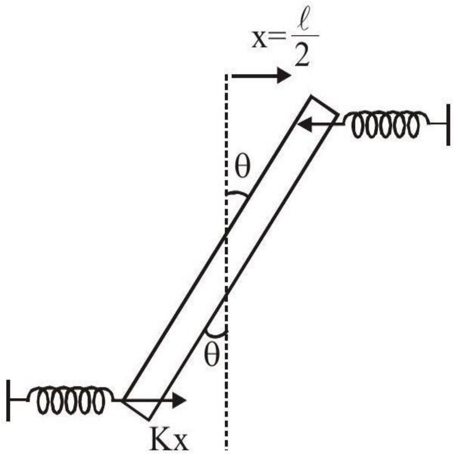
1. (a)



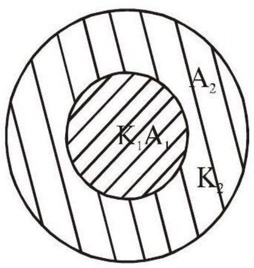
$$\tau = -2Kx\frac{\ell}{2}\cos\theta$$

$$\Rightarrow \tau = \left(\frac{K\ell^2}{2}\right)\theta = -C\theta$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6\,K}{M}}$$

2. (d)



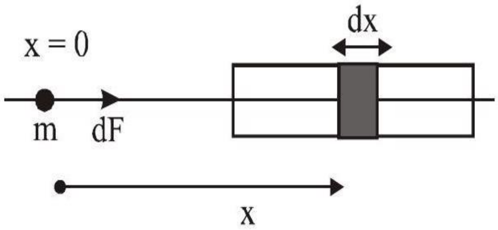
$$K_{eq} = \frac{K_1\,A_1 + K_2\,A_2}{A_1 + A_2}$$
$$= \frac{K_1(\pi R^2) + K_2(3\pi R^2)}{4\pi R^2}$$
$$= \frac{K_1 + 3\,K_2}{4}$$

3. (a). $y = a\sin(\omega t + kx)$

$\Rightarrow$ wave is moving along - ve x-axis with speed

$v = \frac{\omega}{K} \Rightarrow v = \frac{50}{2} = 25$ m/sec.

4. (b)



$dm = (A + Bx^2)dx$

$dF = \dfrac{GMdm}{x^2}$

$= F = \int_a^{a+L} \dfrac{GM}{x^2}(A + Bx^2)dx$

$= GM\left[-\dfrac{A}{x} + Bx\right]_a^{a+L}$

$= GM\left[A\left(\dfrac{1}{a} - \dfrac{1}{a+L}\right) + BL\right]$

5. (b) $P_{\text{refracted}} = \frac{96}{100}P_I$

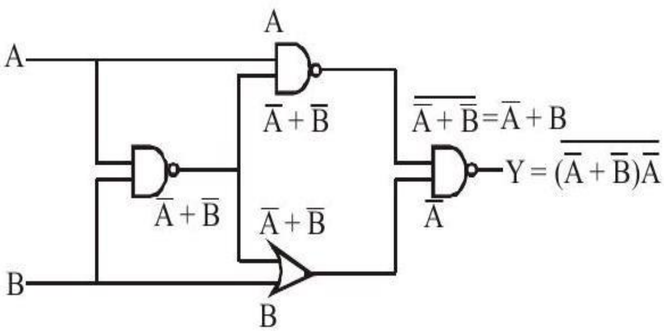
$\Rightarrow K_2\, A_t^2 = \dfrac{96}{100}\, K_1\, A_i^2$

$\Rightarrow r_2\, A_t^2 = \dfrac{96}{100} r_1\, A_i^2$

$\Rightarrow A_t^2 = \dfrac{96}{100} \times \dfrac{1}{\frac{3}{2}} \times (30)^2$

$A_t \sqrt{\dfrac{64}{100} \times (30)^2} = 24$

6. (b)



$Y=\overline{(\bar{A}+\bar{B})\bar{A}}$

$=\overline{\bar{A}+\bar{A}B}$

$=A(\overline{\bar{A}B})$

$=A(A+\bar{B})$

$=A+A\bar{B}=A\bar{B}$

7. (a) $V_i=\frac{1}{2}CE^2$

$$V_f=\frac{(CE)^2}{2\times 4c}=\frac{1}{2}\frac{CE^2}{4}$$

$$\Delta E=\frac{1}{2}CE^2\times\frac{3}{4}=\frac{3}{8}CE^2$$

8. (d). $F=\frac{dV}{dr}=kr=\frac{mv^2}{r}$

$$mvr=\frac{nh}{2\pi}$$

$r^2\propto n\ r^2\propto\sqrt{n}$

$$E=\frac{1}{2}kr^2+\frac{1}{2}mv^2\propto r^2$$

$\propto n$

9. (c) $R_1=\frac{220^2}{25}$

$$R_2=\frac{220^2}{100}$$

$$L=\frac{220}{R_1+R_2}$$

$P_1=i^2R_1$

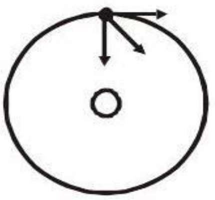
$P_2=i^2\ (R_2=4\ W)$

$$=\frac{220^2}{\left(\frac{220^2}{25}+\frac{220^2}{100}\right)}\times\frac{220^2}{25}$$

$$=\frac{400}{25}=16\ W$$

10. (c) $mv\hat{\imath}+mv\hat{\jmath}$

$=2m\vec{v}^1$



$$\vec{v} = \frac{1}{\sqrt{2}} \times \sqrt{\frac{GM}{R}}$$

11. (c)

