1. **(b)** Mutual induction

$\phi_q = MI_p$

$10^{-3} = M(3)$

$\Rightarrow M = \frac{1}{3} \times 10^{-3}$

$\phi_p = MI_q$

$\Rightarrow \phi_p = 6.67 \times 10^{-4}$ Wb

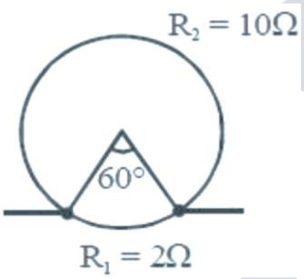
2. **(b)**

$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{(V/\ell)} = \frac{\rho\ell^2}{V}$ (V → Volume of wire)

$\Rightarrow$ Final resistance $= 3 \times (B)^2 = 12\Omega$

$R_{eq} = 2\Omega \parallel 10\Omega$

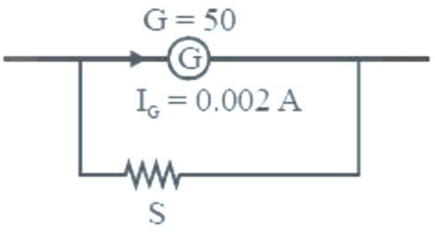
$= \frac{5}{3}\Omega$



3. **(a)** Since shunt resistance is connected in parallel with galvanometer, both will have same voltage drop.

$(0.002)(R_G) = (0.5 - 0.002)(r_s)$

$\Rightarrow r_s \approx 0.2\Omega$



4. **(d)** $x = at + bt^2 - ct^3$

$V = \frac{dx}{dt} = a + 2bt - 3ctt^2$

$a = \frac{dv}{dt} = 2b - 6ct$

Put acceleration $= 0$

$\Rightarrow t = \frac{b}{3c}$

Find V at $t = \frac{b}{3c}$

$$v = a + \frac{b^2}{3c}$$

5. (b) For image to form at object itself, says must retrace their path back to object. Hence must incident on mirror normally.

Case 1: Object will be at focus of lens

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{1}{-18}$$

$$\Rightarrow \mathrm{R} = 18 \text{ cm}$$

Case 2: Retraction at 1st surface:

$$\frac{1}{-27} - \frac{1.5}{\mathrm{V_1}} = \frac{1 - 1.5}{\mathrm{R}}$$

2nd retraction:

$$\frac{1.5}{\mathrm{V_1}} - \frac{\mu}{\infty} = \frac{1.5 - \mathrm{u}}{-\mathrm{R}}$$

From (i) and (ii)

$\mu = \frac{4}{3}$.

6. (a) $\tau = \vec{M} \times \vec{B}$

$$\mathrm{C}\theta = \mathrm{iNAB}$$

$$10^{-6} \times \frac{\pi}{180} = 10^{-3} \times 10^{-4} \times 175 \times B$$

$\mathrm{B} = 10^{-3}$ Tesla

7. (d)

8. (d) By law of conservation of energy

$$\frac{1}{2}\mathrm{kx}^2 = (\mathrm{m_1\ s_1} + \mathrm{m_2\ s_2})\Delta\mathrm{T}$$

$$\Delta\mathrm{T} = \frac{16 \times 10^{-2}}{4384} = 3.65 \times 10^{-5}$$

9. (c) Conservation of momentum

$$\vec{\mathrm{p}}_\mathrm{x} + \vec{\mathrm{p}}_\mathrm{y} = \vec{\mathrm{p}}_\mathrm{final}$$

$$m_x v_x - m_y v_y = (m_x + m_y)v$$

$$\frac{\mathrm{h}}{\lambda_\mathrm{x}} - \frac{\mathrm{h}}{\lambda_\mathrm{y}} = \frac{\mathrm{h}}{\lambda}$$

$$\Rightarrow \lambda = \frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$$

10. (d) Magnification is 2

If image is real, $x_1 = \frac{3f}{2}$

If image is virtual, $x_2 = \frac{f}{2}$

$\frac{x_1}{x_2} = 3:1$

11. (c) Limit of resolution $= \frac{1.22\lambda}{d}$

$$= \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}}$$
$$= 2.9 \times 10^{-7} \text{rad.}$$

12. (a) KE $= \frac{1}{2} I \omega^2 = 1200$ (given)

$\Rightarrow \omega = 40 \text{rad/s}$

$\omega = \omega_0 + \alpha t$

$40 = 0 + (20)t$

$\Rightarrow t = 2 \text{sec.}$

13. (a) ruth table can be formed as

A	B	Equivalent
0	0	0
0	1	1
1	0	1
1	1	1

Hence the equivalent is "OR" gate.

