1. **(d)** For solid sphere

$$\frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mR^2\cdot\frac{v^2}{R^2} = mgh_{sph}$$

For solid cylinder

$$\frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{1}{2}mR^2\cdot\frac{v^2}{R^2} = mgh_{cyl}$$

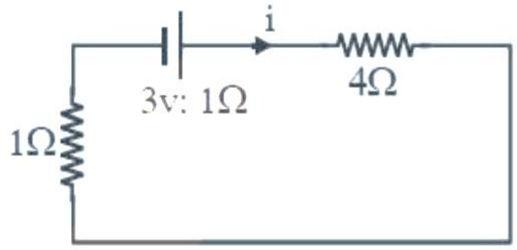
$$\Rightarrow \frac{h_{sph}}{h_{cyl}} = \frac{7/5}{3/2} = \frac{14}{15}$$

2. **(c)** Resistance of wire AB $= 400 \times 0.01 = 4\Omega$

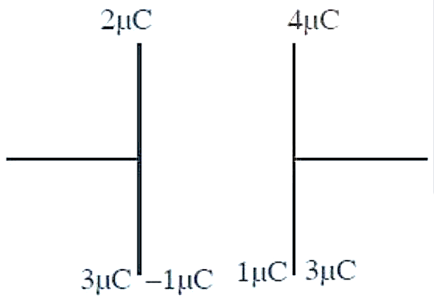
$$i = \frac{3}{6} = 0.5 \text{ A}$$

Now voltmeter reading = i (Resistance of 50 cm length)

$= (0.5 \text{ A})(0.01 \times 50) = 0.25$ volt



3. **(b)**



Charges at inner plates are $1\mu C$ and $-1\mu C$.

∴ Potential difference across capacitor

$$= \frac{q}{C} = \frac{1\mu C}{1\mu F} = \frac{1\times10^{-6}C}{1\times10^{-6}\text{ Farad}} = 1 \text{ V}$$

4. **(a)**

$p = ks^a l^b h^c$

Where k is dimensionless constant

$$MLT^{-1} = (MT^{-2})^a (ML^2)^b (ML^2T^{-1})^c$$

$a + b + c = 1$

$2b + 2c = 1$

$-2a - c = -1$

$a = \frac{1}{2} \quad b = \frac{1}{2} \quad c = 0$

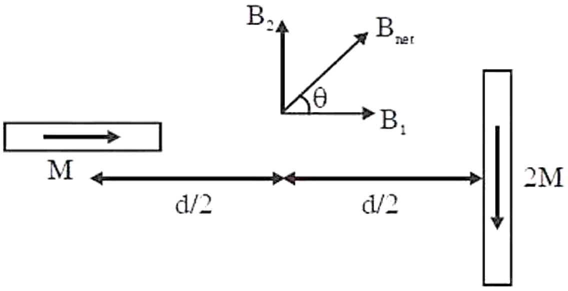
$S^{1/2}|^{1/2} h^0$

5. (b)

$B_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}; \ B_2 = \left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}$

$B_1 = B_2$

$\Rightarrow$ B_{net} is at $45^\circ (\theta = 45^\circ)$

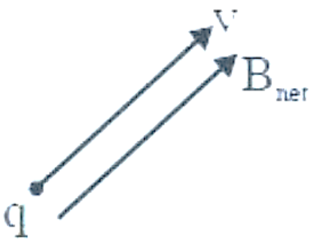


Velocity of charge and B_{net} are parallel so

by

$\vec{F} = q(\vec{b} \times \vec{B})$ force on charge particle is

zero.



6. (d) $A = A_0 e^{-\gamma t}$

$A = \frac{A_0}{2}$ after 10 oscillations

$\because$ After 2 seconds

$\frac{A_0}{2} = A_0 e^{-\gamma(b)}$; $2 = e^{2\gamma}$

$\ell n 2 = 2\gamma$; $\gamma = \frac{\ell n 2}{2}$

$\because A = A_0 e^{-\gamma t}$

$\ln\frac{A_0}{A} = \gamma t$; $\ln 1000\frac{\ell n2}{2}t$

$2\left(\frac{3\ln 10}{\ell n2}\right) = t$; $\frac{6\ln 10}{\ell n2} = t$

$t = 19.931\text{sec}$

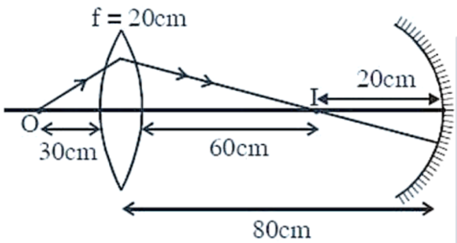
$t \approx 20\text{sec}$

7. **(b)** Image formed by lens

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}; \frac{1}{v} + \frac{1}{30} = \frac{1}{20}$

$v = +60$ cm

If image position does not change even when mirror is removed it means image formed by lens is formed at centre of curvature of spherical mirror.



Radius of curvature of mirror $= 80 - 60 = 20$ cm.

$\Rightarrow$ Focal length of mirror $f = 10$ cm for virtual image, object is to be kept between focus and pole.

$\Rightarrow$ Maximum distance of object from spherical mirror for which virtual image is formed, is 10 cm.