12. (d)

13. (d) Energy $= \frac{1}{2}nRT = \frac{f}{2}PV$

$$= \frac{f}{2}(3 \times 10^6)(b)$$
$$= f \times 3 \times 10^6$$

Considering gas is monoatomic i.e. $f = 3$

E. $= 9 \times 10^6$ J

14. (a)

Sol. $E_m + E_c = 160$

$E_m + 100 = 160$

$$\mu = \frac{E_m}{E_C} = \frac{60}{100}$$
$$\mu = 0.6$$

15. (d) case I $i_g = \frac{E}{220+R_g} = C\theta_0$

Case II

$$i_g = \left(\frac{E}{220 + \frac{5R_g}{5+R_g}}\right) \times \frac{5}{(R_g+5)} = \frac{C\theta_0}{5}$$

$$\Rightarrow \frac{5E}{225R_g + 1100} = \frac{C\theta_0}{5}$$

$$\frac{E}{220 + R_g} = C\theta$$

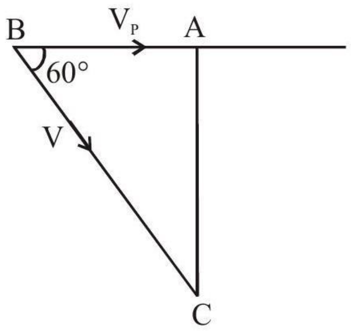
$$\Rightarrow \frac{225R_g + 1100}{1100 + 5R_g} = 5$$

$$\Rightarrow 5500 + 25R_g = 225R_g + 1100$$

$$200R_g = 4400$$

$R_g = 22\Omega$

16. (c)



$$\frac{1}{2} = \frac{V_p \times t}{Vt}$$

$$V_P = \frac{V}{2}$$

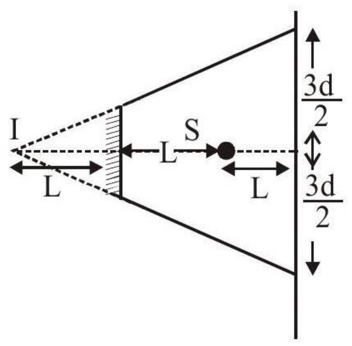
17. (a) $KE = q\Delta V$

$$r = \frac{\sqrt{2mq\Delta V}}{qB}$$

$$r \propto \sqrt{\frac{m}{q}}$$

$$\frac{r_p}{r_\alpha} = \frac{1}{\sqrt{2}}$$

18. (a)



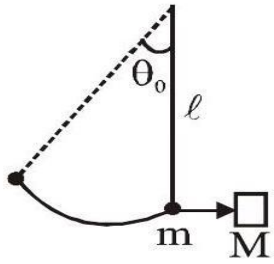
3 d

19. (c) Least count $= \dfrac{\text{Pitch}}{\text{Number of division on circular scale}}$

$$5 \times 10^{-6} = \frac{10^{-3}}{N}$$

$N = 200$

20. (c)



Before colision After collision

$\dot{\text{m}}\text{M}$
$= \sqrt{2\ \text{g}\ell(1-\cos\theta_0)}$

$\text{v}_1 \overset{\text{m}}{\text{M}} \overset{\text{M}}{\text{v}_\text{m}}$

$\text{v}_1 = \sqrt{2\ \text{g}\ell(1-\cos\theta_1)}$

By momentum conservation

$\text{m}\sqrt{2\ \text{g}\ell(1-\cos\theta_0)} = \text{MV}_\text{m} - \text{m}\sqrt{2\text{gl}(1-\cos\theta}$

$\Rightarrow \text{m}\sqrt{2\ \text{g}\ell}\{\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}\} = \text{MV}_\text{m}$

and $e = 1 = \frac{V_m + \sqrt{2g\ell(1-\cos\theta_1)}}{\sqrt{2g\ell(1-\cos\theta_0)}}$

$\sqrt{2\ \text{g}\ell}\left(\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}\right) = \text{V}_\text{m}$

$\text{m}\sqrt{2\ \text{g}\ell}\left(\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}\right) = \text{MV}_\text{M}$

Dividing

$$\frac{\left(\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}\right)}{\left(\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}\right)} = \frac{M}{m}$$

By componendo divided

$$\frac{m-M}{m+M} = \frac{\sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0}} = \frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}$$

$$\Rightarrow \frac{\text{M}}{\text{m}} = \frac{\theta_0 - \theta_1}{\theta_0 + \theta_1} \Rightarrow \mathbf{M} = \frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}$$

