

PART A - PHYSICS

1. A radioactive nucleus A with a half life T , decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime t , the ratio of the number of B to that of A is 0.3. Then, t is given by :

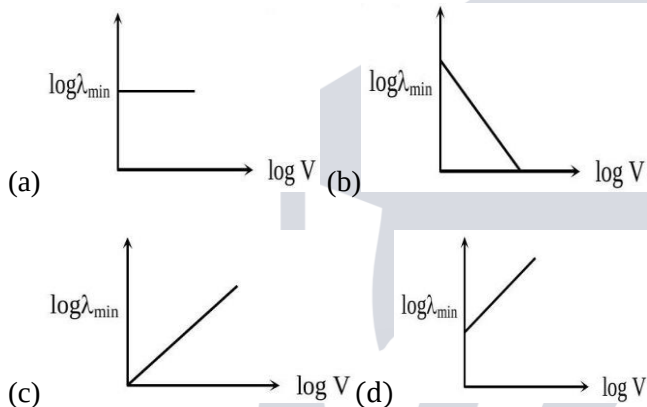
(a) $t = \frac{T}{\log(1.3)}$ (b) $t = \frac{T \log 2}{2 \log 1.3}$
 (c) $t = T \frac{\log 1.3}{\log 2}$ (d) $t = T \log(1.3)$

2. The following observations were taken for determining surface tension T of water by capillary method: diameter of capillary, $D = 1.25 \times 10^{-2}$ m rise of water, $h = 1.45 \times 10^{-2}$ m

Using $g = 9.80 \text{ m/s}^2$ and the simplified relation $T = \frac{r h g}{2} \times 10^3 \text{ N/m}$, the possible error in surface tension is closest to :

- (a) 10% (b) 0.15%
 (c) 1.5% (d) 2.4%

3. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If $\lambda_{\min}$ is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log \lambda_{\min}$ with $\log V$ is correctly represented in:

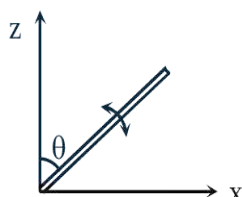


4. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I .

What is the ratio ℓ/R such that the moment of inertia is minimum?

- (a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{\frac{3}{2}}$
 (c) $\frac{\sqrt{3}}{2}$ (d) 1

5. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is:



- (a) $\frac{2g}{3\ell} \cos \theta$ (b) $\frac{3g}{2\ell} \sin \theta$
 (c) $\frac{2g}{3\ell} \sin \theta$ (d) $\frac{3g}{2\ell} \cos \theta$

6. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$$C_p - C_v = a \text{ for hydrogen gas}$$

$$C_p - C_v = b \text{ for nitrogen gas}$$

The correct relation between a and b is

- (a) $a = 28b$ (b) $a = \frac{1}{14}b$
(c) $a = b$ (d) $a = 14b$

7. A copper ball of mass 100gm is at a temperature T . It is dropped in a copper calorimeter of mass 100gm, filled with 170gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C . T is given by: (Given : room temperature = 30°C , specific heat of copper = $0.1\text{ cal/gm}^\circ\text{C}$)

- (a) 825°C (b) 800°C
(c) 885°C (d) 1250°C

8. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m . The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m \ll \omega_c$. Which of the following frequencies is not contained in the modulated wave?

- (a) $\omega_c - \omega_m$ (b) ω_m
(c) ω_c (d) $\omega_m + \omega_c$

9. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains $1 \times 10^5\text{ Pa}$. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be :

- (a) -2.5×10^{25} (b) -1.61×10^{23}
(c) 1.38×10^{23} (d) 2.5×10^{25}

10. In a Young's double slit experiment, slits are separated by 0.5 mm , and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm , is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is

- (a) 15.6 mm (b) 1.56 mm
(c) 7.8 mm (d) 9.75 mm

11. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is:

- (a) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$
(c) $\frac{\lambda_A}{\lambda_B} = 2$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$

12. A magnetic needle of magnetic moment $6.7 \times 10^{-2}\text{ Am}^2$ and moment of inertia $7.5 \times 10^{-6}\text{ kg m}^2$ is performing simple harmonic oscillations in a magnetic field of 0.01 T . Time taken for 10 complete oscillations is:

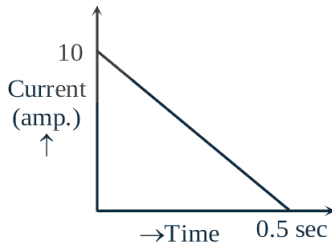
- (a) 8.76 s (b) 6.65 s
(c) 8.89 s (d) 6.98 s

13. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x -axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is:

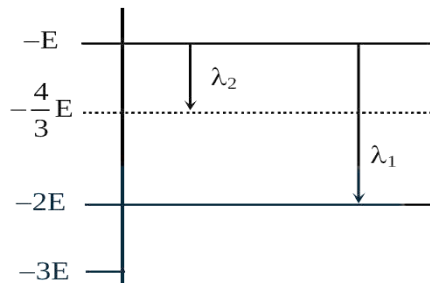
- (a) 90° (b) 30°
(c) 45° (d) 60°

14. In a coil of resistance 100Ω , a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is:

- (a) 275 Wb (b) 200 Wb
(c) 225 Wb (d) 250 Wb

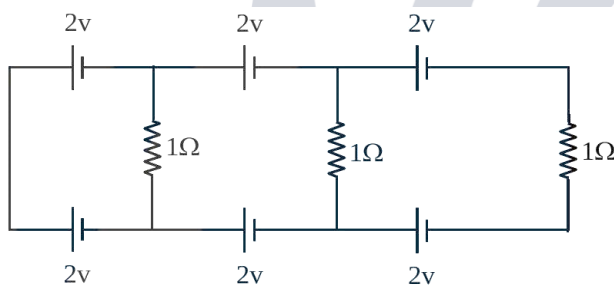


15. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be:
- (a) 18 J (b) 4.5 J
(c) 22 J (d) 9 J
16. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by:
- (a) $r = \frac{1}{3}$ (b) $r = \frac{4}{3}$
(c) $r = \frac{2}{3}$ (d) $r = \frac{3}{4}$

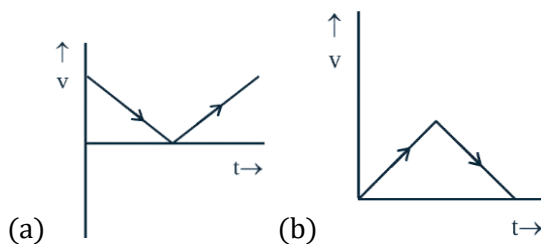


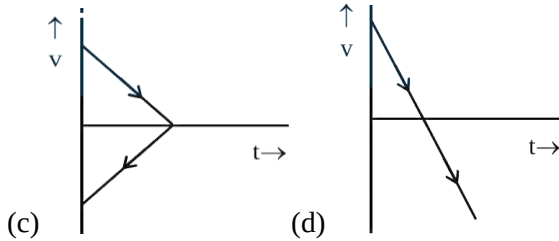
17. In the given circuit, the current in each resistance is:

- (a) 0 A (b) 1 A
(c) 0.25 A (d) 0.5 A



18. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?



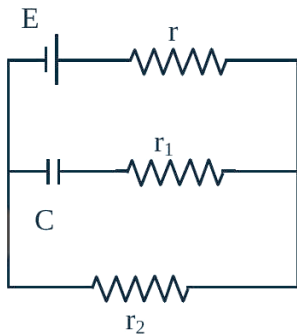


19. A capacitance of $2\mu\text{F}$ is required in an electrical circuit across a potential difference of 1.0kV . A large number of $1\mu\text{F}$ capacitors are available which can withstand a potential difference of not more than 300 V . The minimum number of capacitors required to achieve this is:

- (a) 32 (b) 2
(c) 16 (d) 24

20. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be:

- (a) $CE \frac{r_1}{(r_1+r)}$ (b) CE
(c) $CE \frac{r_1}{(r_2+r)}$ (d) $CE \frac{r_2}{(r+r_2)}$



