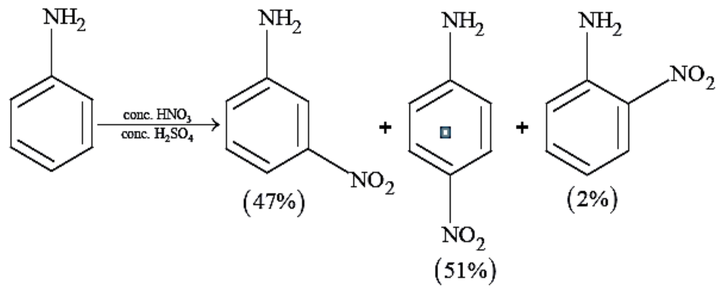
**47. (d)**

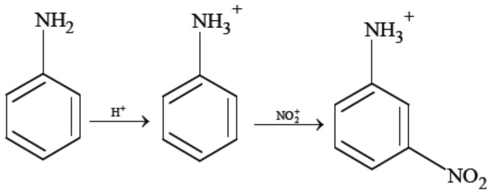



**48. (b)**

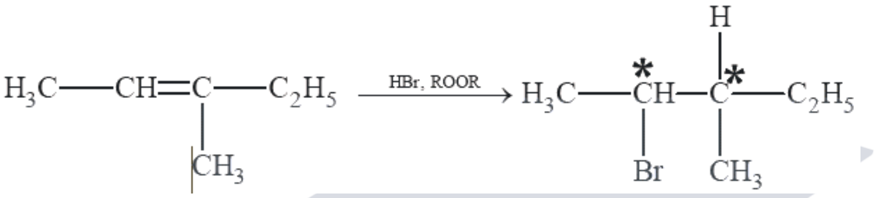
$$Cl_2 + \underset{\text{(cold \& dilute)}}{2NaOH} \longrightarrow NaCl + NaOCl + H_2O$$

**49. (b)**





**50. (c)**



Stereoisomers $= 2^n = 2^2 = 4$

**51. (c)** $k_1 = Ae^{-E_{a1}/RT}$

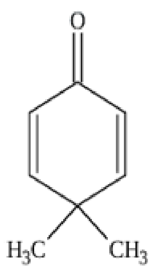
$k_2 = Ae^{-E_{a_2}/RT}$

$$\frac{k_2}{k_1} = e^{-\frac{(E_{a_2}-E_{a_1})}{RT}}$$

$$\frac{k_2}{k_1} = e^{+\frac{10\times10^3}{8.314\times300}}$$

$$\ln\frac{k_2}{k_1} = 4$$

**52. (c)**



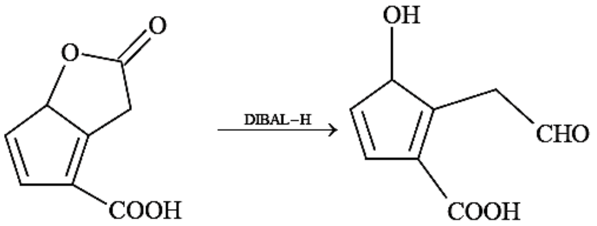
Except all are aromatic in nature, so it has least resonance energy.

**53. (d)** $O^{2-}, F^-, Na^+$ and $Mg^{2+}$, all have 10 electrons each.

**54. (c)**

$$r_n = \frac{0.53n^2}{Z}\text{Å}$$
$n = 2$
$Z = 1$
$r_2 = 0.53 \times 4\text{ÅA} = 2.12\text{Å}$

**55. (d)**




**56. (d)**

$$\overset{+4}{X}eF_4 + \overset{+1}{O_2}F_2 \longrightarrow \overset{+6}{Xe}FF_6 + \overset{0}{O_2}$$

Xenon oxidizes and oxygen gets reduced.

**57. (c)** In FCC structure

$4r = \sqrt{2}a$

$2r = \frac{a}{\sqrt{2}}$ = closest approach between two atoms.

