1. **(c)** Numerical aperature of the microscope is given as

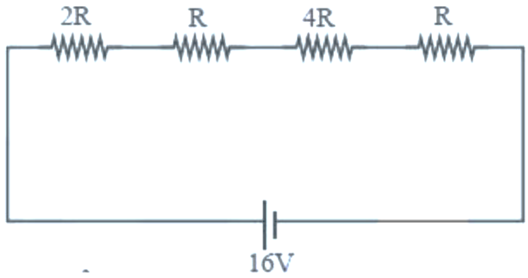
$$NA = \frac{0.61\lambda}{d}$$

Where d = minimum sparaton between two points to be seen as distinct

$$\mathrm{d} = \frac{0.61\lambda}{\mathrm{NA}} = \frac{(0.61) \times (5000 \times 10\ \mathrm{m}^{-10})}{1.25} = 2.4 \times 10^{-7}\ \mathrm{m}$$

$= 0.24\mu\mathrm{m}$

2. **(a)**



$$P = \frac{16^{\hat{2}}}{8R} = 4$$

$\therefore\ \mathrm{R} = 8\Omega$

3. **(b)** Since it is a balanced wheatstone bridge, its equivalent resistance $= \frac{4}{3}\Omega$

$\varepsilon = \mathrm{B}\ell\mathrm{v} = 5 \times 10^{-4}\ \mathrm{V}$

So total resistance

$$\mathrm{R} = \frac{4}{3} + 1.7 \approx 3\Omega$$

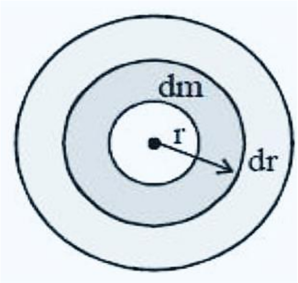
$$\therefore\ \mathrm{i} = \frac{\varepsilon}{\mathrm{R}} \approx 166\mu A \approx 170\mu A$$

4. **(b)** $\mathrm{dl} = (\mathrm{dm})\mathrm{r}^2$

$= (\sigma dA)r^2$

$= \left(\frac{\sigma_0}{r}2\pi dr\right)r^2 = (\sigma_0 2\pi 0 r^2 dr$

$$I = \int DI = \int_a^b \sigma_0 2\pi r^2 dr$$

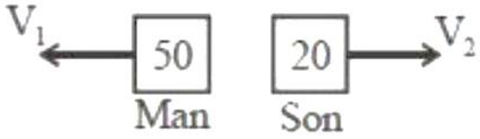


$= \sigma_0 2\pi\left(\frac{b^3 - a^3}{3}\right)$

$m = \int dm = \int \sigma dA$

$= \sigma_0 2\pi \int_a^b dr$

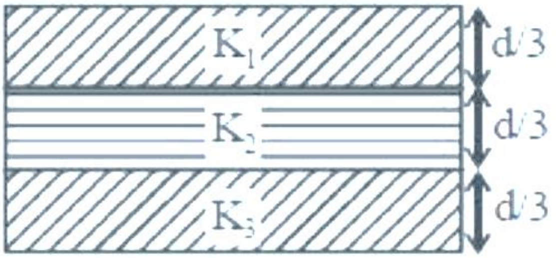
5. (d)



$\Rightarrow 0 = 50\ V_1 - 20\ V_2$ and $V_1 + V_2 = 0.7$

$\Rightarrow V_1 = 0.2$

6. (b)



$C_1 = \frac{3\varepsilon_0 AK_1}{d}$

$C_2 = \frac{3\varepsilon_0 AK_2}{d}$

$C_3 = \frac{3\varepsilon_0 AK_3}{d}$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$\Rightarrow C_{eq} = \frac{3\varepsilon_0 AK_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)}$

II. 

$C_1 = \frac{\varepsilon_0\ K_1\ A}{3\ d}$

$C_2 = \frac{\varepsilon_0\ K_2\ A}{3\ d}$

$C_3 = \frac{\varepsilon_0 K_3 A}{3d}$

$C'_{eq} = C_1 + C_2 + C_3$

$= \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3)$

Now,

$$\frac{E_1}{E_2} = \frac{\frac{1}{2}C_{eq} \cdot V^2}{\frac{1}{2}C'_{eq} V^2} = \frac{9 K_1 K_2 K_3}{(K_1 + K_2 + K_3)(K_1 K_2 + K_2 K_3 + K_3 K_1)}$$

7. (a) $f_{mix} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2} = \frac{2\times3+3\times5}{5} = \frac{21}{5}$