8. **(d)** Current $i = \frac{E}{r+R}$

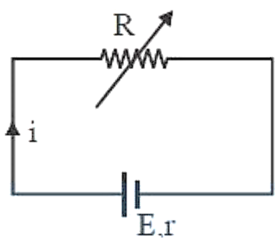
Power generated in R

$P = i^2R$

$$P = \frac{E^2R}{(r+R)^2}$$

For maximum power $\frac{dP}{dR} = 0$

$$E^2\left[\frac{(r+R)^2 \times 1 - R \times 2(r+R)}{(r+R)^4}\right] = 0$$



$\Rightarrow r = R$

9. (c) $T = \frac{30\sec}{20}$ $\Delta T = \frac{1}{20}\sec$,

$L = 55$ cm $\Delta L = 1$ mm $= 0.1$ cm

$$g = \frac{4\pi^2 L}{T^2}$$

percentage error in g is

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right) 100\%$$

$$= \left(\frac{0.1}{55} + \frac{2\left(\frac{1}{20}\right)}{\frac{30}{20}}\right) 100\% \simeq 6.8\%.$$

10. (b)

$$v_{\text{rms}} = \sqrt{\frac{3RT}{m}} \quad v_{\text{escape}} = \sqrt{2gR_e}$$

$$v_{\text{rms}} = v_{\text{escape}}$$

$$\frac{3RT}{m} = 2gR_e$$

$$\frac{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}}{2} \times 10 \times 10^3 = 10^4 \text{k}$$

11. (d) Given initial velocity $u = 0$ and acceleration is constant

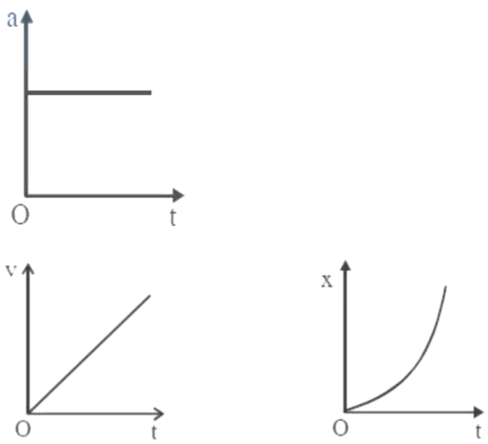
At time t

$$v = 0 + at$$

$$\Rightarrow v = at$$

Also $x = 0(t) + \frac{1}{2}at^2$

$$\Rightarrow x = \frac{1}{2}at^2$$



Graph (a), (b) and (d) are correct.

12. (d)

$\vec{E} = (20x + 10)\hat{\imath}$

$V_1 - V_2 = -\int_{-5}^{1}(20x + 10)dx$

$V_1 - V_2 = -(10x^2 + 10x)_{-5}^{1}$

$V_1 - V_2 = 10(25 - 5 - 1 - 1)$

$V_1 - V_2 = 180\text{ V}$

13. (c) Given:

$\frac{Y_A}{Y_B} = \frac{7}{4}\ \ L_A = 2\text{ m}\ \ A_A = \pi R^2$

$L_B = 1.5\text{ m}\ \ A_B = \pi(2\text{ mm}))^2$

$\frac{F}{A} = Y\left(\frac{\ell}{L}\right)$

given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B}$

$\Rightarrow \frac{(\pi R^2)\left(\frac{7}{4}Y_B\right)}{2} = \frac{\pi(2\text{ mm})^2 \cdot Y_B}{1.5}$

$R = 1.74\text{ mm}$

14. (c) Applying linear momentum conservation

$m_1v_1\hat{\imath} + m_2v_2\hat{\imath} = m_1v_3\hat{\imath} + m_2v_4\hat{\imath}$

$m_1v_1 + 0.5m_1v_2 = m_1(0.5v_1) + 0.5m_1v_4$

$0.5\ m_1v_1 = 0.5\ m_1(v_4 - v_2)$

$v_1 = v_4 - v_2$

15. (d) Mass densities of all nuclei are same so their ratio is 1 .

16. (a) Angular impulse = change in angular momentum.

$\tau\Delta t = \Delta L$

$mg\frac{\ell}{2} \times 0.01 = \frac{m\ell^2}{3}\omega$

$\omega = \frac{3\text{ g} \times 0.01}{2\ell} = \frac{3 \times 10 \times 0.01}{2 \times 0.3} = \frac{1}{2} = 0.5\text{rad/s}$

Time taken by rod to hit the ground

$t = \sqrt{\frac{2\text{ h}}{\text{g}}} = \sqrt{\frac{2\times5}{10}} = 1\text{sec.}$

In this time angle rotate by rod

$\theta = \omega t = 0.5 \times 1 = 0.5$ radian

17. (b) Moment of inertia

$(I) = m\left(\frac{d}{2}\right)^2 \times 2 = \frac{md^2}{2}$

Now by $\tau = 1\alpha$

$(qE)(d\sin\theta) = \frac{md^2}{2}\cdot\alpha$

$\alpha = \left(\frac{2qE}{md}\right)\sin\theta$ for small θ

$F = qE^{-q,m}$

$\Rightarrow \alpha = \left(\frac{2qE}{md}\right)\theta$

$\Rightarrow$ Angular frequency $\omega = \sqrt{\frac{2qE}{md}}$

18. (a) If we use that direction of light propagation will be along $\vec{E}\times\vec{B}$. Then (A) option is correct.

