

1. (d)

$$k = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (7.5 \times 10^{-12})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV}$$

Alternate method

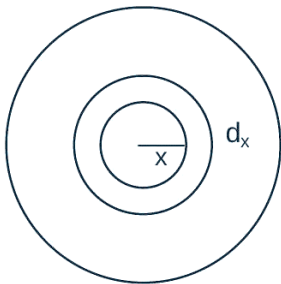
$$\lambda = \sqrt{\frac{150}{V}}$$

$$\Rightarrow 7.5 \times 10^{-12} = \sqrt{\frac{150}{V}}$$

$$V = \frac{80}{3} \text{ kV}$$

$$\text{Energy} = \frac{80}{3} \text{ keV} \approx 25 \text{ keV}$$

2. (d)



$$d\tau = (\mu dN)x$$

$$= \mu \left(\frac{F}{\pi R^2} \times 2\pi \times dx \right) x$$

$$\tau = \int_0^R d\tau$$

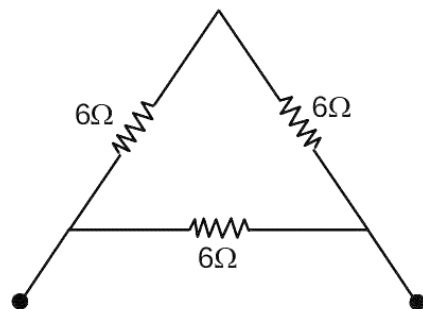
3. (c) $V_{AJ} = IR_{AJ}$

$$\frac{E}{3} = \left(\frac{E}{12r + 3r} \right) \times \left(\frac{x}{L} \times 12r \right)$$

4. (a)

5. (c)

6. (a)



$$R_{\text{net}} = 4\Omega$$

7. (d)

$$W = F \cos \theta$$

$$= m \left(g + \frac{g}{2} \right) \times \left[\frac{1}{2} \times \frac{g}{2} \times t^2 \right]$$

8. (a) For maxima,

$$d \sin \theta = n \lambda$$

$$\lambda = \frac{d \theta}{n} \quad (\because \sin \theta \approx \theta \text{ when } \theta \text{ is small})$$

$$= \frac{2500}{n} \text{ nm}$$

Take $n = 4, 5$ for $\lambda = 625 \text{ nm}$ and 500 nm .

9. (a) Area $\propto \pi (\text{Range})^2 \propto v^4$

$$\therefore \frac{A_1}{A_2} = \left(\frac{1}{2} \right)^4$$

10. (a)

$$\frac{128 \text{ kg}}{\text{m}^3} = \frac{64(2 \text{ kg})}{10 \text{ cm}^3}$$

$$= \frac{64(2 \text{ kg})}{10^6(50 \text{ cm})^3} \times 50^3$$

$$= \frac{64 \times 50 \times 50 \times 50}{100 \times 100 \times 100} \times \frac{2 \text{ kg}}{(50 \text{ cm})^3} = 8 \frac{(2 \text{ kg})}{(50 \text{ cm}^3)}$$

11. (d) Work done by external agent = $U_f - U_i$

$$= 2 \text{ MB}$$

$$12. (a) \frac{\Delta Q}{\Delta t} = \frac{kA}{\ell} (T_2 - T_1)$$

$$\frac{1}{A} \left(\frac{\Delta Q}{\Delta t} \right) = \frac{k}{\ell} (T_2 - T_1)$$