$C_v = \frac{fR}{5} = \frac{21}{5} \times \frac{R}{2} = 17.4\ \text{J/molK}$

8. (b) $hv = \phi + ev v_0$

$v_0 = \frac{h\nu}{e} - \frac{\phi}{e}$

v_0 is zero for $v = 4 \times 10^{14}$ Hz

$0 = \frac{hv}{e} - \frac{\phi}{e}$

$\Rightarrow \phi = hv$

$= \frac{6.63 \times 10^{-34} \times 4 \times 10^{14}}{1.6 \times 10^{-19}} = 1.66\text{eV}$

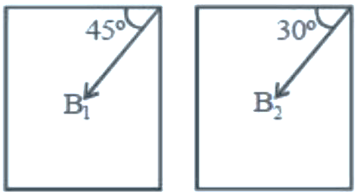
9. (BONUS)

$Mg = \left(\frac{Ay}{\ell}\right)\Delta\ell$

$Mg = (Ay)\alpha\Delta T = 2\pi$

It is closest to 9.

10. (d)



$f_1 = \frac{1}{2\pi}\sqrt{\frac{\mu B_1 \cos 45^\circ}{I}}$ $\quad f_2 = \frac{1}{2\pi}\sqrt{\frac{\mu B_2 \cos 30^\circ}{I}}$

$\frac{f_1}{f_2} = \frac{B_1 \cos 45^\circ}{B_2 \cos 30^\circ}$ $\quad \therefore \frac{B_1}{B_2} \times 0.7$

11. (c) $\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) z^2$

$\frac{1}{1085} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) 2^2$

$\therefore m = 2$

$\therefore n = 5$

12. (a) Heat lost = Heat gain

$\Rightarrow M_2 \times 1 \times 50 = M_1 \times 0.5 \times 10 + M_1 \cdot L_f$

$$\Rightarrow L_f = \frac{50M_2 - 5M_1}{M_1}$$

$$= \frac{50M_2}{M_1} - 5$$

13. (a) Range will be same for time t_1 and t_2, so angles of projection will be 'θ' & '$90° - \theta'$'

$t_1 = \frac{2u\sin\theta}{g} t_2 = \frac{2u\sin(90° - \theta)}{g}$ and $R = \frac{u^2 \sin 2\theta}{g}$

$$t_1 t_2 = \frac{4u^2 \sin\theta \cos\theta}{g^2} = \frac{2}{g}\left[\frac{2u^2 \sin\theta \cos\theta}{g}\right]$$

$$= \frac{2R}{g}$$

14. (a) $V_{gain} = \left(\frac{\Delta I_C}{\Delta I_B}\right)\frac{R_{out}}{R_{in}} = \left(\frac{5\times10^{-3}}{100\times10^{-6}}\right) \times 10^3$

$$= \frac{1}{20} \times 10^8 = 5 \times 10^4$$

$$P_{gain} = \left(\frac{\Delta I_C}{\Delta I_b}\right)(V_{gain})$$

$$= \left(\frac{5 \times 10^{-3}}{100 \times 10^{-6}}\right)(5 \times 10^4)$$

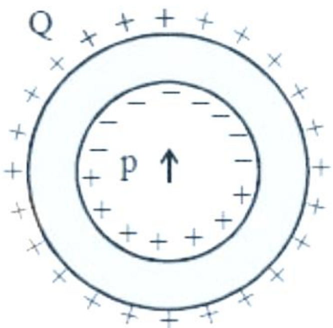
$$= 2.5 \times 10^6$$

15. (b) Total charge of dipole $= 0$, so charge induced on outside surface $= 0$.

But due to non-uniform electric field of dipole, the charge induced on inner surface is non zero and non-uniform.

So, for any observer outside the shell, the resultant electric field is due to Q uniformly distributed on outer surface only and it is equal to.

$$E = \frac{KQ}{r^2}$$



16. (d) $[\varepsilon_0] = M^{-1} L^{-3} T^4 A^2$

$[\mu_0] = ML^{-2}A^{-2}$

$[R] = ML^2T^{-3}A^{-2}$

$$[R] = \left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$$

17. (c) $20 \times 50 \times 10^{-6} = 10^{-3}$Amp.

$$V_1 = \frac{2}{10^{-3}} = 100 + R_1$$

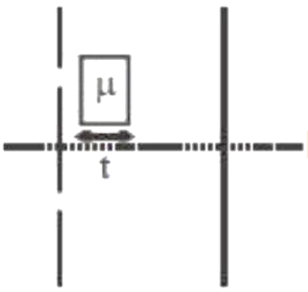
$1900 = R_1$

$$V_2 = \frac{10}{10^{-3}} = (2000 + R_2)$$

$R_2 = 8000$

$$V_3 = \frac{20}{10^{-3}} = 10 \times 10^3 + R_3 = 10 \times 10^3 R_3$$

18. (b)

