1. (b) $\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{j} + 4\hat{j})4$

$= 10\hat{\imath} + 8\hat{\jmath} + 8\hat{\imath} + 8\hat{\jmath}$

$\vec{r}_2 - \vec{r}_1 = 18\hat{\imath} + 16\hat{\jmath}$

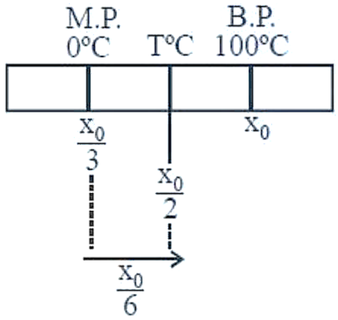
$\vec{r}_2 = 20\hat{\imath} + 20\hat{\jmath}$

$|\vec{r}_f| = 20\sqrt{2}$

2. (a) $t = \frac{x_t - x_0}{x_{100} - x_0} 100°C$

$$\frac{\frac{x_0}{2} - \frac{x_0}{3}}{x - \frac{x_0}{3}} 100°C$$

$= 25°C$



3. (c) $R_g = 20\Omega$

$N_L = N_g = N = 30$

FOM $= \frac{1}{\phi} = 0.005$ A/Div.

Current sentivity $= CS = \left(\frac{1}{0.005}\right) = \frac{\phi}{1}$

$Ig_{max} = 0.005 \times 30$

$= 15 \times 10^{-2} = 0.15$

$15 = 0.15[20 + R]$

$100 = 20 + R$

$R = 80.$

4. (c) $\frac{R_1}{R_2} = \frac{2}{3}$

$$\frac{R_1 + 10}{R_2} = 1$$

$\Rightarrow R_1 + 10 = R_2$

$\frac{2R_2}{3} + 10 = R_2 \,;\; 10 = \frac{R_2}{3}$

$\Rightarrow R_2 = 30\Omega \,\&R_1 = 20\Omega$

$30 \times R$

$\frac{30 + R}{30} = \frac{2}{3}$

$R = 60\Omega$

5. (b) $MR^2\ I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$

$= \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2 = 3$

6. (a) $2.5 = 1 \times 5\sin\theta$

$\text{Sin}\,\theta = 0.5 = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

7. (c) Total length L will remain constant

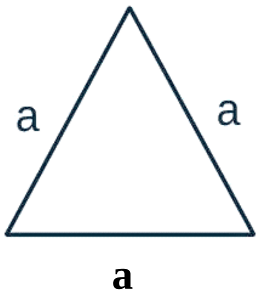
$L = (3a)N$

($N =$ total turns)

And length of winding $= (d)N = \ell$

$(d =$ diameter of wire$)$

Self inductance $= \mu_0 n^2\ A\ell$



$= \mu_0 n^2\left(\frac{\sqrt{3}a^2}{4}\right)dN$

$\propto a^2\ N \propto a$

So self inductance will become 3 times.

8. (b)

$\frac{dp}{dt} = F = kt$

$$\int_P^{3P} dP = \int_0^T ktdt$$

$$2p = \frac{KT^2}{2};\ T = 2\sqrt{\frac{p}{k}}$$

9. (d)

$$\chi = \frac{\mathrm{I}}{\mathrm{H}}$$

$$I = \frac{\text{Magnetic moment}}{\text{Volume}}$$

$$I = \frac{20 \times 10^{-6}}{10^{-6}} = 20\ \mathrm{N/m^2}$$

$$\chi = \frac{20}{60 \times 10^3} = \frac{1}{3} \times 10^{-3}$$

$$= 0.33 \times 10^{-3} = 3.3 \times 10^{-4}$$

10. (a) Angular frequency of pendulum

$$\omega \propto \sqrt{\frac{g_{\text{eff}}}{\ell}}$$

$$\therefore\ \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{\text{eff}}}{g_{\text{eff}}}$$

$$\Delta\omega = \frac{1}{2}\frac{\Delta g}{g} \times \omega$$

[ω_s = angular frequency of support]

$$\frac{\Delta\omega}{\omega} = \frac{1}{2} \times \frac{\Delta \mathrm{g}}{\mathrm{g}}$$

$$= \frac{1}{2} \times \frac{2(\mathrm{A}\omega_5^{\mathrm{s}})}{10}$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = \frac{1 \times 10^{-2}}{10} = 10^{-3}$$