13. (c) $C_{\text{net}} = C_1 + C_2 + C_3$

$$\frac{kAE_0}{d} = k_1 \left(\frac{A}{3} \right) \frac{E_0}{d} + k_2 \left(\frac{A}{3} \right) \frac{E_0}{d} + k_3 \left(\frac{A}{3} \right) \frac{E_0}{d}$$

$$k = \frac{k_1 + k_2 + k_3}{3} = 12$$

14. (d)

$$Q_1 + Q_2 + Q_3 = Q$$

$$\frac{Q_1}{4\pi a^2} = \frac{Q_2}{4\pi b^2} = \frac{Q_3}{4\pi c^2} = k$$

Subs. Q_1, Q_2, Q_3 in (1)

$$k = \frac{Q}{4\pi(a^2 + b^2 + c^2)}$$

$$v = \frac{kQ_1}{a} + \frac{kQ_2}{b} + \frac{kQ_3}{c}$$

15. (b)

$$n_1 = n_2 = n_3$$

$$\Rightarrow 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$$\Rightarrow T_2 T_3 = T_1 T_4 \text{ and } \frac{T_3^2}{T_2} = T_4$$

Solve for T_2 and T_3 .

16. (b) Initially, kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m \frac{GM_E}{r}$

By conservation of M.E.,

$$\frac{-GM_E m}{r} + KE = 0$$

$$KE = \frac{GM_E m}{r} = mv^2$$

17. (a) $\frac{dv}{dt} = \phi - a\sqrt{2gh} = 0$ (for maximum height)

$$h = \frac{\phi^2}{2ga^2} = \frac{10^{-8}}{2 \times 9.8 \times 10^{-8}} = 5.1 \text{ cm}$$

18. (d) $f = \frac{n}{2\ell} \sqrt{\frac{T}{m}}$

On solving, $n = 5$,

5 loops are formed in 1 m.

$\therefore$ Separation between successive nodes $= \frac{1}{5} \text{ m} = 20 \text{ cm}$

19. (b) $f_1 = f_o \left(\frac{340}{340-17} \right)$; $f_2 = f_o \left(\frac{340}{340-34} \right)$

$$\frac{f_1}{f_2} = \frac{306}{323} \text{ or } \frac{18 \times 17}{19 \times 17} \text{ or } \frac{18}{19}$$

20. (b)

21. (c) $R = \overline{P + Q} = \overline{(\bar{x} + y) + (x\bar{y})}$

$$= \overline{(\bar{x} + y)} \cdot (x\bar{y})$$

$$= (x \cdot \bar{y}) \cdot (x\bar{y})$$

$$= x\bar{y}$$

22. (b)

23. (a) In 8 seconds count rate becomes $\frac{1}{16}$ times.

$\therefore$ 4 half lives = 8 s

1 half lie = 2 s

In 6 s or 3 half lives, count rate = $\frac{1600}{2^3} = 200$

24. (a) $E = vB$

$v = Ed = dvB$

25. (d)

26. (c)

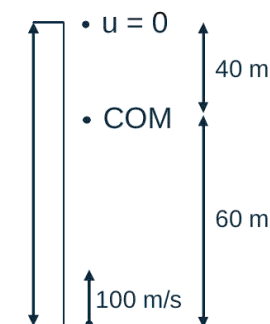
27. (a) By colour coding,

$$R = 50 \times 10^2 \Omega$$

$$I^2 R = P$$

$$I = \sqrt{\frac{P}{R}} = 20 \text{ mA}$$

28. (c)



$$Y_{cm} \text{ from ground} = \frac{0.03 \times 100}{0.05} = 60 \text{ m}$$

$$V_{cm} = \frac{0.02 \times 100}{0.05} = 40 \text{ m/s}$$

$$H = \frac{V_{cm}^2}{2g} = \frac{40 \times 40}{20} = 80 \text{ m}$$

$$\text{Height above building} = 80 - 40 = 40 \text{ m}$$

29. (c) $dM = diA$

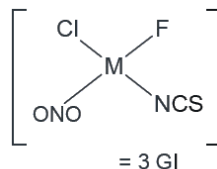
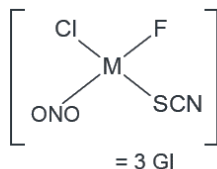
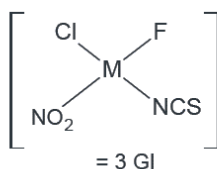
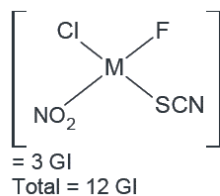
$$= \left(\frac{dq\omega}{2\pi} \right) \pi x^2$$

$$= (\rho dx) \frac{\omega}{2\pi} \pi x^2$$

$$M = \int_0^L dM$$

30. (d)