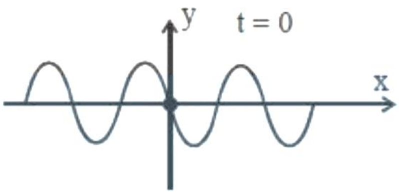


$\Delta x = (\mu - 1)t = 1\lambda$

for one maximum shift

$$t = \frac{\lambda}{\mu - 1}$$

19. (a)

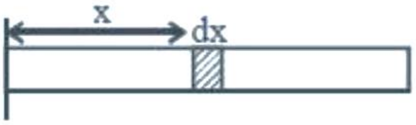


$y = A\sin(kx - wt + \phi)$

at $x = 0, t = 0$ and slope is negative

$\Rightarrow \phi = \pi$

20. (c)

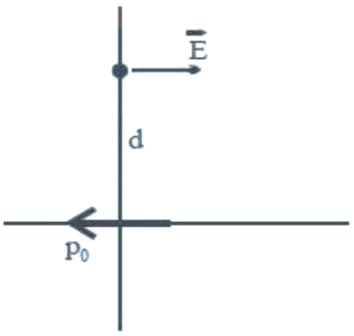


$$T = \int_{x=x}^{x=\ell} dm\omega^2 x = \int_{x=x}^{x=\ell} \frac{m}{\ell} dx\omega^2 xT$$
$$= \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$
$$T = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$

21. (d) V = 0

$$E = -\frac{K\vec{P}}{r^3}$$

$$= -\frac{\vec{p}}{4\pi\varepsilon_0 d^3}$$



22. (a) $\vec{E} = E_0\hat{n}\sin\left(\omega t + (6y - 8z)\right) = E_0\hat{n}\sin\left(\omega t + \vec{k}\cdot\vec{r}\right)$

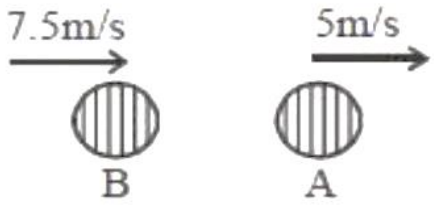
where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $\vec{k}\cdot\vec{r} = 6y - 8z$

$\Rightarrow \vec{k} = 6\hat{j} - 8\hat{k}$

direction of propagation $\hat{s} = -\hat{k}$

$$= \left(\frac{-3\hat{\jmath} + 4\hat{k}}{5}\right)$$

23. (b)



$f_0 = 500$ Hz frequency received by B again $=$

$\longleftarrow 1500$

(B)

(A) & $\Rightarrow$

$7.5\ \mathrm{m/s} \longrightarrow$

$\longrightarrow 5\ \mathrm{m/sec}$

$$f_2 = \left(\frac{1500 + 7.5}{1500 + 5}\right) \times \left(\frac{1500 - 5}{1500 - 7.5}\right) f_0 = 502\ \mathrm{Hz}$$

24. (a) Equation of trajectory is given as

$y = 2x - 9x^2$

Comparing with equation:

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta} \cdot x^2$$

We get, $\tan\theta = 2$

$$\therefore \cos\theta = \frac{1}{\sqrt{5}}$$

Also, $\frac{g}{2u^2\cos^2\theta} = 9$

$$\Rightarrow \frac{10}{2 \times 9 \times \left(\frac{1}{\sqrt{5}}\right)^2} = u^2; u^2 = \frac{25}{9}$$

$$\Rightarrow \mathrm{u} = \frac{5}{3}\ \mathrm{m/s}$$

25. (d) $B = \frac{\mu_0 \mathrm{i}}{2R} = \frac{\mu_0 q\omega}{2R2\pi}$

$\Rightarrow q = 3 \times 10^{-5}\mathrm{C}$

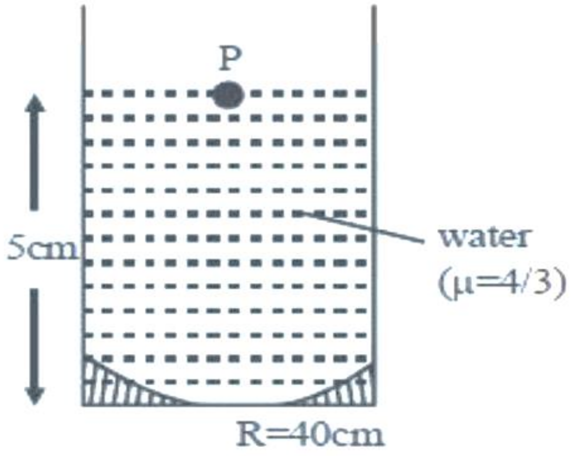
26. (c)
27. (d)
28. (b) Light incident from particle P will be reflected at mirror.

$$u = -5\ \mathrm{cm}, \mathrm{f} = \mathrm{m} - \frac{\mathrm{R}}{2} = -20\ \mathrm{cm}$$

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f};\ v_1=+\frac{20}{3}$ cm

This image will act as object for light getting refracted at water surface.