11. (b) $I = \frac{6}{300} = 0.002$ (D_2 is in reverse bias)

12. (b)

$$U = -\vec{P} \cdot \vec{E}$$

$$= -\mathrm{PE}\cos\theta$$
$$= -(10^{-29})(10^3)\cos 45^\circ$$
$$= -0.707 \times 10^{-26}\ \mathrm{J}$$
$$= -7 \times 10^{-27}\ \mathrm{J}$$

13. (d) $0.1 \times 400 \times (500 - \mathrm{T}) = 0.5 \times 4200 \times (T - 30) + 800(T - 30)$

$\Rightarrow\ 40(500 - T) = (T - 30)(2100 + 800)$

$\Rightarrow 20000 - 40\,T = 2900\,T - 30 \times 2900$

$\Rightarrow 20000 + 30 \times 2900 = T(2940)$

$T = 30.4^{\circ}C$

$$\frac{\Delta T}{T} \times 100 = \frac{6.4}{30} \times 100 \simeq 20\%$$

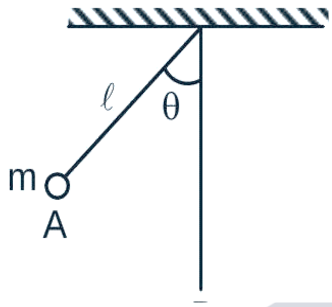
14. BONUS

15. (a) Maximum kinetic energy at lowest point B is given by

$K = mg\ell(1 - \cos\theta)$

where θ = angular amp.

$K_1 = mg\ell(1 - \cos\theta)$



$K_2 = mg(2\ell)(1 - \cos\theta)$

B

$K_2 = 2\,K_1$

16. (a) $\Delta\ell_1 = \Delta\ell_2$

$\ell\alpha_1\Delta T_1 = \ell\alpha_2\Delta T_2$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta T_1}{\Delta T_2}\ ;\ \frac{4}{3} = \frac{T - 30}{180 - 30}$$

$T = 230^{\circ}C$

17. (c)

$$\frac{F}{A} = y \cdot \frac{\Delta\ell}{\ell}; [Y] = \frac{F}{A}$$

Now from dimension

$$F = \frac{ML}{T^2}; L = \frac{F}{M} \cdot T^2$$

$$L^2 = \frac{F^2}{M^2}\left(\frac{V}{A}\right)^4 \quad \because\ T = \frac{V}{A}$$

$$L^2 = \frac{F^2}{M^2\,A^2}\frac{V^4}{A^2} \quad F = MA$$

$$L^2 = \frac{V^4}{A^2}$$

$$[Y] = \frac{[F]}{[A]} = F^1\,V^{-4}\,A^2$$

18. (b)

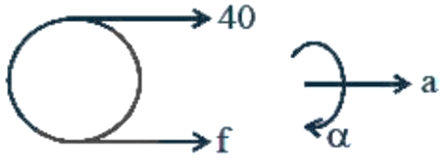
$0 + f = m(R\alpha)$

$40 \times R - f \times R = mR = 2^{\alpha}$

$40 - f = mR\alpha$

From (i) and (ii)

$\alpha = \dfrac{40}{mR} = 16$



19. (d) Intensity of EM wave is given by

$$\mathrm{I} = \frac{\text{Power}}{\text{Area}} = \frac{1}{2}\varepsilon_0 \mathrm{E}_0^2 \mathrm{C}$$

$$= \frac{27 \times 10^{-3}}{10 \times 10^{-6}} = \frac{1}{2} \times 9 \times 10^{-2} \times \mathrm{E}^2 \times 3 \times 10^8$$

$$E = \sqrt{2} \times 10^3 \mathrm{kV/m}$$

$$= 1.4 \mathrm{kV/m}$$

20. (b) Potential difference across AB will be equal to battery equivalent across CD.