Magnitude of E = CB

$E = 3\times10^8\times1.6\times10^{-6}\times\sqrt{5}$

$E = 4.8\times10^2\sqrt{5}$

$\vec{E}$ and $\vec{B}$ are perpendicular to each other

$\Rightarrow \vec{E}\cdot\vec{B} = 0$

$\Rightarrow$ Either direction of $\vec{E}$ is $\hat{\imath} - 2\hat{\jmath}$ or $-\hat{\imath} + 2\hat{\jmath}$ from given option

Also wave propagation direction is parallel to $\vec{E}\times\vec{B}$ which is $-\hat{k}$

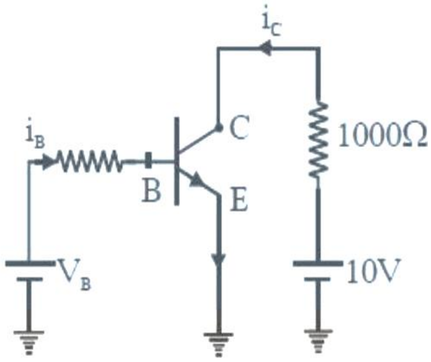
$\Rightarrow \vec{E}$ is along $(-\hat{\imath} + 2\hat{\jmath})$

19. (b) At saturation state, V_{CE} becomes zero

$\Rightarrow i_c = \frac{10\text{ V}}{1000\Omega} = 10\text{ mA}$

Now current gain factor $\beta = \frac{i_c}{i_B}$

$\Rightarrow i_B = \frac{10\text{ mA}}{250} = 40\mu A$



20. (c) $\frac{1}{2}mV^2 = -q\left(V_f - V_i\right)$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\Delta \mathrm{V} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{\mathrm{r}_0}{\mathrm{r}}\right)$$

$$\frac{1}{2}mv^2 = \frac{-q\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r}\right)$$

$$v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$$

21. (c) $|\vec{A}_1| = 3, |\vec{A}_2| = 5$, and $|\vec{A}_1 + \vec{A}_2| = 5$.

$$|\vec{A}_1 + \vec{A}_2| = |\vec{A}_1|^2 + |\vec{A}_2|^2 + 2|\vec{A}_1||\vec{A}_2|\cos\theta$$

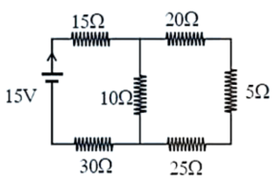
$$\cos\theta = -\frac{3}{10}$$

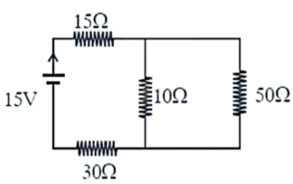
$$(2\vec{\mathrm{A}}_1 + 3\vec{\mathrm{A}}_2)\cdot(3\vec{\mathrm{A}}_1 - 2\vec{\mathrm{A}}_2)$$

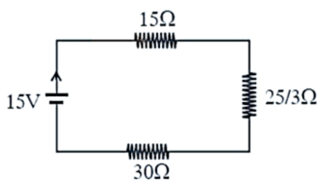
$$= 6|\vec{A}_1|^2 + 9\vec{A}_1\cdot\vec{A}_2 - 4\vec{A}_1\cdot\vec{A}_2 - 6|\vec{A}_2|^2$$

$$= -118.5$$

22. (c)







$$R_{\mathrm{eq}} = 15 + \frac{25}{3} + 30 = \frac{45 + 25 + 90}{3} = \frac{160}{3}$$

$I = \frac{E}{R_{eq}} = \frac{15\times3}{160} = \frac{9}{32}$ amp.

23. (a) Range $= \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$50 \times 10^3 = \sqrt{2 \times 6400 \times 10^3 \times h_T} + \sqrt{2 \times 6400 \times 10^3 \times 70}$

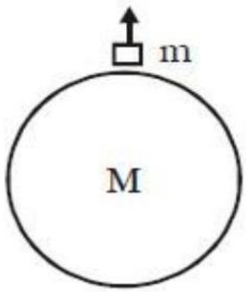
By solving $h_T = 32$ m.

24. (b) Minimum energy required $(E) = -$ (Potential energy of object at surface of earth)

$= -\left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$

Now $M_{\text{earth}} = 64 M_{\text{moon}}$

$\rho \cdot \frac{4}{3}\pi R_e^3 = 64 \cdot \frac{4}{3}\pi R_m^3 \Rightarrow R_e = 4R_m$



Now $\frac{E_{\text{moon}}}{E_{\text{earth}}} = \frac{M_{\text{moon}}}{M_{\text{earth}}} \cdot \frac{R_{\text{earth}}}{R_{\text{moon}}} = \frac{1}{64} \times \frac{4}{1}$

$\Rightarrow E_{\text{moon}} = \frac{E}{16}$

25. Given phase difference $= \frac{\pi}{4}$ and $\omega = 100$rad/s

$\Rightarrow$ Reactance (X) $=$ Resistance (R) Now by checking option.

Option (A)

$R = 1000\Omega$ and $X_c = \frac{1}{10^{-6}\times100} = 10^4\Omega$

Option (B)

$R = 10^3\Omega$ and $X_L = 10 \times 10^{-3} \times 100 = 1\Omega$

Option (C)

$R = 10^3\Omega$ and $X_L = 10^{-3}100 = 10^{-1}\Omega$

Option (D)

$R = 10^3\Omega$ and $X_C = \frac{1}{10\times10^{-6}\times100} = 10^3\Omega$

26. (d)

Let mass of B and C is m each. By momentum conservation

$$2mv_0 = mv - \frac{mv}{2}$$

$v = 4v_0$

$P_A = 2mv_0 p_B = 4mv_0 p_c = 2mv_0$

De-Broglie wavelength $\lambda = \frac{h}{p}$

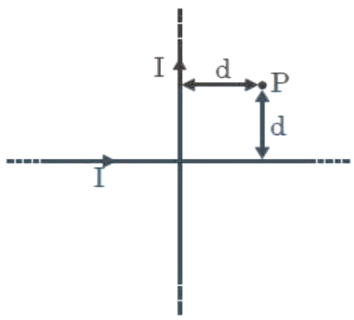
$$\lambda_A = \frac{h}{2mv_0}; \lambda_B = \frac{h}{4mv_0}; \lambda_C = \frac{h}{2mv_0}$$