31. (d)

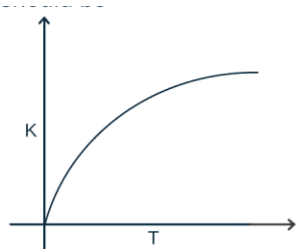


32. (c)
33. (b)
34. (a)
35. (a)
36. (a)
37. (a)
38. (a)

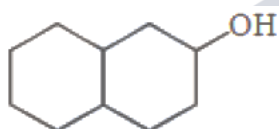
$$K = Ae^{-E_a/RT}$$

(i) is true but (ii) is false

(ii) should be

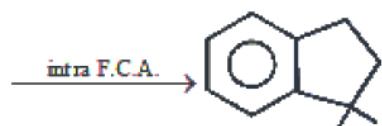
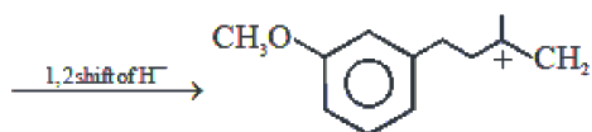
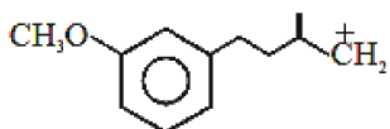
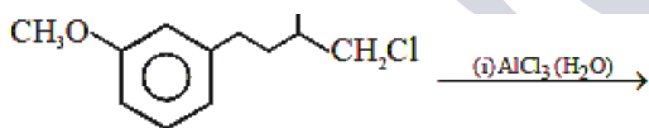


39. (c) answer should be

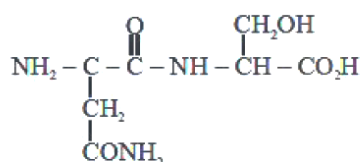


40. (c)

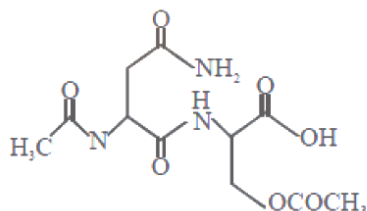
41. (a) Density of DCM is higher than H_2O .
42. (a) Adipic acid will give unstable 7 membered anhydride.
43. (b) Electron withdrawing group enhances the rate of hydrolysis.
44. (C) $K.E = h\nu - h\nu_0$,
45. (b)
46. (c) Atomic & ionic radii decreases
47. (b)
48. (d)



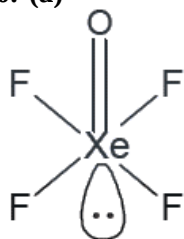
49. (b)



P is



50. (a)

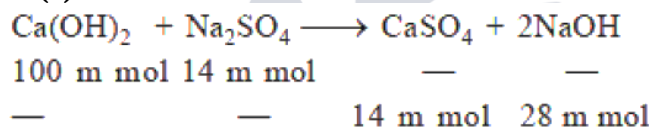


51. (b)

52. (b)

53. (d)

54. (a)



$$w_{\text{CaSO}_4} = 14 \times 10^{-3} \times 136 = 1.9 \text{ gm}$$

$$[\text{OH}^-] = \frac{28}{100} = 0.28 \text{ M}$$

$$55. (b) P_A = \chi_A P_A^0 = Y_A P_T$$

$$P_T = \chi_A P_A^0 + \chi_B P_B^0 = 0.4 \times 7 \times 10^3 + 0.6 \times 12 \times 10^3 = 10^4$$

$$0.4 \times 7 \times 10^3 = Y_A \times 10^4$$

$$Y_A = 0.28$$

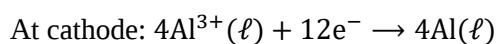
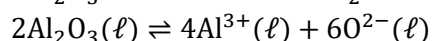
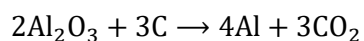
$$Y_B = 1 - 0.28 = 0.72$$

56. (d) NaBH_4 don't reduce ester.