21. (d) For first lens

$$\frac{1}{\text{V}} - \frac{1}{-20} = \frac{1}{5}$$

$$\text{V} = \frac{20}{3}$$

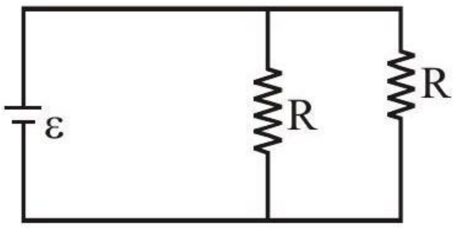
For second lens

$$V = \frac{20}{3} - 2 = \frac{14}{3}$$

$$\frac{1}{V} - \frac{1}{\frac{14}{3}} = \frac{1}{-5}$$

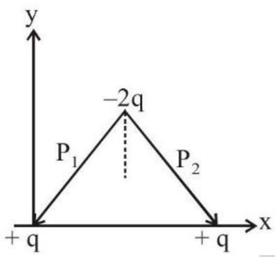
$V = 70$ cm

22. (a) Ideal inductor will behave like zero resistance long time after switch is closed



$$I = \frac{2\varepsilon}{R} = \frac{2 \times 15}{5} = 6 \text{ A}$$

23. (c)



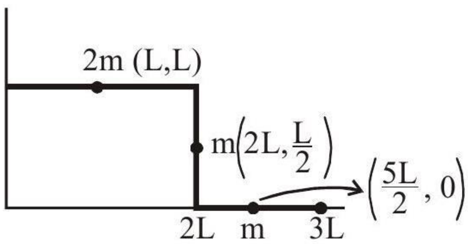
$|P_1| = q(d)$

$|P_2| = qd$

$|\text{ Resultant }| = 2P\cos 30^\circ$

$$2qd\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}qd$$

24. (a)



$$X_{cm} = \frac{2\text{ mL} + 2\text{ mL} + \frac{5\text{ mL}}{2}}{4\text{ m}} = \frac{13}{8}\text{ L}$$

$$Y_{cm} = \frac{2\text{ m} \times L + \text{m} \times \left(\frac{L}{2}\right) + \text{m} \times 0}{4\text{ m}} = \frac{5\text{ L}}{8}$$

25. (d) Magnetic field at ' O ' will be done to 'PS' and ' QN ' only

i.e. $B_0 = B_{PS} + B_{QN} \rightarrow$ Both inwards

Let current in each wire = i

$$\therefore \quad B_0 = \frac{\mu_0 i}{4\pi d} + \frac{\mu_0 i}{4\pi d}$$
$$\text{or} \quad 10^{-4} = \frac{\mu_0 i}{2\pi d} = \frac{2 \times 10^{-7} \times i}{4 \times 10^{-2}}$$
$$\therefore \quad i = 20\text{ A}$$

26. (a) For the given wire : $dR = C\frac{d\ell}{\sqrt{\ell}}$, where C = constant.

Let resistance of part AP is R_1 and PB is R_2

$\therefore \frac{R'}{R'} = \frac{R_1}{R_2}$ or $R_1 = R_2$ By balanced

WSB concept.

$$\text{Now} \int dR = c\int \frac{d\ell}{\sqrt{\ell}}$$
$$\therefore R_1 = C\int_0^{\ell} \ell^{-1/2}\, d\ell = C \cdot 2 \cdot \sqrt{\ell}$$
$$R_2 = C\int_{\ell}^{1} \ell^{-1/2}\, d\ell = C \cdot (2 - 2\sqrt{\ell})$$

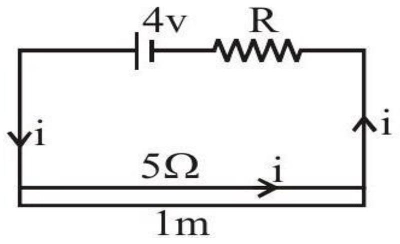
Putting $R_1 = R_2$

$$C2\sqrt{\ell} = C(2 - 2\sqrt{\ell})$$
$$\therefore 2\sqrt{\ell} = 1$$
$$\sqrt{\ell} = \frac{1}{2}$$
$$\text{i.e. } \ell = \frac{1}{4}\text{ m} \Rightarrow 0.25\text{ m}$$

27. (c) Since P-V indicator diagram is given, so work done by gas is area under the cyclic diagram.

$\therefore \Delta W$ = Work done by gas $= \frac{1}{2} \times 4 \times 5\text{ J} = 10\text{ J}$

28. (c)



Let current flowing in the wire is i.