**58. (c)**

$$(X)\,Na_2C_2O_4 \xrightarrow[H_2SO_4]{conc.} CO_2\uparrow + CO\uparrow + Na_2SO_4$$

$$\Big\downarrow CaCl_2$$

$$\underset{\text{white ppt.}}{CaC_2O_4\downarrow} \xrightarrow[KMnO_4]{acidic} CO_2\uparrow$$

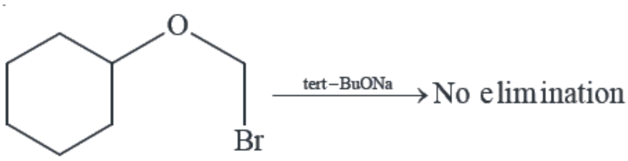
**59. (b)**

Permissible limit for $SO_4{}^{2-} = 500\text{ppm}$

Permissible limit for $NO_3{}^{-} = 50\text{ppm}$

Permissible limit for $F^{-} = 1\text{ppm}$

**60. (d)**




**61. (d)**

Here, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0 \Rightarrow a = 1$

For $a = 1$, the equations become

$x + y + z = 1$

$x + y + z = 1$

$x + by + z = 0$

These equations give no solution for $b = 1$

$\Rightarrow$ S is singleton set

**62. (a)**

$$\begin{aligned} &(p \to q) \to ((\sim p \to q) \to q) \\ &= (p \to q) \to ((p \vee q) \to q) \\ &= (p \to q) \to ((\sim p \wedge \sim q) \vee q) \\ &= (p \to q) \to ((\sim p \vee q) \wedge (\sim q \vee q)) \\ &= (p \to q) \to (\sim p \vee q) \\ &= (p \to q) \to (p \to q) \\ &= T \end{aligned}$$

**63. (d)**

$5(\sec^2 x - 1 - \cos^2 x) = 2(2\cos^2 x - 1) + 9$

$5\left(\frac{1}{t} - 1 - t\right) = 2(2t - 1) + 9$

$5(1 - t - t^2) = 4t^2 + 7t$

$\Rightarrow 9t^2 + 12t - 5 = 0$

$t = \frac{1}{3}, -\frac{5}{3}$

$\Rightarrow \cos^2 x = \frac{1}{3}$

$\Rightarrow \cos 2x = \frac{2}{3} - 1 = -\frac{1}{3}$

$\Rightarrow \cos 4x = -\frac{7}{9}$

**64. (b)**

$P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$

$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

$\mathrm{P(A) + P(C) - 2P(A \cap C)} = \frac{1}{4}$

$\Rightarrow \sum \mathrm{P(A)} - \sum \mathrm{P(A \cap B)} = \frac{3}{8}$

$\Rightarrow \mathrm{P(A \cup B \cup C)} = \sum \mathrm{P(A)} - \sum \mathrm{P(A \cap B)} + \mathrm{P(A \cap B \cap C)}$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}.$

**65. (a)** Determinant simplifies to $3k = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$

$= 3\begin{vmatrix} 1 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$

$= -3\mathrm{z}$

$\Rightarrow \mathrm{k} = -\mathrm{z}$

**66. (d)** $\Delta = \frac{1}{2}\begin{vmatrix} \mathrm{k} & -3\mathrm{k} & 1 \\ 5 & \mathrm{k} & 1 \\ -\mathrm{k} & 2 & 1 \end{vmatrix} = 28$

$\Rightarrow \mathrm{k} = 2$ (since $\mathrm{k} \in \mathrm{I}$)

$\Rightarrow$ Orthocentre is $\left(2, \frac{1}{2}\right)$

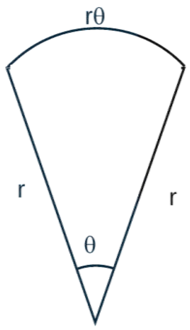
**67. (c)** Length of wire $= r(\theta + 2) = 20$ m

Area $A = \frac{\theta}{2} r^2$

$\Rightarrow \mathrm{A(r)} = 10\mathrm{r} - \mathrm{r}^2$

$\Rightarrow$ Area is maximum if $r = 5$.