So, object distance $d=5+\frac{20}{3}=\frac{35}{3}$ cm

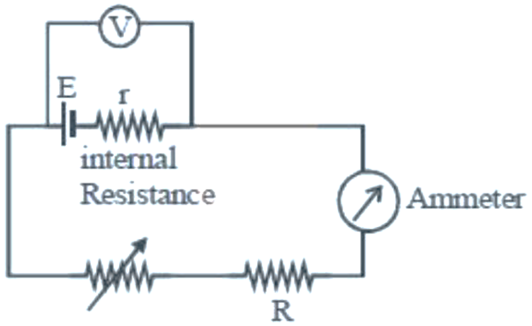


Below water surface.

After refraction, final image is at

$$d'=d\left(\frac{\mu_2}{\mu_1}\right)=\left(\frac{35}{3}\right)\left(\frac{1}{4/3}\right)$$
$$=\frac{35}{4}=8.75\text{ cm}$$
$$\approx 8.8\text{ cm}$$

29. (c)



$V=E-Ir$

When $V=V_0=0\Rightarrow 0=E-Ir$

$\therefore E=r$

When $I=0,\ V=E=1.5$ V

$\therefore\ r=1.5\Omega$

30. (a) Angular momentum conservation

$MV_0L=MV_1(L-\ell)$

$$V_1 = V_0\left(\frac{L}{L-\ell}\right)$$

$$w_g + w_p = \Delta KE$$

$$-mg\ell + w_p = \frac{1}{2}m(V_1^2 - V_0^2)$$
$$w_p = mg\ell + \frac{1}{2}mV_0^2\left(\left(\frac{L}{L-\ell}\right)^2 - 1\right)$$
$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1-\frac{L}{L-\ell}\right)^{-2} - 1\right)$$

Now, $\ell \ll L$

By, Binomial approximation

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1+\frac{L}{L-\ell}\right)^{-2} - 1\right)$$
$$= mg\ell + \frac{1}{2}mV_0^2\left(\frac{2\ell}{L}\right)$$
$$W_P = mg\ell + mV_0^2\frac{\ell}{L}$$

Here, V_0 = maximum velocity $= \omega \times A = \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$

So, $w_p = mg\ell + m\left(\theta_0\sqrt{gL}\right)^2\frac{\ell}{L}$

$== mg\,\ell(1+\theta_0^2)$

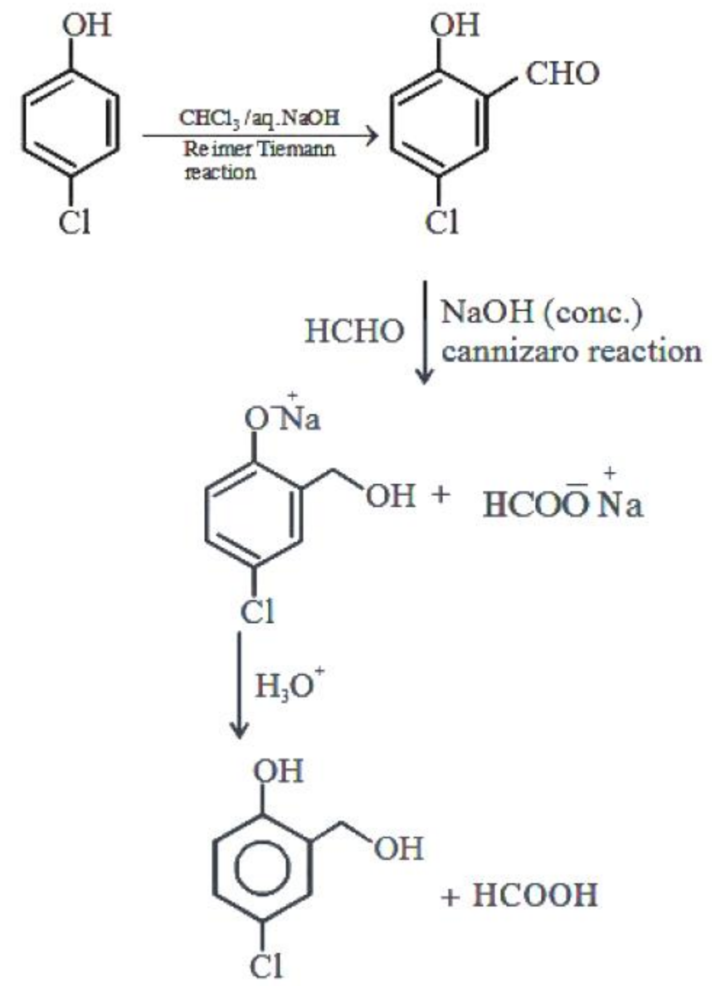
31. (c) $X_{solvent} = 0.8 = 8/10$

$N_{Total} = 10, n_{solutent} = 8, n_{solute} = 2$

Wt of solvent $= 8 \times 18$

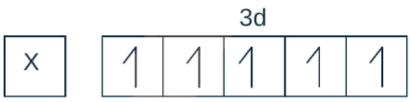
Molality $= \frac{2\times1000}{8\times18}$

