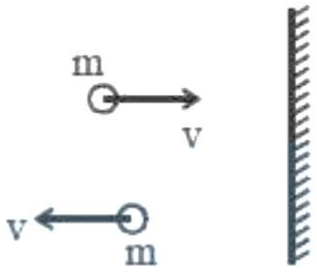
1. **(b)** Pressure is defined as normal force per unit area.
Force is calculated as change in momentum/ time.

By this answer is $2\ \mathrm{N/m^2}$

None of the option matches so this question must be

Bonus.

Detailed solution is as following:



Magnitude of change in momentum per collision $= 2$

mv

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{N(2mv)}{1}$$
$$= \frac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1}$$
$$= 2\ \mathrm{N/m^2}$$

2. **(d)** According to work energy theorem.
Work done by force on the particle = Change in KE

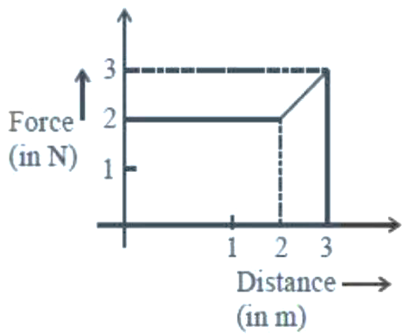
Work done = Area under $F - x$ graph $= \int F \cdot dx =$

$$2 \times 2 + \frac{(2+3) \times 1}{2}$$

$W = KE_{\text{final}} - KE_{\text{initial}} = 6.5$

$KE_{\text{initial}} = 0$

$KE_{\text{final}} = 6.5\ \mathrm{J}$



3. **(c)** There will be 3 phenomenon

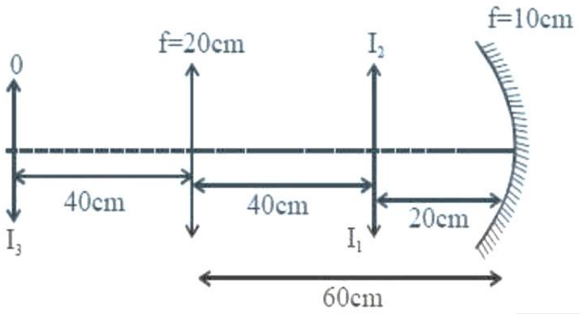
(i) Refraction from lens

(ii) Reflection from mirror

(iii) Refraction from lens

After these phenomena. Image will be on object and will have same size.

None of the option depicts so this question is Bonus.



1st refraction $u = -40$ cm; $f = +20$ cm

$\Rightarrow v = +40$ cm (image I_1) and $m_1 = -1$

for reflection

$u = -20$ cm; $f = -10$ cm
$\Rightarrow v = -20$ cm (image I_2) and $m_2 = -1$

2nd refraction

$u = -40$ cm; $f = +20$ cm

$\Rightarrow v = +40$ cm (image I_3) and $m_3 = -1$

Total magnification $= m_1 \times m_2 \times m_3 = -1$ and final image is formed at distance 40 cm from convergent lens and is of same size as the object.

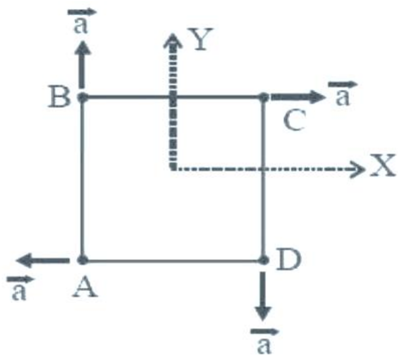
4. (a)

$\vec{a}_A = -a\hat{i}; \vec{a}_B = a\hat{j}$
$\vec{a}_C = a\hat{i}\,;\ \vec{a}_D = -a\hat{j}$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2m\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m} = \frac{a}{5}\hat{\imath} - \frac{a}{5}\hat{\jmath} = \frac{a}{5}(\hat{\imath} - \hat{\jmath})$$



5. (a) $-ms\frac{dT}{dt} = e\sigma A\left(T^4 - T_0^4\right)$

$$-\frac{dT}{dt} = \frac{e\sigma A}{ms}\left(T^4 - T_0^4\right); \quad -\frac{dT}{dt} = \frac{4e\sigma ATT_0^3}{ms}(\Delta T)$$

$$T = T_0 + (T_i - T_0)e^{-kt}$$

where $k = \frac{4e\sigma AT_0^3}{ms}$

$$k = \frac{4e\sigma AT_0^3}{\rho vs}; \left|\frac{dT}{dt}\right| \propto k$$

$$\therefore \left|\frac{dT}{dt}\right| \propto \frac{1}{\rho s}$$

$$\rho_A S_A = 2000 \times 8 \times 10^2 = 16 \times 10^5$$

$$\rho_B S_B = 4000 \times 10^3 = 4 \times 10^6$$

$$\rho_A S_A < \rho_B S_B$$

$$\left|\frac{dT}{dt}\right|_A > \left|\frac{dT}{dt}\right|_B$$

6. (d)

$$T_0 = 2\pi\sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{10}}$$

$$A = A_0 e^{-1/\gamma}$$

$\therefore$ for $A = \frac{A_0}{e}, t = \gamma$

$$t = \gamma = \frac{2\text{ m}}{b} = \frac{2\text{ m}}{\frac{B^2\ell^2}{R}} = 10^4 \text{ s}$$

$\therefore$ No of oscillation $\frac{t}{T_0} = \frac{10^4}{2\pi/\sqrt{10}} \approx 5000.$

7. (d) $LIDI = I^2R$

$$L \times \frac{E}{10}\left(-e^{-t/2}\right) \times \frac{-1}{2} = \frac{E}{10}\left(1 - e^{-t/2}\right) \times 10$$

$e^{-1/2} = 1 - e^{-1/2}$; $t = 2\ln 2$

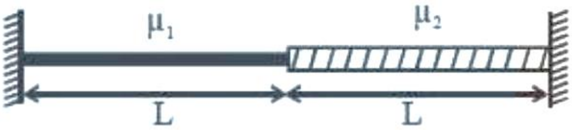
8. **(b)** Let mass per unit length of wires are μ_1 and μ_2 respectively.

∵ Materials are same, so density ρ is same.