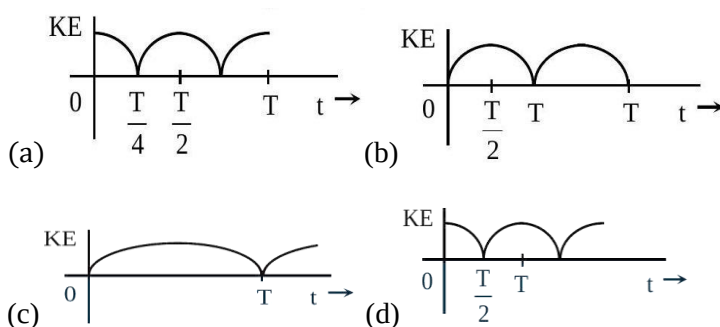
21. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be:

- (a) 180° (b) 45°
(c) 90° (d) 135°

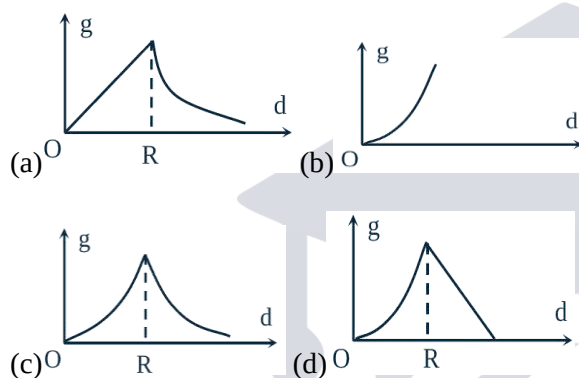
22. Which of the following statements is false?

- (a) Kirchhoff's second law represents energy conservation.
(b) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.
(c) In a balanced wheat-stone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.
(d) A rheostat can be used as a potential divider.

23. A particle is executing simple harmonic motion with a time period T . At time $t = 0$, it is at its position of equilibrium. The kinetic energy - time graph of the particle will look like:



24. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10GHz. What is the frequency of the microwave measured by the observer? (speed of light = $3 \times 10^8 \text{ ms}^{-1}$)
- (a) 15.3GHz (b) 10.1GHz
(c) 12.1GHz (d) 17.3GHz
25. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of:
- (a) $\frac{1}{81}$ (b) 9
(c) $\frac{1}{9}$ (d) 81
26. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω , it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10 V is:
- (a) $4.005 \times 10^3\Omega$ (b) $1.985 \times 10^3\Omega$
(c) $2.045 \times 10^3\Omega$ (d) $2.535 \times 10^3\Omega$
27. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius):



28. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion? Suppose we want to bring the cube to its original size by heating. The temperature should be raised by:
- (a) $3PK\alpha$ (b) $\frac{P}{3\alpha K}$
(c) $\frac{P}{\alpha K}$ (d) $\frac{3\alpha}{PK}$
29. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is:
- (a) real and at a distance of 6 cm from the convergent lens.
(b) real and at a distance of 40 cm from convergent lens.
(c) virtual and at a distance of 40 cm from convergent lens.
(d) real and at a distance of 40 cm from the divergent lens.
30. A body of mass $m = 10^{-2} \text{ kg}$ is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10 \text{ ms}^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be:
- (a) $10^{-1} \text{ kg m}^{-1} \text{ s}^{-1}$ (b) $10^{-3} \text{ kg m}^{-1}$
(c) $10^{-3} \text{ kg s}^{-1}$ (d) $10^{-4} \text{ kg m}^{-1}$

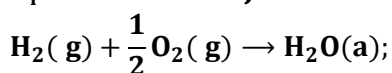
PART B – CHEMISTRY

31. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2 . The molar mass of M_2CO_3 in $g\text{mol}^{-1}$ is:

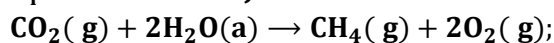
- (a) 84.3 (b) 118.6
(c) 11.86 (d) 1186

32. Given $C_{(\text{graphite})} + O_2(g) \rightarrow CO_2(g)$;

$$\Delta_r H^0 = -393.5 \text{ kJ mol}^{-1}$$



$$\Delta_r H^0 = -285.8 \text{ kJ mol}^{-1}$$



$$\Delta_r H^0 = +890.3 \text{ kJ mol}^{-1}$$

Based on the above thermochemical equations, the value of $\Delta_r H^0$ at 298 K for the reaction $C_{(\text{graphite})} + 2H_2(g) \rightarrow CH_4(g)$ will be:

- (a) +144.0 kJ mol^{-1} (b) -74.8 kJ mol^{-1}
(c) -144.0 kJ mol^{-1} (d) +74.8 kJ mol^{-1}

33. The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be:

(K_f for benzene = $5.12 \text{ K kg mol}^{-1}$)

- (a) 80.4% (b) 74.6%
(c) 94.6% (d) 64.6%

34. The most abundant elements by mass in the body of a healthy human adult are: Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is:

- (a) 37.5 kg (b) 7.5 kg
(c) 10 kg (d) 15 kg

35. ΔU is equal to

- (a) Isobaric work (b) Adiabatic work
(c) Isothermal work (d) Isochoric work

36. The formation of which of the following polymers involves hydrolysis reaction?

- (a) Bakelite (b) Nylon 6, 6
(c) Terylene (d) Nylon 6

37. Given

$$E_{Cl_2/Cl^-}^0 = 1.36 \text{ V}, E_{Cr^{3+}/Cr}^0 = -0.74 \text{ V}$$

$$E_{Cr_2O_7^{2-}/Cr^{3+}}^0 = 1.33 \text{ V}, E_{MnO_4^-/Mn^{2+}}^0 = 1.51 \text{ V}.$$

Among the following, the strongest reducing agent is

- (a) Mn^{2+} (b) Cr^{3+}
(c) Cl^- (d) Cr

38. The Tyndall effect is observed only when following conditions are satisfied:

- (a) The diameter of the dispersed particles is much smaller than the wavelength of the light used.
(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(a) (b) and (d) (b) (a) and (c)

(c) (b) and (c) (d) (a) and (d)

39. In the following reactions, ZnO is respectively acting as a/an:

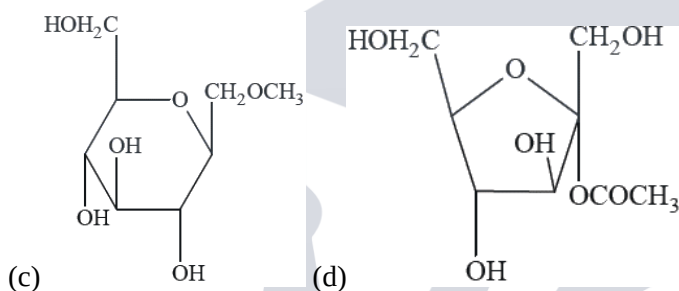
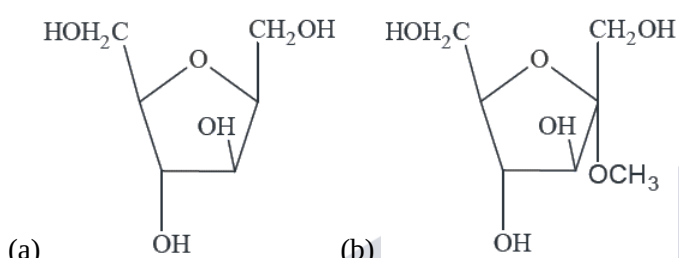
(a) $\text{ZnO} + \text{Na}_2\text{O} \rightarrow \text{Na}_2\text{ZnO}_2$

(b) $\text{ZnO} + \text{CO}_2 \rightarrow \text{ZnCO}_3$

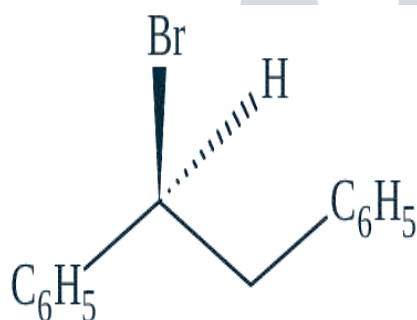
(a) base and base (b) acid and acid

(c) acid and base (d) base and acid

40. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?



