1. (b) Maximum current will flow from zener if input voltage is maximum.

When zener diode works in breakdown state, voltage across the zener will remain same.

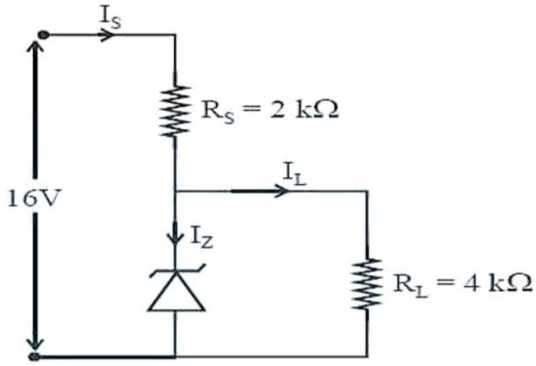
$\therefore V_{\text{across } 4k\Omega} = 6\ V$

$\therefore$ Current through 4 KΩ $= \frac{6}{4000}$ A $= \frac{6}{4}$ mA

Since input voltage $= 16$ V

$\therefore$ Potential difference across 2 KΩ $= 10$ V

$\therefore$ Current through 2kΩ $= \frac{10}{2000} = 5$ mA



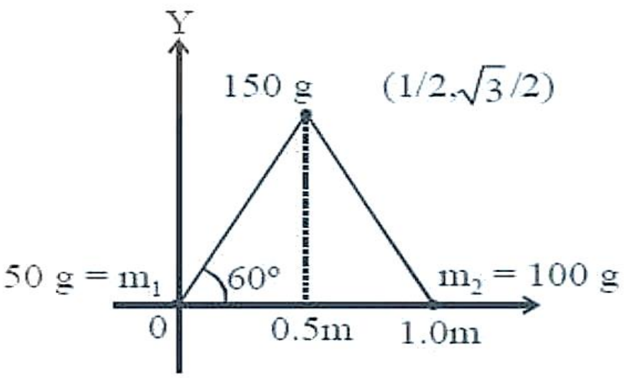
$\therefore$ Current through zener diode $= (I_S - I_L) = 3.5$ mA

2. (d) The co-ordinates of the centre of mass

$$\vec{r}_{cm} = \frac{0 + 150 \times \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) + 100 \times \hat{i}}{300}$$

$$\vec{r}_{cm} = \frac{7}{12}\hat{i} + \frac{\sqrt{3}}{4}\hat{j}$$

$\therefore$ Co-ordinate $\left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)$ m



3. (d) Since mass of the object remains same

$\therefore$ Weight of object will be proportional to ' g ' (acceleration due to gravity) Given:

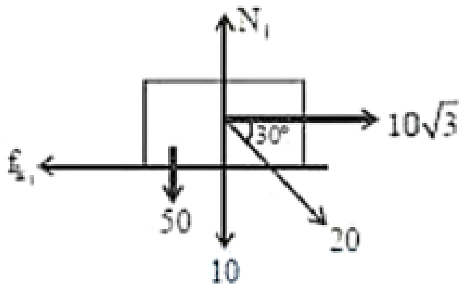
$$\frac{W_{\text{earth}}}{W_{\text{planet}}} = \frac{9}{4} = \frac{g_{\text{earth}}}{g_{\text{planet}}}$$

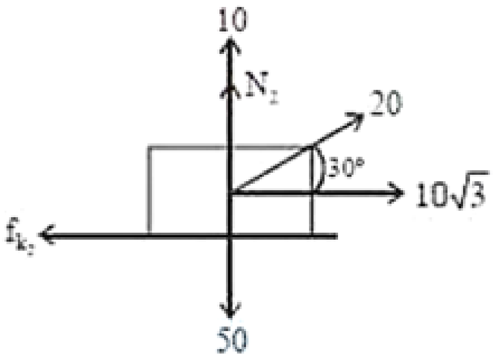
Also, $g_{\text{surface}} = \frac{GM}{R^2}$ (M is mass planet, G is universal gravitational constant, R is radius of planet)

$$\therefore \frac{9}{4} = \frac{\mathrm{GM}_{\text{earth}} \mathrm{R}_{\text{planet}}^2}{\mathrm{GM}_{\text{planet}} \mathrm{R}_{\text{earth}}^2} = \frac{M_{\text{earth}}}{M_{\text{planet}}} \times \frac{R_{\text{planet}}^2}{R_{\text{earth}}^2} = 9\frac{R_{\text{planet}}^2}{R_{\text{earth}}^2}$$

$$\therefore \mathrm{R}_{\text{planet}} = \frac{\mathrm{R}_{\text{earth}}}{2} = \frac{\mathrm{R}}{2}$$

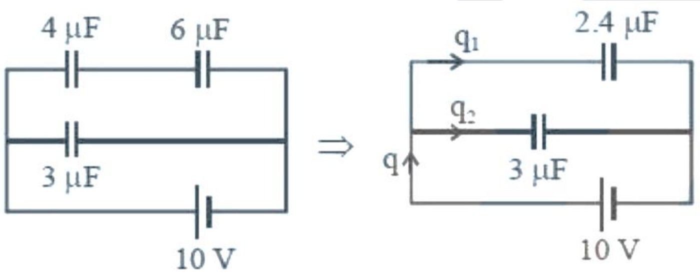
4. (d)





$$\mathrm{N}_1 = 60$$
$$\mathrm{a}_1 = \frac{10\sqrt{3} - 0.2 \times 60}{5}$$
$$\mathrm{a}_1 - \mathrm{a}_2 = 0.8$$

5. (b)



So total charge flow $= q = 5.4\mu\mathrm{F} \times 10\ \mathrm{V}$

$= 54\mu\mathrm{F}$

The charge will be distributed in the ratio of capacitance

$$\Rightarrow \frac{\mathrm{q}_1}{\mathrm{q}_2} = \frac{2.4}{3} = \frac{4}{5}$$

$$\therefore 9\mathrm{X} = 54\mu\mathrm{C}$$

$\therefore$ X = $6\mu C$

$\therefore$ Charge on $4\mu F$ capacitor will be $= 4X = 4 \times 6\mu C$

$= 24\mu C$

6. **(c)** For a diatomic gas, $C_p = \frac{7}{2}R$

Since gas undergoes isobaric process

$$\Rightarrow \Delta Q = n\frac{7}{2}R\Delta T = \frac{7}{2}(nR\Delta T) = 35 \text{ J}$$

7. **(b)**

Efficiency of Carnot engine $= 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

Given,

$$\frac{1}{6} = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}} \Rightarrow \frac{T_{\text{sink}}}{T_{\text{source}}} = \frac{5}{6}$$