27. **(c)** Limit of resolution of telescope $= \frac{1.22\lambda}{D}$

$$\theta = \frac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}} = 305 \times 10^{-9} \text{ radian}$$

28. **(c)** Magnetic field at point P

$$\vec{B}_{\text{net}} = \frac{\mu_0 \mathrm{i}}{2\pi \mathrm{d}}(-\hat{\mathrm{k}}) + \frac{\mu_0 \mathrm{i}}{2\pi \mathrm{d}}(\hat{\mathrm{k}}) = 0$$



29. **(d)** isochoric → Process d

Isobaric → Process a

Adiabatic slope will be more than isothermal so

Isothermal → Process b

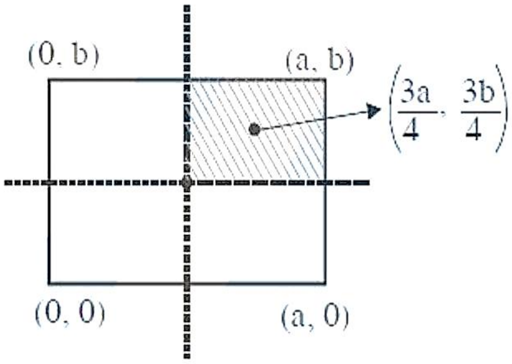
Adiabatic → Process c

Order → dabc

30. **(d)** $x = \frac{M\frac{a}{2} - \frac{M}{4} \times \frac{3a}{4}}{M - \frac{M}{4}}$

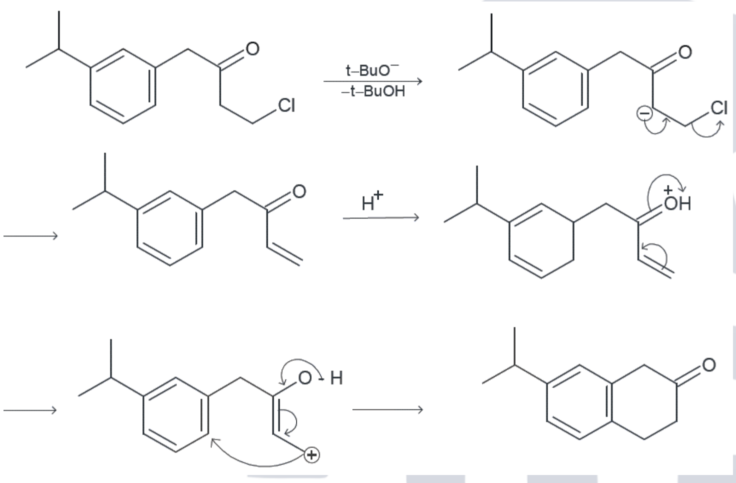
$$= \frac{\frac{a}{2} - \frac{3a}{16} \times \frac{3a}{4}}{\frac{3}{4}} = \frac{\frac{5a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

$$y=\frac{M\frac{b}{2}-\frac{M}{4}\times\frac{3b}{4}}{M-\frac{M}{4}}=\frac{5b}{12}$$



31. (b) In case of interstial compounds there is presence of small atoms (or impurity) in the lattice of metal. So interstial compounds are hard, high melting point and chemically inert.

32. (b)



33. (d)

34. (c)

$$K.E=\frac{1}{2}mu^2=\frac{1}{2}\frac{m^2u^2}{m}=\frac{1}{2}\frac{P^2}{m}$$

$$\frac{hc}{\lambda_1}=Wo+(K.E)_1=Wo+\frac{1}{2}\frac{P^2}{m}$$

$$\frac{hc}{\lambda_2}=Wo+(K.E)_2=Wo+\frac{1}{2}\frac{(1.5P)^2}{m}$$

On solving

$$\lambda_2=\frac{4}{9}\lambda_1$$

Since the K.E is very high is comparison to work function, then we can assume that $K.E+Wo=K.E$

35. (b)

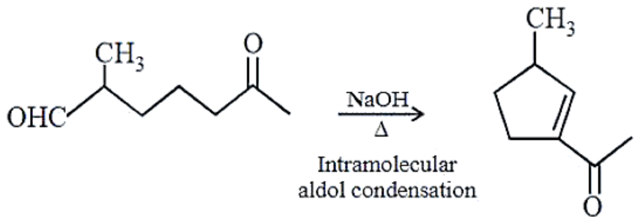
$[ICl_4]^- \rightarrow sp^3\ d^2$

$[IF_6]^- \rightarrow sp^3\ d^2$

$[ICl_2]^- \rightarrow sp^3d$

$[BrF_2]^- \rightarrow sp^3d$

36. (a)



37. (a) Applying steady state of approximation

$$\frac{d}{dt}[B] = K_1[A] - K_2[B]$$
$$0 = K_1[A] - K_2[B]$$
$$\frac{K_1}{K_2}[A] = [B]$$

38. (b) $2SO_2(g) + O_2(g) \longrightarrow 2SO_3(g)$

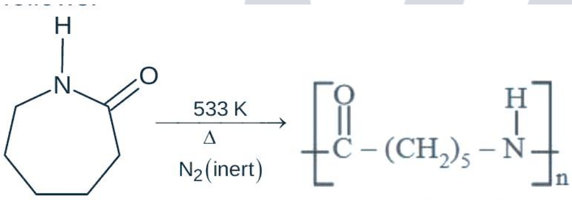
$$K_{eq} = \frac{[SO_3]^2}{[O_2][SO_2]^2}$$

$$= \frac{K_2}{K_1} = \frac{10^{129}}{10^{104}} = 10^{25}$$

39. (d)

40. (a)

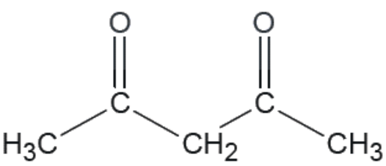
41. (c) The direct monomer of Nylon-6 is caprolactum which polymerises to give Nylon-6 as follows:



Nylon-6

42. (b) The IUPAC name of element having atomic number 119 is "Ununennium". So its symbol is uue.