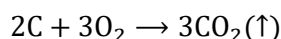
57. (d)

58. (d) Higher the K_a value lower the $\text{p}K_a$ value.

59. (c) Hall-Herolut's process is given by



At anode: $6\text{O}^{2-}(\ell) \rightarrow 3\text{O}_2(\text{g}) + 12\text{e}^-$



60. (d)

$$\text{N}_2^{\oplus} \Rightarrow \text{B.O.} = 2.5 \Rightarrow [\pi - \text{Bond} = 2 \& \sigma - \text{Bond} = \frac{1}{2}]$$

$$\text{N}_2 \Rightarrow \text{B.O.} = 3.0 \Rightarrow [\pi - \text{Bond} = 2 \& \sigma - \text{Bond} = 1]$$

$$\text{O}_2^{\oplus} \Rightarrow \text{B.O.} = 2.5 \Rightarrow [\pi - \text{Bond} = 1.5 \& \sigma - \text{Bond} = 1]$$

$$\text{O}_2 \Rightarrow \text{B.O.} = 2 \Rightarrow [\pi - \text{Bond} = 1 \& \sigma - \text{Bond} = 1]$$

61. (c) P(7 or 8)

$$= P(\text{H})P(7 \text{ or } 8) + P(\text{T})P(7 \text{ or } 8) = \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{11}{72} + \frac{1}{9} = \frac{19}{72}$$

62. (a) Let P be the point nearest to $(\frac{3}{2}, 0)$, then normal at P will pass through $(\frac{3}{2}, 0)$.

Let Co - ordinates of P be $s(\frac{t^2}{4}, \frac{t}{2})$

Hence equation of normal is $y + tx = \frac{t}{2} + \frac{t^3}{4}$

The line passes through $(\frac{3}{2}, 0)$

$$\frac{3t}{2} = \frac{t}{2} + \frac{t^3}{4} \Rightarrow t = 2 \text{ } (-2, 0 \text{ are rejected})$$

hence nearest point is (1,1)

$$\text{distance} \sqrt{(\frac{3}{2} - 1)^2 + (1 - 0)^2} = \frac{\sqrt{5}}{2}$$

$$63. (\text{b}) \vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i} - 7\hat{j} + 7\hat{k}$$

Equation of plane is $-7(x - 4) - 7(y + 1) + 7(z - 2) = 0$

$$\Rightarrow -7x - 7y + 7z + 7 = 0 \Rightarrow x + y - z = 1$$

$$64. (\text{b}) \mu = \frac{1+3+8+x+y}{5}$$

$$25 = 12 + x + y \Rightarrow x + y = 13$$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

$$9.2 = \frac{1 + 9 + 64 + x^2 + y^2}{5} - 25$$

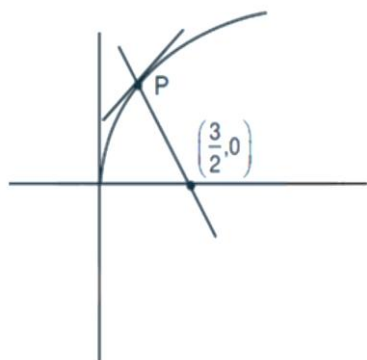
$$34.2 \times 5 = 74 + x^2 + y^2$$

$$171 = 74 + x^2 + y^2$$

$$97 = x^2 + y^2$$

$$(x + y)^2 = x^2 + y^2 + 2xy$$

$$169 - 97 = 2xy \Rightarrow xy = 36$$



$$T = 4,9$$

$$\text{So ratio is } \frac{4}{9} \text{ or } \frac{9}{4}$$

65. (c) $5, 5r, 5r^2$ sides of triangle,

$$5 + 5r > 5r^2$$

$$5 + 5r^2 > 5r$$

$$5r + 5r^2 > 5$$

$$\text{From } r^2 - r - 1 < 0$$

$$\left[r - \left(\frac{1 - \sqrt{5}}{2}, \frac{1 + \sqrt{5}}{2} \right) \right]$$