32. (c)



33. (a)

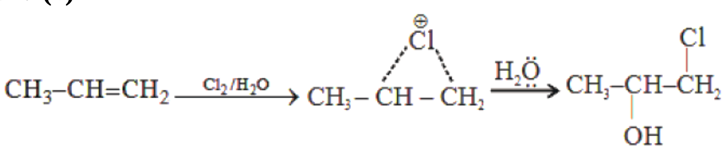
34. (d) When Fe^{2+} oxidizes to Fe^{3+}



35. (a) Smog photochemical is NO, NO_2, O_3.

36. (c)

37. (c)



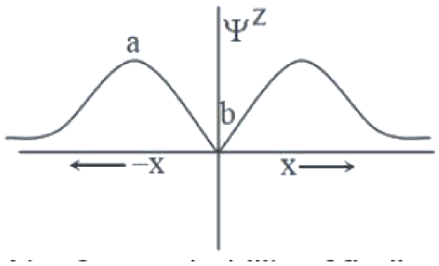
38. (c) Smaller the size of cation, more polarisability high thermal decomposition of carbonates.

39. (a) $15 \rightarrow 1s^2, 2s^2, 2p^6, 3s^2, 3p^3\}$ Period- III, Group-V

40. (b) Peptization is process to form collidal solution by adding peptizing agent.

41. (c)

42. (b)



At a & c, probability of finding electron is maximum

43. (c) $2Cu^{\oplus} \rightarrow Cu^{+2} + Cu$

44. (c) Mol. wt is of 1 mol

$AB_2\ A + 2B = 25$

$A_2B_2\ 2A + 2B = 30$

45. (d) Basic unit is silicate $(SiO_4)^{4-}$

46. (d) $\Delta C_p = \frac{\Delta H_2 - \Delta H_1}{T_2 - T_1}$

47. (b) Thermosetting are cross-linked polymer.

48. (a)

49. (a) Canary yellow ppt comes in test of PO_4^{3-} ion.

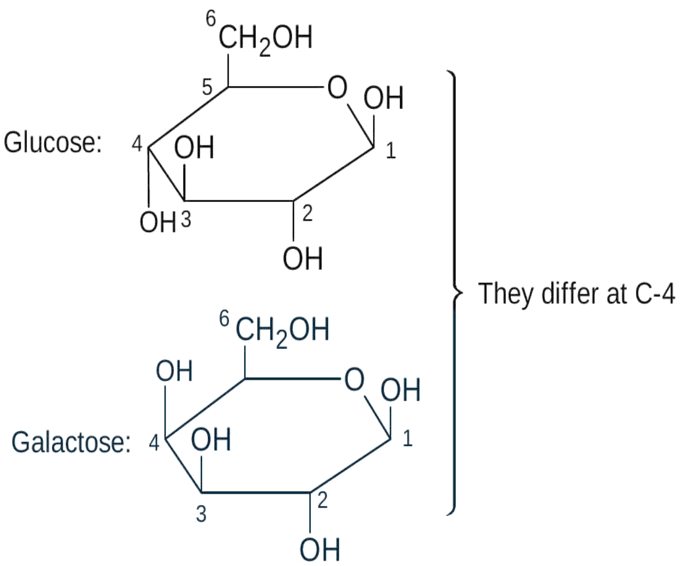
50. (a) Concentration (on dressing), i.e. washing of ore.

51. (d) $-\frac{1}{x}\left(\frac{dA}{dt}\right) = \frac{1}{y}\left(\frac{dB}{dt}\right)$

$$\log_{10}\left[-\frac{d(A)}{dt}\right] = \left[\log_{10}\left(\frac{dB}{dt}\right)\right] + \log\frac{x}{y}\Big]\log\frac{x}{y} = \log 0.3$$

$$\left(\frac{x}{y} = 2\right)$$

52. (a)



53. (a) Not always double stranded α-helix.

54. (d)

$Al(OH)_3 \rightleftharpoons Al^{3+} + 3OH^-$

$s\,(3\,s + 0.2)$

$2.4 \times 10^{-24} = s(3\,s + 0.2)^3$

55. (b)

56. (c) Lower the standard reduction potential, more the ability to get reduced higher the oxidizing power.