43. (c)



Due to presence of active methylene group and stabilization of enol by intramoelcular H bond forming 6 membered ring structure.

44. (d) Mass of fatty acid $= 0.027$ g

Radius of plate $= 10$ cm

Density of fatty acid $= 0.9\ g/cm^3$

Volume of fatty acid $= \frac{0.027}{0.9} = 0.03\ \text{cm}^3$

Area of plate $= \pi r^2 = 3 \times 10^2 = 300\ \text{cm}^2$

Height of fatty acid $= \frac{\text{Volume}}{\text{Area}} = \frac{0.03}{300} = 10^{-4}\ \text{cm} = 10^{-6}\ \text{m}$

45. (b)
46. (d)
47. (b)
48. (b)

$\Delta U = nCv \times \Delta T$

$= 5 \times 28 \times 100 = 14\ \text{kJ}$

$\Delta PV = nR\Delta T$

$= 5 \times 8 \times 100 = 4\ \text{kJ}$

49. (d)

$[Fe(CN)_6]^{4-}$

$[Fe(H_2O)_6]^{2+}$

$Fe^{2+} \longrightarrow 3d^6$

$t_{2g}^4 eg^2$

$\mu = \sqrt{n(n+2)} = \sqrt{24} = 4.9BM$

$Fe^{2+} \longrightarrow 3\ d^6$

$t_{2g}^6 eg^0$

$\mu = \sqrt{n(n+2)}$

$\mu = 0$

50. (d)

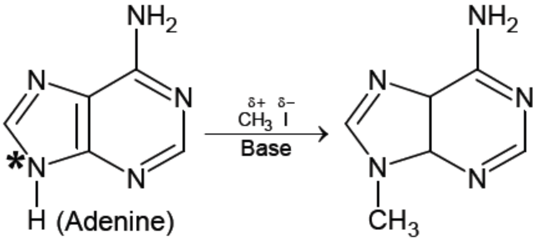
Barfoed's test → Detecting presence of monosaccharides

Fehling's test → For aldehdyes

Benedict's test → For reducing sugars

Seliwanoff's test → Differentiate between aldose and ketose

51.



The nitrogen atom marked with * contain high electron density. It is attached to one H atom which can be removed by base $OH^-$and the CH_3 group gets attached there.

52. **(a)** Polysubstitution is a major drawback of Friedal craft's alkylation. It is so because activating behaviour of benzene ring increases with increase in number of alkyl group on benzene ring.

53. **(a)** $2H_2O_2 \longrightarrow 2H_2O + \underset{22.4 \text{ of } O_2 \text{ (STP)}}{O_2} \uparrow$

11.2 L of O_2 is given by 34 g of H_2O_2 present in 1 L of H_2O_2 solution.

So % strength of $H_2O_2 == \frac{34}{1000} \times 100 = 3.4 \text{ g}/100 \text{ mL} = 3.4\%$

54. **(c)** From Henry's law

$P_{\text{gas}} = K_H \cdot X_{\text{gas}}$

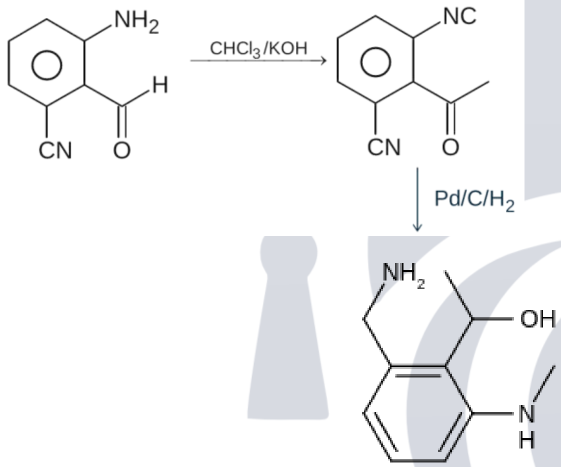
$P_{\text{gas}} = K_H(1 - X_{H_2O})$

$P_{\text{gas}} = K_H - K_H X_{H_2O}$

Comparing this equation with straight line equation $y = mx + c$

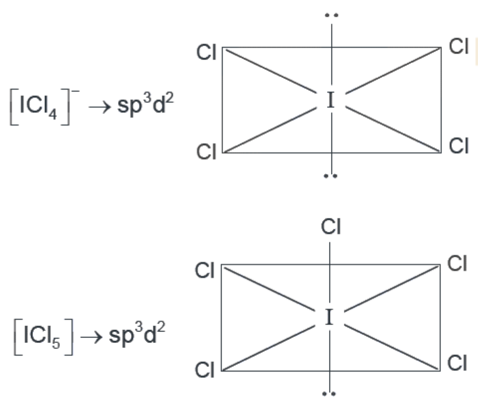
55. **(b)**

56. The reaction involves carbylamine reaction of 1° amine with $CHCl_3/KOH$ followed by reduction with $Pd/C/H_2$.



57. **(b)** Maximum prescribed concentration of copper in drinking water is 3ppm. Above this concentration water becomes toxic.