$\therefore\ i = \left(\frac{4}{R+5}\right) A$

If resistance of 10 m length of wire is x

then $x = 0.5\Omega = 5 \times \frac{0.1}{1}\Omega$

$\therefore\ \Delta V = \text{P. d. on wire} = i.x$

$$5 \times 10^{-3} = \left(\frac{4}{R+5}\right) \cdot (0.5)$$

$$\therefore \frac{4}{R+5} = 10^{-2} \text{ or } R + 5 = 400\Omega$$

$$\therefore R = 395\Omega$$

29. (b) K.E. acquired by charge $= K = qV$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = \frac{\sqrt{2\,m_B q_B V_B}}{\sqrt{2\,m_A q_A V_A}} = \sqrt{\frac{4\,m \cdot q \cdot 2500}{m \cdot q \cdot 50}} = 2\sqrt{50}$$

$= 2 \times 7.07 = 14.14$

30. (a) At any instant ' t '

Total energy of charge distribution is constant

i.e. $\frac{1}{2}mV^2 + \frac{KQ^2}{2R} = 0 + \frac{KQ^2}{2R_0}$

$$\therefore\ \frac{1}{2}mV^2 = \frac{KQ^2}{2R_0} - \frac{KQ^2}{2R}$$

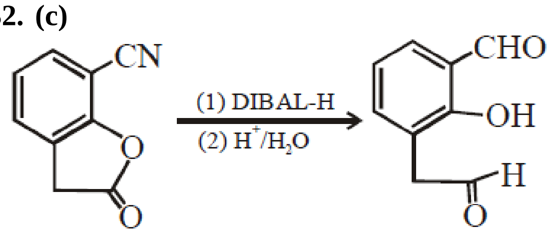
$$\therefore\ V = \sqrt{\frac{2}{m}\frac{KQ^2}{2} \cdot \left(\frac{1}{R_0} - \frac{1}{R}\right)}$$

$$\therefore V = \sqrt{\frac{KQ^2}{m}\left(\frac{1}{R_0} - \frac{1}{R}\right)} = C\sqrt{\frac{1}{R_0} - \frac{1}{R}}$$

Also the slope of v-s curve will go on decreasing

31. (a) $I_2 + 10HNO_3 \longrightarrow 2HIO_3 + 10NO_2 + 4H_2O$ In HIO_3 oxidation state of iodine is +5

32. (c)



DIBAL-H will reduce cyanides & esters to aldehydes.

33. (b)

$$\begin{array}{lcccc} & A & + \ 2B & \rightleftharpoons \ 2C & + \ D \\ t=0 & a_0 & 1.5a_0 & 0 & 0 \\ t=t_{eq} & a_0 - x & 1.5a_0 - 2x & 2x & x \end{array}$$

At equilibrium [A] = [B]

$a_0 - x = 1.5a_0 - 2x \Rightarrow x = 0.5a_0$

$t = t_{eq}$ $0.5a_0$ $0.5a_0$ a_0 $0.5a_0$

$$K_C = \frac{[C]^2[D]}{[A][B]^2} = \frac{(a_0)^2(0.5a_0)}{(0.5a_0)(0.5a_0)^2} = 4$$

34. (c)

Sol. $A(s) \rightleftharpoons B(g) + C(g)$ $K_{P_1} = x = P_B \cdot P_C$

P_1 P_1 $x = P_1(P_1 + P_2)$

$D(s) \rightleftharpoons C(g) + E(g)$ $K_{P_2} = y = P_C \cdot P_E$

P_2 P_2 $y \overset{2}{=} (P_1 + P_2)(P_2)$

Adding (a) and (b)

$x + y = (P_1 + P_2)^2$

Now total pressure

$P_T = P_C + P_B + P_E$

$= (P_1 + P_2) + P_1 + P_2 = 2(P_1 + P_2)$

$P_T = 2(\sqrt{x + y})$

35. (b) For same freezing point, molality of both solution should be same.

$m_x = m_y$

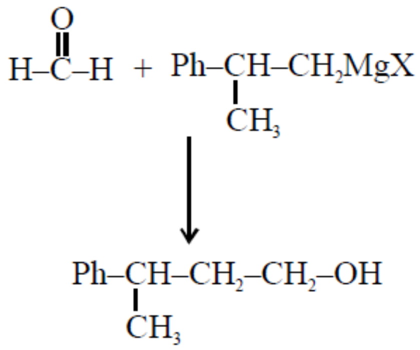
$$\frac{4 \times 1000}{96 \times M_x} = \frac{12 \times 1000}{88 \times M_y}$$

or, $M_y = \frac{96 \times 12}{4 \times 88} M_x = 3.27$ A

36. (d) PHBV is a polymer of 3-hydroxybutanoic acid and 3-Hydroxy pentanoic acid.

37. (a) M.P. of Napthalene $\simeq 80^\circ C$

38. (a)



39. (a) $V_A = 2\ V_B$

$Z_A = 3Z_B$

$$\frac{P_A V_A}{n_A RT_A} = \frac{3 \cdot P_B \cdot V_B}{n_B \cdot RT_B}$$

$2P_A = 3P_B$

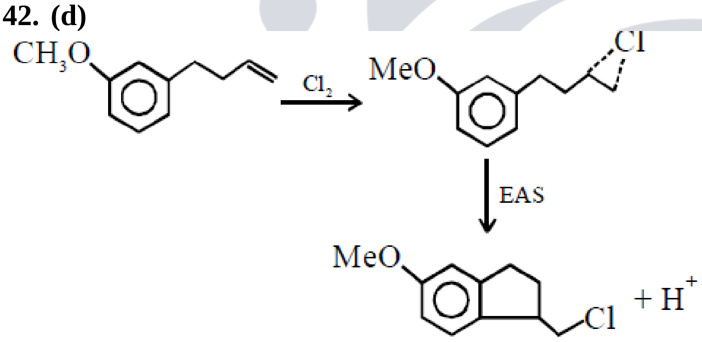
40. (c) $Z = 120$

Its general electronic configuration may be represented as [Nobal gas] ns^2 , like other alkaline earth metals.