14. (d) Radiation pressure for 100% reflection $= \frac{2I}{c}$

Radiation pressure for 0% reflection $= \frac{I}{c}$

Hence, in given case, radiation pressure $= (0.25)\left(\frac{2I}{c}\right) + (0.75)\left(\frac{I}{c}\right)$

$= (1.25)\left(\frac{I}{C}\right)$

$$\text{Force} = \text{P} \times (\text{ Area })$$
$$= 20.83 \times 10^{-8} \text{ N}$$

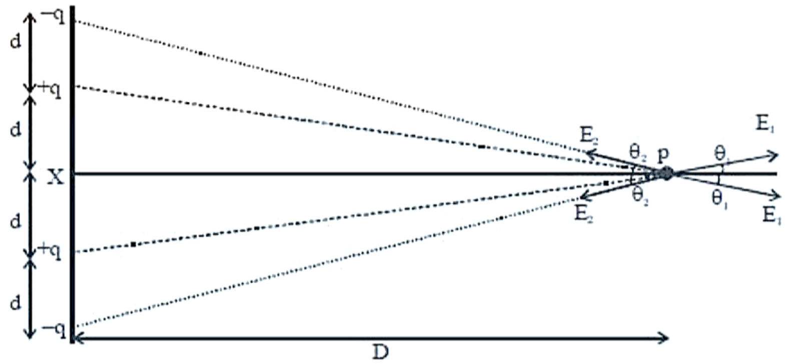
15. (c) Total Area $= A_1 + A_2 + A_7$

$$= A + A + \cdots \ldots \ldots .7 \text{ times}$$
$$= 37.03 \text{ m}^2$$

Addition of 7 terms all having 2 terms beyond decimal, so final answer must have 2 terms beyond decimal (as per rules of significant digits)

16. (c) The physical size of antenna of receiver and transmitter both are inversely proportional to carrier frequency.

17. (d)



Electric field at $p = 2\mathrm{E}_1\cos\theta_1 - 2\mathrm{E}_2\cos\theta_2$

$$= \frac{2Kq}{(d^2+D^2)} \times \frac{D}{(d^2+D^2)^{1/2}} - \frac{2Kq}{[(2d)^2+D^2]} \times \frac{D}{[(2d)^2+D^2]^{1/2}}$$

$$= 2KqD\left[(d^2+D^2)^{-3/2} - (4d^2+D^2)^{-3/2}\right]$$

$$= \frac{2KqD}{D^3}\left[\left(1+\frac{d^2}{D^2}\right)^{-3/2} - \left(1+\frac{4d^2}{D^2}\right)^{-3/2}\right]$$

Applying binomial approximation $\because$ d < D

$$= \frac{2KqD}{D^3}\left[1 - \frac{3}{2}\frac{d^2}{D^2} - \left(1 - \frac{3\times 4d^2}{2D^2}\right)\right]$$

$$= \frac{2KqD}{D^3}\left[\frac{12}{2}\frac{d^2}{D^2} - \frac{3}{2}\frac{d^2}{D^2}\right]$$

$$= \frac{9kqd^2}{D^4}$$

18. (d) For A:

$$\frac{C_p}{C_v} = \gamma = 1 + \frac{2}{f} = \frac{29}{22}$$

It gives $f = 6.3 \approx 6$ (3 translational, 2 rotational and 1 vibrational)

For B:

$$\frac{C_p}{C_v} = \gamma = 1 + \frac{2}{f} = \frac{30}{21}$$

$\Rightarrow f = 4.67$

$\Rightarrow \ \approx 5$ (3 translational, 2 rotational, no vibrational)

19. (d) $\vec{r} = (15t^2)\hat{\imath} + (4 - 20t^2)\hat{\jmath}$

$$\vec{v} = \frac{d\vec{r}}{dt} = (30t)\hat{\imath} - (40t)\hat{\jmath}$$

$\vec{a} = \frac{d\vec{v}}{dt} = (30)\hat{i} - (40)\hat{j}$

$|\vec{a}| = 50$

20. (d) For circular motion of particle:

$$\frac{mV^2}{r} = mE$$
$$= m\left(\frac{GM}{r^2}\right)$$

Where $M = \int_0^r (4\pi x^2 dx)\left(\frac{k}{x^2}\right)$

$= 4\pi kr$

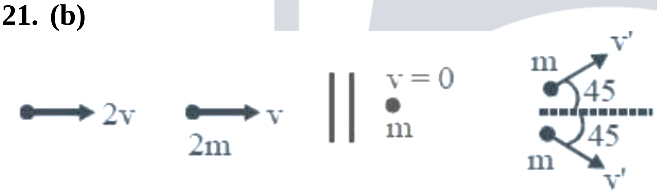
$\Rightarrow \frac{mV^2}{r} = m\left(\frac{G(4\pi k)}{r}\right)$

$\Rightarrow$ V = constant

$T = \frac{2\pi R}{V}$

$\Rightarrow \frac{T}{R} =$ Constant

21. (b)