From (2)

$$r^2 - r + 1 > 0 \Rightarrow r \in R$$

From (3)

$$r^2 + r - 1 > 0$$

$$\text{So, } \left(r + \frac{\sqrt{1+\sqrt{5}}}{2} \right) \left(r + \frac{1-\sqrt{5}}{2} \right) > 0$$

$$r \in \left(-\infty, \frac{1 + \sqrt{5}}{2} \right) \cup \left(-\frac{1 - \sqrt{5}}{2}, \infty \right)$$

$$\text{From (4), (5), (6) } r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2} \right)$$

$$66. (d) \sum_{i=1}^{20} \left(\frac{{}^{20}C_{1-i}}{{}^{20}C_1 + {}^{20}C_{t-1}} \right)^3$$

$$\text{Now } \frac{{}^{20}C_{1-1}}{{}^{20}C_1 + {}^{20}C_{1-1}} = \frac{{}^{20}C_{1-1}}{{}^{21}C_1} = \frac{1}{21}$$

Let given sum be S, so

$$S = \sum_{i=1}^{20} \frac{(i)^3}{21^3} = \frac{1}{(21)^3} \left(\frac{20 \cdot 21}{2} \right)^2 = \frac{100}{21}$$

Given $S = \frac{k}{21} \Rightarrow k = 100$

67. (c) $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

Let $\cos^2 2\theta = t$

$$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$$

$$\Rightarrow t = \frac{1}{2} \Rightarrow \cos^2 2\theta = \frac{1}{2}$$

$$\Rightarrow 2\cos^2 2\theta - 1 = 0 \Rightarrow \cos 4\theta = 0$$

$$\Rightarrow 4\theta = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{8} \Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \in \left[0, \frac{\pi}{2}\right]$$

Sum of values of θ is $\frac{\pi}{2}$

68. (d)

69. (a) $\frac{dy}{dx} + (3\sec^2 x)y = \sec^2 x$

This is linear differential equation

Integrating factor = $e^{\int 3\sec^2 x dx} = e^{3\tan x}$

Hence $y \cdot e^{3\tan x} = e^{\int 3\tan x \cdot \sec^2 x dx}$

$$\Rightarrow ye^{3\tan x} = \frac{e^{3\tan x}}{3} + c$$

$$\Rightarrow y = Ce^{-3\tan x} + \frac{1}{3}$$

Given $y\left(\frac{\pi}{4}\right) = \frac{4}{3} \Rightarrow \frac{4}{3} = Ce^{-3} + \frac{1}{3}$

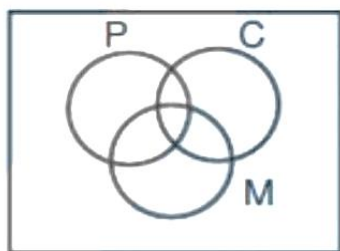
$$\Rightarrow C = e^3$$

Hence $y\left(\frac{\pi}{4}\right) = e^3 \cdot e^3 + \frac{1}{3} = e^6 + \frac{1}{3}$

70. (d) $n(p) = \left[\frac{140}{3}\right] = 46$

$$n(C) = \left[\frac{140}{5}\right] = 28$$

$$n(M) = \left[\frac{140}{2}\right] = 70$$



$$n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(M \cap P) + n(P \cap M \cap C)$$

$$= 46 + 28 + 70 - \left[\frac{140}{15} \right] - \left[\frac{140}{10} \right] - \left[\frac{140}{6} \right] + \left[\frac{140}{30} \right]$$

$$= 144 - 9 - 14 - 23 + 4 = 102$$

$$\text{So required number of student} = 140 - 102 = 38$$

71. (a) In the expansion of $(1 + x^{\log_2 x})^5$

$$\text{third term say } T_3 = {}^5C_2 (x^{\log_2 x})^2 = 2560$$

$$\Rightarrow (x^{\log_2 x})^2 = 256$$

taking logarithm to the base 2 on both sides

$$\Rightarrow 2(\log_2 x)^2 = 8 \Rightarrow (\log_2 x) = \pm 2$$

$$\Rightarrow x = 4, \frac{1}{4}$$

$$\text{Here } x = \frac{1}{4}$$

72. (a,b,c,d)