41. (a) Rate constant $(K) = 0.05\mu g/$ year means zero order reaction

$$t_{1/2} = \frac{a_0}{2\ K} = \frac{5\mu g}{2\times 0.05\mu g/\ \text{year}} = 50\ \text{year}$$

42. (d)



43. (a) Smaller the value of critical temperature of gas, lesser is the extent of adsorption. so least adsorbed gas is H_2

44. (b) At higher temperature, rotational degree of freedom becomes active.

$$C_P = \frac{7}{2}R \ \ (\text{Independent of P})$$

$$C_V = \frac{5}{2}R \ \ (\text{Independent of V})$$

Variation of U vs T is similar as C_V vs T.

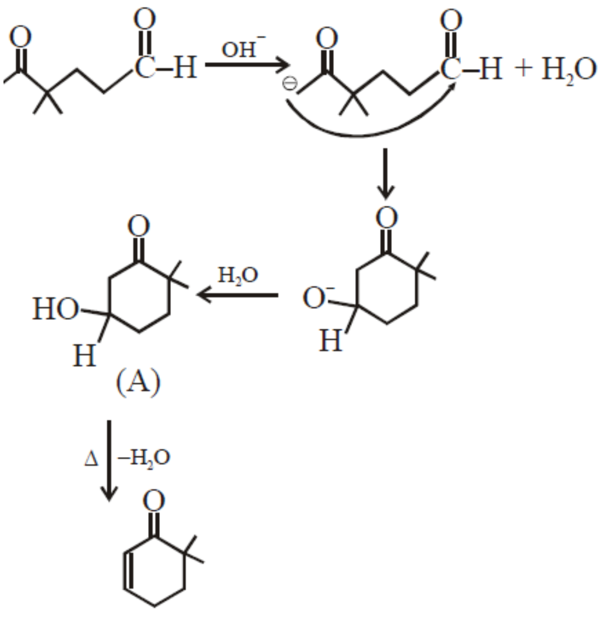
45. (a) Chiefly NO_2, O_3 and hydrocarbon are responsible for build up smog.

46. (b) Chiefly NO_2, O_3 and hydrocarbon are responsible for build up smog.

47. (b) carbon

48. (c) Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17ppm or more.

49. (d)



(B)

50. (c) $h\nu = \phi + h\nu^\circ$

$$\frac{1}{2}mv^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$h\nu = \phi + \frac{1}{2}mv^2$$

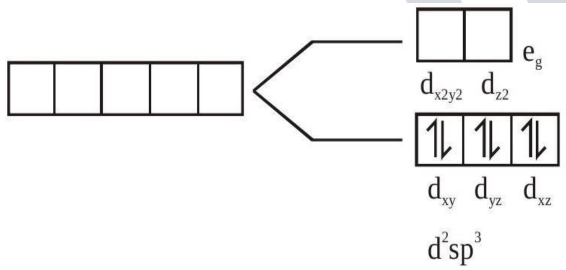
$$\phi = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{4000 \times 10^{-10}} - \frac{1}{2} \times 9 \times 10^{-31} \times (6 \times 10^5)^2$$

$$\phi = 3.35 \times 10^{-19}\ \mathrm{J} \Rightarrow \phi \simeq 2.1\mathrm{eV}$$

51. (d) Histidine

52. (d) $K_3[Co(CN)_6]$

$Co^{+3} \rightarrow [Ar]_{18} 3\ d^6$



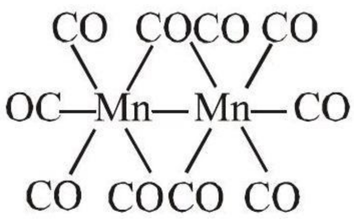
53. (a) ppm of $CaCO_3$

$(10^{-3} \times 10^3) \times 100 = 100\mathrm{ppm}$

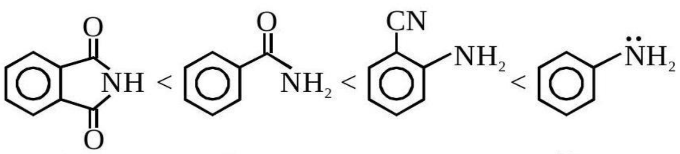
54. (b) $CH \equiv CH > CH_3 - C \equiv CH > CH_2 = CH_2$

(Acidic strength order)

55. (b) Compounds having at least one bond between carbon and metal are known as organometallic compounds.