Linear momentum conservation

$$m2v + 2mv = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$$

$v' = 2\sqrt{2}v$

22. (c) In 1st situation

$V_b\rho_b g = V_s\rho_w g$

$$\frac{V_s}{V_b} = \frac{\rho_b}{\rho_w} = \frac{4}{5}$$

Here V_b is volume of block

V_s is submerged volume of block

ρ_b is density of block

ρ_w is density of water & Let ρ_o is density of oil

Finally in equilibrium condition

$$V_b\rho_b g=\frac{V_b}{2}\rho_o g+\frac{V_b}{2}\rho_w g$$

$$2\rho_b=\rho_0+\rho_w$$
$$\Rightarrow \frac{\rho_o}{\rho_w}=\frac{3}{5}=0.6$$

23. (a) Doppler effect:

$$f=\left(\frac{v+u_0}{v-u_s}\right)(f_0)$$
$$2000=\left(\frac{340+(-20)}{340-(-20)}\right)(f_0)$$
$$f_0=2250Hz$$

24. (c) Let mass attains height ' h ' on wedge and at that time, both attain velocity V_f. Conservation of momentum:

$$mv=(m+4m)V_f$$

COE:

$$\frac{1}{2}mv^2=\frac{1}{2}(m+4m)V_f^2+mgh$$

From (A) and (B)

$$h=\frac{2\,V^2}{5\,g}$$

25. (b) T.E. $=-(13.6)\left(\frac{z^2}{n^2}\right)eV$

$z=n=2$

$\Rightarrow$ Ionisation energy = – T.E.

$=13.6eV$

26. (b) At steady state:

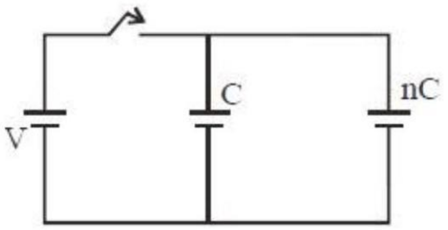
$$\left(\frac{\Delta q}{\Delta t}\right)_1=\left(\frac{\Delta q}{\Delta t}\right)_2$$
$$\frac{3kA(\theta_2-\theta)}{d}=\frac{kA(\theta-\theta_1)}{3\,d}$$
$$\Rightarrow \theta=\frac{\theta_1+9\theta_2}{10}$$

27. (d) Conservation of angular momentum about rotation axes:

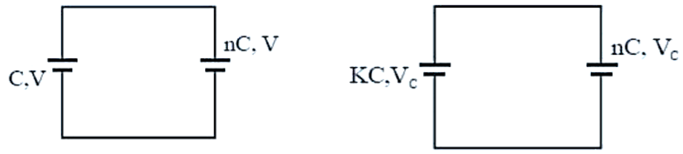
$L_i=L_f$

$$\left(\frac{M\ell^2}{12}\right)\omega_0=\left[\frac{M\ell^2}{12}+2\left(m\left(\frac{\ell}{2}\right)^2\right)\right]\omega_f$$
$$\Rightarrow \omega_f=\left(\frac{M}{M+6\,m}\right)\omega_o$$

28. (c)



After fully charging, battery is disconnected.



Total chare of the system $= CV + nCV$

$= (n+1)CV$

After the insertion of dielectric of constant K

New potential (common)

$$V_C = \frac{\text{Total charge}}{\text{Total capacitance}}$$
$$= \frac{(n+1)CV}{KC+nC} = \frac{(n+1)V}{K+n}$$

29. (d)

$$\rho = \frac{2m}{ne^2\tau}$$
$$= 3.34 \times 10^{-8}\,\Omega\text{m}$$

30. (d) We have:

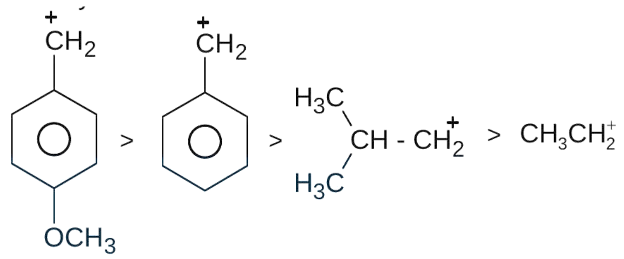
$$f = \frac{nv}{2\ell}$$

$$240 = \frac{3 \times v}{2 \times 2}$$

$$\Rightarrow v = 320 \text{ m/s}$$

Fundamental frequency $= \frac{v}{2\ell} = 80$ Hz.

31. (c) Stability of carbonium ions involved in the reactions follow the order:



∴ Reactivity α Stability of carbonium ions

32. (a)

Malachite $= CuCO_3 \cdot Cu(OH)_2$

Bauxite $= Al_2O_3 \cdot 2H_2O$

Calamine $= ZnCO_3$

Siderite $= FeCO_3$

33. (a) Work done on system $= +10\ kJ$

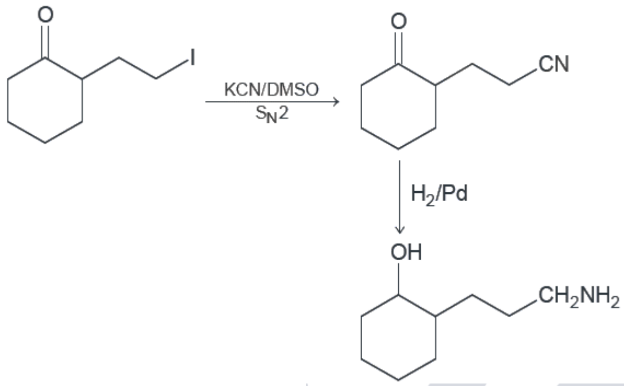
Heat escaped $= -2\ kJ$

$\Delta U = q + w$

$= 10 - 2 = 8\ kJ$

34. (b) Kieselguhr is amorphous form of silica

35. (b)