$\therefore\ \mu_1 = \frac{\rho\pi r^2 L}{L} = \mu$ and $\mu_2 = \frac{\rho 4\pi r^2 L}{L} = 4\mu$

Tension in both are same $= T$, let speed of wave in wires are V_1 and V_2

$$V_1 = \frac{V_1}{2L} = \frac{V}{2L} \& f_{02} = \frac{V_2}{2L} = \frac{V}{4L}$$

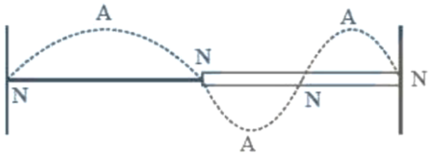


radius $= r$

radius $= 2r$

Frequency at which both resonate is L.C.M. of both frequencies i.e. $\frac{V}{2L}$.

Hence number of loops in wires are 1 and 2 respectively



So, ratio of number of antinodes is $1:2$.

9. **(d)** Tensile stress in wire will be

$$= \frac{\text{Tensile force}}{\text{Cross section Area}}$$
$$= \frac{mg}{\pi R^2} = \frac{4 \times 3.1\pi}{\pi \times 4 \times 10^{-6}} Nm^{-2} = 3.1 \times 10^6 Nm^{-2}$$

10. **(d)** $A = 10^{-4}\ m^2$

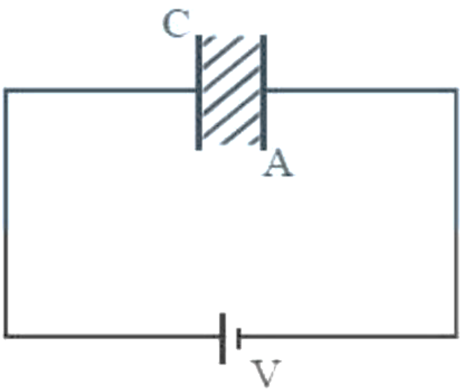
$E_{max} = 10^6\ V/m$

$C = 15\mu F$

$$C = \frac{k\varepsilon_0 A}{d};\ \frac{Cd}{\varepsilon_0 A} = k$$

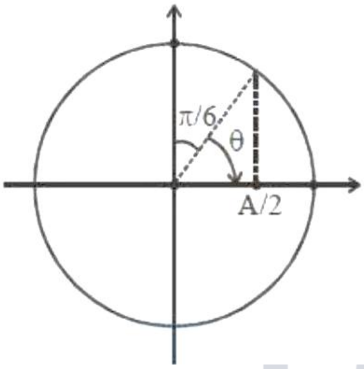
$$k = \frac{15 \times 10^{-12} \times 500 \times 10^{-6}}{8.86 \times 10^{-12} \times 10^4} = \frac{15 \times 5}{8.86} = 8.465$$

$k \approx 8.5$



11. (b) $V(t) = 220\sin(100\pi t)$ volt time taken,

$$t = \frac{\theta}{\omega} = \frac{\frac{\pi}{3}}{100\pi} = \frac{1}{300}\text{sec}$$



12. (a) To minimize attenuation, wavelength of carrier waves is close to 1500 nm.

13. (d) Reynolds number $= \frac{\rho v d}{\eta}$

Volume flow rate $= v \times \pi r^2$

$$v = \frac{100 \times 10^{-3}}{60} \times \frac{1}{\pi \times 25 \times 10^{-4}}$$

$$v = \frac{2}{3\pi}\text{m/s}$$

Reynolds number $= \frac{10^3 \times 2 \times 10 \times 10^{-2}}{10^{-3} \times 3\pi} = 2 \times 10^4$

Order 10^4

14. (b) Energy of catapult $= \frac{1}{2} \times \left(\frac{\Delta \ell}{\ell}\right)^2 \times Y \times A \times \ell$

$=$ Kinetic energy of the ball $= \frac{1}{2}mv^2$

Therefore, $\frac{1}{2} \times \left(\frac{20}{42}\right)^2 \times Y \times \pi \times 3^2 \times 10^{-6} \times 42 \times 10^{-2} = \frac{1}{2} \times 2 \times 10^{-2} \times (20)^2$

$Y = 3 \times 10^6 \text{Nm}^2$

15. (d) $\Theta \frac{h}{\lambda_1} = P_1 \oint P_2 = \frac{h}{\lambda_2}$

$\vec{P}_1 = \frac{h}{\lambda_1}\hat{i}$ and $\vec{P}_2 = \frac{h}{\lambda_2}\hat{j}$

Using momentum conservation

$$\vec{P} = \vec{P}_1 + \vec{P}_2 = \frac{h}{\lambda_1}\hat{\imath} + \frac{h}{\lambda_2}\hat{\jmath}$$

$$|\vec{P}| = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{h}{\lambda} = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

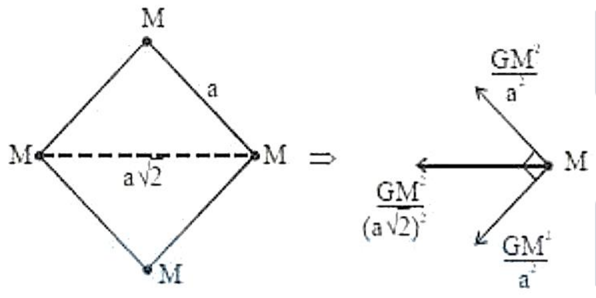
$$\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

16. (b) Net force on particle towards centre of circle is

$$F_c = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\sqrt{2}$$
$$= \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

This force will act as centripetal force.

Distance of particle from centre of circle is



$\frac{a}{\sqrt{2}}$.

$$r = \frac{a}{\sqrt{2}}, \ F_c = \frac{mv^2}{r}$$

$$\frac{mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$v^2 = \frac{GM}{a}\left(\frac{1}{2\sqrt{2}} + 1\right)$$

$$v^2 = \frac{GM}{a}(1.35); v = 1.16\sqrt{\frac{GM}{a}}$$

17. (a) If we approximate the angle θ_2 as 30° initially then answer will be closer to 57000 . but if we solve thoroughly, answer will be close to 55000 .

So both the answers must be awarded. Detailed solution as following.