56. (b) Nucleophilicity order



B A C D

57. (b) V^{2+} and Co^{2+}

$$V^{2+} \rightarrow [V(H_2O)_6]Cl_2\ ; \quad [Ar]_{18}\ \underbrace{\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}\boxed{}}_{3d^3}$$

3 unpaired e⁻, spin only magnetic moment

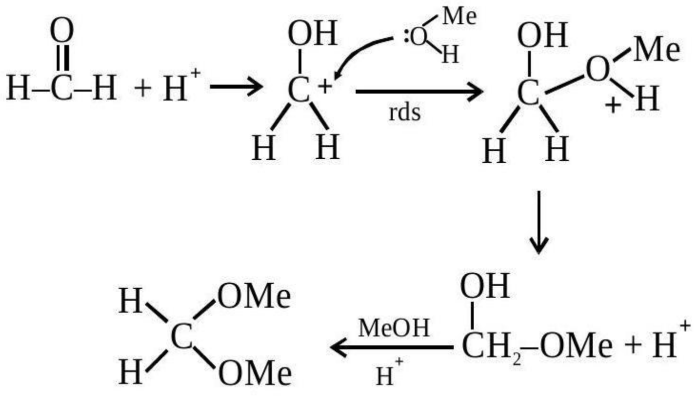
= 3.89 B.M.

$$Co^{2+} \rightarrow [Co(H_2O)_6]Cl_2\ ; \quad [Ar]_{18}\ \underbrace{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}_{3d^7}$$

3 unpaired e⁻, spin only magnetic moment

= 3.89 B.M.

58. (a)



acetal

rate $\propto \dfrac{1}{\text{steric crowding of aldehyde}}$

t-butanol can show formation of carbocation in acidic medium.

59. (BONUS)

$H_2C_2O_4 + 2NaOH \longrightarrow Na_2C_2O_4 + 2H_2O$

m_{eq} of $H_2C_2O_4 = m_{eq}$ NaOH

$50 \times 0.5 \times 2 = 25 \times M_{NaOH} \times 1$

$\therefore\ M_{NaOH} = 2M$

Now 1000 ml solution $= 2 \times 40$ gram NaOH

$\therefore$ 50ml solution $= 4$ gram NaOH

60. (a) $Rb + O_{2\,(excess)} \longrightarrow RbO_2$ $2RbO_2 + 2H_2O \longrightarrow 2RbOH + H_2O_2 + O_2$

61. (d) $(2x)^{2y} = 4e^{2x-2y}$

$2y\ell n2x = \ell n4 + 2x - 2y$

$$y = \frac{x + \ell n2}{1 + \ell n2x}$$

$$y' = \frac{(1 + \ell n2x) - (x + \ell n2)\frac{1}{x}}{(1 + \ell n2x)^2}$$

$$y'(1 + \ell n2x)^2 = \left[\frac{x\ell n2x - \ell n2}{x}\right]$$

62. (c) $\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$

$\mu(\mu^2 - 1) - 1(\mu - 1) + 1(1 - \mu) = 0$

$\mu^3 - \mu - \mu + 1 + 1\mu = 0$

$\mu^3 - 3\mu + 2 = 0$

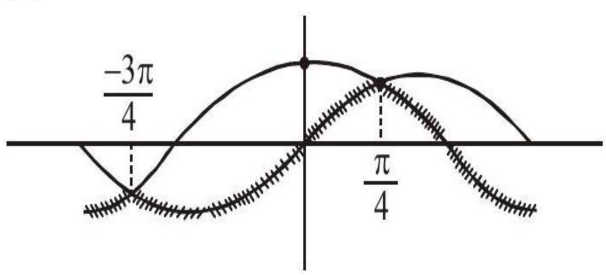
$\mu^3 - 1 - 3(\mu - 1) = 0$

$\mu = 1, \mu^2 + \mu - 2 = 0$

$\mu = 1, \mu = -2$

sum of distinct solutions $= -1$

63. (a)



64. (d) Let terms are $\frac{a}{r}, a, ar \rightarrow$ G.P

$\therefore a^3 = 512 \Rightarrow a = 8$

$\frac{8}{r} + 4, 12, 8r \rightarrow$ A.P.

$24 = \frac{8}{r} + 4 + 8r$

$r = 2, r = \frac{1}{2}$

$r = 2(4,8,16)$

$r = \frac{1}{2}(16,8,4)$

Sum $= 28$

65. (b) $I = \int \cos(\ell nx)dx$

$I = \cos(\ln x) \cdot x + \int \sin(\ell nx)dx$

$\cos(\ell nx)x + \left[\sin(\ell nx) \cdot x - \int \cos(\ell nx)dx\right]$

$I = \frac{x}{2}[\sin(\ell nx) + \cos(\ell nx)] + C$

66. (a)

Sol. $S_K = \frac{K+1}{2}$

$\Sigma S_k^2 = \frac{5}{12}\ A$

$\Sigma_{K=1}^{10}\left(\frac{K+1}{2}\right)^2 = \frac{2^2 + 3^2 + - - + 11^2}{4} = \frac{5}{12}\ A$