36. (d) $T.E = -K.E = \frac{PE}{2}$

The energy of electrons increases as they are away from the nucleus. The probability of finding the $1\ s$ electron may be higher at a_0 but the energy is not. The probability density of finding the electron is not zero at any place in the atom. So, the energy may be higher when it is far from a_0.

37. (a) General coordination number of CN^- in transition element is 6 and in inner transition element is 8-12. because inner transition metal ions can make available more number of vacant orbitals of nearly same energy than transition metal ions. The high effective nuclear charge of inner-transition metal ions make them form complex with high coordination number.

38. (a)

39. (d) CO is diamagnetic in nature due to absence of any unpaired electron.

40. (b) Due to $-OH$ group of serine it give cerric ammonium nitrate test whereas due to $-NH_2$ group lysine give +ve carbylamine test.

41. (c) Extraction of Fe is done form haematite ore this is true but reason is wrong as haematite is Fe_2O_3.

42. (a) Moles $= \frac{MVml}{1000} = \frac{10^{-3} \times 10}{1000} = 10^{-5}$ mole

$10^{-5}\ N_A$ molecules covering area $= 0.24\ cm^2$

1------------------------------------- $= \frac{0.24}{10^{-5}\ N_A}\ cm^2$

$$\frac{0.24}{10^{-5}\times 6\times 10^{23}}=a^2$$

$a^2=4\times 10^{-20}\ \mathrm{cm}^2$

$a=2\times 10^{-10}\ \mathrm{cm}$

$a=2\times 10^{-12}\ \mathrm{m}$

$a=2\mathrm{pm}$

43. (d) $E_a=(D\rightarrow C)$

$=15-0=15\ \mathrm{kJ\ mol^{-1}}$

$E_a=(A+B)\rightarrow C=15\ \mathrm{kJ\ mol^{-1}}$

$E_a=(A+B)\rightarrow D=10\ \mathrm{kJ\ mol^{-1}}$

$E_a=C\rightarrow(A+B)=20\ \mathrm{kJ\ mol^{-1}}$ (high activation enthalpy in the reaction)

$\therefore$ Activation enthalpy to form C is $5\ \mathrm{kJ\ mol^{-1}}$ more than that of form D.

44. (c) $P=\frac{RT}{(V-b)}$

$P(V-b)=RT$

$\left(P+\frac{a}{V^2}\right)(V-b)=RT$

At high pressure.

$P(V-b)=RT$

$PV-Pb=RT$

$\frac{PV}{RT}-\frac{Pb}{RT}=1$

$Z=1+\frac{Pb}{RT}$

$Z>1, Z\alpha b$

The value of ' b ' for the gases follows the order

$\mathrm{Ne}<\mathrm{Ar}<\mathrm{Kr}<\mathrm{Xe}$

45. (b) $1\ \mathrm{F}$ deposits $1\ \mathrm{g}$ equivalent of Ni or $1/2$ mole of Ni

$\therefore 0.1\ \mathrm{F}$ will deposit $\frac{1}{20}=0.05$ moles of Ni

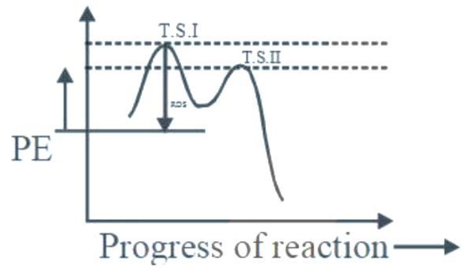
46. (d) Ketones < Aldehyde $\longrightarrow$ Rate of Nucleophilic addition reaction

Only aldehydes are responsible for formation of acetals. Excess of MeOH is used to drive the reaction towards forward direction.

47. (a) Solid state - chain

In vapour state dimeric

48. (d) For S_N1 carbocation is formed.



The first transition state should have higher energy than the second transition state. Reaction intermediate is also formed.

49. (c)

50. (d) Ac Th Pu U NpPu Am Cm Bk Cf Es Fm Md No Lr

+3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +2 +3

+4 +4 +4 +4 +4 +4

+5 +5 +5 +5 +5

+6 +6 +6

$+6+7+7$

$\therefore$ Np and Pu has maximum no. of possible oxidation states

51. (b) $K_f = 4\ \text{KKgmol}^{-1}$

$i = 3$

$\text{Molality} = 0.03$

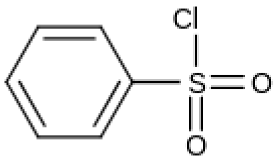
$\Delta T_f = iKK$

$= 3(4)(0.03)$

$\Delta T_f = 0.36\ K$

52. (a) It is an organic chemical in the catecholamine family that functions in the brain and body as hormone and neurotransmitter.