$$V_{AB} = V_{CD} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} = \frac{\frac{1}{1} + \frac{2}{1} + \frac{3}{1}}{\frac{1}{1} + \frac{1}{1} + \frac{1}{1}}$$

$$= \frac{6}{3} = 2\ \mathrm{V}$$

21. (d) $\because g = \frac{GM}{R^2}$

$$\frac{g_p}{g_e} = \frac{M_p}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

Also, $T \propto \frac{1}{\sqrt{g}}$

$$\Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$$

$$\Rightarrow T_p = 2\sqrt{3}s$$

22. (C) Analysis of graph says

(1) Amplitude varies as $8 - 10$ V or 9 ± 1

(2) Two time period $100\mu s$ (signal wave) and $8\mu s$ (carrier wave)

Hence signal is $\left[9 \pm 1\sin\left(\frac{2\pi t}{T_1}\right)\right]\sin\left(\frac{2\pi t}{T_2}\right)$

$= 9 \pm 1\sin(2\pi \times 10^4 \mathrm{t})\sin 2.5\pi \times 10^5 \mathrm{t}$

23. (a) $\mathrm{VT} = \mathrm{K}$

$\Rightarrow V\left(\frac{PV}{nR}\right) = k$

$\Rightarrow PV^2 = K$

$\because C = \frac{R}{1-x} + C_V$ (For polytropic process)

$$C = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2}$$

$\therefore \Delta Q = nC\Delta T$

24. (c) $100 \times S_A \times [100 - 90] = 50 \times S_B \times (90 - 75)$

$2\,S_A = 1.5\,S_B$

$S_A = \frac{3}{4}\,S_B$

Now, $100 \times S_A \times [100 - T] = 50 \times S_B(T - 50)$

$2 \times \left(\frac{3}{4}\right)(100 - T) = (T - 50)$

$300 - 3\,T = 2\,T - 100$

$400 = 5\,T$

$T = 80$

25. (d)

For M → L steel

$\frac{1}{\lambda} = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{K \times 5}{36}$

for N → L

$\frac{1}{\lambda'} = K\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{K \times 3}{16}$

$\lambda' = \frac{20}{27}\lambda$

26. (c) $i = e$

$r_1 = r_2 = \frac{A}{2} = 30°$

by Snell's law

$1 \times \sin i = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

$i = 60$

27. (b) For diffraction

Location of 1^{st} minima

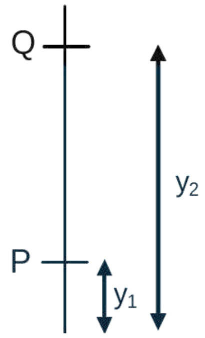
$y_1 = \frac{D\lambda}{a} = 0.2469D\lambda$

Location of 2^{nd} minima

$y_2 = \frac{2D\lambda}{a} = 0.4938D\lambda$

Now for interference

Path for interference



Path difference at P.

$\frac{dy}{D} = 4.8\lambda$

Path difference at P.

$\frac{dy}{D} = 9.6\lambda$

So orders of maxima in between P and Q is

5,6,7,8,9

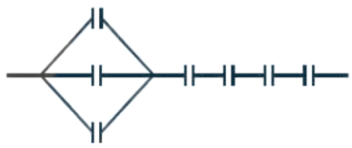
So 5 bright fringes all present between $P\&Q$.

28. (b)

$C_{eq} = \frac{6}{13}\mu F$

Therefore three capacitors most be in parallel to get 6 in

$$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$



$C_{eq} = \frac{3C}{13} = \frac{6}{13}\mu F$

29. (b) $\vec{F}_{net} = d\vec{E} + q(\vec{V} \times \vec{B})$

$= (2q\hat{\imath} + 3q\hat{\imath}) + q(\vec{v} \times \vec{B})$

$W = \vec{F}_{net} \cdot \vec{S}$

$= 2q + 3q$

$= 5q$

30. (c) $\frac{hc}{\lambda_1} = \phi + eV_1$

$\frac{hc}{\lambda_2} = \phi + eV_2$

(i) - (ii)

$$hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = e(V_1 - V_2)$$

$$\Rightarrow V_1 - V_2 = \frac{hc}{e}\left(\frac{\lambda_2 - \lambda_1}{\lambda_1 - \lambda_2}\right)$$

$$= (1240 \text{ nm V}) \frac{100 \text{ nm}}{300 \text{ nm} \times 400 \text{ nm}}$$

$$= \frac{12.4}{12} \approx 1 \text{ V.}$$

31. (c) In order to be spontaneous $\Delta G°$ should be -ve

$\Delta G° = \Delta H° - T\Delta S°$

$0 = 491.1 \times 10^3 - T \times 198$

$T = \frac{491100}{198} = 2480$

If temp is above 2480 K, the reaction will be spontaneous.