Exact solution

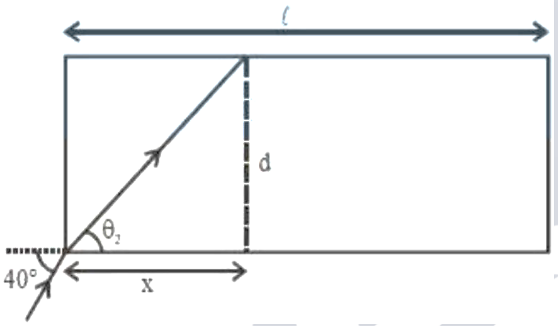
By Snell's law $1.\sin 40^\circ = (1.31)\sin\theta_2$

$$\sin\theta_2 = \frac{.64}{1.31} = \frac{64}{131} \approx .49$$

$$\text{Now } \tan\theta_2 = \frac{64}{\sqrt{(131)^2-(64)^2}} = \frac{64}{\sqrt{13065}} \approx \frac{64}{114.3} = \frac{\mathrm{d}}{\mathrm{x}}$$

Now number of reflections

$$= \frac{2\times 64}{114.3\times 20\times 10^{-6}} = \frac{64\times 10^5}{114.3}$$
$$\approx 55991 \approx 55000$$



Approximate solution

By Snell's law $1.\sin 40^\circ = (1.31)\sin\theta_2$

$$\mathrm{Sin}\,\theta_2 = \frac{0.64}{1.31} = \frac{64}{131} \approx 0.49$$

If assume $\Rightarrow \theta_2 \approx 30^\circ$

$$\mathrm{Tan}\,30^\circ = \frac{\mathrm{d}}{\mathrm{x}} \Rightarrow \mathrm{x} = \sqrt{3}\,\mathrm{d}$$

Now number of reflections

$$= \frac{\ell}{\sqrt{3}\,\mathrm{d}} = \frac{2}{\sqrt{3}\times 20\times 10^{-6}} = \frac{10^5}{\sqrt{3}}$$

$$\approx 57735 \approx 57000$$

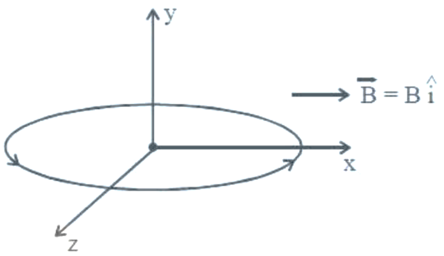
18. (d) Magnetic moment of coil $=$ NIA $\hat{j}$

$= \mathrm{NI}(\pi r^2)\hat{j}$

Torque on loop (coil) $= \vec{M}\times\vec{B}$

$= \mathrm{NI}(\pi r^2)\mathrm{B}\sin 90^\circ(-\hat{k})$

$= NI\pi r^2 B(-\hat{k})$



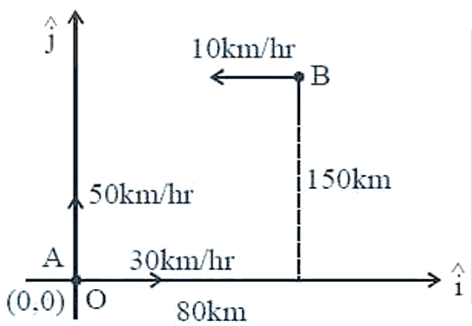
19. **(c)** If we take the position of ship ' A ' as origin then positions and velocities of both ships can be given as:

$\vec{v}_A = (30\hat{\imath} + 50\hat{\jmath})\text{km/hr}$

$\vec{v}_B = -10\hat{\imath}\text{km/hr}$; $\vec{r}_A = 0\hat{\imath} + 0\hat{\jmath}$

$\vec{r}_B = (80\hat{\imath} + 150\hat{\jmath})\text{km}$

Time after which distance between them will be minimum



$t = -\dfrac{\vec{r}_{BA} \cdot \vec{v}_{BA}}{|\vec{v}_{BA}|^2}$;

Where $\vec{r}_{BA} = (80\hat{\imath} + 150\hat{\jmath})\text{km}$

$\vec{V}_{BA} = -10\hat{\imath} - (30\hat{\imath} + 50\hat{\jmath})(-40\hat{\imath} - 50\hat{\jmath}) km/hr$

$\therefore\ t = -\dfrac{(80\hat{\imath} + 150\hat{\jmath}) \cdot (-40i - 50\hat{h})}{|-40i - 50\hat{h}|^2}$

$= \dfrac{3200 + 7500}{4100}\text{hr} = \dfrac{10700}{4100}\text{hr} = 2.6\text{hrs}$

20. **(a)** The direction of propagation of an EM wave is direction of $\vec{E} \times \vec{B}$.

$\hat{\imath} = \hat{\jmath} \times \hat{B}$

$\Rightarrow \hat{B} = \hat{k}$

$C = \dfrac{E}{B} \Rightarrow B = \dfrac{E}{C} = \dfrac{6}{3 \times 10^8}$

$B = 2 \times 10^{-8}$ T along z direction.

21. **(d)** Suppose ' m ' gram of water evaporates then, heat required

$\Delta Q_{\text{req}} = mL_v$

Mass that converts into ice = (150 − m)

So, heat released in this process

$\Delta Q_{\text{rel}} = (150 - m)L_f$

Now,

$\Delta \mathrm{Q}_{\text{rel}} = \Delta \mathrm{Q}_{\text{req}}$

$(150 - m)L_f = mL_v$

$M(L_f + L_v) = 150L_f$

$m = \dfrac{150L_f}{L_f + L_v}; m = 20 \text{ g}$

22. (a) Energy released for tension $n = 2$ to $n = 1$ of hydrogen atom

$E = 13.6Z^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$Z = 1, n_1 = 1, n_2 = 2$

$E = 13.6 \times 1 \times \left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)$

$E = 13.6 \times \dfrac{3}{4} \text{eV}$

For He^+ ion $z = 2$

(A) $\mathrm{n} = 1$ to $\mathrm{n} = 4$

$E = 13.6 \times 2^2 \times \left(\dfrac{1}{1^2} - \dfrac{1}{4^2}\right) = 13.6 \times \dfrac{15}{4} \text{eV}$

(B) $n = 2$ to $n = 4$

$E = 13.6 \times 2^2 \times \left(\dfrac{1}{1^2} - \dfrac{1}{4^2}\right) = 13.6 \times \dfrac{3}{4} \text{eV}$

(C) $n = 2$ to $n = 5$

$E = 13.6 \times 2^2 \times \left(\dfrac{1}{2^2} - \dfrac{1}{5^2}\right) = 13.6 \times \dfrac{21}{25} \text{eV}$

(D) $n = 2$ to $n = 5$

$E = 13.6 \times 2^2 \times \left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) = 13.6 \times \dfrac{5}{9} \text{eV}$

23. (a) When red is replace with green 1^{st} digit changes to 5 so new resistance will be 500Ω.