57. (c)

58. (c)

59. (b) Ligand filed exert mass repulsion along x, y axis as compared to Z-axis so $d_{x^2-y^2}$ and d_{xx} will have increase in energy y

60. (b) $W = -P_{\text{ext}}\,(V_2 - V_1)$

61. (c) Tangents $y^2 = 12x \Rightarrow y = 2x + \frac{3}{m}$

$$\frac{x^2}{1} - \frac{y^2}{8} = 1 \Rightarrow y = mx \pm \sqrt{m^2 - 8}$$

Common tangent gives

$$\therefore \frac{3}{m = \pm\sqrt{m^2 - 8}}$$

$$\frac{x^2}{1} - \frac{y^2}{8} = 1$$

$$m^4 - 8m^2 - 9 = 0$$

$$e = 3$$

$$m = \pm 3$$

$$ae = 3$$

$$\therefore y = 3x + 1\; P\left(-\frac{1}{3}, 0\right)$$

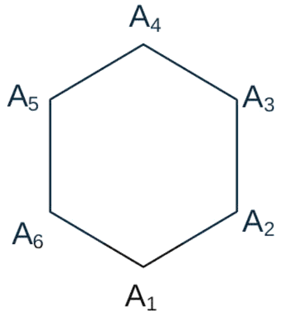
$$S = (3,0)$$

$$S' = (-3,0)$$

$$y = -3x - 1$$

P divides S' ' in 5: 4.

62. (c) Only two equilateral triangle are possible $A_1A_3A_5$ and $A_2A_5A_6$ $\frac{2}{^6C_3} = \frac{2}{20} = \frac{1}{10}$



63. (b) $\lim_{x\to 2}\frac{\int_6^{f(x)} 4t^3 dt}{x-2}$

$= \lim_{x\to 2}\frac{4\cdot f^3(x)\cdot f'(x)}{1}$

$= 4f^3(2)f'(2) = 18$

64. (c)

65. (a)
$(1+x)(1-x)^{10}(1+x+x^2)^9$
$(1-x^2)(1-x^3)^9$
${}^9C_6 = 84$

66. (a) Since ${}^{21}C_0 + \cdots .. + {}^{21}C_{10} + {}^{21}C_{11} + \cdots .... + {}^{21}C_{21} = 2^{21}$

$\Rightarrow$ but we have to select 10 objects and ${}^{21}C_0 + \cdots .. + {}^{21}C_{10} = {}^{21}C_{11} + \cdots .... + {}^{21}C_{21}$

$({}^{21}C_0 + \cdots .. + {}^{21}C_{10}) = 2^{20}$

67. (b) $np = 8$

$npq = 4$

$q = \frac{1}{2} \Rightarrow p = \frac{1}{2}$

$n = 16$

$p(x = r) = {}^{16}C_r\left(\frac{1}{2}\right)^{16}$

$p(x \le 2) = \frac{{}^{16}C_0 + {}^{16}C_1 + {}^{16}C_2}{2^{16}}$

$= \frac{137}{2^{16}}$

68. (d)

69. (d) $f(x) = x\sqrt{kx - x^2}$

$f'(x) = \frac{3kx - 4x^2}{2\sqrt{kx - x^2}}$

For increasing $f(x) \ge 0$

$3kx - 4x^2 \ge 0$

$kx - x^2 \ge 0$

$4x^2 - 3kx \le 0$

$x^2 - kx \le 0$

$x(x-k) \le 0$ so $x \in [0,3)$

$4x\left(x - \frac{3k}{4}\right) \le 0$

+ve $x \ge 3$

$3-\frac{3k}{4}\leq 0$

$k\geq 4$

minimum value of k is $m=4$

$f(x)=x\sqrt{kx-x^2}$

$=3\sqrt{4\times 3-3^2}=3\sqrt{3}, M=3\sqrt{3}$

70. (b) $x_1+\cdots\ldots+x_4=44$

$x_5+\cdots\ldots+x_{10}=96$

$\bar{x}=14, \sum x_i=140$

Variance $=\frac{\sum x_i^2}{n}-\bar{x}^2=4$

Standard deviation $=2$

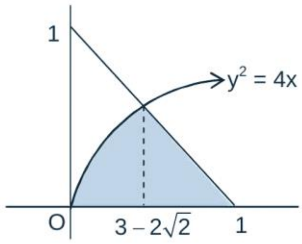
71. (b) $\{(x,y): y^2\leq 4x, x+y\leq 1, x\geq 0, y\geq 0\}$

A $\int_0^{3-2\sqrt{2}} 2\sqrt{x}dx+\frac{1}{2}(1-(3-2\sqrt{2}))(1-(3-2\sqrt{2}))$