32. (a)

33. (d) Th is a metal having large size and oxalate is a bidentate ligand hence its co-ordination number in given complex is 10 .

34. (c) Since $LiAlH_4$ is a strong reducing agent, it will reduce $-NO_2, \overset{I}{O} - O - H$ and ketonic group but cannot reduce normal alkene $> C = C <$

35. (a) $\Delta G° = \Delta H° - T\Delta S°$

$\Delta G^0 = A - BT$

In endothermic reaction $\Delta H = +ve$. Hence, $A = +ve$

36. (d) $\sqrt{3}a = 4R$

$R = \frac{\sqrt{3}a}{4}$

$2(R + r) = a$

$2\left(\frac{\sqrt{3}a}{4} + r\right) = a$

$\frac{\sqrt{3}a}{2} + 2r = a$

$$2r = a - \frac{\sqrt{3}a}{2} = \frac{2a - \sqrt{3}a}{2}$$

$$r = \frac{2a - 1.7329}{4} = \frac{.268a}{4} = .067a$$

37. (a) SiH_4 has complete octet hence it is not an electron deficient hydride.

38. (d) $\Delta G^\circ = -nFE^\circ_{cell} = -2.303RT\log K_{eq}$

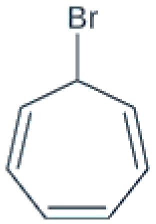
$$2\,F^0_{cell} = 2.303RT\log K_{eq}$$

$$2E^\circ_{cell} = \frac{2.303RRT}{F}\log 10 \times 10^{15}$$

$$E^\circ_{cell} = \frac{.059}{2} \times 16 = 8 \times .059 \Rightarrow .472$$

39. (b) Electronegativity increases from left to right in a period and decreases down the group.

40. (b)



this compound will produce aromatic cation which is table

41. (d) $\lambda = \frac{h}{mv}$

According to Einstein's theory of photoelectric effect:

$$hv = hv_0 + KE$$

$$hv = hv_0 + \frac{1}{2}mvv^2$$

$$2h(v - v_0) = mv^2$$

$$\frac{2\,h(v - v_o)}{m} = v^2$$

$$v \propto (v - v_0)^{\frac{1}{2}}$$

$$\lambda \propto \frac{h}{m(v - v_o)^{\frac{1}{2}}}$$

$$\lambda \propto \frac{1}{(v - v_0)^{\frac{1}{2}}}$$

42. (d)

43. (b) After protonation at b or c or d the conjugate acid is stabilized by resonance.

44. (b) Acid rain reacts with marble. Hence, The Taj Mahal which made up of marble is discoloured.

45. (d) Inert pair effect gradually increases down the group. Hence, stability of lower oxidation state increases down the group.

46. (c) $\Delta G^0 = -2.303 RT \log K_{eq}$

$= -2.303 \times 8.314 \times 298 \log 10^{-14}$

$= -2.303 \times 8.314 \times 298 \times -14$

$= 79{,}881.87$

$\simeq 80 KJmol^{-1}$

47. (b) Calcination takes place in absence of air. Hence step 2 is not defining it.
48. (c)
49. (b)
50. (d)
51. (b) $\underset{1-\alpha}{K_2HgI_4}\Big] \rightleftharpoons \underset{2\alpha}{2}\, K^+ + \overset{\alpha}{[HgI_4]^{2-}}$