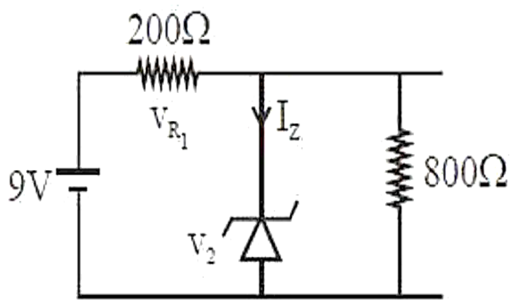
24. (a) $9 = V_z + V_{R_1}$

$\mathrm{V_Z} = 5.6 \text{ V}$

$V_{R_1} = 9 - 5.6$

$V_{R_1} = 3.4$

$I_{R_1} = \frac{V_{R_1}}{R} = \frac{3.4}{200}; I_{R_1} = 17\text{ mA}$



$V_z = V_{R_2} = I_{R_2}(R_2)$

$\frac{5.6}{800} = I_{R_2}\,;\ I_{R_2} = 7\text{ mA}$

$I_z = (17 - 7)\text{mA} = 10\text{ mA}$

25. (a) $M = \int_0^R \rho_0 r(2\pi r dr) = \frac{\rho_0 \times 2\pi \times R^3}{3}$

$\underset{\text{(MOlabout COM)}}{I_0} = \int_0^R \rho_0 r(2\pi r dr) \times r^2 = \frac{\rho_0 \times 2\pi R^5}{5}$

By parallel axis theorem

$$\begin{aligned} I &= I_0 + MR^2 \\ &= \frac{\rho_0 \times 2\pi R^5}{5} + \frac{\rho_0 \times 2\pi R^3}{3} \times R^2 = \rho_0 2\pi R^5 \times \frac{8}{15} \\ &= MR^2 \times \frac{8}{5} \end{aligned}$$

26. (c) Dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$[\varepsilon_0] = [M^{-1}\,L^{-3}\,T^4\,A^2]$

$[\mu_0] = [MLT^{-2}\,A^{-2}]$

Dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}} = \left[\frac{M^{-1}\,L^{-3}\,T^4\,A^2}{MLT^{-2}\,A^{-2}}\right]^{\frac{1}{2}}$

$= [M^{-2}\,L^{-4}\,T^6\,A^4]^{1/2}$

$= [M^{-1}\,L^{-2}\,T^3\,A^2]$

27. (a)

$$E_{eq} = \frac{\frac{E_1}{2R_1} + \frac{E_2}{R_2} + \frac{E_3}{2R_1}}{\frac{1}{2R_1} + \frac{1}{R_2} + \frac{1}{2R_1}}$$

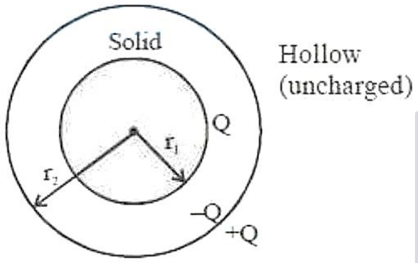
$$= \frac{\frac{2}{2} + \frac{4}{2} + \frac{4}{2}}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = \frac{5}{\frac{3}{2}} = \frac{10}{3} = 3.3$$

28. (d) As given in the first condition:

Both conducting spheres are shown.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1}\right) - \left(\frac{kQ}{r_2}\right)$$

$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$



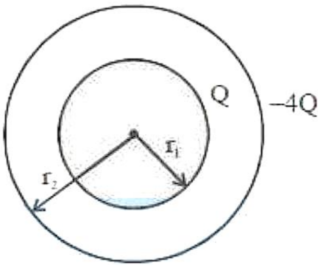
In the second condition:

Shell is now given charge $-4Q$.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1} - \frac{4kQ}{r_2}\right) - \left(\frac{kQ}{r_2} - \frac{4kQ}{r_2}\right)$$

$$= \frac{kQ}{r_1} - \frac{kQ}{r_2}$$

$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$



Hence, we also obtain that potential difference does not depend on charge of outer sphere.

∴ P. d. remains same.

29. (b)

Given $\frac{a_1}{a_2} = \frac{1}{3}$

Ratio of intensities, $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{1}{9}$

Now, $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{1+3}{1-3}\right)^2 = 4$

30. (d) $\tan\theta = \frac{qE}{mg} = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10}$

$\tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}(0.5)$

31. (b) For cubic unit cell, only FCC has octahedral and tetrahedral voids.

$Z_B = 4, Z_A = 4\times\frac{1}{4} = 2, Z_o = 8$

Formula $= A_2\ B_2O_8 = AB_2O_4$

32. (c)

33. (d) For isoelectronic species the size is compared by nuclear charge.

Size $\alpha \frac{1}{Z(\text{ Nuclear Charge })}$

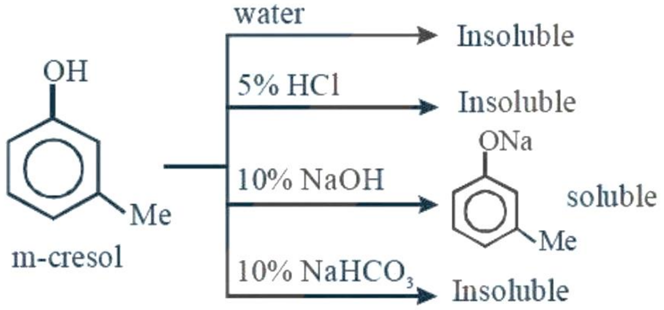
34. (c)

35. (b) Since $CN^-$and NH_3 are strong field ligands, low spin complexes are formed.

Complex	Configuration	No. of unpaired electrons
$[V(CN)_6]^{4-}$	$t_{2g}^3e_g^0$	3
$[Cr(NH_3)_6]^{2+}$	$t_{2g}^4e_g^0$	2
$[Ru(NH_3)_6]^{3+}$	$t_{2g}^5e_g^0$	1
$[Fe(CN)_6]^{4-}$	$t_{2g}^6e_g^0$	0

Magnetic moment is directly proportional to number of unpaired electrons.

36. (d)



Oleic acid is also soluble in $NaHCO_3$

o-toluidine is not soluble in $NaOH$ as well as $NaHCO_3$

Benzamide is also not soluble in $NaOH\&NaHCO_3$

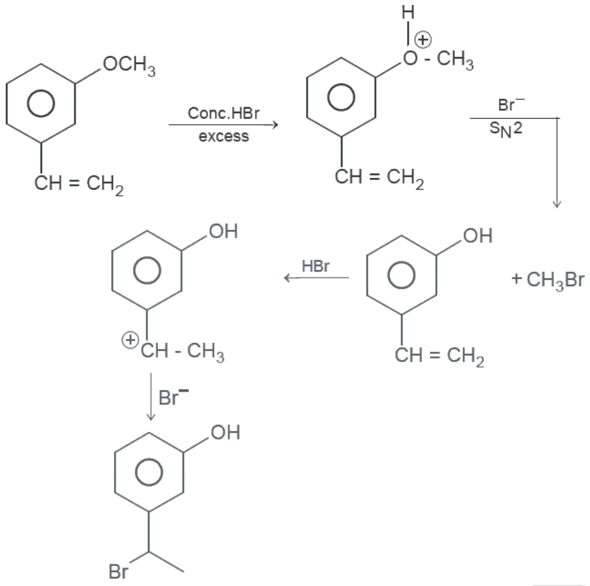
$\therefore$ m-cresol is the right answer.