Also,

$$\frac{2}{6} = 1 - \frac{T_{\text{sink}} - 62}{T_{\text{source}}} \Rightarrow \frac{62}{T_{\text{source}}} = \frac{1}{6}$$

$\therefore$ $T_{\text{source}} = 372 \text{ K} = 99^\circ\text{C}$

Also, $T_{\text{sink}} = \frac{5}{6} \times 372 = 310 \text{ K} = 37^\circ\text{C}$

(Note: Temperature of source is more than temperature of sink)

8. **(b)** $2\pi r_n = n\lambda_n$

$$\lambda_3 = \frac{2\pi(4.65 \times 10^{-10})}{3}$$

$\lambda_3 = 9.7\text{Å}$

9. **(c)** Loudness of sound is given by

$dB = 10\log\frac{I}{I_0}$ (I is intensity of sound, I_0 is reference intensity of sound)

$$\therefore\ 120 = 10\log\left(\frac{I}{I_0}\right)$$

$\Rightarrow I = 1 \text{ W/m}^2$

Also, $I = \frac{P}{4\pi r^2} = \frac{2}{4\pi r^2}$

$$\therefore\ r = \sqrt{\frac{2}{4\pi}} = \sqrt{\frac{1}{2\pi}}\text{m} = 0.399 \text{ m}$$

≈ 40 cm

10. **(a)** Since unpolarized light falls on P_1

$\Rightarrow$ Intensity of light transmitted from $P_1 = \frac{I_0}{2}$.

Pass axis of P_2 will be at an angle of 30° with P_1

$\therefore$ Intensity of light transmitted from

$$P_2 = \frac{I_0}{2}\cos^2 30^\circ = \frac{3I_0}{8}$$

Pass axis of P_3 is at an angle of 60° with P_2

$\therefore$ Intensity of light transmitted from

$$P_3 = \frac{3I_0}{8}\cos^2 60^\circ = \frac{3I_0}{32}$$

$$\therefore \left(\frac{I_0}{I}\right) = \frac{32}{3} = 10.67$$

11. (c)

12. (a) $R = \frac{mv}{qB}$

$$= \frac{\sqrt{2m(K.E.)}}{qB}$$

$$R = \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times (100 \times 1.6 \times 10^{-19})}}{1.6 \times 10^{-19} \times 1.5 \times 10^{-3}}$$

$R = 2.248$ cm

$$\sin\theta = \frac{2}{2.248}\ ;\ \tan\theta = \frac{QU}{TU}$$

$$\frac{2}{1.026} = \frac{QU}{6}$$

$QU = 11.69$

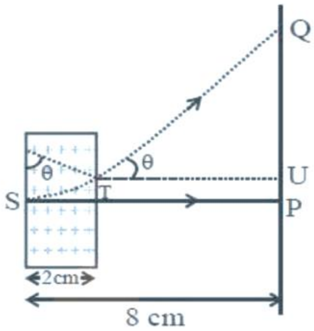
$PU = R(1 - \cos\theta)$

$= 1.22$

$d = QU + PU$

13. (c) For same range angle of projection will be θ will be $\theta \& 90 - \theta$

$$R = \frac{u^2 2\sin\theta\cos\theta}{g}$$



$$h_1 = \frac{u^2 \sin^2 \theta}{g}$$

$$h_2 = \frac{u^2 \sin^2 (90-\theta)}{g}$$

$$\frac{R^2}{h_1\ h_2} = 16$$

14. (b) $v = b\sqrt{x}$

$\frac{dv}{dt} = \frac{b}{2\sqrt{x}}\frac{dx}{dt}; a = \frac{bv}{2\sqrt{x}}$

$a = \frac{b(b\sqrt{x})}{2\sqrt{x}};\ \frac{dv}{dt} = a = \frac{b^2}{2};\ v = \frac{b^2}{2}\tau$

15. (c) $Q = P \times t$

$Q = mc\Delta T + mL$

$$P = \frac{V_{\text{rms}}^2}{R}$$

$$4200 \times 80 + 2260 \times 10^3 = \frac{(200)^2}{20} \times t$$

$t = 1298\text{sec}$

$t \simeq 22\text{ min}$

16. (a) $N_A = N_{OA}e^{-\lambda t} = \frac{N_{OA}}{2^{t/t_{1/2}}} = \frac{N_{OA}}{2^6}$

∴ Number of nuclei decayed

$$= N_{OA} - \frac{N_{OA}}{2^6} = \frac{63\ N_{OA}}{64}$$

$$N_B = N_{OBe^{-\lambda t}} = \frac{N_{OB}}{2^{t/t_{1/2}}} = \frac{N_{OB}}{2^3}$$

$\therefore$ Number of nuclei decayed

$$= N_{OB} - \frac{N_{OB}}{2^3} = \frac{7\,N_{OB}}{8}$$

Since, $N_{OA} = N_{OB}$

$\therefore$ Ratio of decayed numbers of nuclei

$$A\&B = \frac{63\,N_{OA} \times 8}{64 \times 7\,N_{OB}} = \frac{9}{8}$$

17. (a) Modulation index is given by

$$m = \frac{A_m}{A_c} = \frac{2}{4} = 0.5$$

& (a) carrier wave frequency is given by $= 2\pi f_c = 2 \times 10^4 \pi$

$f_c = 1\text{kHz}$

lower side band frequency

$\Rightarrow f_c - f_m$

$\Rightarrow 10\text{kHz} - 1\text{kHz} = 9\text{kHz}$

18. (c) $\because$ Length of cylinder remains unchanged

so $\left(\frac{F}{A}\right)_{\text{Compressive}} = \left(\frac{F}{A}\right)_{\text{Thermal}}$

$\frac{F}{\pi r^2} = Y\alpha T$ (α is linear coefficient of expansion)

$$\therefore\ \alpha = \frac{F}{YT\pi r^2}$$

$\therefore$ The coefficient of volume expansion $\gamma = 3\alpha$

$$\therefore\ \gamma = 3\frac{F}{YT\pi r^2}$$

19. (b) $q = \int \text{Idt}$

$$q = \int_0^{L/R} \frac{E}{R}\left[1 - e^{\frac{-Rt}{L}}\right] dt$$

$$q = \frac{EL}{R^2}\frac{1}{e}\ ;\ q = \frac{EL}{2.7R^2}$$

20. (d) $f = 660$ Hz, $v = 330$ m/s

$$f_1 = f\left(\frac{v-u}{v}\right);\ f_2 = f\left(\frac{v+u}{v}\right)$$

$$f_2 - f_1 = \frac{f}{v}[v + u - (v - u)]$$

$$10 = f_2 - f_1 = \frac{f}{v}[2u]$$

$$u = 2.5 \text{ m/s}$$

21. (c) Magnetic field when electromagnetic wave propagates in $+z$ direction.