Total number of particle $= 1 + 2\alpha$

Hence, Van't Hoff factor $= \frac{1+2\alpha}{1}$

$= \frac{1 + 2 \times 0.4}{1} = 1 + 0.8 \Rightarrow 1.8$

52. (a) Apply law of equivalence:
$25 \times N = 30 \times 0.1 \times 2$

$N_{HCl} = \frac{30 \times 0.2}{25} = \frac{6}{5} \times 0.2 = \frac{1.2}{5}$

For the 2^{nd} titration

$\frac{1.2}{5} \times V_{HCl} = 30 \times 0.2$

$V_{HCl} = \frac{6 \times 5}{1.2} = \frac{30}{1.2} = 25 ml$

53. (c) For zero order reaction
$C_0 - C_t = Kt$

$0.5M - 0.2M = Kt$

$0.3 = Kt$

' K ' can be calculated by

$t_{1/2} = \frac{C_0}{2\,K}$

$6 = \frac{0.2}{2K}$

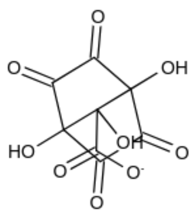
$K = \frac{0.2}{12} = \frac{2 \times 10^{-1}}{12} = \frac{1}{60}$

Putting the value of K in eq (1)

$$t = \frac{0.3}{K} = \frac{0.3}{1/60} = 60 \times 0.3 = 18\text{Hr}$$

54. (d)

55. (a) Attack on $\vec{C} = C/C$ is more preferred than benzene ring.
56. (d) First Markonikov's addition one alkene followed by intramolecular Friedel-Craft alkylation takes place.
57. (c) SO_2 gas causes stiffness of flower buds
58. (a)
59. (a) Go through structure of $[Co_2(CO)_8]$



60. (b) $MnO_2 \xrightarrow{4KOH, O_2} \underset{(Green)}{2} K_2MnO_4 + 2H_2O$

$3\ K_2MnO_4 \xrightarrow{4HCl} \underset{(Purple)}{2KMnO_4} + MnO_2 + 2H_2O$

$2KMnO_4 \xrightarrow{H_2O, KI} 2MnO_2 + 2KOH + KIO_3$

61. (a) $\frac{x\cos 4x \sin^2 2x}{\sin^2 x \cdot \cos^2 2x \cdot \sin 4x}$

$$= \frac{4x}{\sin 4x} \cdot \frac{\cos 4x}{\cos^2 2x} \cos^2 x$$

$\Rightarrow 1$ as $x \to 0$

62. (c) $(\cot^{-1}(x) - 5)(\cot^{-1}(x) - 2) > 0$

$\Rightarrow \cot^{-1}(x) \in (-\infty, 2) \cup (5, \infty)$

Put $0 < \cot^{-1}(x) < \pi$

$\Rightarrow \cot^{-1}(x) \Rightarrow (0,2)$

$\Rightarrow x \in (\cot 2, \infty)$

63. (a) 2 b = 5 and 2ae = 13

$(ae)^2 = a^2 + b^2$

$$\Rightarrow a^2 = (ae)^2 - b^2 = \frac{169}{4} - \frac{25}{4}$$

$\Rightarrow a = 6$

$$e = \frac{ae}{a} = \frac{13}{12}$$

64. (d) $y^2 = -4(x - a^2)$

Vertices of triangle are $(a^2, 0)$ and $(0, 2a)$ and $(0, -2a)$

Area $= \frac{1}{2}(a^2)(4a) = 250$

$\Rightarrow a^3 = 125$

65. (a) Points on the given lines are $(\lambda + 3, 3\lambda - 1, -\lambda + 6)$ and $(7\alpha - 5, -6\alpha + 2, 4\alpha + 3)$

$\Rightarrow \lambda + 3 = 7\alpha - 5$

$3\lambda - 1 = -6\alpha + 2$

$\Rightarrow \alpha = 1, \lambda = -1$

Point R is $(2, -4, 7)$

Image of R under xy - plane is $(2, -4, -7)$

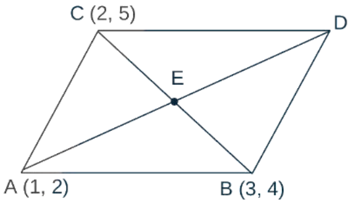
66. (c) Contra positive of $p \to q$ is $\sim q \to \sim p$