37. (b) $B_2H_6 + 3H_2O \longrightarrow 2H_3BO_3 + 3H_2$

$B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

38. (d) $Sm^{3+}(4f^5) =$ yellow colour, other ions have stable electron configurations with half filled or full-filled electron configuration.

39. (d)



40. (a) $n_{eq.}CaCO_3 = n_{eq}Ca(HCO_3)_2 + n_{eq}Mg(HCO_3)_2$ (n_{eq} = Number of equivalent) or, $\frac{W}{100} \times 2 = \frac{0.81}{162} \times 2 + \frac{0.73}{146} \times 2$

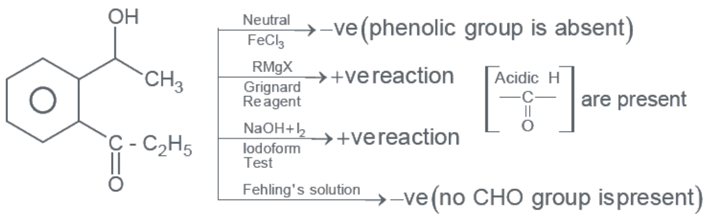
$\therefore$ w = 1.0

Volume of water = 100 mL

Mass of water = 100 g

$\therefore$ Hardness $= \frac{1.0}{100} \times 10^6 = 10000$ppm

41. (c)



42. (b)

$r = K[A]^x[B]^y$

$0.045 = K(0.05)^x(0.05)^y$

$0.090 = K(0.10)^x(0.05)^y$

$0.72 = K(0.20)^x(0.10)^y$

Dividing (1) by (2) we get

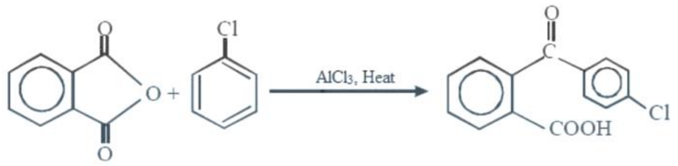
$$\frac{0.045}{0.090} = \left(\frac{0.05}{0.10}\right)^x \Rightarrow x = 1$$

Dividing (2) by (3)

$$\frac{0.090}{0.720}=\left(\frac{0.10}{0.20}\right)^x\left(\frac{0.05}{0.10}\right)^y \Rightarrow y=2$$

Hence, $r=K[A][B]^2$

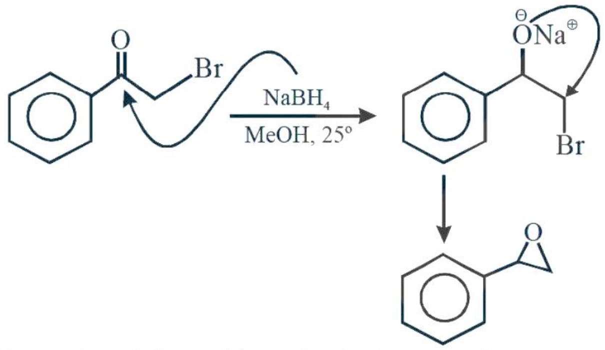
43. (a)



Fridel-craft acylation. $-Cl$ group is an ortho & para directing

44. (b) Plastics are non-biodegradable.

45. (d)



Reduction followed by substitution reaction

46. (b) $\Delta H = n\int_{T_1}^{T_2} C_{p.m} dT = 3\times\int_{300}^{1000}(23+0.01\ T)dT$

$$=3[23(1000-300)]+\frac{0.01}{2}[(1000)^2-(300)^2]$$

$$=61950\ J\approx 62\ kJ$$

47. (d) Hydration enthalpy depends upon ionic potential (charge/size). As ionic potential increases hydration enthalpy increases.

$$\Delta_{hyd} H^0 \alpha \frac{q}{r}$$

48. (c)

$$\frac{x}{m}=Kp^{1/n}$$

Taking log from both sides

$$\therefore \log\frac{x}{m}=\log K+\frac{1}{n}\log P$$

$$\text{slope}=\frac{1}{n}=\frac{2}{3}$$

$$\therefore \frac{x}{m}=Kp^{2/3}$$

49. (a)

$$P_{total}=X_A P_A^0+X_B P_B^0=0.5\times 400+0.5\times 600=500\text{mmHg}$$

Now, mole fraction of A in vapour

$Y_A = \frac{P_A}{P_{total}} = \frac{0.5\times400}{500} = 0.4$ and mole fraction of B in vapour

$Y_B = 1 - 0.4 = 0.6$

50. **(b)** For strongest oxidising agent, standard reduction potential should be highest. Peroxy oxygen (-O - $O-$) is reduced to oxide (O^{2-}) in the change

51. **(c)** According to Aufbau principle, the energy sequence is $3p < 3d < 4p < 4d$

52. **(a)**

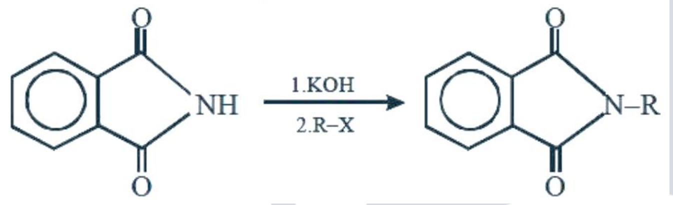
$Zr_3(PO_4)_4(s) \rightleftharpoons 3Zr^{4+}(aq.) + 4PO_4^{3-}(aq.)$

3 SM 4 SM

$K_{sp} = [Zr^{4+}]^3[PO_4^{3-}]^4 = (3S)^3 \cdot (4S)^4 = 6912 S^7$

$\therefore S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

53. **(b)** Gabriel phthalimide synthesis:



For branched chain RX, elimination reaction takes place.

54. **(a)** According to the first law of thermodynamics $q = \Delta U - W$

For cyclic process: $\Delta U = 0 \Rightarrow q = -w$

For isothermal process: $\Delta U = 0 \Rightarrow q = -w$

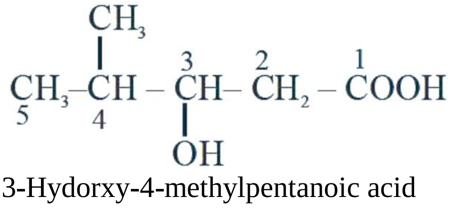
For adiabatic process: $q = 0 \Rightarrow \Delta U = W$

For isochoric process: $w = 0 \Rightarrow \Delta U = q$

55. **(a)** Both nitrogen & oxygen are donating atoms.