$$B = B_0 \sin(kz - \omega t)$$

where

$$B_0 - \frac{60}{3 \times 10^8} = 2 \times 10^{-7}$$

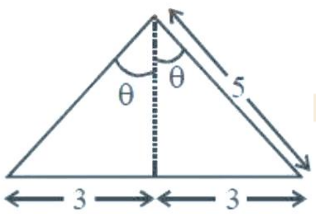
$$k = \frac{2\pi}{\lambda} = 0.5 \times 10^3$$

$$\omega = 2\pi f = 1.5 \times 10^{11}$$

22. (d) $B = \frac{\mu_0 I}{4\pi d} 2\sin\theta$

$d = 4$ cm

$$\sin\theta = \frac{3}{5}$$



23. (a) $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right); \frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$$\frac{1}{\lambda_1} = R\left(\frac{7}{9 \times 16}\right);\ \frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{5}{4 \times 9}\right)$$

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{5}{36}}{\frac{7}{9 \times 16}} = \frac{20}{7}$$

24. (d) $N\sin\theta = m\frac{r}{2}\omega^2$

$N\cos\theta = mg$

$$\tan\theta = \frac{r\omega^2}{2g}$$

$$\frac{r}{2\frac{\sqrt{3}r}{2}} = \frac{r\omega^2}{2g}\ ;\ \omega^2 = \frac{2g}{\sqrt{3}r}$$

25. (c) We have

$$V_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_\ell)g$$

$\Rightarrow V_T \propto r^2$

Since mass of the sphere will be same

$$\therefore \rho\frac{4}{3}\pi R^3 = 27\cdot\frac{4}{3}\pi r^3\rho$$

$$\Rightarrow r = \frac{R}{3}$$

$$\therefore\ \frac{v_1}{v_2} = \frac{R^2}{r^2} = 9$$

26. (a) Given number density of molecules of gas as a function of r is

$n(r) = n_0 e^{-\alpha r^4}$

$\therefore$ Total number of molecule $= \int_0^\infty n(r)dV = \int_0^\infty n_0 e^{-\alpha r^4} 4\pi r^2 dr$

$\therefore$ Number of molecules is proportional to $n_0\alpha^{-3/4}$

27. (d) $\sin\ c = \frac{\mu_1}{\mu_2}$

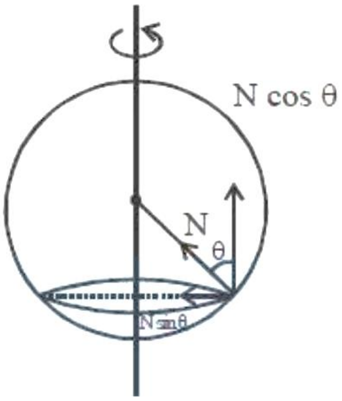
$\mu_1\sin\ \theta = \mu_2\sin(90^\circ - C)$

$$\sin\ \theta = \frac{\mu_2\sqrt{1 - \frac{\mu_1^2}{\mu_2^2}}}{\mu_1}$$

$$\theta = \sin^{-1}\sqrt{\frac{\mu_1^2 - \mu_2^2}{\mu_1^2}}$$

For TIR

$$\theta < \sin^{-1}\sqrt{\frac{u_2^2}{\mu_1^2} - 1}$$



28. (c) $v = 2f(I_2 - I_1)$

$v = 2 \times 480 \times (70 - 30) \times 10^{-2}$

$v = 960 \times 40 \times 10^{-2}$

$v = 38400 \times 10^{-2}\ \text{m/s}$

$\text{v} = 384\ \text{m/s}$

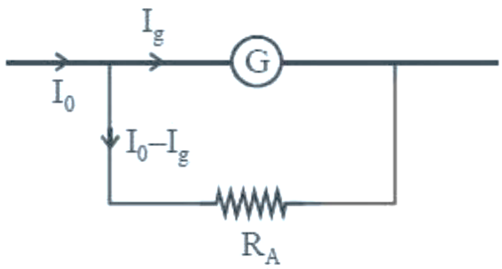
29. (d) $k_1 = \frac{C}{\ell_1}$

$k_2 = \frac{C}{\ell_2}$

$\frac{k_1}{k_2} = \frac{C\ell_2}{\ell_1 C} = \frac{\ell_2}{n\ell_2} = \frac{1}{n}$

30. (b) When galvanometer is used an ammeter shunt is used in parallel with galvanometer.

$\therefore$



$\therefore I_g G = (I_0 - I_g)R_A$

$\therefore\ R_A = \left(\frac{I_g}{I_0 - I_g}\right)G$

When galvanometer is used as a voltmeter, resistance is used in series with galvanometer.

$I_g(G + R_v) = V = GI_0$ (given $V = GI_0$)

$\therefore R_V = \frac{(I_0 - I_g)G}{I_g}$

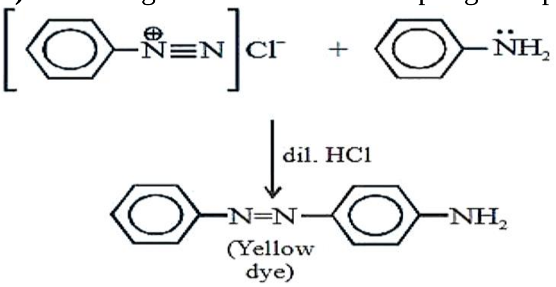
$\therefore\ R_A R_V = G^2 \,\&\, \frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

31. (c)

32. (c) $(CH_3)_3CCl$ will give Cl^- and most stable carbocation. Hence $(CH_3)_3CCl$ likely to give a precipitate with $AgNO_3$ solution.

33. (b) $\Delta G^\circ = -2.303RT\log K_{eq}$

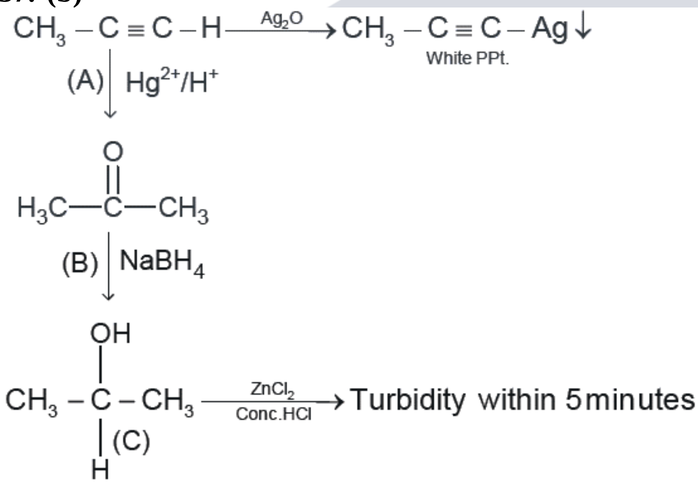
34. (b) According to NCERT $C - N$ coupling take place, when diazonium ion is treated with aniline.