67. (a) E is $\left(\frac{5}{2}, \frac{9}{2}\right)$

Slope of $AD = \frac{5}{3}$

Equation of AD is $y - 2 = \frac{5}{3}(x - 1)$

$\Rightarrow 5x - 3y + 1 = 0$



68. (b)

69. (c) $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$

$= \frac{1}{\left(x^m + \frac{1}{x^m}\right)\left(y^n + \frac{1}{y^n}\right)}$

Put $x^m + \frac{1}{x^m} \geq 2$

$\Rightarrow \frac{1}{\left(x^m + \frac{1}{x^m}\right)} \leq \frac{1}{2}$

$\Rightarrow$ Maximum Value $= \frac{1}{4}$

70. (d) $\sum_{r=1}^{101} {}^{101}C_r s_{r-1}$

$= \sum_{r=1}^{101} {}^{101}C_r \frac{q^r - 1}{q - 1}$

$= \frac{1}{q-1}\left(\sum_{r=1}^{101} {}^{101}C_r q^r - \sum_{r=1}^{101} {}^{101}C_r\right)$

$= \frac{1}{q-1}\left((1+q)^{101} - 1 - 2^{101} + 1\right)$

$= \frac{\alpha}{2^{100}}\left(\frac{(1+q)^{101} - 2^{101}}{q-1}\right)$

$\Rightarrow \alpha = 2^{100}$

71. (c) Using quadratic formula,

$x = \frac{(\cos\theta\sin\theta + 1) \pm \sqrt{(\cos\theta\sin\theta + 1)^2 - 4\sin\theta\cos\theta}}{2\sin\theta}$

$= \frac{(\cos\theta\sin\theta + 1)^2 \pm (\cos\theta\sin\theta - 1)}{2\sin\theta}$

$= \cos\theta, \operatorname{cosec}\theta$

$\alpha = \cos\theta, \beta = \operatorname{cosec}\theta$

$\therefore \sum_{n=0}^{\infty} \alpha^n + \frac{(-1)^n}{\beta^n}$

$= \sum_{n=0}^{\infty} (\operatorname{cosec})^n + \sum_{n=0}^{\infty} (-\sin\theta)^n$

$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

72. (b) There are 30 white balls and 10 red balls

$\mathrm{P(\text{ white ball })} = \frac{30}{40} = \frac{3}{4} = \mathrm{p}$

$\Rightarrow \mathrm{q} = \frac{1}{4}$

$\frac{\text{mean(x)}}{\text{standarddeviation (x)}} = \frac{\mathrm{np}}{\sqrt{\mathrm{npq}}}$

$= \sqrt{\frac{np}{q}} = \sqrt{\frac{16 \times \left(\frac{3}{4}\right)}{\frac{1}{4}}} = 4\sqrt{3}$

73. (b) $z = x + iy$

$\sqrt{x^2 + y^2} + x + iy = 3 + i$

$\Rightarrow \mathrm{y} = 1$

$\sqrt{x^2 + 1} + x = 3 \Rightarrow x^2 + 1 = 9 - 6x + x^2$

$\Rightarrow \mathrm{x} = \frac{4}{3}$

$|z| = \sqrt{x^2 + y^2} = \frac{5}{3}$

74. (d) $R_1 \to R_1 + R_2 + R_3$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-a-c & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_3 \to C_3 - C_1, C_2 \to C_2 - C_1$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$= (a+b+c)^3$

$= (a+b+c)(x+a+b+c)^2$

75. (d) Equation of angle bisector of DA and OB is $y = x$

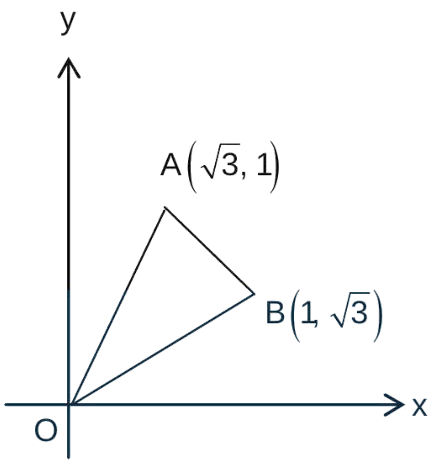
Given that, $\left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