$\frac{11 \times 12 \times 23}{6} - 1 = \frac{5}{3}\ A$

$505 = \frac{5}{3}\ A,\ \ A = 303$

67. (a) $S = \{1,2,3 - \cdots - - - 100\}$

= Total non empty subsets-subsets with product of element is odd

$= 2^{100} - 1 - 1[(2^{50} - 1)]$
$= 2^{100} - 2^{50}$
$= 2^{50}(2^{50} - 1)$

68. (d) $\sum_{i=1}^{50}(x_i - 30) = 50$

$\Sigma x_i = 50 \times 30 = 50$

$\Sigma x_i = 50 + 50 + 30$

Mean $= \bar{x} = \frac{\Sigma x_i}{n} = \frac{50\times30+50}{50} = 30 + 1 = 31$

69. (a) Centre of circles are opposite side of line

$(3 + 4 - \lambda)(27 + 4 - \lambda) < 0$

$(\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7,31)$

distance from S_1

$\left|\frac{3 + 4 - \lambda}{5}\right| \geq 1 \Rightarrow \lambda \in (-\infty, 2] \cup [(12, \infty]$

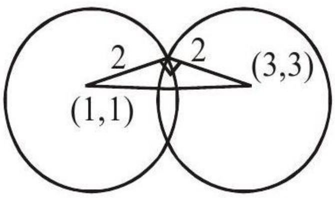
distance from S_2

$\left|\frac{27 + 4 - \lambda}{5}\right| \geq 2 \Rightarrow \lambda \in (-\infty, 21] \cup [41, \infty)$

so $\lambda \in [12,21]$

70. (d) $\frac{T_5}{T_5^1} = \frac{{}^{10}C_4\left(2^{1/3}\right)^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2\left(3^{1/3}\right)}\right)^{10-4}\left(2^{1/3}\right)^4} = 4.(36)^{1/3}$

71. (d)



Area $= 2 \times \frac{1}{2} \cdot 4 = 2$

72. (b)

73. (d) $\frac{17-\beta}{-8} \times \frac{2}{3} = -1$

$\beta = 5$

74. (b) $I = \int_0^a f(x)g(x)dx$

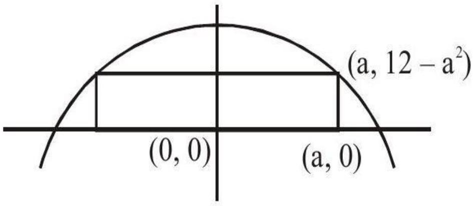
$I = \int_0^a f(a - x)g(a - x)dx$

$I = \int_0^a f(x)(4 - g(x)dx$

$I = 4\int_0^a f(x)dx - I$

$\Rightarrow \mathrm{I} = 2\int_0^a \mathrm{f(x)dx}$

75. (c) $\mathrm{f(a)} = 2\mathrm{a}(12 - \mathrm{a})^2$



$f'(a) = 2(12 - 3a^2)$

maximum at a = 2

maximum area = f(b) = 32

76. (c)

77. (c) $\lim_{x\to\pi/4}\frac{\cot^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}$

$$\lim_{x\to\pi/4}\frac{(1-\tan^4 x)}{\cos(x+\pi/4)}$$

$$2\lim_{x\to\pi/4}\frac{(1-\tan^2 x)}{\cos(x+\pi/4)}$$

$$R\lim_{x\to\pi/4}\frac{\cos^2 x - \sin^2 x}{\frac{\cos x - \sin x}{\sqrt{2}}}\frac{1}{\cos^2 x}$$

$$4\sqrt{2}\lim_{x\to\pi/4}(\cos x + \sin x) = 8$$

78. (d) $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$$\Rightarrow \frac{5x}{1-6x^2} = 1$$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$x = -1 \text{ or } x = \frac{1}{6}$$

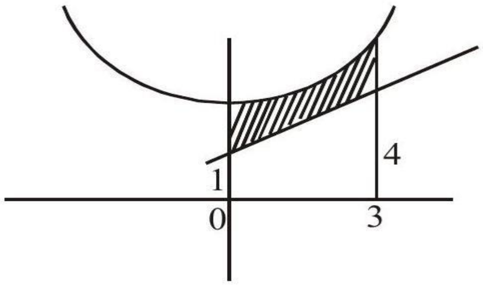
$$x = \frac{1}{6} \quad \because x > 0$$

79. (c) For unique solution

$$\Delta \neq 0 \Rightarrow \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0 \Rightarrow \alpha + \beta \neq -2$$

80. (b)



Req. area $= \int_0^3 (x^2+2)dx - \frac{1}{2}\cdot 5\cdot 3 = 9+6-\frac{15}{2} = \frac{15}{2}$