56. **(a)** The priority of COOH is higher that OH.

$\therefore$ COOH is the functional group.



3-Hydorxy-4-methylpentanoic acid

57. **(b)** n-factors of $KMnO_4 = 5$, n-factor of $FeSO_4 = 1$

n-factors of $FeC_2O_4 = 3, Fe_2(SO_4)_3$ does not react n-factors of $Fe_2(C_2O_4)_3 = 6$,

$n_{eq}\ KMnO_4 = n_{eq}\ [FeC_2O_4 + Fe_2(C_2O_4)_3 + FeSO_4]$

or, $x \times 5 = 1 \times 3 + 1 \times 6 + 1 \times 1$

$x = 2$

58. (c)

59. (d)

60. (b) Ellingham diagram which are the curves of the graph between ΔG and T helps in predicting the feasibility of thermal reduction of ores.

61. (d) Vector perpendicular to plane containing the vectors $\hat{\imath} + \hat{\jmath} + \hat{k} d\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is parallel to vector

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$$

∴ Required magnitude of projection

$$= \frac{|(2\hat{\imath} + 3\hat{\jmath} + \hat{k}) \cdot (\hat{\imath} - 2\hat{\jmath} + \hat{k})|}{|\hat{\imath} - 2\hat{\jmath} + \hat{k}|}$$

$$= \frac{|2 - 6 + 1|}{|\sqrt{6}|} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

62. (c) We have

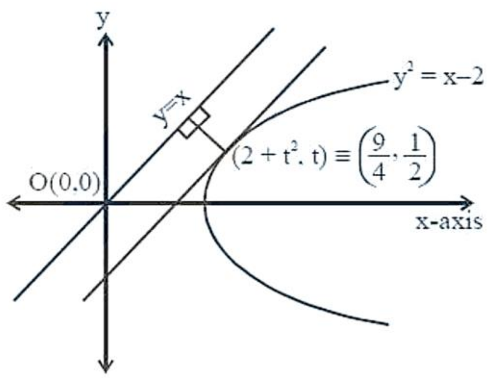
$$2y\frac{dy}{dx} = 1 \Rightarrow \left.\frac{dy}{dx}\right]_{P(2+t^2,t)} = \frac{1}{2t} = 1$$

$$\Rightarrow\ t = \frac{1}{2}$$

$$\therefore\ P\left(\frac{9}{4}, \frac{1}{2}\right)$$

So, shortest distance

$$= \frac{\left|\frac{9}{4} - \frac{2}{4}\right|}{\sqrt{2}} = \frac{7}{4\sqrt{2}}$$

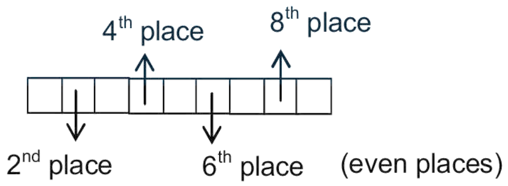


63. (a) $(x-1)^2+1=0 \Rightarrow x=1+i, 1-i$

$\therefore \left(\frac{\alpha}{\beta}\right)^{\mathrm{n}}=1 \Rightarrow (\pm \mathrm{i})^{\mathrm{n}}=1$

$\therefore$ n (least natural number) $=4$

64. (a)



Number of such numbers $= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{2!4!}=180$

65. (c)

$\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx=\int \frac{2 \sin \frac{5x}{2} \cos \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} dx$

$=\int \frac{\sin 3x+\sin 2x}{\sin x} dx$

$=\int \frac{3 \sin x-4 \sin ^3 x+2 \sin x \cos x}{\sin x} dx$

$=\int\left(3-4 \sin ^2 x+2 \cos x\right) dx$

$=\int(3-2(1-\cos 2x)+2 \cos x) dx$

$=\int(1+2 \cos 2x+2 \cos x) dx$

$=x+\sin 2x+2 \sin x+c$

66. (c)

$\mathrm{P}+\mathrm{OP}+\mathrm{AO}=4$

$\sqrt{\mathrm{h}^2+(\mathrm{k}-1)^2}+\sqrt{\mathrm{h}^2+\mathrm{k}^2}+1=4$

$\sqrt{\mathrm{h}^2+(\mathrm{k}-1)^2}+\sqrt{\mathrm{h}^2+\mathrm{k}^2}=3$

$h^2+(\mathrm{k}-1)^2=9+\mathrm{h}^2+\mathrm{k}^2-6 \sqrt{\mathrm{h}^2+\mathrm{k}^2}$

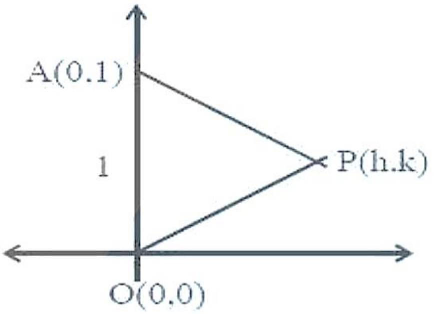
$-2 \mathrm{k}-8=-6 \sqrt{\mathrm{h}^2+\mathrm{k}^2}$

$\mathrm{k}+4=3 \sqrt{\mathrm{h}^2+\mathrm{k}^2}$

$k^2+16+8k=9\left(h^2+k^2\right)$

$9 \mathrm{~h}^2+8 \mathrm{k}^2-8 \mathrm{k}-16=0$

Locus of P is $9x^2+8y^2-8y-16=0$



67. (c) $0 < \alpha + \beta = \frac{\pi}{2}$ and $\frac{-\pi}{4} < \alpha - \beta < \frac{\pi}{4}$

If $\cos(\alpha+\beta) = \frac{3}{5}$ then $\tan(\alpha+\beta) = \frac{4}{3}$ and if $\sin(\alpha-\beta) = \frac{5}{13}$ then $\tan(\alpha-\beta) = \frac{5}{12}$ (since $\alpha - \beta$ here lies in the first quadrant)

Now $\tan(2\alpha) = \tan\{(\alpha+\beta)+(\alpha-\beta)\}$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)} = \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\cdot\frac{5}{12}} = \frac{63}{16}$$

68. (d)

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3\alpha & -\sin 3\alpha \\ \sin 3\alpha & \cos 3\alpha \end{bmatrix}$$

Similarly $A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \cos 32\alpha = 0$ and $\sin 32\alpha = 1$