$2\beta - 1 = \pm 3$

$\Rightarrow \beta = 2, -1$

Sum of values of $\beta = 1$

76. (c) $+18d = 0 \Rightarrow a = -18d$



$$\frac{t_{49}}{t_{29}} = \frac{a+48d}{a+28d} = \frac{-18d+48d}{-18d+28d}$$

$$= \frac{30d}{10d} = 3$$

77. (d) Put $2x - 1 = t^2$

$$\Rightarrow \int \frac{x+1}{\sqrt{2x-1}} dx$$

$$= \int \left(\frac{t^2+3}{2}\right) dt = \frac{t^3}{6} + \frac{3t}{2} + C$$

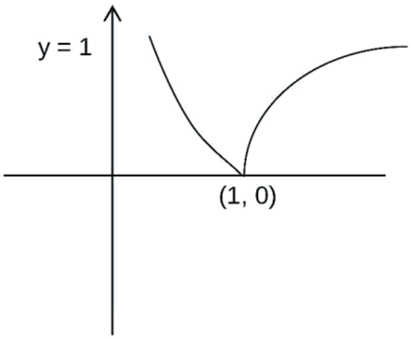
$= t\left(\frac{t^2}{6}+\frac{3}{2}\right)+C$

$= \sqrt{2x-1}\left(\frac{x+4}{3}\right)+C$

78. (BONUS)

$y = \left|1-\frac{1}{x}\right|$

Neither one - one nor Onto



79. (a) At $x=\pi+$ or $x=\pi-$

$f(x) = \sin x - x + 2(x-\pi)\cos x$

$f(x)$ is differentiable

For $x = 0+$

$f(x) = \sin x - x + 2(x-\pi)\cos x$
$f'(x) = \cos x - 1 + 2\cos x - 2(x-\pi)\sin x$

$f'(0+) = 2$

For $x = 0-$

$f(x) = -\sin x + x + 2(x-\pi)\cos x$

$f'(x) = -\cos x + 1 + 2\cos x - 2(x-\pi)\sin x$

$f'(0-) = 2$

LHD $=$ RHD

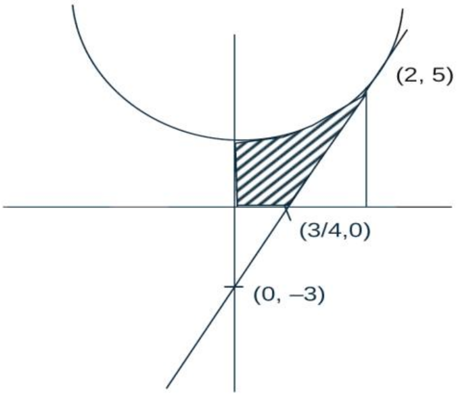
$\therefore$ f is differentiable at $x = 0$.

80. (b) Equation of tangent at (2,5) is $\frac{y+5}{2} = x(2)+1$ or $y = 4x-3$

Required Area $= \int_0^2 (x^2+1)dx - \frac{1}{2}\left(2-\frac{3}{4}\right)\cdot(5)$

$= \left.\frac{x^3}{3}+x\right]_0^2 - \frac{25}{8}$

$= \frac{8}{3} - \frac{9}{8} = \frac{37}{24}$



81. (a) $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \frac{a+b+c}{18}$

$\Rightarrow a = 7k, b = 6k, c = 5k$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{5}$$

$$\cos B = \frac{19}{25}, \cos C = \frac{5}{7}$$

$$\frac{1}{5\alpha} = \frac{19}{35\beta} = \frac{5}{7\gamma}$$

$$\Rightarrow \frac{7}{35\alpha} = \frac{19}{35\beta} = \frac{25}{35\gamma}$$

$\alpha : \beta : \gamma = 7 : 19 : 25$

82. (b) $u = x - y$

$$\frac{du}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow 1 - \frac{du}{dx} = u^2$$

$$1 - u^2 = \frac{du}{dx}$$

$$\frac{du}{1 - u^2} = dx$$

$$\Rightarrow \frac{1}{2}\log\left|\frac{1+u}{1-u}\right| = x + c$$

$$\Rightarrow \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| = x + c$$

Put $x = 1 \Rightarrow c = -1$

$$\Rightarrow \log\left|\frac{1+x-y}{1-x+y}\right| = 2(x-1)$$

83. **(b)** Consider $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Given that $2b=2ae$