41. The major product obtained in the following reaction is:



(+)

(a) $\text{C}_6\text{H}_5\text{CH} = \text{CHC}_6\text{H}_5$

(b) $(+)\text{C}_6\text{H}_5\text{CH}(\text{O}^t\text{Bu})\text{CH}_2\text{C}_6\text{H}_5$

(c) $(-)\text{C}_6\text{H}_5\text{CH}(\text{O}^t\text{Bu})\text{CH}_2\text{C}_6\text{H}_5$

(d) $(\pm)\text{C}_6\text{H}_5\text{CH}(\text{O}^t\text{Bu})\text{CH}_2\text{C}_6\text{H}_5$

42. Which of the following species is not paramagnetic?

(a) CO

(b) O_2

(c) B_2

(d) NO

43. On treatment of 100 mL of 0.1M solution of $\text{CoCl}_3 \cdot 6\text{H}_2\text{O}$ with excess AgNO_3 ; 1.2×10^{22} ions are precipitated. The complex is:

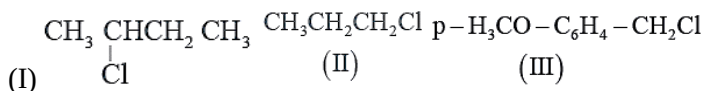
(a) $[\text{Co}(\text{H}_2\text{O})_3\text{Cl}_3] \cdot 3\text{H}_2\text{O}$

- (b) $[\text{Co}(\text{H}_2\text{O})_6]\text{Cl}_3$
 (c) $[\text{Co}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2 \cdot \text{H}_2\text{O}$
 (d) $[\text{Co}(\text{H}_2\text{O})_4\text{Cl}_2]\text{Cl} \cdot 2\text{H}_2\text{O}$

44. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is:

- (a) 6.9 (b) 7.0
 (c) 1.0 (d) 7.2

45. The increasing order of the reactivity of the following halides for the $\text{S}_{\text{N}}1$ reaction is:

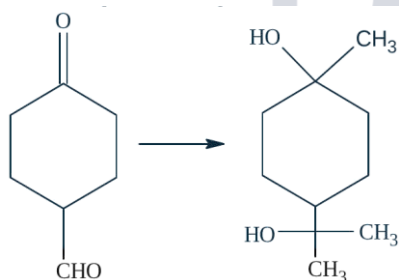


- (a) (II) < (I) < (III) (b) (I) < (III) < (II)
 (c) (II) < (III) < (I) (d) (III) < (II) < (I)

46. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect, is:

- (a) both form soluble bicarbonates
 (b) both form nitrides
 (c) nitrates of both Li and Mg yield NO_2 and O_2 on heating
 (d) both form basic carbonates

47. The correct sequence of reagents for the following conversion will be:

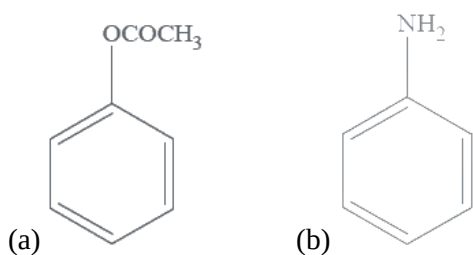


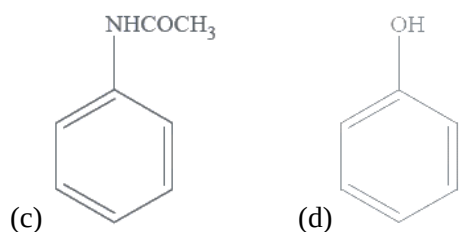
- (a) CH_3MgBr , $\text{H}^+/\text{CH}_3\text{OH}$, $[\text{Ag}(\text{NH}_3)_2]^+\text{OH}^-$
 (b) CH_3MgBr , $[\text{Ag}(\text{NH}_3)_2]^+\text{OH}^-$, $\text{H}^+/\text{CH}_3\text{OH}$
 (c) $[\text{Ag}(\text{NH}_3)_2]^+\text{OH}^-$, CH_3MgBr , $\text{H}^+/\text{CH}_3\text{OH}$
 (d) $[\text{Ag}(\text{NH}_3)_2]^+\text{OH}^-$, $\text{H}^+/\text{CH}_3\text{OH}$, CH_3MgBr

48. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are:

- (a) ClO_2^- and ClO_3^- (b) Cl^- and ClO^-
 (c) Cl^- and ClO_2^- (d) ClO^- and ClO_3^-

49. Which of the following compounds will form significant amount of meta product during mono-nitration reaction?





50. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is :

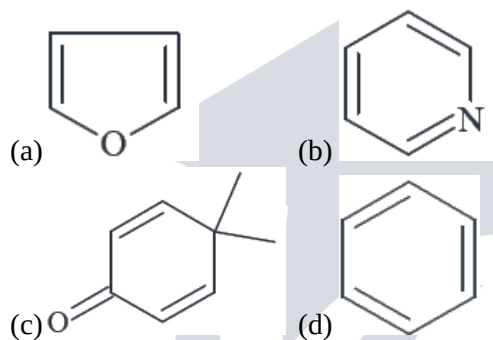
- (a) Zero (b) Two
(c) Four (d) Six

51. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1} . If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to:

($R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$)

- (a) 12 (b) 6
(c) 4 (d) 8

52. Which of the following molecules is least resonance stabilized?



53. The group having isoelectronic species is:

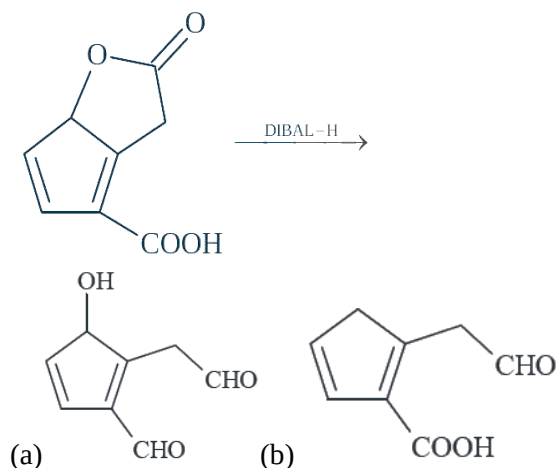
- (a) O^- , F^- , Na , Mg^+ (b) O^{2-} , F^- , Na , Mg^{2+}
(c) O^- , F^- , Na^+ , Mg^{2+} (d) O^{2-} , F^- , Na^+ , Mg^{2+}

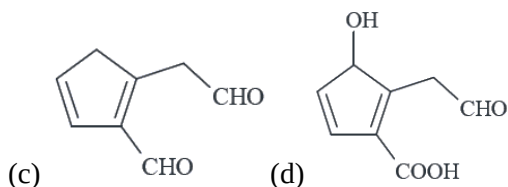
54. The radius of the second Bohr orbit for hydrogen atom is:

(Planck's Const. $h = 6.6262 \times 10^{-34} \text{ Js}$; mass of electron $= 9.1091 \times 10^{-31} \text{ kg}$; charge of electron $e = 1.60210 \times 10^{-19} \text{ C}$; permittivity of vacuum $\epsilon_0 = 8.854185 \times 10^{-12} \text{ kg}^{-1} \text{ m}^{-3} \text{ A}^2$)

- (a) $4.76 \text{ \AA}$ (b) $0.529 \text{ \AA}$
(c) $2.12 \text{ \AA}$ (d) $1.65 \text{ \AA}$

55. The major product obtained in the following reaction is:





56. Which of the following reactions is an example of a redox reaction?

- (a) $\text{XeF}_2 + \text{PF}_5 \rightarrow [\text{XeF}]^+ \text{PF}_6^-$
 (b) $\text{XeF}_6 + \text{H}_2\text{O} \rightarrow \text{XeOF}_4 + 2\text{HF}$
 (c) $\text{XeF}_6 + 2\text{H}_2\text{O} \rightarrow \text{XeO}_2\text{F}_2 + 4\text{HF}$
 (d) $\text{XeF}_4 + \text{O}_2\text{F}_2 \rightarrow \text{XeF}_6 + \text{O}_2$

57. A metal crystallizes in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be:

- (a) $2\sqrt{2}a$ (b) $\sqrt{2}a$
 (c) $\frac{a}{\sqrt{2}}$ (d) $2a$

58. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4 . 'X' reacts with the acidified aqueous CaCl_2 solution to give a white precipitate which decolourises acidic solution of KMnO_4 . 'X' is:

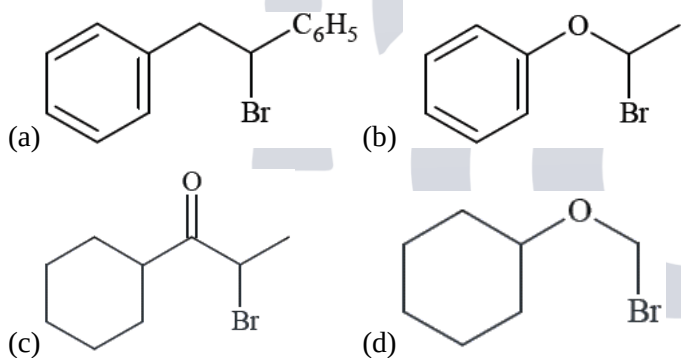
- (a) HCOONa (b) CH_3COONa
 (c) $\text{Na}_2\text{C}_2\text{O}_4$ (d) $\text{C}_6\text{H}_5\text{COONa}$

59. A water sample has ppm level concentration of following anions $\text{F}^- = 10$; $\text{SO}_4^{2-} = 100$; $\text{NO}_3^- = 50$

The anion/anions that make/makes the water sample unsuitable for drinking is/are:

- (a) both SO_4^{2-} and NO_3^- (b) only F^-
 (c) only SO_4^{2-} (d) only NO_3^-

60. Which of the following, upon treatment with tert-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?



PART C - MATHEMATICS

61. If S is the set of distinct values of 'b' for which the following system of linear equations $x + y + z = 1$

$$x + ay + z = 1$$

$$ax + by + z = 0$$

has no solution, then S is:

- (a) an empty set
 (b) an infinite set
 (c) a finite set containing two or more elements
 (d) a singleton

62. The following statement $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is:

- (a) a tautology
- (b) equivalent to $\sim p \rightarrow q$
- (c) equivalent to $p \rightarrow \sim q$
- (d) a fallacy

63. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is:

- (a) $-\frac{3}{5}$
- (b) $\frac{1}{3}$
- (c) $\frac{2}{9}$
- (d) $-\frac{7}{9}$

64. For three events A, B and C, $P(\text{Exactly one of A or B occurs}) = P(\text{Exactly one of B or C occurs}) = P(\text{Exactly one of C or A occurs}) = \frac{1}{4}$ and $P(\text{All the three events occur simultaneously}) = \frac{1}{16}$. Then the probability that at least one of the events occurs, is:

- (a) $\frac{7}{32}$
- (b) $\frac{7}{16}$
- (c) $\frac{7}{64}$
- (d) $\frac{3}{16}$

65. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k, \text{ then } k \text{ is equal to:}$$

- (a) $-z$
- (b) z
- (c) -1
- (d) 1

66. Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point:

- (a) $(2, -\frac{1}{2})$
- (b) $(1, \frac{3}{4})$
- (c) $(1, -\frac{3}{4})$
- (d) $(2, \frac{1}{2})$

67. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is:

- (a) 12.5
- (b) 10
- (c) 25
- (d) 30

68. The area (in sq. units) of the region $\{(x, y): x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is:

- (a) $\frac{59}{12}$
- (b) $\frac{3}{2}$
- (c) $\frac{7}{3}$
- (d) $\frac{5}{2}$

69. If the image of the point $P(1, -2, 3)$ in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to:

- (a) $3\sqrt{5}$
- (b) $2\sqrt{42}$
- (c) $\sqrt{42}$
- (d) $6\sqrt{5}$

70. If for $x \in (0, \frac{1}{4})$, the derivative of $\tan^{-1}(\frac{6x\sqrt{x}}{1-9x^3})$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals:

- (a) $\frac{9}{1+9x^3}$
- (b) $\frac{3x\sqrt{x}}{1-9x^3}$
- (c) $\frac{3x}{1-9x^3}$
- (d) $\frac{3}{1+9x^3}$