58. **(b)**



59. **(d)** cis- $[PtCl_2(NH_3)_2]$ is used in chemotherapy to inhibits the growth of tumors

60. **(a)** % packing efficiency $= \frac{\text{Vol. occupied by atom}}{\text{Vol. of unit cell}} \times 100 = \frac{\frac{4}{3}\pi r^3}{a^3} \times 100$

Let radius of corner atom is r and radius of central atom is $2r$

So, $\sqrt{3}a = 2(2r) + 2r = 6r$

$$a = \frac{6r}{\sqrt{3}} = 2\sqrt{3}r$$

Now

$$\% \text{ P.E } = \frac{\frac{4}{3}\pi r^3 + \frac{4}{3}\pi(2r)^3}{(2\sqrt{3}r)^3} \times 100$$
$$= \frac{\frac{4}{3}\pi(r^3 + 8r^3)}{8 \times 3\sqrt{3}r^3} \times 100$$
$$= \frac{4\pi \times 9r^3}{3 \times 8 \times 3\sqrt{3}r^3} \times 100 = 90.6\% \approx 90\%$$

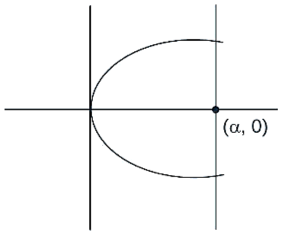
61. (c)

$$S(\lambda) = 2\int_0^\lambda \sqrt{x}dx = \frac{4}{3}\lambda^{3/2}$$

$$\frac{S(\lambda)}{S(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{3/2}}{4^{3/2}} = \frac{2}{5}$$

$$\Rightarrow \lambda = 4\left(\frac{4}{25}\right)^{1/3}$$

62. (a) Since $g(x)$ is even with $f(0) = 0$



$f(x)$ is odd function

$g(x) = f(x+5)$

$g(-x) = f(-x+5)$

$g(x) = -f(x-5)$

Replace x by $x+5$

$\Rightarrow \mathrm{f(x)} = -\mathrm{g(x+5)}$

$\int_0^x f(t)dt = -\int_0^x g(t+5)dt$

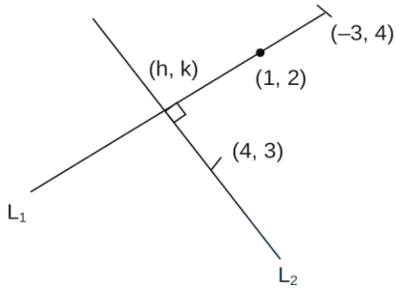
$= -\int_5^{x+5} \mathrm{g(t)dt}$

$= \int_{x+5}^{5} g(t)dt$

63. (b) Equation of L_1 is $x + 2y = 5$ and equation of L_2 is $2x - y = 5$

Their point of intersection is (3,1)

$\Rightarrow \frac{k}{h} = \frac{1}{3}$



64. (b) $f_1(x) = \frac{a^x + a^{-x}}{2}$ and $f_2(x) = \frac{a^x - a^{-x}}{2}$

$f_1(x+y) + f_1(x-y)$

$= \frac{1}{2}(a^{x+y} + a^{-x-y} + a^{x-y} + a^{-x+y})$

$= \frac{1}{2}\left(a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})\right)$

$= 2 \cdot \left(\frac{a^x + a^{-x}}{2}\right)\left(\frac{a^y + a^{-y}}{2}\right)$

$= 2f_1(x)f_1(y)$

65. (d)

$$f(x) = \begin{cases} -x-1 & x \in [-1,0) \\ x & x \in [0,1) \\ 2x & x \in [1,2) \\ x+2 & x \in [2,3) \end{cases}$$

$f(x)$ is discontinuous at $x = 0,1$

66. (a)

$z = \frac{\sqrt{3} + i}{2} = e^{\frac{i}{6}}$

$(1 + iz + z^5 + iz^8)^9$

$= \left(1 + e^{i\pi/2}e^{i\pi/6} + e^{i5\pi/6} + e^{i\pi/2}e^{i8\pi/6}\right)^9$

$= \left(1 + e^{i2\pi/6} + e^{i5\pi/6} + e^{i\frac{11\pi}{6}}\right)^9$

$= \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^9 = \left(e^{i\pi/3}\right)^9 = e^{i3\pi} = -1$

67. (d)

(A) $(p \vee q) \rightarrow (p \vee (\sim q))$

$=\sim(p \vee q) \vee (p \vee \sim q)$
$= (\sim p \wedge \sim q) \vee (p \vee \sim q)$
$\neq T$

(B) $(p \wedge q) \rightarrow p$

$=\sim(p \wedge q) \vee p = (\sim p \vee \sim q) \vee p$

$= (\sim p \vee p) \vee \sim q$

$= \mathrm{T}$

(C) $\sim p \vee (p \vee q)$

$= (\sim p \vee p) \vee q = T$

(D) $\sim(p \vee q) \vee (\sim p \vee q)$

$= (\sim p \vee \sim q) \vee (\sim p \vee q)$

$=\sim p \vee T = T$

68. (d) $\frac{dy}{dx} = \frac{2y}{x^2}$

$\Rightarrow \ln y = -\frac{2}{x} + \ln C$

Passes through $(1,1)$

$0 = -2 + \ln C$

$\Rightarrow \ell n y = \frac{-2}{x} + 2$

$x\ell n|y| = 2(x-1)$

69. (a)

$f = f(f(f(x))) + (f(x))^2$

$\frac{dy}{dx} = f'(f(f(x)))f'(f(x))f'(x) + 2f(x)f'(x)$

Put $x = 1$

$\frac{dy}{dx} = 27 + 6 = 33$

70. (c) Starting with $5 = 6^3 = 216$

Starting with $44 = 6^2 = 36$

Starting with $45 = 6^2 = 36$

Starting with $43 = 18$ (Not using 0,1,2)