35. (a) $2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 \uparrow + \frac{1}{2}O_2 \uparrow$

36. (c) Since carbon in diamond is sp^3 hybridized and its $C - C$ bond order is 1 . In graphite and fullerene there is both $C - C$ and $C = C$ in conjugation, hence there is partial double bond character between carbon atoms.

37. (b)



38. (b) If $\Delta n_g = 0$

$K_P = K_C$

If $\Delta n_g \neq 0,\ K_P \neq K_C$

39. (d) en and $C_2O_4^{2-}$ are a bidentate ligand. So coordination number of $[Co(Cl)(en)_2]Cl$ is 5 and $K_3[Al(C_2O_4)_3]$ is 6

40. (d) Glycogen is a multibranched polysaccharide.

41. (b) NO_2 and Hydrocarbons are primary precursors of photochemical smog.

42. (a) No. of atoms in simple cubic $= 1$, bcc $= 2$ & fcc $= 4$

43. (d)

$$2KMnO_4 \xrightarrow[\Delta]{513K} K_2MnO_4 + MnO_2 + O_{2(g)}$$

(X) (Y)

$$MnO_2 + 4NaCl + 4H_2SO_4 \rightarrow MnCl_2 + 4NaHSO_4 + 2H_2O + \underset{\substack{(Z)\\ \text{pungent}\\ \text{gas}}}{Cl_{2(g)}}$$

44. (d) Due to resonance there is partial double bond character between carbon and chlorine, hence it do not undergoes nucleophilic substitution reaction.

45. (d) K_{sp} of $Cd(OH)_2 = 4\,s^3 = 4 \times (1.84 \times 10^{-5})^3$

If pH = 12

pOH = 2

$[OH^-] = 10^{-2}M$

$K_{sp} = [Cd^{2+}][OH^-]^2$

$4 \times (1.84 \times 10^{-5})^3 = [Cd^{2+}][OH^-]^2$

$$[Cd^{2+}] = \frac{4 \times (1.84)^3 \times 10^{-15}}{10^{-4}}$$

$Cd^{2+} = 4 \times 6.22 \times 10^{-11} = 2.49 \times 10^{-10}M$

46. (a)

(a) EDTA (ethylene diamine tetra acetate) is used for lead poisoning

(b) Cis platin is used as a anti cancer drug

(c) D-penicillamine is used for copper poisoning

(d) desferrioxime B is used for iron poisoning

47. (d) Due to lanthanoide contraction Mo & W have similar atomic radii.

48. (a)

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

$$\left(\frac{25}{M}\right)\ x \times \frac{25}{M}\ \frac{y}{2} \times \frac{25}{M}$$

$$C\ \ x \times \frac{25}{M} = 2$$

$$H\ \ y \times \frac{25}{M} = 1$$

$$C_{2y}H_y \equiv 24ygmC + ygmH$$

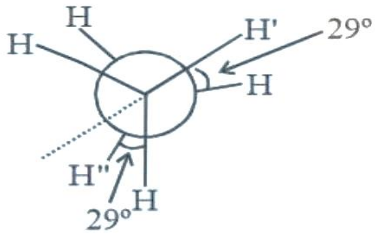
24:1 ratio by mass

49. (a) Stronger the acidic strength greater will be its electrical conductivity.

K_a value of formic acid $>$ benzoic acid $>$ acetic acid.

50. (a) Greater the nuclear charge, stronger will be the attraction, hence lower will be energy of 2s

51. (c) Dihedral angle is then angle between bond pairs present on adjacent atoms.



$H' - C - C - H''$

Hence angle between

$H' - C - C - H''$ is
$(120° + 29°) = 149°$

52. (a) Since $NaOH$ is a strong base hence it reacts with Al_2O_3 and SiO_2 to form salts.

53. (a)

$$2\,N_2O_5(g) \longrightarrow 4NO_2(g) + O_2(g)$$

$t = 0$ 3.0M

$t = 30$ 2.75M

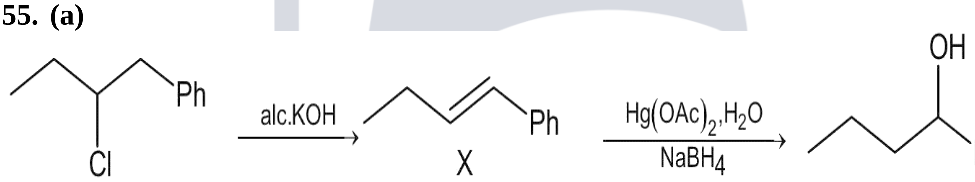
$$\frac{-\Delta[N_2O_5]}{\Delta t} = \frac{0.25}{30}$$

$$\frac{1}{2} \times \frac{-\Delta[N_2O_5]}{\Delta t} = \frac{1}{4} \times \frac{\Delta[NO_2]}{\Delta t}$$

$$\frac{\Delta[NO_2]}{\Delta t} = \frac{0.25}{30} \times 2 = 1.66 \times 10^{-2} M/min$$

54. (d) Isobutylene on polymerization will form given polymer.

55. (a)



OMDM follow Markonikovs addition rule.

56. (a) According to IUPAC rules, select the largest chain including functional group, if alkene and alkyne are present at equivalent position then priority is given to alkene.

57. (c) Definitions and property of colloidal will explain & solve above question.

58. (d) Since boron has higher nuclear charge because it has greater atomic number and lower 1^{st} I.E. then beryllium due to fully filled s-orbital.

59. (c) $\pi = CRT = \left(\frac{6}{60} + \frac{18}{180}\right) \times .0821 \times 300$

$= 0.2 \times .082 \times 300 = 4.926$ atm.