Normal to there 2 curves are

$$y = m(x - c) - 2bm - bm^3$$

$$y = mx - 4am - 2am^3$$

If they have a common normal

$$(c + 2b) - 4a = (2a - b)m^2 (m = 0 \text{ corresponds to axis})$$

$$\Rightarrow m^2 = \frac{c}{2a - b} - 2 > 0$$

$$\frac{c}{2a - b} > 2$$

According to the question answer is (1,2,3,4) but maybe in question they want common normal other than x - axis, hence answer is (b)

$$73. (b) x + y - z = 5$$

$$x + 2y + 3z = 9,$$

$$x + 3y + \alpha z = \beta$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 0 \end{vmatrix} = 0 \Rightarrow (2\alpha - 9) + (3 - \alpha) + (3 - 2) = 0 \Rightarrow \alpha = 5$$

$$\text{Now, } D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \beta \end{vmatrix} = 0 \Rightarrow 2\beta - 27 + 9\beta - 5(3 - 2) = 0 \Rightarrow \beta = 13$$

$\Rightarrow$ at $\alpha = 5, b = 13$ above 3 planes from common line

$$\begin{aligned} 74. (b) \lim_{x \rightarrow 1^+} \frac{(1 - |x| + \sin |1 - x|) \sin \left([1 - x] \frac{\pi}{2} \right)}{|1 - x| [1 - x]} \\ = \lim_{x \rightarrow 1^+} \frac{(1 - x + \sin(x - 1)) \sin \left(-\frac{\pi}{2} \right)}{(x - 1)(-1)} \\ = \lim_{x \rightarrow 1} \frac{-(x - 1) + \sin(x - 1)}{(x - 1)} = -1 + 1 = 0 \end{aligned}$$

$$\begin{aligned} 75. (a) |A| &= \begin{vmatrix} -2 & 4 + d & (\sin \theta - 2) \\ 1 & (\sin \theta) + 2 & d \\ 5 & (2\sin \theta) - d & (-\sin \theta) + 2 + 2d \end{vmatrix} \\ &= \begin{vmatrix} -2 & 4 + d & (\sin \theta - 2) \\ 1 & (\sin \theta) & d \\ 1 & 0 & 0 \end{vmatrix} \quad (\text{New } R_3 = R_3 - 2R_2 + R_1) \\ &= (4 + d)d - \sin^2 \theta + 4 = (d + 2)^2 - \sin^2 \theta \end{aligned}$$

Because minimum value of $|A| = 8 \Rightarrow (d + 2)^2 = 9 \Rightarrow d = 1$ or -5

76. (BONUS)

$$\left| \frac{3z_1}{2z_2} \right| = 2$$

$$\begin{aligned} \text{Let } \frac{3z_1}{2z_2} &= 2\cos \theta + 2(\sin \theta)i \\ \Rightarrow \frac{2z_2}{3z_1} &= \frac{1}{2}\cos \theta - \frac{1}{2}(\sin \theta)i \end{aligned}$$

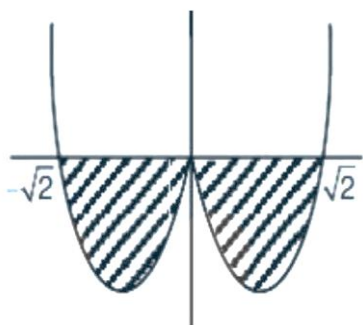
$$\text{Given, } z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} = \frac{5}{2}\cos \theta + \frac{3}{2}(\sin \theta)i$$

Which is neither purely real nor purely imaginary and $|z|$ depends on θ .