$=\frac{2[x^{3/2}]_0^{3-2\sqrt{2}}}{3/2}+\frac{1}{2}(2\sqrt{2}-2)(2\sqrt{2}-2)$

$=\frac{8\sqrt{2}}{3}+\left(-\frac{10}{3}\right)$

$a=\frac{8}{3}, b=-\frac{10}{3}$



$a-b=6$

72. (c) $y^2dx+xdy=\frac{dy}{y}$

$\frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$

$\text{IF}=e^{\int\frac{1}{y^2}dy}=e^{-\frac{1}{y}}$

$e^{\frac{1}{y}}\cdot x=\int e^{\frac{1}{y}}\cdot\frac{1}{y^3}dy+C$

$xe^{\frac{1}{y}} = e^{\frac{1}{y}} + \frac{e^{-\frac{1}{y}}}{y} + C$

$C = -\frac{1}{e}$

$x = \frac{3}{2} - \frac{1}{\sqrt{e}}$ when $y = 2$

73. (a) $375x^2 - 25x - 2 = 0$

$\alpha + \beta = \frac{25}{375}, \alpha\beta = \frac{-2}{375}$

$\Rightarrow (\alpha + \alpha^2 + \cdots$ upto inf inite terms $) + (\beta + \beta^2 + \cdots$ ... . upto inf inite terms $)$

$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} = \frac{1}{12}$

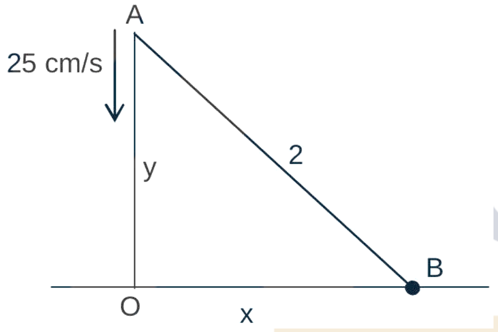
74. (d)

$x^2 + y^2 = 4\left(\frac{dy}{dt} = -25\right)$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$\sqrt{3}\frac{dx}{dt} - 1(25) = 0$

$\frac{dx}{dt} = \frac{25}{\sqrt{3}}$ cm/sec



75. (c) $1 + \sin^4 x = \cos^2 3x$

$\sin x = 0$ and $\cos 3x = 1$

$0, 2\pi, -2\pi, -\pi, \pi$

76. (c) $e^y = xy = e$ differentiate w.r.t. x

$e^y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$

$e^y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$

$\frac{dy}{dx}(x + e^y) = -y, \left.\frac{dy}{dx}\right|_{(0,1)} = -\frac{1}{e}$ again differentiate w.r.t.x

$e^y \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot e^y \cdot \frac{dy}{dx} + x \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$

$(x + e^y)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \cdot e^y + 2\frac{dy}{dx} = 0$

$e\frac{d^2y}{dx^2} + \frac{1}{e^1}e + 2\left(-\frac{1}{e}\right) = 0$

$\therefore \frac{d^2y}{dx^2} = \frac{1}{e^2}$

77. (d)

78. (b) $3x^2 + 4y^2 = 12$

$\frac{x^2}{4} + \frac{y^2}{3} = 1$

$x = 2\cos\theta, y = \sqrt{3}\sin\theta$

Equation of normal is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

$2x\sin\theta - \sqrt{3}\cos\theta y = \sin\theta\cos\theta$

Slope $\frac{2}{\sqrt{3}}\tan\theta = -2 \;\therefore \tan\theta = -\sqrt{3}$

Equation of tangent is it passes through (4,4)

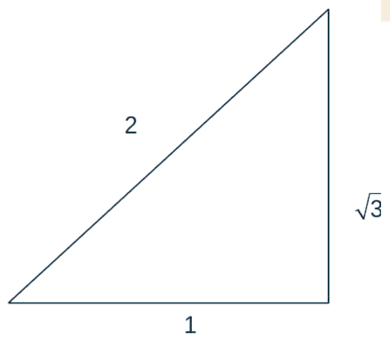
$3x\cos\theta + 2\sqrt{3}\sin\theta y = 6$

$12\cos\theta + 8\sqrt{3}\sin\theta = 6$

$\cos\theta = -\frac{1}{2}, \sin\theta = \frac{\sqrt{3}}{2} \therefore \theta = 120^\circ$

Hence point P is $\left(2\cos 120^\circ, \sqrt{3}\sin 120^\circ\right)$

$\mathrm{P}\left(-1, \frac{2}{2}\right), \mathrm{Q}(4,4)$



$\mathrm{PQ} = \frac{5\sqrt{5}}{2}$

79. (a)

$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})$

$$= 2\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 2 & 2 \end{vmatrix}$$

$$= 2(8\hat{\imath} - 8\hat{\jmath} + 4\hat{k})$$

Required vector $= \pm 12\frac{(2\hat{\imath}-2\hat{\jmath}-\hat{k})}{3}$

$= \pm 4(2\hat{\imath} - 2\hat{\jmath} - \hat{k})$

80. (c)