60. (c) $Mg(HCO_3)_2(aq) \longrightarrow Mg(OH)_2 \downarrow + 2CO_2 \uparrow$

61. (b) $(y^2 - x^3)dx - xydy = 0\ x \neq 0$

$$\Rightarrow y^2 - x^3 - xy\frac{dy}{dx} = 0$$

or, $xy\frac{dy}{dx} - y^2 = -x^3$

$$y\frac{dy}{dx} - \frac{1}{x}y^2 = -x^2$$

Let $y^2 = \mu$

$2y\frac{dy}{dx}=\frac{d\mu}{dx}$

Putting this value in equation (i)

$\frac{1}{2}\frac{d\mu}{dx}-\frac{1}{x}\mu=-x^2$

$\frac{d\mu}{dx}+\left(-\frac{2}{x}\right)\mu=-2x^2$

I.F. $=e^{\int\frac{-2}{x}dx}=e^{-2\ln x}=\frac{1}{x^2}$

Sol. of equation (ii)

$\mu\times\frac{1}{x^3}=\int-2x^2\times\frac{1}{2}dx-C$

$\frac{\mu}{x^2}=-2x-C$

$y^2=-2x^3-cx^2$

$y^2+2x^3+cx^2=0$

62. (b) $[\vec{a}\cdot\vec{b}\cdot\vec{c}]=0$

$\begin{vmatrix}\alpha & 3 & 1\\ 2 & 1 & -\alpha\\ \alpha & -2 & 3\end{vmatrix}=0$

$\alpha(3-2\alpha)+1(-\alpha^2-6)+3(-4-\alpha)=0$

$3\alpha-2\alpha^2-\alpha^2-6-12-3\alpha=0$

$-3\alpha^2-18=0$

$\alpha^2+6=0$ not possible for real α

S is empty set

63. (c) $\int\frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx=\int\frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$

Let, $x-\alpha=t$

$\Rightarrow\int\frac{\sin(t+2\alpha)}{\sin t}dt=\int\cos 2\alpha dt+\int\cot(t)\sin 2\alpha dt$

$=t\cdot\cos 2\alpha+\ell n|\sin t|\cdot\sin 2\alpha+C$

$=(x-\alpha)\cos 2\alpha+\ln|\sin(x-\alpha)|\cdot\sin 2\alpha+C$

64. (c) Given, $\cos 2x+2\sin x=2\alpha-7$

$\Rightarrow 1-2\sin^2 x+\alpha\sin x=2\alpha-7$

$\Rightarrow 2\sin^2 x-\alpha\sin x+2\alpha-8=0$

$\Rightarrow \sin x = \dfrac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha - 8)}}{4}$

$\Rightarrow \sin x = \dfrac{\alpha \pm (\alpha - 8)}{4}$

$\Rightarrow \sin x = \dfrac{\alpha + \alpha - 8}{4}, \dfrac{\alpha - \alpha + 8}{4}$

$\sin \mathrm{x} = 2$ (Not possible)

For solution

$-1 \leq \dfrac{2\alpha - 8}{4} \leq 1$

$-4 \leq 2\alpha - 8 \leq 4$

$\Rightarrow 4 \leq 2\alpha \leq 12$

$\Rightarrow \alpha \in [2,6]$

65. (c) $f(x) = 5 - |x - 2|$

$f(x)$ attains maximum value when $|x - 2| = 0 \Rightarrow x = 2 = \alpha$

$g(x) = |x + 1|$

$g(x)$ attains minimum value of $x = -1 = \beta$

$\mathrm{Lim}_{x \to -\alpha\beta} \dfrac{(x-1)(x^2 - 5x + 6)}{x^2 - 6x + 8}$

$= \mathrm{Lim}_{x \to 2} \dfrac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$

$= \dfrac{(2-1)(2-3)}{(2-4)} = \dfrac{1}{2}$

66. (c) Let $\mathrm{z} = \mathrm{x} + 10\mathrm{i}$

given $\frac{2z-n}{2z+n} = 2i - 1$

$\Rightarrow \dfrac{2(\mathrm{x} + 10\mathrm{i}) - \mathrm{n}}{2(\mathrm{x} + 10\mathrm{i}) + \mathrm{n}} = 2\mathrm{i} - 1$

$\Rightarrow (2\mathrm{x} - \mathrm{n}) + 20\mathrm{i} = (2\mathrm{i} - 1)[(2\mathrm{x} + \mathrm{n}) + 20\mathrm{i}]$

Comparing real and imaginary part

$\Rightarrow 2\mathrm{x} - \mathrm{n} = 2(-20) - (2\mathrm{x} + \mathrm{n})$ and $20 = 2(2\mathrm{x} + \mathrm{n}) - 20$

$\Rightarrow 2\mathrm{x} - \mathrm{n} = -40 - 2\mathrm{x} - \mathrm{n}$ and $20 = 4\mathrm{x} + 2\mathrm{n} - 20$

$\Rightarrow 4\mathrm{x} = -40$ and $4\mathrm{x} + 2\mathrm{n} = 40$

$\Rightarrow \mathrm{x} = -10$ and $-40 + 2\mathrm{n} = 40$

$\Rightarrow \mathrm{n} = 40$

$\Rightarrow \mathrm{n} = 40$ and $\mathrm{Re}(\mathrm{z}) = -10$

67. (b) $\left(\frac{1}{60} - \frac{x^8}{81}\right)\left(2x^2 - \frac{3}{x^2}\right)^6$

Term independent of x will be $\frac{1}{60} \times$ independent of x in

$\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{8} \times$ Term of x^{-8} in $\left(2x^2 \frac{3}{x^3}\right)^6$

T_{r+1} in $\left(2\mathrm{x}^2 - \frac{3}{\mathrm{x}^2}\right)^6$ will be $\mathrm{T}_{r+1} = {}^6\mathrm{C}_r(2\mathrm{x}^2)^{6-r}\left(-\frac{3}{\mathrm{x}^2}\right)^r$

$= {}^6\mathrm{C}_r 2^{6-r}(-1)^r \times 3^r \times \mathrm{x}^{12-2r-2r}$

Case I : For term independent of $x, 12 - 4r = 0 \Rightarrow r = 3$

$\mathrm{T}_4 = {}^6\mathrm{C}_3 \times 2^3 \times 3^3 \times {}^6 = -20 \times 2^3 \times 3^3$

Case II : For term of x^{-8}

$12 - 4r = -8$
$4r = 20 \Rightarrow r = 5$

$\mathrm{T}_6 = {}^6\mathrm{C}_5 \cdot 2^1(-1) \cdot 3^5 \cdot \mathrm{x}^{-8}$

Required Answer $= \frac{1}{60} \times (-20)2^3 \times 3^3 - \frac{1}{81} \times 6 \times 2 \times (-1) \times 3^5$