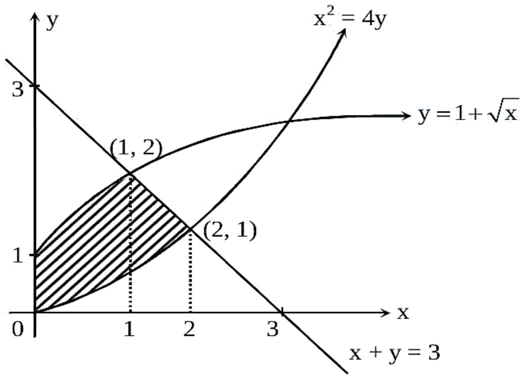
Maximum area $\mathrm{A} = 25$ sq. m




**68. (d)** Required area

$= \int_0^1 (1 + \sqrt{x})dx + \frac{1}{2}(3 \times 1) - \int_0^2 \frac{x^2}{4} dx$

$= \frac{5}{2}$ sq. units




**69. (b)** $\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda$

Let midpoint of PQ be M which lines on the plane

$\Rightarrow M(x, y, z) = (1+\lambda, 4\lambda - 2, 5\lambda + 3)$

$2(1+\lambda) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

$\Rightarrow -6\lambda + 6 = 0 \Rightarrow \lambda = 1$

$\Rightarrow M(2,2,8), P(1,-2,3)$

$PM = \sqrt{1+16+25} = \sqrt{42}$

$PQ = 2\sqrt{42}$.

**70. (a)** Here, $y = 2\tan^{-1} 3x^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{9x^{\frac{1}{2}}}{1+9x^3} = \sqrt{x}g(x)$$

$$\Rightarrow g(x) = \frac{9}{1+9x^3}$$

**71. (a)**

$$(2+\sin x)dy + \cos x(y+1)dx = 0$$
$$(y+1)(2+\sin x) = C$$
$$\Rightarrow (1+1)(2+0) = C = 4$$
$$(y+1)\cdot(2+\sin x) = 4$$

Put $x = \frac{\pi}{2}$

$$y = \frac{1}{3}$$

**72. (c)**

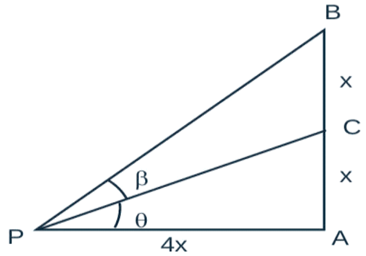
$$\tan(\theta + \beta) = \frac{1}{2}$$

and $\tan\theta = \frac{1}{4}$

$$\Rightarrow \frac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}\tan\beta} = \frac{1}{2}$$

$\Rightarrow 9\tan\beta = 2$

$\Rightarrow \tan\beta = \frac{2}{9}$




**73. (b)**

$A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$(3A^2 + 12A) = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$\text{Adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$.

**74. (b)**

$(15a - 3b)^2 + (15a - 5c)^2 + (3b - 5c)^2 = 0$

Let, $15a = 3b = 5c = 45\lambda$

$\Rightarrow a = 3\lambda; b = 15\lambda; c = 9\lambda$

$\Rightarrow 2c = a + b$

b, c, a are in A.P.

**75. (b)**

Equation of plane is $\begin{vmatrix} x-1 & y+1 & z+1 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 0$

$5x + 7y + 3z + 5 = 0$

Distance from $(1,3,-7) = \frac{|5+21-21+5|}{\sqrt{83}} = \frac{10}{\sqrt{83}}$

**76. (b)**

$$I_n = \int \tan^n x dx, n > 1$$

$$= \int \tan^{n-2} x(\sec^2 x - 1)dx$$

$$= \frac{\tan^{n-1} x}{n-1} - I_{n-2} + C$$

$$\Rightarrow I_n + I_{n-2} = \frac{\tan^{n-1} x}{n-1} + C$$

$$\Rightarrow I_6 + I_4 = \frac{\tan^5 x}{5} + C$$

Given, $I_4 + I_6 = a\tan^5 x + bx^3 + C$

$\Rightarrow a = \frac{1}{5}, b = 0$

**77. (b)** Eccentricity, $e = \frac{1}{2}$

Let 2a be the length of major axis and 2 b be the length of minor axis

$\Rightarrow \frac{a}{e} = 4$

$\Rightarrow a = 2$

Also, $b = \sqrt{3}$, as $e = \frac{1}{2}$

$\Rightarrow$ Equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow$ Equation of normal at $\left(1, \frac{3}{2}\right)$ is $4x - 2y = 1$