$\Rightarrow 32\alpha = (4n+1)\frac{\pi}{2}, n \in I$

$\alpha = (4n+1)\frac{\pi}{64}, n \in I$

$\alpha = \frac{\pi}{64}$ for $n = 0$

69. (a) $f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x| < 1$

$$f\left(\frac{2x}{1+x^2}\right) = \ln\left(\frac{1-\frac{2x}{1+2x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$= \ln\left(\frac{(x-1)^2}{(x+1)^2}\right) = 2\ln\left|\frac{1-x}{1+x}\right| = 2f(x)$$

70. (c) Consider $\cot^{-1}\left(\frac{\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x}{\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\sin x}\right)$

$$= \cot^{-1}\left(\frac{\sin\left(x+\frac{\pi}{3}\right)}{\cos\left(x+\frac{\pi}{3}\right)}\right)$$

$$= \cot^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right) = \frac{\pi}{2} - \tan^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right)$$

$$\begin{cases} \frac{\pi}{2} - \left(x+\frac{\pi}{3}\right) = \left(\frac{\pi}{6}-x\right); 0 < x < \frac{\pi}{6} \\ \frac{\pi}{2} - \left(\left(x-\frac{\pi}{3}\right)-\pi\right) = \left(\frac{7\pi}{6}-x\right); \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

$$\therefore\ 2y = \begin{cases} \left(\frac{\pi}{6}-x\right)^2; 0 < x < \frac{\pi}{6} \\ \left(\frac{7\pi}{6}-x\right)^2; \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

$$\therefore\ 2\frac{dy}{dx} = \begin{cases} 2\left(\frac{\pi}{6}-x\right)\cdot(-1); 0 < x < \frac{\pi}{6} \\ 2\left(\frac{7\pi}{6}-x\right)\cdot(-1); \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

71. (c)

$|\sqrt{x}-2| + \sqrt{x}(\sqrt{x}-4) + 2 = 0$

$|\sqrt{x}-2| + (\sqrt{x})^2 - 4\sqrt{x} + 2 = 0$

$|\sqrt{x}-2|^2 + |\sqrt{x}-2| - 2 = 0$

$|\sqrt{x}-2| = -2$ (not possible) or $|\sqrt{x}-2| = 1$

$\sqrt{x}-2 = 1, -1$

$\sqrt{x} = 3, 1$

$x = 9, 1$

Sum $= 10$

72. (b) $\lim_{x\to 0} \frac{\left(\frac{\sin^2 x}{x^2}\right)(\sqrt{2}+\sqrt{1+\cos x})}{\left(\frac{1-\cos x}{x^2}\right)}$

$$= \frac{(1)^2\cdot(2\sqrt{2})}{\frac{1}{2}} = 4\sqrt{2}$$

73. (d) $2\cdot {}^{20}C_0 + 5\cdot {}^{20}C_1 + 8\cdot {}^{20}C_2 + 11\cdot {}^{20}C_3 + \cdots \ldots + 62\cdot {}^{20}C_{20}2$

$= \sum_{r=0}^{20}(3r+2)\,{}^{20}C_r$

$= 3\sum_{r=0}^{20} r\cdot {}^{20}C_r + 2\sum_{r=0}^{20} {}^{20}C_r$

$= 3\sum_{r=0}^{20} r\binom{20}{r}\ {}^{19}C_{r-1} + 2.2^{20}$

$= 60.2^{19} + 2.2^{20} = 2^{25}$

74. (c) $\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{1}{(x^2+1)^2}$

(Linear differential equation)

$\therefore$ I.F. $= e^{\ln(x^2+1)} = (x^2+1)$

So, general solution is $y \cdot (x^2+1) = \tan^{-1} x + c$

As $y(0) = 0 \Rightarrow c = 0$

$$\therefore\ y(x) = \frac{\tan^{-1} x}{x^2+1}$$

As, $\sqrt{a}, y(1) = \frac{\pi}{32}$

$$\Rightarrow \sqrt{a} = \frac{1}{4} \Rightarrow a = \frac{1}{16}$$

75. (a) $g(f(x)) = \ln(f(x)) = \ln\left(\frac{2-x\cdot\cos x}{2+x\cdot\cos x}\right)$

$$\therefore\ I = \int_0^{\pi/4}\left(\ell n\left(\frac{2-x\cdot\cos x}{2+x\cdot\cos x}\right) + \ell\left(\frac{2+x\cdot\cos x}{2-x\cdot\cos x}\right)\right)dx$$

$$= \int_0^{\pi/2}(0)dx = 0 = \log_e(1)$$

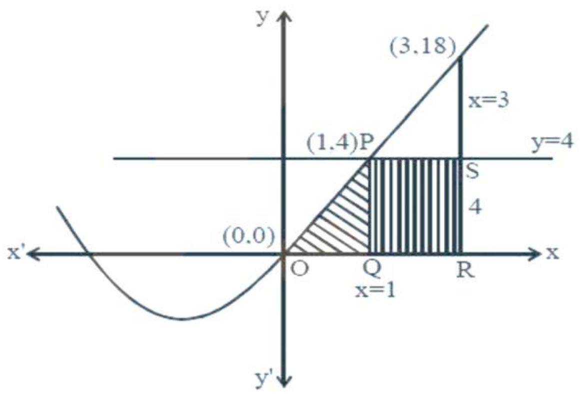
76. (b)

Required Area

$= \int_0^1 (x^2+3x)dx +$ Area of

rectangle PQRS

$$= \frac{11}{6} + 8 = \frac{59}{6}$$



77. (d) Let 7 observations be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$\bar{x} = 8 \Rightarrow \sum_{i=1}^{7} x_i = 56$

Also $\sigma^2 = 16$

$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - (\bar{x})^2$

$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64$

$\Rightarrow \left(\sum_{i=1}^{7} x_i^2\right) = 560$

Now, $x_1 = 2, x_2 = 4, x_3 = 10, x_4 = 12, x_5 = 14$
$\Rightarrow x_6 + x_7 = 14$, (from (1)) and $x_6^2 + x_7^2 = 100$ (from (2))

$\therefore x_6^2 + x_7^2 = (x_6 + x_7)^2 - 2x_6x_7 \Rightarrow x_6x_7 = 48$

78. (d)

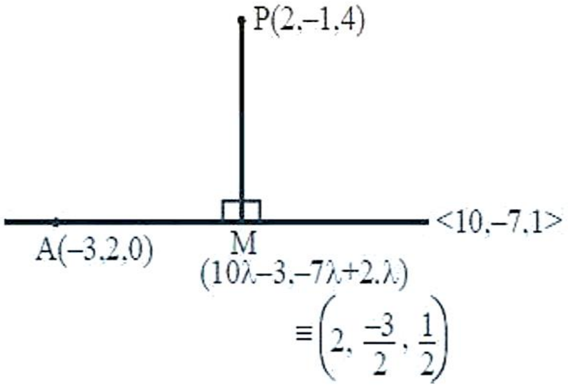
Now, $\overrightarrow{MP} \cdot (10\hat{\imath} - 7\hat{\jmath} + \hat{k}) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

$\therefore$ Length of perpendicular

$(= \text{PM}) = \sqrt{0 + \frac{1}{4} + \frac{49}{4}}$

$= \sqrt{\frac{50}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{\sqrt{2}},$



which is greater than 3 but less

than 4.