$= -72 + 36 = -36$

68. (d) $y^2 = 4\lambda x$ and $y = \lambda x$

$\lambda^2 x^2 = 4\lambda x$

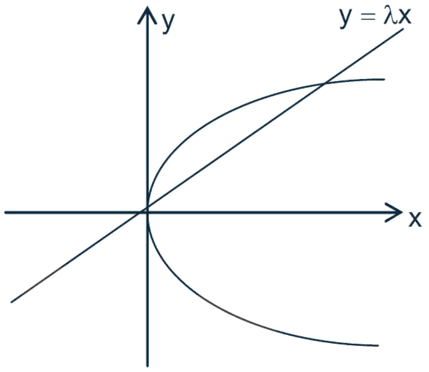
$x = 0$ and $x = \frac{4}{\lambda}$

Area $= \int_0^{4/\lambda}(\sqrt{4\lambda x} - \lambda x)dx = \frac{1}{9}$

$$\Rightarrow 2\sqrt{\lambda} \times \left(\frac{x^{3/2}}{3/2}\right)_0^{4/\lambda} - \lambda\left(\frac{x^2}{2}\right)_0^{4/\lambda} = \frac{1}{9}$$

$$\frac{4}{3}\sqrt{\lambda} \times (2^2)^{3/4}\frac{x}{\lambda^{3/2}} - \frac{x}{2} \times \frac{16}{\lambda} = \frac{1}{9}$$

$$\Rightarrow \frac{32}{3\lambda} - \frac{8}{\lambda} = \frac{1}{9}$$



$\Rightarrow \frac{8}{3\lambda} = \frac{1}{9}$

$\lambda = 24$

69. (a) $[\sin\,\theta]x + [-\cos\,\theta]y = 0$

$[\cot\,\theta]x + y = 0$

Case I

When $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

$\sin\,\theta \in \left(\frac{\sqrt{3}}{2}, 1\right)$

$\cos\,\theta \in \left(-\frac{1}{2}, 0\right) - \cos\,\theta \in \left(0, \frac{1}{2}\right)$

$\cot\,\theta \in \left(-\frac{1}{\sqrt{3}}, 0\right)$

$[\sin\,\theta] = 0\ [-\cos\,\theta] = 0\ [\cot\,\theta] = -1$

Equation (1) and (2) will

$\left.\begin{array}{r} 0x + 0y = 0 \\ -x + y = 0 \end{array}\right]$ system will have infinitely many solution

Case II

When $\theta \in \left(\pi, \frac{7\pi}{6}\right) \sin\,\theta \in \left(-\frac{1}{2}, 0\right)$

$\cos\,\theta \in \left(-1, \frac{-\sqrt{3}}{2}\right)$

$\cot\,\theta \in (\sqrt{3}, \infty)$

$[\sin\,\theta] = -1, [\cos\,\theta] = -1$

$[\cot\,\theta] = \{1,2,3, \dots \dots$.

$-x - y = 0$

$\mathrm{I}x+y=0\ \mathrm{I}=\{1,2,\ldots\ldots\}$

It will have unique solution in all cases $x=0, y=0$

70. (c) Total problems $=50$

P (Solving) $=\frac{4}{5}$

$P($ Not solving $)=\frac{1}{5}$

P (unable to solve less than two problems)

$=$ P (not solving one problem) +P (not solving zero problem)

$$= {}^{50}C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{50}+{}^{50}C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^{49}$$

$$=\frac{4^{50}}{5^{50}}+50\cdot\frac{4^{49}}{5.5^{49}}$$

$$=\left(\frac{4}{5}\right)^{50}+10\cdot\left(\frac{4}{5}\right)^{49}$$

$$=\left(\frac{4}{5}\right)^{49}\left(\frac{4}{5}+10\right)$$

$$=\frac{54}{5}\cdot\left(\frac{4}{5}\right)^{49}$$

71. (a) $a_1, a_2, \ldots\ldots a_n$ are in A.P.

$a_1+a_7+a_{16}=40$

$\Rightarrow a+a+6d+a+15d=40$

$\Rightarrow 3\mathrm{a}+21\mathrm{~d}=40$

$\Rightarrow \mathrm{a}+7\mathrm{~d}=\frac{40}{3}$

$515=\frac{15}{2}[2a+14d]$

$=15[\mathrm{a}+7\mathrm{~d}]$

$=15\times\frac{40}{3}$

$=200$

72. (d) Given $2\mathrm{a}=4$ and $2\mathrm{be}=4$

$\Rightarrow \mathrm{a}=2$, $\mathrm{be}=2$

$\Rightarrow b^2\mathrm{e}^2=4$

$\Rightarrow b^2 - a^2 = 4$

$\Rightarrow \mathrm{b}^2 = 8$

$\Rightarrow$ equation of ellipse

$\frac{x^2}{4} + \frac{y^2}{8} = 1$

73. (b) Equation of plane containing both lines is

$$\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$$

$(\mathrm{x}-1)(-4+1) + (\mathrm{y}-1)(1+2) + \mathrm{z}(1+2) = 0$

$-3(x-1) + 3(y-1) + 3z = 0$

$-x + 1 + y - 1 + z = 0$

$-x + y + z = 0$ distance from point (2,1,4) is $\left|\frac{-2+1+4}{\sqrt{1^2+1^2+1^2}}\right| = \sqrt{3}$

74. (a) For $\mathrm{A} = \mathrm{C}, \mathrm{A} - \mathrm{C} = \phi$

$\Rightarrow \phi \subseteq \mathrm{B}$

But $\mathrm{A} \not\subseteq \mathrm{B}$

$\Rightarrow$ option A is NOT true

Let $x \in (Cx \in (C \cup A) \cap (C \cup B))$

$\Rightarrow x(C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow (\mathrm{x} \in \mathrm{C}$ or $\mathrm{x} \in \mathrm{A})$

and

$(\mathrm{x} \in \mathrm{C}$ or $\mathrm{x} \in \mathrm{B})$

$\Rightarrow \mathrm{x} \in \mathrm{C}$ or $\mathrm{x} \in (\mathrm{A} \cap \mathrm{B})$

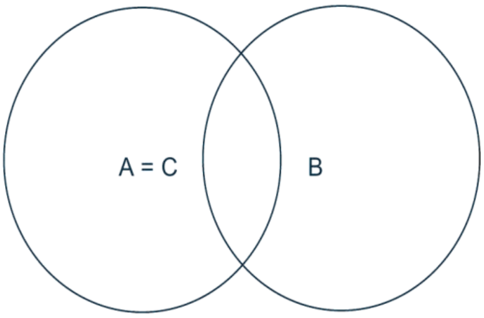
$\Rightarrow \mathrm{x} \in \mathrm{C}$ or $\mathrm{x} \in \mathrm{C}$