Starting with $432 = 4$

Total = 310

71. (c)

Equation of PQ is $\frac{x-2}{6}=\frac{y+3}{3}=\frac{z-4}{6}$

$R(4,y,z)$ lies on this

$\Rightarrow \frac{1}{3}=\frac{y+3}{3}=\frac{z-4}{6}$

$\Rightarrow \mathrm{R}(4,-2,6)$

$\mathrm{QR}=\sqrt{16+4+36}=2\sqrt{14}$

72. (b) For infinitly many solution

$\begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} - 0$

$\Rightarrow \mathrm{k}=\frac{-1}{2}$

Also consider

$\mathrm{x}-2\mathrm{y}+\mathrm{k}=1$ and $2\mathrm{x}+\mathrm{y}+\mathrm{z}=2$

$\Rightarrow 2\mathrm{x}-4\mathrm{y}-\mathrm{z}-2$

$2x+y+z=2$

$\Rightarrow 4\mathrm{x}-3\mathrm{y}=4$

73. (b)

$\mathrm{AM}=\frac{41+45+54+57+43+\mathrm{x}}{6}=48$

$\Rightarrow \mathrm{x}=48$

$\sigma^2+48^2=\frac{1}{6}(41^2+45^2+54^2+57^2+43^2+48^2)$

$\sigma^2=\frac{14024}{6}-2304$

$=\frac{100}{3}$

74. (c)

$P_1: x+y+z=1$

$P_2: 2x+3y+4z=5$

Required plane is $P_1+\lambda P_2=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z=1+5\lambda$

which is perpendicular to $x-y+z=0$

$\Rightarrow 1+2\lambda-(1+3\lambda)+1+4\lambda=0$

$\Rightarrow \lambda=\frac{-1}{3}$

(i) $\Rightarrow x-z+2=0$

$\vec{r}\cdot(\hat{\imath}-\hat{k})+2=0$

75. (a) Given $2b=a+c$

Let $A=\theta, B=\pi-3\theta, C=2\theta$

$2\sin B=\sin A+\sin C$

$2\sin 3\theta=\sin\theta+\sin 2\theta$

$2(3-4\sin^2\theta)=(1+2\cos\theta)$

$\Rightarrow 8\cos^2\theta-2\cos\theta-3=0$

$\Rightarrow \cos\theta=\frac{3}{4}$

$\sin A:\sin B:\sin C$
$\Rightarrow 1:4-3\sin^2\theta:2\cos\theta$

$\Rightarrow 1:\frac{5}{4}:\frac{6}{4}$

$\Rightarrow 4:5:6$

76. (b) $\mathrm{be}=5\sqrt{3}$
$b^2e^2=75$

$(b-a)(b+a)=75\Rightarrow b+a=15$

$\Rightarrow b=10, a=5$

$\mathrm{LR}=\frac{2\mathrm{a}^2}{\mathrm{b}}=5$

77. (c)
$1-\frac{1}{2^n}>\frac{9}{10}$
$\Rightarrow \frac{1}{10}>\frac{1}{2^n}$

$\Rightarrow 2^n>10$

$\Rightarrow \mathrm{n}=4$

78. (c) Let equation of hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

passes through $(4,6)$

$\Rightarrow \frac{16}{a^2} - \frac{36}{b^2} = 1$

Also $e^2 = 1 + \frac{b^2}{a^2} \Rightarrow b^2 = 3a^2$

From (i) and (ii)

$a^2 = 4, b^2 = 12$

Equation $\frac{x^2}{4} - \frac{y^2}{12} = 1$

Tangent at $(4,6)$ is $x - \frac{y}{2} = 1$

Or

$2x - y = 2$

79. (a) $D = 4(1+3m)^2 - 4(1+m^2)(1+8m)$

$= 4(1 + 9m^2 + 6m - 1 - 8m - m^2 - 8m^3)$

$= -8m(4m^2 - 4m + 1)$

$= -8m(2m-1)^2 < 0$

$\therefore$ Infinitely many values of m.

80. (a) $S = \sum_{k=1}^{20} \frac{k}{2^k}$

$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots .. + \frac{20}{2^{20}}$

$S\frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \cdots .... + \frac{20}{2^{21}}$

$\frac{1}{2}S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots ... + \frac{1}{2^{20}} - \frac{20}{2^{21}}$

$= \frac{\frac{1}{2}\left(1 - \frac{1}{2^{20}}\right)}{\frac{1}{2}} - \frac{20}{2^{21}}$

$= 1 - \frac{21}{2^{21}}$

$s = 2 - \frac{11}{2^{19}}$

81. (a)

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$= (2+x)\hat{i} - (3-x)\hat{j} - 5\hat{k}$

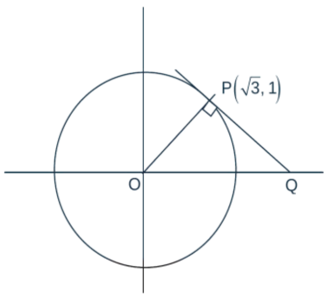
$|\vec{a} \times \vec{b}| = \sqrt{2(x^2 - x + 19)}$

$= \sqrt{2}\sqrt{(x-1/2)^2 + 19^{-1/4}} \geq \frac{5\sqrt{3}}{\sqrt{2}}$

82. (d) Slope of OP $= \frac{1}{\sqrt{3}}$

Equation of PQ is

$$y - 1 = -\sqrt{3}(x - \sqrt{3})$$
$$\Rightarrow y + \sqrt{3}x = 4$$
$$\Rightarrow Q\left(\frac{4}{\sqrt{3}}, 0\right)$$
$$\text{Area } = \frac{2}{\sqrt{3}}$$



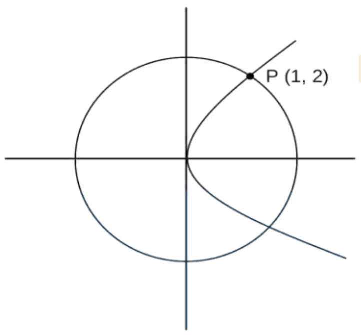
83. (b)

$x^2 + 4x = 5$

$$\Rightarrow x = -5, x = 1$$
$$\Rightarrow P(1,2)$$

Tangent at P is $y = x + 1$

$\left(\frac{3}{4}, \frac{7}{4}\right)$ lies on this.