53. (a) Benzene sulphonyl chloride $C_6H_5SO_2Cl$



54. (c)

55. (c) HF has strong hydrogen bond. $\therefore$ It has highest boiling point among hydrogen halides. The strong hydrogen bond is due to more difference in electronegativity between F and H atoms.

56. (c) $20\%\text{w/wKI}$

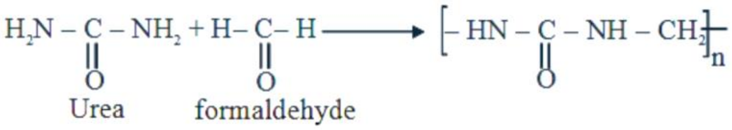
Mass of solute (KI) $= 20$ g

Mass of solvent $= 100 - 20 = 80$ g

Molar mass of KI $= 38 + 128 = 166$

Molality $= \frac{\text{gm(solute)}}{\text{mw} \times \text{Kg(solvent)}} = \frac{20 \times 1000}{166 \times 80} = 1.506 = 1.51$

57. (a)



58. (c)

(i) VBT does not explain colour exhibited by complex of transition metal because splitting of d-orbitals in explained by CFT.

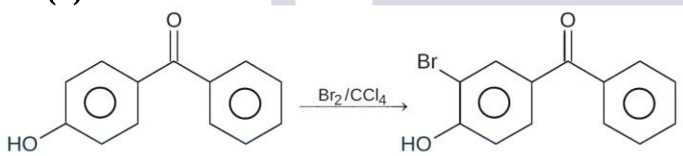
(ii) VBT does not distinguish between strong and weak field complex.

(iii) VBT does not give quantitative interpretation of magnetic properties.

∴ According to question statement I and III are correct.

59. (a) The nature of the curve is such that the pH change of the reaction becomes larger for very small change in volume of titrant.

60. (d)



– R effect, deactivate the ring for EAS. OH → +R effect activate the ring towards EAS

61. (c) Equation of tangent to the parabola $y^2 = x$

At (α, β) is $T = 0$

$$y\beta = \frac{x + \alpha}{2}$$

$$\Rightarrow y\beta = \frac{x + \beta^2}{2} (\because \beta^2 = \alpha)$$

$$\Rightarrow y = \frac{1}{2\beta} x + \frac{\beta}{2}$$

$$\left(m = \frac{1}{2\beta}, c = \frac{\beta}{2}\right)$$

This is also a tangent to ellipse $x^2 + 2y^2 = 1$

$$\therefore C = \pm\sqrt{a^2 m^2 + b^2}$$

$$\Rightarrow \frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2} + \frac{1}{2}}$$

$\Rightarrow \frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$

$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$

$\Rightarrow (\beta^2 - 1)^2 = 2$

$\Rightarrow \beta^2 - 1 = \sqrt{2}$

$\Rightarrow \beta^2 = \sqrt{2} + 1$

$\alpha = \beta^2 = \sqrt{2} + 1$

62. (a) $\frac{n(n+1)}{2} + 99 = (n-2)^2$

$\Rightarrow n^2 + n + 198 = 2n^2 - 8n + 8$

$\Rightarrow n^2 - 9n - 190 = 0$
$\Rightarrow (n-19)(n+10) = 0$

$\Rightarrow n = 19$

$\therefore$ Number of balls is $\frac{19 \times 20}{2} = 190$

63. (c)

$\lim_{x\to 2} \int_6^{f(x)} \frac{2tdt}{(x-2)} dx$ {given that $f(2) = 6$ }

$\frac{0}{0}$ from, so we use $L -$ Hospital Rule

$= \lim_{x\to 2} \frac{f'(x) \cdot 2f(x)}{1}$

$= f'(2) \cdot 2f(2)$

$= 12f'(2)$

64. (c) system of equations has non trival solution

$\therefore D = 0 = \begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow k = \frac{9}{2}$

So equation are $2x + 3y - z = 0$

$x + \frac{9}{2}y - 2z = 0$
$2x - y + z = 0$

(1) $-(3) \Rightarrow 4y - 2z = 0$

$\Rightarrow 2y = z$

$\Rightarrow \frac{y}{z} = \frac{1}{2}$

From equation (1) and (4)

$2x + 3y - 2y = 0$

$\Rightarrow 2x + y = 0$

$\Rightarrow \frac{x}{y} = \frac{-1}{2}$ or $\frac{z}{x} = -4$

$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{1}{2}$

65. (b) Circle $x^2 + y^2 = 4$

$\Rightarrow c_1(0,0); r_1 = 2$

and circle $x^2 + y^2 + 6x + 8y - 24 = 0$

$\Rightarrow c_2(-3,4); r_2 = 7$

$\Rightarrow d = c_1c_2 = 5$

also $d = |r_1 - r_2|$

circles touch internally

Equation of common tangent $S_1 - S_2 = 0$

$\Rightarrow 6x + 8y - 20 = 0$

$\Rightarrow 3x + 4y - 10 = 0$

Point $(6, -2)$ satisfy it.