77. (d)

$$\int_a^b (x^4 - 2x^2) dx$$

From figure min area is $(-\sqrt{2}, \sqrt{2})$



78. (c) Let point P is (α, β) and centroid of $\triangle PQR$ is (h, k) , then $3h = \alpha + 1 + 3$ and $3k = \beta + 4 - 2$
 $\Rightarrow \alpha = 3h - 4$ and $\beta = 3k - 2$

Because (α, β) lies on $2x - 3y + 4 = 0$

$$\Rightarrow 2(3h - 4) - 3(k - 2) + 4 = 0$$

$$\Rightarrow \text{locus is } 6x - 9y + 2 = 0 \text{ whose slope is } \frac{2}{3}$$

79. (b) From the graph we can easily conclude that $f(x)$ is non - derivable at $x = -2, -1, 0, 1, 2$

80. (c) Tangent at $(1, -1)$ is $x(1) + y(-1) + 2(x+1) - 3(y-1) - 12 = 0$

$$= 3x - 4y = 7$$

$$\text{Required circle is } (x-1)^2 + (y+1)^2 + \lambda(3x - 4y - 7) = 0$$

It pass through $(4, 0)$

$$\Rightarrow 9 + 1 + \lambda(12 - 7) = 0 \Rightarrow \lambda = -2$$

$$\Rightarrow \text{required circle is } x^2 + y^2 - 8x + 10y + 16 = 0$$

$$\Rightarrow \text{Radius} = \sqrt{16 + 25 - 16} = 5$$

81. (c)

$$f(x) = x^3 + ax^2 + bx + c$$

$$f'(x) = 3x^2 + 2ax + b$$

$$f''(x) = 6x + 2a$$

$$f'''(x) = 6a = f'(1) = 3 + 2a + b \Rightarrow a + b = -3$$

$$b = f'' = 12 + 2a \Rightarrow 2a - b = -12$$

$$c = f'''(3) \Rightarrow c = 6 \text{ and } a = -5, b = 2$$

$$\Rightarrow f(x) = x^3 - 5x^2 + 2x + 6$$

$$\Rightarrow f(2) = 8 - 20 + 4 + 6 = -2$$

82. (b) Because $b = 2\vec{a}$, so $3 - \lambda_2 = 2\lambda_1$

Because a is perpendicular to c so $6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$

$$\Rightarrow (\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1) \text{ where } \lambda_1 \in \mathbb{R}$$

$$\Rightarrow \left(-\frac{1}{2}, 4, 0\right) \text{ satisfied above triplet.}$$

83. (a) Let A is $(1 - 3\mu, \mu - 1, 2 + 5\mu)$

$$\overrightarrow{AB} = (3\mu + 2)\mathbf{i} + (3 - \mu)\mathbf{j} + (4 - 5\mu)\mathbf{k}$$

$\hat{k}$ which is parallel to plane $x - 4y + 3z = 1$

$$\Rightarrow 1(3\mu + 2) - 4(3 - \mu) + 3(4 - 5\mu) = 0$$

$$= -8\mu + 2 = 0 \Rightarrow \mu = \frac{1}{4}$$

84. (c) $\int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$

$$= \int \frac{(t^n - t)^{\frac{1}{n}} dt}{t^{n+1}} \quad (\text{Put } \sin \theta = t)$$