$2y = 2\sin\ x\sin\ (x+2) - 2\sin^2\ (x+1)$

$2y = \cos\ 2 - \cos\ (2x+2) - (1 - \cos\ (2x+2))$

$= \cos\ 2 - 1$

$2y = -2\sin^2\ \frac{1}{2}$

$y = -\sin^2\ \frac{1}{2} \leq 0$

81. (a) $f(x) = \sqrt{x}, g(x) = \tan x, h(x) = \frac{1-x^2}{1+x^2}$

$\text{fog}(x) = \sqrt{\tan x}$

$\text{hofog}(x) = h(\sqrt{\tan x}) = \dfrac{1-\tan x}{1+\tan x}$

$= -\tan\left(\frac{\pi}{4} - x\right)$

$\phi(x) = \tan\left(\frac{\pi}{4} - x\right)$

$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$

$= \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$

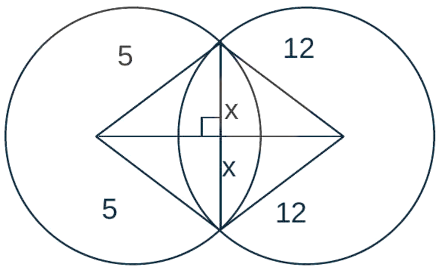
82. (b)

Let length of common chord $= 2x$

$\sqrt{25 - x^2} + \sqrt{144 - x^2} = 13$

After solving $x = \frac{12\times 5}{13}$

$2x = \dfrac{120}{13}$



83. (d)

$$\int_0^{\pi/2} \frac{\cot x dx}{\cot x + \operatorname{cosec} x}$$

$$\int_0^{\pi/2} \frac{\cos x}{\cos x + 1} = \int \frac{2\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2} - 1}$$

$$\int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2\frac{x}{2}\right) dx$$

$$\left[x - \tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$\frac{1}{2}[\pi - 2]\ m = \frac{1}{2}, n = -2$$

84. (c)

$2\{2a + 3d\} = 16$

$3(2a + 5d) = -48$

$$2a + 3d = 8$$
$$2a + 5d = -16$$
$$d = -12$$
$$S_{10} = 5\{44 - 9 \times 12\}$$
$$= -320$$

85. (c) $|B| = 5(-5) - 2\alpha(-\alpha) - 2\alpha$

$= 2\alpha^2 - 2\alpha - 25$

$1 + |A| = 0$

$\alpha^2 - \alpha - 12 = 0$

Sum $= 1$

86. (a) $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$

$x = 3\lambda + 2, y = 2\lambda - 1, z = -\lambda + 1$

Intersection with plane $2x + 3y - z + 13 = 0$

$2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$

$13\lambda + 13 = 0$

$\lambda = -1$

$\therefore P(-1,-3,2)$

Intersection with plane

$3x + y + 4z = 16$

$3(3\lambda + 2) + (2\lambda - 1) + 4(-\lambda + 1) = 16$

$\lambda = 1$

$Q(5,1,0)$

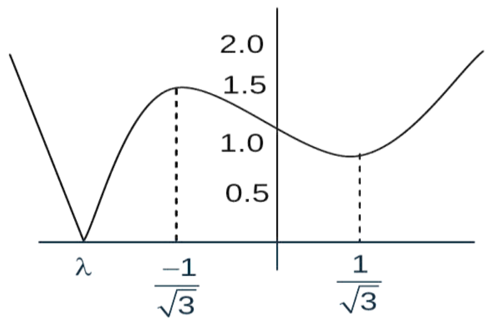
$PQ = \sqrt{6^2 + 4^2 + 2^2} = \sqrt{56} = 2\sqrt{14}$

87. (BONUS)(b)

Volume of paralleopiped $= \left\|\begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix}\right\|$

$f(\lambda) = |\lambda^3 - \lambda + 1|$

Its graphs as follows



where $\lambda \approx -1.32$

For minimum value of volume of paralelopiped and corresponding value of λ; the minimum value is zero, $\because$ cubic always has at least one real root.

Hence answer to the question must be root of cubic $\lambda^3 - \lambda + 1 = 0$. None of the options satisfies the cubic.

88. (b) $A = A', B = -B'$

$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$

$A' - B' = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$

$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$

After adding equation (1) and (2)

$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

89. (a) $P \rightarrow (\sim q \vee r)$

$\sim p \vee (\sim q \vee r)$

$\sim p \rightarrow F\, p \rightarrow T)$

$\sim q \rightarrow F\} \Rightarrow q \rightarrow T\}$

$\sim r \rightarrow F\, r \rightarrow F$

90. (c) $$\underbrace{\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \cdots . + \left[\frac{1}{3} - \frac{66}{100}\right]}_{(-1)67} + \underbrace{\left[-\frac{1}{3} - \frac{67}{100}\right] + \cdots + \left[-\frac{1}{3} - \frac{99}{100}\right]}_{-2(33)} = -133$$