66. (a) Let the three numbers in A.P. are $a - d, a, a + d$

Given that $a - d + a + a + d = 33$

$\Rightarrow a = 11$ and $(a - d)(a)(a + d) = 1155$

$\Rightarrow a(a^2\ d^2) = 1155$

$\Rightarrow 11(121 - d^2) = 1155$

$\Rightarrow d^2 = 16$

$\Rightarrow d = \pm 4$

If $d = 4$ then first term $a - d = 7$

If $d = -4$ then first term $a - d = 15$

$T_{11} = 7 + 10(4) = 47,\ T_{11} = 15 + 10(-4) = -25$

67. (a) $I = \int_0^1 x\cot^{-1}(1 - x^2 + x^4)dx$

$I=\int_0^1 x\tan^{-1}\left(\frac{1}{1-x^2+x^4}\right)dx$

$I=\int_0^1 x\tan^{-1}\left\{\frac{x^2-(x^2-1)}{1+x^2(x^2-1)}\right\}dx$

$I=\int_0^1 x\{\tan^{-1}x^2-\tan^{-1}(x^2-1)\}dx$

It $x^2=t\Rightarrow 2xdx=dt$

$I=\frac{1}{2}\int_0^1\{\tan^{-1}t-\tan^{-1}(t-1)\}dt$

$=\frac{1}{2}\int_0^1\tan^{-1}tdt-\frac{1}{2}\int_0^1\tan^{-1}(t^{-1})dt$

$=\frac{1}{2}\int_0^1\tan^{-1}tdt-\frac{1}{2}\int_0^1\tan^{-1}(-t)dt$

$=\frac{1}{2}\int_0^1\tan^{-1}tdt+\frac{1}{2}\int_1^1\tan^{-1}(t)dt$

$=\int_0^1\tan^{-1}(t)dt$

$=(t\cdot\tan^{-1}t)_0^1-\int_0^1\frac{t}{1+t^2}dt$

$=\left(\frac{\pi}{4}\right)-\frac{1}{2}[\log(1+t^2)]_0^1$

$=\frac{\pi}{4}-\frac{1}{2}\log_e 2$

68. (d)

$\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ$

$$\begin{aligned}&=\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ\\&=\sin 30^\circ\{\sin 10^\circ\sin(60^\circ-10^\circ)\sin(60^\circ+10^\circ)\}\\&=\sin 30^\circ\left\{\frac{1}{4}\sin 3(10^\circ)\right\}\\&=\frac{1}{2}\left(\frac{1}{4}\times\frac{1}{2}\right)\\&=\frac{1}{16}\end{aligned}$$

69. (c)

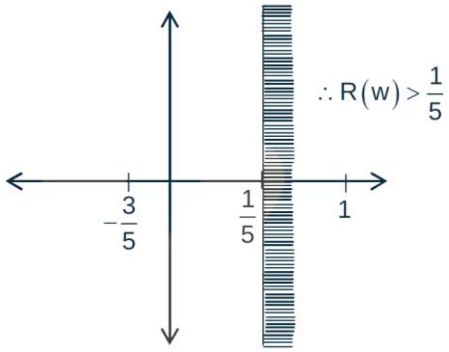
$W=\frac{5+3z}{5(1-z)}$

$\Rightarrow 5w-5wz=5+3z$

$\Rightarrow z=\frac{5w-5}{3+5w}$

given $|z|<1$

$\Rightarrow \left|\frac{5w-5}{3+5w}\right| < 1$



$\Rightarrow |w-1| < \left|\frac{3}{5}+w\right|$

70. (d) Given: $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15} \Rightarrow \frac{r}{n-r+1} = \frac{2}{15}$

$\Rightarrow 15r = 2n - 2r + 2$

$\Rightarrow 17r = 2n + 2$

also given $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70} \Rightarrow \frac{r+1}{n-r} = \frac{3}{14}$

$\Rightarrow 3n - 3r = 14r + 14$

$\Rightarrow 17r = 3n - 14$

Solving (1) and (2)

$n = 16, r = 2$

Average of coefficient $= \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3}$

$= \frac{16+120+560}{3}$

$= 232$

71. (d) $\cos x \frac{dy}{dx} - y\sin x = 6x$

$\Rightarrow \frac{dy}{dx} - y\tan x = 6x\sec x$

$= e^{-\int \tan x dx} = e^{-\log_e \sec x} = \frac{1}{\sec x}$

: solution of equation

$\Rightarrow y \cdot \frac{1}{\sec x} = \int 6x\sec x \cdot \frac{1}{\sec x} dx$

$\Rightarrow \frac{y}{\sec x} = 3x^2 + c$

given $y\left(\frac{\pi}{3}\right) = 0$

So, $0 = 3\frac{\pi^2}{9} + C$

$\Rightarrow C = -\frac{\pi^2}{3}$

Now from (1)