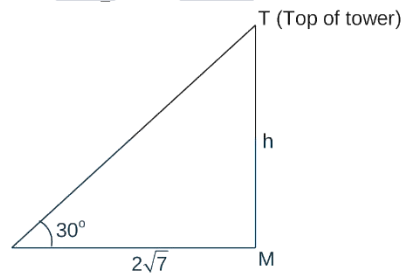
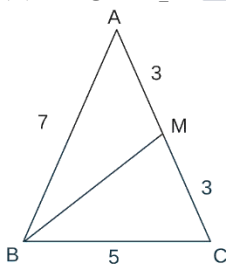
$$= \int \frac{t \left(1 - \frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^{n+1}} dt = \int \frac{\left(1 - \frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^n} dt$$

$$\text{Put } 1 - \frac{1}{t^{n-1}} = z \Rightarrow \frac{(n-1)}{t^n} dt = dz,$$

$$\Rightarrow I = \frac{1}{n-1} \int z^{\frac{1}{n}} dz = \frac{z^{\frac{1}{n}+1}}{\left(\frac{1}{n}+1\right)(n-1)} + c = \frac{n(1 - t^{1-n})^{n+1}}{n^2 - 1} + c$$

$$= \frac{n}{n^2 - 1} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + c$$

85. (b) Length of median $BM = \frac{1}{2} \sqrt{2(BC^2 + BA^2) - (AC)^2} = \frac{1}{2} \sqrt{2(25 + 49) - 36}$



$$= \frac{1}{2} \sqrt{112} = \sqrt{\frac{112}{4}} = \sqrt{28} = 2\sqrt{7}$$

Let h be height of tower, given $\tan 30^\circ = \frac{h}{2\sqrt{7}} \Rightarrow h = 2\sqrt{\frac{7}{3}} = \sqrt{\frac{28}{3}}$

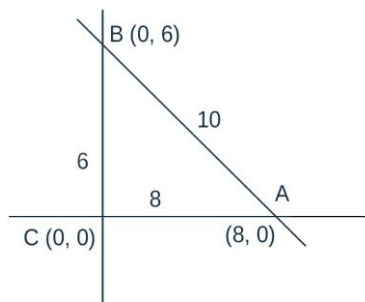
86. (a) Hyperbola is $\frac{x^2}{5} - \frac{y^2}{4} = 1$

Equation of its tangent in slope form is $y = mx \pm \sqrt{5m^2 - 4}$

Hence tangent with slope 1 is $y = x \pm 1$

87. (b) $I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$

$$= \left(\frac{8(60) + 6(0) + 10(0)}{24}, \frac{10(0) + 6(8) + 8(0)}{24} \right) = (2, 2)$$



88. (d) Two -digit numbers of the form $7\lambda + 2$ are 16, 23, 93

Two digit numbers of the form $7\lambda + 5$ are 12, 19, ... , 96

Sum of all the above numbers equals to $\frac{12}{2}(16 + 93) + \frac{13}{2}(12 + 96) = 654 + 702 = 1356$

89. (b) $y = kx^2, x = ky^2$

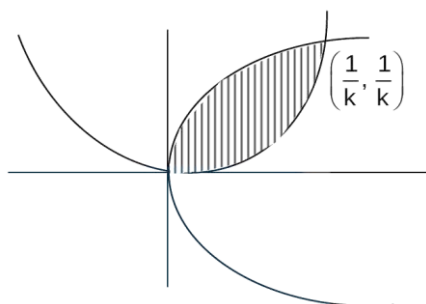
$$\Rightarrow x = k(k^2x^4) \Rightarrow x = 0 \text{ or } x^3 = \left(\frac{1}{k}\right)^3 \Rightarrow x = \frac{1}{k}, 0$$

Point of intersection are $\left(\frac{1}{k}, \frac{1}{k}\right)$ and $(0, 0)$

$$\text{Area} = \int_0^{1/k} \left(\sqrt{\frac{x}{k}} - kx^1 \right) dx = 1 \Rightarrow \left(\frac{1}{\sqrt{k}} \frac{x^{3/2}}{3/2} - \frac{kx^3}{3} \right)^{1/k}$$

$$= 1 \Rightarrow \frac{2}{3k^2} = 1 \Rightarrow k^2 = \frac{1}{3}$$

$$k = \frac{1}{\sqrt{3}}$$



90. (a) $P(n) = n^2 + 41$

$$P(3) = 9 - 3 + 41 = 47$$

$$P(5) = 25 - 5 + 41 = 61$$

Hence $P(3)$ and $P(5)$ are both prime