**78. (b)**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{4-a^2} = 1$$

$\Rightarrow a^2 = 8,1, (a^2 \neq 8)$

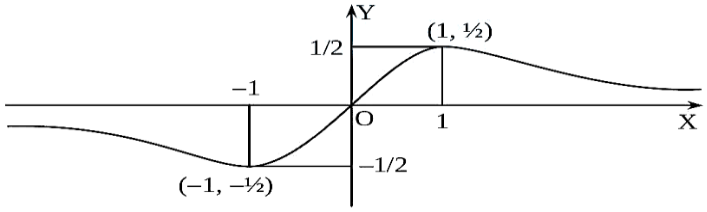
$\Rightarrow \frac{x^2}{1} - \frac{y^2}{3} = 1.$

Hence equation of tangent at $P(\sqrt{2}, \sqrt{3})$ is $\frac{x\sqrt{2}}{1} - \frac{y\sqrt{3}}{3} = 1$

$\Rightarrow \sqrt{6}x - y = \sqrt{3}$

**79. (c)**

For, $f(x) = \frac{x}{1+x^2}$ the curve has graph as shown




Which is onto but not one-one for, $f: R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$

**80. (b)**

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos x - \cot x}{8\left(x - \frac{\pi}{2}\right)^3}$$

Put $x - \frac{\pi}{2} = t; x = t + \frac{\pi}{2}$

$$\lim_{t \to 0} \frac{1}{8} \cdot \frac{-\sin t + \tan t}{t^3}$$

$$\lim_{t \to 0} \frac{1}{8} \cdot \frac{\sin t(1 - \cos t)}{t \cdot \cos t \cdot t^2}$$

$$= \frac{1}{8} \cdot 1 \cdot \frac{1}{2} = \frac{1}{16}$$

**81. (b)** $|\vec{c}|^2 + |\vec{\mathbf{a}}|^2 - 2\vec{a} \cdot \vec{c} = 9$

$\Rightarrow |\vec{c}|^2 - 2\vec{a} \cdot \vec{c} = 0$

and $|\vec{a} \times \vec{b}| \cdot |\vec{c}| \cdot \sin 30^\circ = 3$

$\Rightarrow 3 \times |\vec{c}| \times \frac{1}{2} = 3$

$\Rightarrow |\vec{c}| = 2$

$\therefore \vec{a} \cdot \vec{c} = 2$

**82. (b)** $\frac{dy}{dx}$ at $x = 0$ is 1

$\Rightarrow$ Slope of normal at $(0,1)$ is -1

$\Rightarrow$ Equation of normal is $x + y = 1$

**83. (a)** Consider two sequences: 0,4,8 and 2, 6, 10

Take both numbers from either of these sequences.

Hence, probability $= \frac{{}^3C_2 + {}^3C_2}{{}^{11}C_2} = \frac{6}{55}$.

**84. (a)** X : 4L, 3M; Y : 3L, 4M

Possible combinations

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| X | 3 L | 2L, 1M | 1L, 2M | 3M |
| Y | 3M | 1 L, 2M | 2L, 1M | 3 L |

$\therefore$ Number of ways $= {}^4C_3 \cdot {}^4C_3 + {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 + {}^3C_3 \cdot {}^3C_3$

$= 485$

**85. (d)**

Let $S = ({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + \cdots + ({}^{21}C_{10} - {}^{10}C_{10})$

$\Rightarrow S = ({}^{21}C_0 + {}^{21}C_1 + \cdots + {}^{21}C_{10}) - ({}^{10}C_0 + {}^{10}C_1 + \cdots + {}^{10}C_{10})$

$\Rightarrow S = 2^{20} - 2^{10}$.