79. (b) The contrapositive of a statement $p \rightarrow q$ is $\sim q \rightarrow \sim p$

Here, p : your are born in India

q : you are citizen of India

So, contrapositive of above statement is "If you are not a citizen of India, then you are not born in India".

80. (d) S_A = sum of numbers between 100 and 200 which are divisible by 7 .

$\Rightarrow S_A = 105 + 112 + \cdots .. + 196$

$S_A = \frac{14}{2}[105 + 196] = 2107$

S_B = Sum of numbers between 100 and 200 which are divisible by 13.

$S_B = 104 + 117 + \cdots .. + 195 = \frac{8}{2}[104 + 195] = 1196$

S_C = Sum of numbers between 100 and 200 which are divisible by 7 and 13 .

$S_C = 182$

$\Rightarrow$ H.C. F. $(91, n) > 1 = S_A + S_B - S_C = 3121$

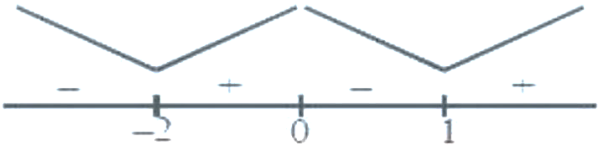
81. (a) $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

$f(x) = 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2)$

$= 36x(x - 1)(x + 2)$

Point of minima $= \{-2,1\} = S_1$



Point of maxima $= \{0\} = S_2$

82. (b) $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$

$= 2[\,{}^6C_0x^6 + {}^6C_2x^4(x^3 - 1) + {}^6C_4x^2(x^3 - 1)^2 + {}^6C_6(x^3 - 1)^3]$

$= 2[\,{}^6C_0x^6 + {}^6C_2x^7 - {}^6C_2x^4 + {}^6C_4x^8 + {}^6C_4x^2 - 2\,{}^6C_4x^5 + (x^9 - 1 - 3x^6 + 3x^3)]$

$\Rightarrow$ Sum of coefficient of even powers of x

$= 2[1 - 15 + 15 + 15 - 1 - 3] = 24$

83. (d) Now, $\left|\frac{15-3t}{5}\right| = |t|$

$\Rightarrow \frac{15-3t}{5} = t$ or $\frac{15-3t}{5} = -t$

$\therefore\ t = \frac{15}{8}$ or $t = \frac{-15}{2}$

So, $P\left(\frac{15}{8}, \frac{15}{8}\right) \in 1^{st}$ quadrant

or $P\left(\frac{-15}{2}, \frac{15}{2}\right) \in \|^{nd}$ Quadrant

84. (a)

$4a^2 + b^2 = 8$

$$\left.\frac{dy}{dx}\right)_{(1,2)} = -\frac{4x}{y} = -2$$

$$\Rightarrow -\frac{4a}{b} = \frac{1}{2}$$

$b = -8a$

$\Rightarrow b^2 = 64a^2$

$$68a^2 = 8, a^2 = \frac{2}{17}$$

85. (a) $\cos\alpha = \frac{3}{5}, \tan\beta = \frac{1}{3}$

$$\Rightarrow \tan\alpha = \frac{4}{3}$$

$$\Rightarrow \tan(\alpha-\beta) = \frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{3}\cdot\frac{1}{3}} = \frac{9}{13}$$

$$\Rightarrow \sin(\alpha-\beta) = \frac{9}{5\sqrt{10}}$$

$$\Rightarrow \alpha-\beta = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$$

86. (d) The required plane is $(2x-y-4)+\lambda(y+2z-4)=0$ it passes through $(1,1,0)$

$\Rightarrow (2-1-4)+\lambda(1-4)=0$

$\Rightarrow -3-3\lambda = 0 \Rightarrow \lambda = -1$

$\Rightarrow x-y-z=0$

87. (d) $p = \frac{n}{\sqrt{2}}$, but $\frac{n}{\sqrt{2}} < 4 \Rightarrow n = 1,2,3,4,5$

Length of chord $AB = 2\sqrt{16-\frac{n^2}{2}}$

$= \sqrt{64-2n^2} = \ell$ (say)

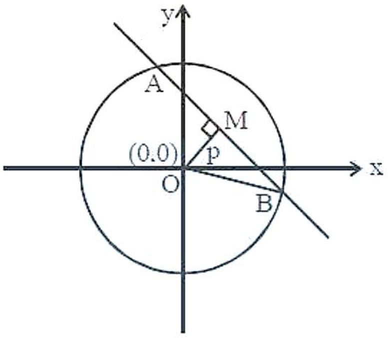
For $n = 1, \ell^2 = 62$

$n = 2, \ell^2 = 56$

$n = 3, \ell^2 = 46$

$n = 4, \ell^2 = 32$

$n = 5, \ell^2 = 14$



$\therefore$ Required sum

$= 62 + 56 + 46 + 32 + 14 = 210$

88. (c) For non - trivial solution

$D = 0$

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0 \Rightarrow 2c^3 + 3c^2 - 1 = 0$$

$\Rightarrow (c+1)^2(2c-1) = 0$

$\therefore$ Greatest value of c is $\frac{1}{2}$

89. (c)

$\phi(x) = f(x) + f(2-x)$

$\phi'(x) = f(x) - f'(2-x)$

Since $f''(x) > 0$

$\Rightarrow f(x)$ is increasing $\forall x \in (0,2)$

Case $-\mathrm{I}$: When $x > 2 - x \Rightarrow x > 1$

$\Rightarrow \phi'(x) > 0 \forall x \in (1,2)$

$\therefore \phi(x)$ is increasing on $(1,2)$

Case - II : When $x < 2 - x \Rightarrow x < 1$

$\Rightarrow \phi'(x) < 0 \forall x \in (0,1)$

$\therefore \phi(x)$ is decreasing on $(0,1)$

90. (d) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$

(as $A \subset B \Rightarrow P(A \cap B) = P(A)$)

$\Rightarrow P(A \mid B) \geq P(A)$