$\Rightarrow \frac{y}{\sec x} = 3x^2 - \frac{\pi^2}{3}$

At $x = \frac{\pi}{6}$

$\Rightarrow \frac{\sqrt{3y}}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3}$

$\Rightarrow \mathrm{y} = -\frac{\pi^2}{2\sqrt{3}}$

72. (d) Two lines are perpendicular

$\therefore \mathrm{m_1\ m_2} = -1$

$\Rightarrow \left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$

$\Rightarrow a^3 - a^2 + 2 = 0$

$\Rightarrow (\mathrm{a}+1)(\mathrm{a}^2 - 2\mathrm{a} + 2) = 0$

$\therefore a = -1$

So lines are $\left.\begin{matrix} L_1 : x - 2y + 1 = 0 \\ L_2 : 2x + y - 1 = 0 \end{matrix}\right\}$

Solving these equation we get point of intersection

$\mathrm{P}\left(\frac{1}{5}, \frac{3}{6}\right)$

Now distance of P from origin

$\mathrm{OP} = \sqrt{\frac{1}{25} + \frac{9}{25}} = \sqrt{\frac{2}{5}}$

73. (b) Given $\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \tan\theta = \frac{1}{2} = \frac{\mathrm{r}}{\mathrm{h}}$

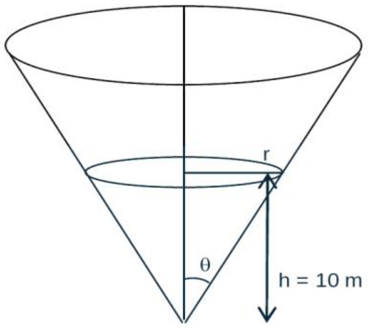
$\Rightarrow \mathrm{r} = \frac{\mathrm{h}}{2}$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi\frac{h^3}{4}$

$\frac{\mathrm{dv}}{\mathrm{dt}}=\frac{\pi}{12}(3\,\mathrm{h}^2)\frac{\mathrm{dh}}{\mathrm{dt}}$

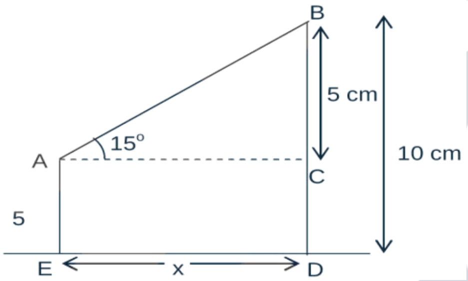
$5=\frac{\pi}{4}(100)\frac{\mathrm{dh}}{\mathrm{dt}}\Rightarrow\frac{\mathrm{dh}}{\mathrm{dt}}=\frac{1}{5\pi}$



74. **(c)** In $\triangle$ ABC $\Rightarrow \tan 15^\circ=\frac{5}{x}$

$\Rightarrow 2-\sqrt{3}=\frac{5}{x}$

$\Rightarrow x=5(2+\sqrt{3})$



75. **(b)** Let any point on given line is

$D(3\lambda-2,1,4\lambda)$

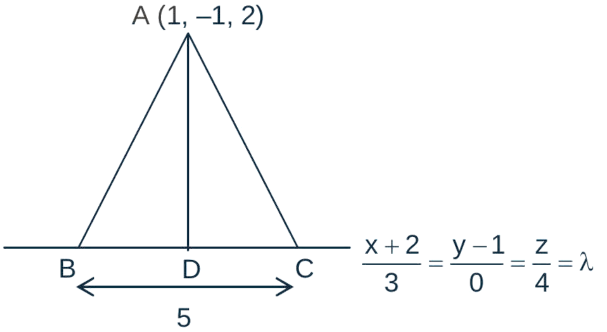
Now $AD\perp BC$

D.R. of BC

$\Rightarrow a_1=3,\ b_1=0, c_1=y$

D.R. of AD

$\Rightarrow a_2=3\lambda-3,\ b_2=2, c_2=4\lambda-2$



$\Rightarrow a_1 a_2 + b_1\ b_2 + c_1 c_2 = 0$

$\Rightarrow 3(3\lambda - 3) + 0 + 4(4\lambda - 2) = 0$

$25\lambda = 17$

$\Rightarrow \lambda = \frac{17}{25}$

Co - ordinate of point D

$\left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$AD = \sqrt{\frac{576}{625} + 4 + \frac{324}{25}} = \frac{2}{5}\sqrt{34}$

Area of $\triangle ABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 5 \times \frac{2}{6}\sqrt{34}$

$= \sqrt{34}$

76. (d) $A^T A = 3I_3$

$$\Rightarrow \begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\Rightarrow 8x^2 = 3 \Rightarrow x = \pm\sqrt{\frac{3}{8}}$