$\mathrm{A} \cup \mathrm{B} \subseteq \mathrm{C})$

$\Rightarrow \mathrm{x} \in \mathrm{C}$

$\Rightarrow (C \cup A) \cap (C \cup B) \subseteq C$

Now $x \in C \Rightarrow x \in (C \cup A)\ x \in (C \cup B)$



$\Rightarrow x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow C \subseteq (C \cup A) \cap (C \cup B)$

$\Rightarrow$ from (1) and (2)

$C = (C \cup A) \cap (C \cup B)$

$\Rightarrow$ option B is true

Let $x \in A$ and $x \notin B$

$\Rightarrow \mathrm{x} \in (\mathrm{A} - \mathrm{B})$

$\Rightarrow x \in C$ (as $A - B \subseteq C$)

Let $x \in A$ and $x \in B$

$\Rightarrow \mathrm{x} \in (\mathrm{A} \cap \mathrm{B})$

$\Rightarrow x \in C$ (as $A \cap B \subseteq C$)

Hence $x \in A \Rightarrow x \in C$

$\Rightarrow A \subseteq C$

$\Rightarrow$ Option C is true

As $C \supseteq (A \cap B)$

$\Rightarrow B \cap C \supseteq (A \cap B)$

As $A \cap B \neq \phi$

$\Rightarrow B \cap C \neq \phi$

75. (d) α, β, γ are in G.P.
$\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root.

Both roots will be common.

$$\frac{\alpha}{1} = \frac{2\beta}{1} = \frac{\gamma}{-1} = \lambda$$
$$\alpha(\beta + \gamma) = \lambda\left(\frac{\lambda}{2} - \lambda\right) = \frac{-\lambda^2}{2} = \beta\gamma$$

76. (d) $x - y - 3 = 0$

will be chord of contact of parabola

Let the required point is $P(x_1, y_1)$ chord of contact for point P is

$$\frac{y + y_1}{2} = xx_1 - 4\frac{(x + x_1)}{2} + 3$$

$$y + y_1 = 2x_1x - 4x - 4x_1 + 6$$

As equation (i) and (ii) are same line

$$\frac{2x_1 - 4}{1} = \frac{-1}{-1} = \frac{-4x_1 - y_1 + 6}{-3}$$

$$\Rightarrow 2x_1 - 4 = 1$$

$$-4x_1 - y_1 + 6 = -3$$

$$x_1 = \frac{5}{2}$$

$$-10 - y_1 + 9 = 0$$

$$y_1 = -1$$

77. (c) $AB = 30\text{ m} = NP$

In $\triangle$ ANMtan $45^\circ = \frac{MN}{AN} = 1$

$$\Rightarrow MN = AN$$

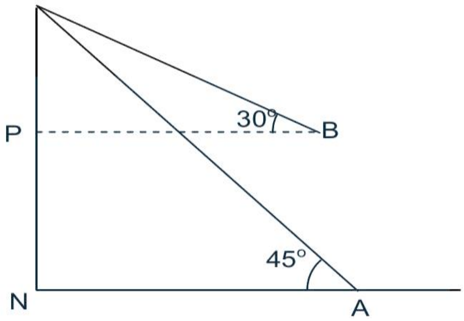
$$PM = MN - 30$$

$$= AN - 30$$

In $\triangle$ BPMtan $30^\circ = \frac{PM}{PB} = \frac{AN-30}{AN}$

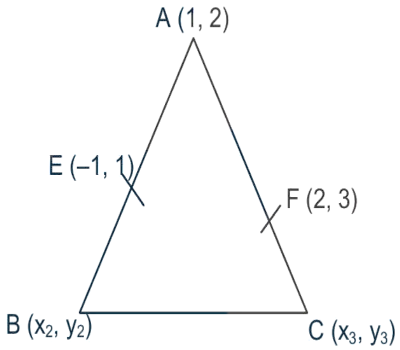
$$\frac{1}{\sqrt{3}} = \frac{AN - 30}{AN}$$

$$AN = \sqrt{3}AN - 30\sqrt{3}$$



$$AN = \frac{30\sqrt{3}}{\sqrt{3} - 1} = \frac{30\sqrt{3}(\sqrt{3} + 1)}{2} = 15(3 + \sqrt{3})$$

78. (c)



$\frac{\alpha_2+1}{2}=-1, \frac{y_2+2}{2}=1$

$\frac{x_3+1}{2}=2$ and $\frac{y_3+2}{2}=3$

$x_2=-3, y_2=0$

$x_3=3, y_3=4$

$B(-3,0)$

$C(3,4)$

Centroid $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\left(\frac{1-3+3}{3}, \frac{2+D+4}{3}\right)=\left(\frac{1}{3}, 2\right)$

79. (d)

Coin	+15	+12	-6
Probability	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{26}{36}$

Probability of doublet $=\frac{6}{36}$

Probability of sum of $9=\frac{4}{36}$

Other probability $=\frac{26}{36}$

Expected gain/loss $=15\times\frac{6}{36}+12\times\frac{4}{36}-6\times\frac{26}{36}$

$=\frac{90}{36}+\frac{48}{36}-\frac{156}{36}=\frac{-1}{2}$

80. (a)

$(1+x)^{20}={}^{20}C_0+{}^{20}C_1+{}^{20}C_2x^2+\cdots\ldots+{}^{20}C_{20}x^{20}$

Differential equation w.r.t. x

$20(1+x)^{19}={}^{20}C_1\cdot 1+2.\,{}^{20}C_2x+\cdots\ldots+20\,{}^{20}C_{20}x^{19}$

Multiply equation (2) by x

$20x(1+x)^{19} = {}^{20}C_1x + 2.\,{}^{20}C_2x^2 + \cdots \ldots + 20\,{}^{20}C_{20}x^{20}$

Differential equation (3) w.r.t. x

$20[(1+x)^{19} + 19x(1+x)^{18}] = 1.\,{}^{20}C_1 + 2^2 \cdot {}^{20}C_2x + \cdots ..(20^2)\,{}^{20}C_{20}x^{19}$

Put $x = 1$ in equation (iv)

$20(2^{19} + 19.2^{18}) = 1^2\,{}^{20}C_1 + 2^2\,{}^{20}C_2 + \cdots \ldots + (20^2)\,{}^{20}C_{20}$

$= 20 \times 2^{18}(2+19) = 20 \times 21 \times 2^{18}$

$= 420 \times 2^{18}$

$A = 420, \beta = 18$

81. (a) Given, $y = \tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\tan x - 1}{\tan x + 1}\right)$

$\Rightarrow y = -\tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$

$\Rightarrow y = -\tan^{-1}\left[\tan\left(\frac{\pi}{4} - x\right)\right]$

$\because 0 < x < \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} < -x < 0$