**86. (a)** $p = \frac{15}{25}, q = \frac{10}{25}, n = 10$

$\sigma^2 = npq = 10 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{5}$.

**87. (a)** Partially differentiating, we get

$f'(x) - x = \text{constant} = \lambda$

$\Rightarrow f(x) = \dfrac{x^2}{2} + \lambda x + k$

$f(0) = 0 \Rightarrow k = 0$

$\dfrac{1}{2} + \lambda = 3 \Rightarrow \lambda = \dfrac{5}{2}$

$\sum_{n=1}^{10} f(n) = a\sum_{n=1}^{10} n^2 + b\sum_{n=1}^{10} n$

$= \dfrac{1}{2}\dfrac{n(n+1)(2n+1)}{6} + \dfrac{5}{2} \times \dfrac{n(n+1)}{2}$

$= \frac{1}{2} \cdot \frac{10 \times 11 \times 21}{6} + \frac{5}{2} \times \frac{10 \times 11}{2} = 330.$

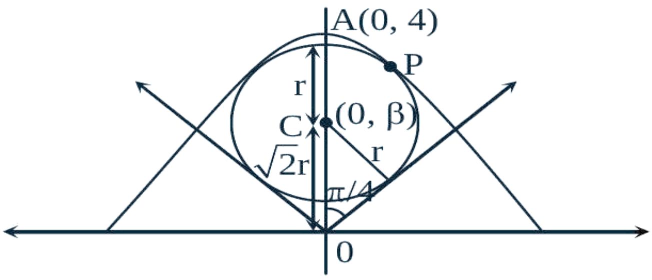
**88. (c)**

Let $P\left(\frac{t}{2}, 4 - \frac{t^2}{4}\right)$ be point where circle touches the parabola $y = 4 - x^2$

$\Rightarrow$ Normal at P: $ty - x + \frac{t^3}{4} - \frac{7t}{2} = 0$ to the parabola passes through centre (c) of the circle $(0, \beta)$.

$\Rightarrow t^3 - 14t + 4\beta t = 0$

Also, radius $r = \frac{|\beta|}{\sqrt{2}}$




$\Rightarrow t^4 + 4t^2 + 8\beta t^2 - 32t^2 - 128\beta + 256 + 16\beta^2 = 16r^2$

$\Rightarrow \mathrm{t}^4 + (8\beta - 28)\mathrm{t}^2 - 128\beta + 256 + 8\beta^2 = 0$

From equation (a) and (b), we get

Either $\beta = 8 \pm 4\sqrt{2}$ for $\mathrm{t} = 0$

or $\beta = \frac{-\sqrt{2}\pm\sqrt{17}}{\sqrt{2}}$ for $\mathrm{t}^2 = 14 - 4\beta$

As, $r = \frac{|\beta|}{\sqrt{2}} \Rightarrow r = 4\sqrt{2} \pm 4, \frac{\sqrt{17}-\sqrt{2}}{2}$

$\Rightarrow$ Minimum possible radius, $\mathrm{r} = \frac{\sqrt{17}-\sqrt{2}}{2}$

[But of the given options $\mathbf{r} = 4(\sqrt{2} - 1)$ is minimum]

**89. (d)** $\mathrm{x}^2 + \mathrm{nx} + \frac{\mathrm{n}^2-31}{3} = 0$

Let I and I + 1 be the roots of the equation

$2\mathrm{I} + 1 = -\mathrm{n}$

$$I(I+1) = \frac{n^2 - 31}{3}$$

Eliminating I from (1) and (2), we get

$$\frac{\mathrm{n}^2 - 1}{4} = \frac{\mathrm{n}^2 - 31}{3}$$

$\Rightarrow \mathrm{n}^2 = 121$

$\Rightarrow \mathrm{n} = 11.$

**90. (b)**

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\operatorname{cosec}^2 x - \operatorname{cosec} x \cdot \cot x) dx$$
$$= 2$$