71. If $(2 + \sin x) \frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y(\frac{\pi}{2})$ is equal to:

- (a) $\frac{1}{3}$
- (b) $-\frac{2}{3}$
- (c) $-\frac{1}{3}$
- (d) $\frac{4}{3}$

72. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$, then $\tan \beta$ is equal to:

- (a) $\frac{6}{7}$ (b) $\frac{1}{4}$
(c) $\frac{2}{9}$ (d) $\frac{4}{9}$

73. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $\text{adj}(3A^2 + 12A)$ is equal to:

- (a) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (b) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$
(c) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (d) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

74. For any three positive real numbers a, b and c , $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then:

- (a) b, c and a are in G.P.
(b) b, c and a are in A.P.
(c) a, b and c are in A.P.
(d) a, b and c are in G.P.

75. The distance of the point $(1, 3, -7)$ from the plane passing through the point $(1, -1, -1)$, having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$, is:

- (a) $\frac{20}{\sqrt{74}}$ (b) $\frac{10}{\sqrt{83}}$
(c) $\frac{5}{\sqrt{83}}$ (d) $\frac{10}{\sqrt{74}}$

76. Let $I_n = \int \tan^n x dx$, ($n > 1$). If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to:

- (a) $(-\frac{1}{5}, 1)$ (b) $(\frac{1}{5}, 0)$
(c) $(\frac{1}{5}, -1)$ (d) $(-\frac{1}{5}, 0)$

77. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $(1, \frac{3}{2})$ is:

- (a) $2y - x = 2$ (b) $4x - 2y = 1$
(c) $4x + 2y = 7$ (d) $x + 2y = 4$

78. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point:

- (a) $(3\sqrt{2}, 2\sqrt{3})$ (b) $(2\sqrt{2}, 3\sqrt{3})$
(c) $(\sqrt{3}, \sqrt{2})$ (d) $(-\sqrt{2}, -\sqrt{3})$

79. The function $f: \mathbb{R} \rightarrow [-\frac{1}{2}, \frac{1}{2}]$ defined as $f(x) = \frac{x}{1+x^2}$, is:

- (a) invertible.
(b) injective but not surjective.
(c) surjective but not injective.
(d) neither injective nor surjective.

80. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals:

- (a) $\frac{1}{24}$ (b) $\frac{1}{16}$
(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

81. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30° . Then $\vec{a} \cdot \vec{c}$ is equal to:

- (a) $\frac{25}{8}$ (b) 2
(c) 5 (d) $\frac{1}{8}$

82. The normal to the curve $y(x-2)(x-3) = x+6$ at the point where the curve intersects the y -axis passes through the point:

- (a) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$
(c) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (d) $\left(\frac{1}{2}, \frac{1}{3}\right)$

83. If two different numbers are taken from the set $\{0, 1, 2, 3, \dots, 10\}$; then the probability that their sum as well as absolute difference are both multiple of 4, is:

- (a) $\frac{6}{55}$ (b) $\frac{12}{55}$
(c) $\frac{14}{45}$ (d) $\frac{7}{55}$

84. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is:

- (a) 485 (b) 468
(c) 469 (d) 484

85. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \dots + ({}^{21}C_{10} - {}^{10}C_{10})$ is:

- (a) $2^{21} - 2^{11}$ (b) $2^{21} - 2^{10}$
(c) $2^{20} - 2^9$ (d) $2^{20} - 2^{10}$

86. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is:

- (a) $\frac{12}{5}$ (b) 6
(c) 4 (d) $\frac{6}{25}$

87. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy, \forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to:

- (a) 330 (b) 165
(c) 190 (d) 255

88. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is:

- (a) $2(\sqrt{2} + 1)$ (b) $2(\sqrt{2} - 1)$
(c) $4(\sqrt{2} - 1)$ (d) $4(\sqrt{2} + 1)$

89. If, for a positive integer n , the quadratic equation, $x(x + 1) + (x + 1)(x + 2) + \dots + (x + \overline{n - 1})(x + n) = 10n$ has two consecutive integral solutions, then n is equal to:

- (a) 12 (b) 9
(c) 10 (d) 11

90. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$ is equal to:

- (a) -2 (b) 2
(c) 4 (d) -1