$\Rightarrow -\frac{\pi}{4} < \frac{\pi}{4} - x < 0$

$\Rightarrow y = -\left(\frac{\pi}{4} - x\right)\ \left\{\because \tan^{-1}(\tan x) = x \forall x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right\}$

$\Rightarrow y = x - \frac{\pi}{4}\ \frac{dy}{d\left(\frac{x}{2}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

82. (c) Equation of required circle will be $(x-3)^2 + (y \pm r)^2 = r^2$

$x^2 - 6x + 9 + y^2 \pm 2ry + r^2 = r^2$

$x^2 + y^2 - 6x \pm 2ry + 9 = 0$

Length of y intercept $= 2\sqrt{f^2 - c}\ = \pm rf$

$8 = 2\sqrt{r^2 - 9}$

$16 = r^2 - 9$

$r = 5$

So equation of required circle will be

$x^2+y^2-6x\pm 10y+9=0$ two circles

$x^2+y^2-6x+10y+9=0$

$x^2+y^2-6x-10y+9=0$

83. (d) $y=mx+\frac{4}{m}$

(i) is always tangent to $y^2=16x$

If it is tangent to the $xy=-4$

$x\left(mx+\frac{4}{m}\right)=-4$

$m^2x^2+4x=-4m$

$m^2x^2+4x=-4m$

$m^2x^2+4x+4m=0$

for tangent $D=0$

$16-16m^3=0$

$\Rightarrow \mathrm{m}=1$ put in equation (i)

$y=x+4$

84. (b) Equation of angle bisectors

$$\frac{2x-y+2z-4}{\sqrt{2^2+(-1)^2+2^2}}=\pm\left(\frac{x+2y+2z-2}{\sqrt{1^2+2^2+2^2}}\right)$$

Case I: take positive sign

$2x-y+2z-4=x+2y+2z-2$
$x-3y-2=0$

Case II : take negative sign

$2\mathrm{x}-\mathrm{y}+2\mathrm{z}-4=-(\mathrm{x}+2\mathrm{y}+2\mathrm{z}-2)$

$2x-y+2z-4=-x-2y+2z+2$

$3x+y+4z-6=0$

85. (b)

$\int_{\alpha}^{\alpha+1}\frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right)$

$\Rightarrow \int_{\alpha}^{\alpha+1}\frac{(x+\alpha+1)-(x+\alpha)}{(x+\alpha)(x+\alpha+1)}dx=\log_e\left(\frac{9}{8}\right)$

$$\Rightarrow \int_{\alpha}^{\alpha+1} \frac{dx}{x+\alpha} - \int_{\alpha}^{\alpha+1} \frac{dx}{x+\alpha+1} = \log_e\left(\frac{9}{8}\right)$$

$$\Rightarrow \log_e\left(\frac{x+\alpha}{x+\alpha+1}\right)\Bigg|_{\alpha}^{\alpha+1} = \log_e\left(\frac{9}{8}\right)$$

$$\Rightarrow \log_e\left(\frac{2\alpha+1}{2\alpha+2}\right) - \log\left(\frac{2\alpha}{2\alpha+1}\right) = \log_e\left(\frac{9}{8}\right)$$

$$\Rightarrow \log\left[\left(\frac{2\alpha+1}{2\alpha+2}\right)\left(\frac{2\alpha+1}{2\alpha}\right)\right] = \log_e \frac{9}{8}$$

$$\Rightarrow \frac{(2\alpha+1)^2}{4\alpha(\alpha+1)} = \frac{9}{8}$$

$$\Rightarrow 8[4\alpha^2+4\alpha+1] = 9[4\alpha^2+4\alpha]$$

$$\Rightarrow 32\alpha^2+32\alpha+8 = 36\alpha^2+36\alpha$$

$$\Rightarrow 4\alpha^2+4\alpha-8=0$$

$$\Rightarrow \alpha^2+\alpha-2=0$$

$$= (\alpha+2)(\alpha-1)=0$$

$$\Rightarrow \alpha = 1, -2$$

86. (d) Given 5 boys and n girls

Total ways of farming team of 3

Members under given condition

$$= {}^5C_1 \cdot {}^nC_2 + {}^5C_2 \cdot {}^nC_1$$

$$\Rightarrow {}^5C_1 \cdot {}^nC_2 + {}^5C_2 \cdot {}^nC_1 = 1750$$

$$\Rightarrow \frac{5n(n-1)}{2} + 10n = 1750$$

$$\Rightarrow \frac{n(n-1)}{2} + 2n = 350$$

$$\Rightarrow n^2+3n = 700$$

$$\Rightarrow n^2+3n-700 = 0$$

$$\Rightarrow n = 25$$

87. (b) $\theta \in \left(0, \frac{\pi}{3}\right)$

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$

$C_1 \to C_1 + C_2$

$\Rightarrow \begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$

expanding along first column

$\Rightarrow 2[1-0] - 1[-4\cos 6\theta] = 0$

$\Rightarrow 2 + 4\cos 6\theta = 0$

$\Rightarrow \cos 6\theta = -\frac{1}{2}$

$\Rightarrow 6\theta = \frac{2\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{9}$

88. (a, d) OP = 4

given OP makes 60° with x + y = 0

Let slope of $OP = m$

$\Rightarrow \tan 60° = \left|\frac{m+1}{1-m}\right|$

$\Rightarrow \frac{m+1}{m-1} = \sqrt{3}$ or $-\sqrt{3}$

$\Rightarrow m + 1 = \sqrt{3}m - \sqrt{3}$ or $m + 1 = \sqrt{3} - \sqrt{3}m$

$\Rightarrow m(\sqrt{3}-1) = \sqrt{3}+1$ or $m(1+\sqrt{3}) = \sqrt{3}-1$

$\Rightarrow m = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ or $m = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow \tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ or $\tan\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow$ equation of line $x\cos\alpha + y\sin\alpha = P$

$\Rightarrow (\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$ or $(\sqrt{3}-1)x + (\sqrt{3}+1)y = 8\sqrt{2}$

89. (a)

$$\text{Lim}_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$$

$$= \text{Lim}_{x\to 0} \frac{x + 2\sin x}{x^2 + 2\sin x + 1 - \sin^2 x + x - 1}\left(\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right)$$

$$= \text{Lim}_{x\to 0} \frac{x + 2\sin x}{x^2 + 2\sin x - \sin^2 x + x}$$

Applying L'H Rule

$$= \text{Lim}_{x\to 0} \frac{2 \cdot (1 + 2\cos x)}{2x + 2\cos x - 2\sin x \cos x + 1}$$

$$= \frac{2(3)}{2+1} = 2$$

90. (c) $\sim (p \rightarrow \sim q) = p \wedge q$