84. (c)

$\mathrm{h} = 2(3\cos\theta) = 6\cos\theta, r = 3\sin\theta$

$\mathrm{V} = \pi \mathrm{r}^2\ \mathrm{h}$

$= \pi(4\sin^2\theta)(6\cos\theta)$

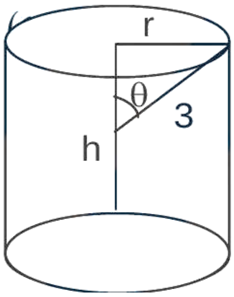
$= 54\pi\sin^2\theta\cos\theta$

$\frac{dv}{d\theta} = 0 \Rightarrow 2\sin\theta\cos\theta - \sin^3\theta = 0$

$\Rightarrow 2\sin\theta - 3\sin^3\theta = 0$

$\Rightarrow \sin\theta = \pm\sqrt{\frac{2}{3}}$

$\cos\theta = \sqrt{\frac{1}{3}}$



$h = \frac{6}{\sqrt{3}}$

85. (d)

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & b^2-4 & c^2-4 \end{vmatrix}$

$= (b-2)(c-2)\begin{vmatrix} 1 & 1 \\ b+2 & c+2 \end{vmatrix}$

$|A| = (b-2)(c-2)(c-b)$

$2, b, c$ are in $AP \Rightarrow 2, 2+d, 2+2d$

$\Rightarrow |A| = (d)(2d)(d) = 2d^3 \in [2,16]$

$\Rightarrow d^3 \in [1,8]$

$\Rightarrow 2d \in [2,4]$
$\Rightarrow 2 + 2d \in [4,6]$

86. (b) 1^∞ Form

$$k = \lim_{x\to 0}\left(\frac{f(3+x)-f(2x)-f(3)(f(\mathrm{b}))}{x(1+f(2-x)-f(\mathrm{b}))}\right)$$
$$= \lim_{x\to 0}\frac{f'(3+x)+f'(2-x)}{(1+f(2-x)-f(\mathrm{b}))-xf'(2-x)}$$
$$= 0$$
$$\Rightarrow e^k = 1$$

87. (b)

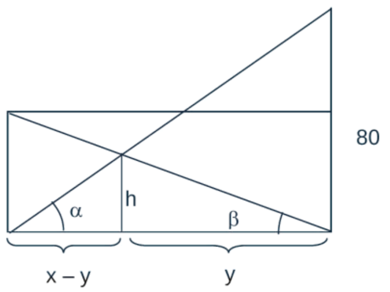
$$I = \int \frac{dx}{x^3(1+x^6)^{2/3}}$$
$$= \int \frac{dx}{x^7\left(1+\frac{1}{x^6}\right)^{2/3}}$$

Put $1+\mathrm{x}^{-6} = \mathrm{t} \Rightarrow \frac{\mathrm{dx}}{\mathrm{x}^7} = \frac{-\mathrm{dt}}{6}$

$$\mathrm{I} = \frac{1}{6}\int \frac{-\mathrm{dt}}{\mathrm{t}^{2/3}} = \frac{-1}{2}\left[1+\frac{1}{\mathrm{x}^6}\right]^{1/3} + \mathrm{C}$$
$$= \frac{-1}{2}\frac{(1+x^6)^{1/3}}{x^2} = xf(x)(1+x^6)^{1/3} + C$$
$$\Rightarrow \mathrm{f(x)} = \frac{-1}{2\mathrm{x}^3}$$

88. (c)

$$\frac{h}{y} = \frac{20}{x}, \frac{h}{x-y} = \frac{80}{x}$$
$$\frac{h}{20} = \frac{y}{x}, \frac{h}{80} = \frac{x-y}{x}$$
$$\frac{\mathrm{h}}{20} + \frac{\mathrm{h}}{80} = 1$$
$$h\left(\frac{100}{1600}\right) = 1$$
$$\mathrm{h} = 16$$



89. All options are incorrect

$${}^6C_3x - \frac{3}{2}(1+\log x)\cdot x^{1/4} = 200$$

$$x^{\frac{1}{4}-\frac{3}{2}(1+\log x)} = 10$$

$$\Rightarrow \frac{1}{4} - \frac{3}{2}(1 + \log_{10} x) \cdot \log_{10} x = 1$$

$\Rightarrow 6t^2 + 5t + 4 = 0, t = \log_{10} x$

$D < 0$

So no real solution

90. (a) $b^2 = ac$

Also roots of $ax^2 + 2bx + c = 0$ are equal

$\Rightarrow x = \frac{-b}{a}$, common root

$$\Rightarrow d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{a}\right) + \int\ = 0$$

$d^2 - 2aeb + fa^2 = 0, b^2 = ac$

$\Rightarrow$ dac $-2eab + fa^2 = 0$

$\Rightarrow dc - 7eb + fa = 0$

Dividing by ac

$$\Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = 2 \cdot \frac{e}{b}$$