81. (b) $3m^2x^2 + m(m-4)x + 2 = 0$

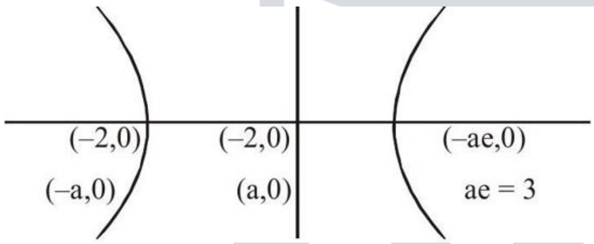
$\lambda + \frac{1}{\lambda} = 1, \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1, \alpha^2+\beta^2 = \alpha\beta$

$(\alpha+\beta)^2 = 3\alpha\beta$

$\left(-\frac{m(m-4)}{3\,m^2}\right)^2 = \frac{3(b)}{3\,m^2}, \frac{(m-4)^2}{9\,m^2} = \frac{6}{3\,m}$

$(m-4)^2 = 18, m = 4 \pm \sqrt{18}, 4 \pm 3\sqrt{2}$

82. (c)



$ae = 3, e = \frac{3}{2},\ b^2 = 4\left(\frac{9}{4} - 1\right), b^2 = 5$

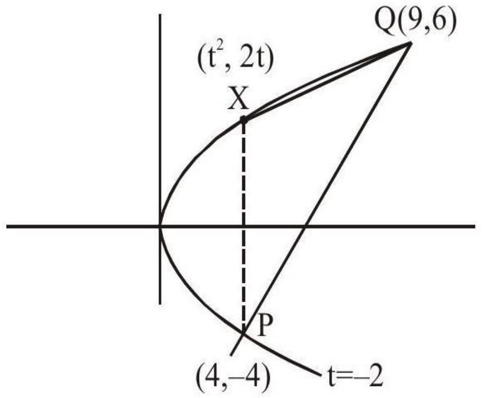
$\frac{x^2}{4} - \frac{y^2}{5} = 1$

83. (b) $\frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$

$z\bar{z} + z\alpha - \alpha\bar{z} - \alpha^2 + z\bar{z} - z\alpha + \bar{z}\alpha - \alpha^2 = 0$

$|z|^2 = \alpha^2,\ a = \pm 2$

84. (a)



$y^2 = 4x$

$2yy' = 4$

$y' = \frac{1}{t} = 2, t = \frac{1}{2}$

Area $= \frac{1}{2}\begin{vmatrix} \frac{1}{4} & 1 & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix} = \frac{125}{4}$

85. (a)

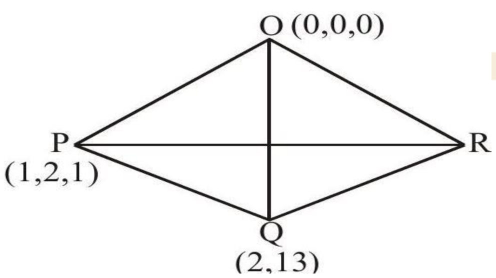
86. (a) $y = 3\cos\theta + 5\left(\sin\theta\frac{\sqrt{3}}{2} - \cos\theta\frac{1}{2}\right)$

$\frac{5\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$

$y_{max} = \sqrt{\frac{75}{4} + \frac{1}{4}} = \sqrt{19}$

87. (a) $\overrightarrow{OP} \times \overrightarrow{OQ} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) \times (2\hat{\imath} + \hat{\jmath} + 3\hat{k})$

$5\hat{\imath} - \hat{\jmath} - 3\hat{k}$



$\overrightarrow{PQ} \times \overrightarrow{PR} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) \times (-2\hat{\imath} - \hat{\jmath} + \hat{k})$

$\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

$\cos\theta = \frac{5+5+9}{(\sqrt{25+9+1})^2} = \frac{19}{35}$

88. (b) $\frac{dy}{dx} = \frac{y}{x} = \ell nx$

$e^{\int \frac{1}{x}dx} = x$

$xy = \int x\ell nx + C$

$\ln x \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2}$

$xy = \frac{x}{2}\ell nx - \frac{x^2}{4} + C$, for $2y(b) = 2\ell n2 - 1$

$\Rightarrow C = 0$

$y = \frac{x}{2}\ell nx - \frac{x}{4}$

$y(e) = \frac{e}{4}$

89. (d) $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$

$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 9+9+9 & 3+3 & 1 \end{bmatrix}$

$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3+3 & 1 & 0 \\ 6.9 & 3+3+3 & 1 \end{bmatrix}$

$P^n = \begin{bmatrix} 1 & 0 & 0 \\ 3n & 1 & 0 \\ \frac{n(n+1)}{2}3^2 & 3n & 1 \end{bmatrix}$

$P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5.3 & 1 & 0 \\ 15.9 & 5.3 & 1 \end{bmatrix}$

$Q = P^5 + I_3$

$Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$

$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$

Aliter

$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$

$P = I + X$

$X = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$

$$X^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}$$

$X^3 = 0$

$P^5 = I + 5X + 10X^2$

$Q = P^5 + I = 2I + 5X + 10X^2$

$$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 15 & 0 & 0 \\ 15 & 15 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 90 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow Q = \begin{pmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{pmatrix}$$

90. (d) No. of ways $= 10C_3 = 120$