$\Rightarrow b=ae$ and $\frac{2b^2}{a}=8$

$a(1-e^2)=4, a^2e^2=a^2(1-e^2)$

$\Rightarrow e^2=\frac{1}{2}$

$\Rightarrow a=8, b=4\sqrt{2}$

Hence equation of ellipse is $\frac{x^2}{64}+\frac{y^2}{32}=1$

$(4\sqrt{3}, 2\sqrt{2})$ lies on this

84. **(b)** Sum of all elements of $S=210$

So X be a nice set if $x=\{S-\{7\}, S-\{1,6\}, S-\{2,5\}, S-\{3,4\}, S-\{1,2,4\}\}$

$P(x)=\frac{5}{2^{20}}$

85. **(b)** $A(7,0,6)$ and $B(3,4,2)$

$\overrightarrow{AB}=-4\hat{\imath}+4\hat{\jmath}-4\hat{k}$

Also $2\hat{\imath}-5\hat{\jmath}$ is parallel to the plane

$\Rightarrow$ Normal perpendicular to the required plane is $\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -1 & 1\\ 2 & -5 & 0\end{vmatrix}=5\hat{\imath}+2\hat{\jmath}-3\hat{k}$

Equation of the plane is $5(x-7)+2(y-0)-3(z-6)=0$

$5x+2y-3z=17$

$(2,\alpha,\beta)$ lies on this

$10+2\alpha-3\beta=17$

$\Rightarrow 2\alpha-3\beta=7$

86. **(c)** $(10+x)^{50}+(10-x)^{50}$

$a_0=(10^{50})(2)$

$a_2={}^{50}C_2(10))^{48}(2)$

$\frac{a_2}{a_0}=\frac{{}^{50}C_2(10)^{48}(2)}{10^{52}(2)}=12.25$

87. **(c)** $k=\{4,8,12,16,20\}$

$f(k)$ can takes the values $\{3,6,9,12,15,18\}$

Number of ways = ${}^6C_5 \cdot 5!$

∴ Total number of onto functions

$= {}^6C_5 \cdot 5!(15!)$

$= (6!)(15!)$

88. (d) $k^2 + 4a^2 = r^2$ and

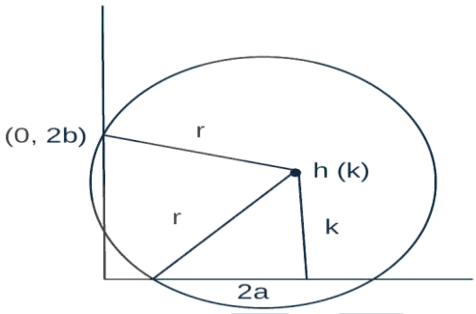
$(h-0)^2 + (k-2b)^2 = r^2$

$\Rightarrow h^2 + (k-2b)^2 = k^2 + 4a^2$

$\Rightarrow h^2 = 4bk - 4b^2 + 4a^2$

Locus is $x^2 = 4(by - b^2 + a^2)$

∴ Parabola.



89. (a) $f(x) = x(x^3 + a^2)^{\frac{-1}{2}} - (d-x)(b^2 + (d-x)^2)^{-\frac{1}{2}}$

$$f'(x) = \frac{a^2}{(x^2+a^2)\sqrt{x^2+a^2}} + \frac{b^2}{(b^2+(d-x)^2)\sqrt{b^2+(d-x)^2}}$$

$= +\text{ve}$

90. (c) $|ABA^{\top}| = |A| \cdot B| \cdot |A^{\top}| = |A|^2|B|$

$|AB^{-1}| = 8 \Rightarrow |A| = 8|B|$

$$|BA^{-1}B| = \frac{|B|^2}{|A|} = \frac{|B|^2}{8|B|} = \frac{|B|}{8} = \frac{1}{16}$$