$\Rightarrow 6y^2 = 3 \Rightarrow y = \pm\sqrt{\frac{1}{2}}$

4 matrices are possible

77. (b) Equation of tangent to the parabola $y^2 = 4x$ at

(1,2) is $2y = 4\left(\frac{x+1}{2}\right)$

$\Rightarrow y = x + 1$

Equation of normal $y = -x + 3$

Let centre be $C(3 - r, r)$

Now $PC^2 = r^2$

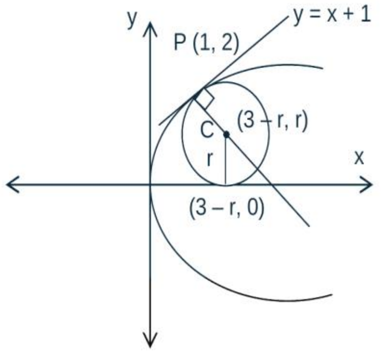
$\Rightarrow (3 - r - 1)^2 + (r - 2)^2 = r^2$

$\Rightarrow 2(2 - r)^2 = r^2$

$\Rightarrow r^2 - 8r + 8 = 0$

$\Rightarrow r = 4 \pm 2\sqrt{2}$

For $r = 4 + 2\sqrt{2}$ $(3 - r < 0)$ Not possible



So $r = 4 - 2\sqrt{2}$

Area $= \pi r^2 = \pi(16 + 8 - 16\sqrt{2})$

$= 8\pi(3 - 2\sqrt{2})$

78. (a)

$f(x) = \begin{cases} a|\pi - x| + 1, x \le 5 \\ b|\pi - x| + 3, x > 5 \end{cases}$

Contributes at $x = 5$

$\therefore$ L.H.L. = R.H.L $= f(5)$

$\Rightarrow b|\pi - 5| + 3 = a|\pi - 5| + 1$

$\Rightarrow -b(\pi - 5) + 3 = -a(5 - \pi) + 1$

$\Rightarrow (a - b)(\pi - 5) = -2$

$\Rightarrow a - b = \frac{-2}{\pi - 5} = \frac{2}{5 - \pi}$

79. (d) $f(x)=[x]-\left[\frac{x}{4}\right]$

$\lim_{x\to4^+} f(x)=\lim_{x\to4^+}\left([x]-\left[\frac{x}{4}\right]\right)=4-1=3$

$\lim_{x\to4^-} f(x)=\lim_{x\to4^-}\left([x]-\left[\frac{x}{4}\right]\right)=3-0=3$

$f(4)=3$

$\therefore$ Continuous at $x=4$

80. (d)

$\int e^{\sec x}\left(\sec x+\tan x f(x)+(\sec x\tan x+\sec^2 x)\right)dx=e^{\sec x}f(x)+C$

Diff. both side w.r.t.x

$=e^{\sec x}\left(\sec x+\tan x+f(x)+(\sec x\tan x+\sec^2 x)\right)$

$=e^{\sec x}\cdot\sec x\tan x f(x)+e^{\sec x}f(n)$

$\Rightarrow f'(x)=\sec^2 x+\tan x\sec x$

$\Rightarrow f(x)=\tan x+\sec x+C$

81. (d) $(m^2+1)x^2-3x+(m+1)^2=0$

$\Rightarrow \alpha+\beta=\frac{3}{m^2+1}$

$\alpha\beta=\frac{(m+1)^2}{m^2+1}$

$\because \alpha+\beta$ is maximum

$\therefore m^2+1$ is minimum

$\Rightarrow m=0$

$\therefore \alpha+\beta=3$ and $\alpha\beta=1$

$|\alpha^3-\beta^3|=|(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)|$

$\left|\sqrt{(\alpha+\beta)^2-4\alpha\beta}\{(\alpha+\beta)^2-\alpha\beta\}\right|$

$=|\sqrt{9-4}(9-1)|$

$=8\sqrt{5}$

82. (c) Let population $=100$

$n(A)=25$

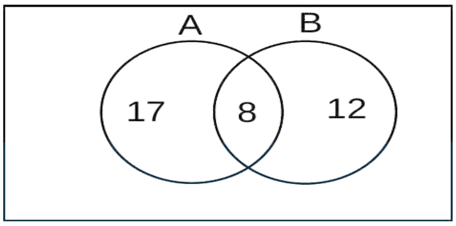
$n(B)=20$

$n(A\cap B)=8$

$n(A \cap \bar{B}) = 17$

$\mathrm{n}(\bar{\mathrm{A}} \cap \mathrm{B}) = 12$

Now % of the population who look advertisement



$$= \frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$$

$= 13.9$

83. (d) Equation of plane $P_1 + \lambda P_2 = 0$

$(x + y + z - 6) + \lambda(2x + 3y + z + 5) = 0$

$\Rightarrow x(1 + 2\lambda) + y(1 + 3\lambda) + 2(1 + \lambda) - 6 + 5\lambda = 0$

This plane is $\perp$ to xy - plane

$\therefore 1 + \lambda = 0$

So, equation of plane

$-x - 2y - 11 = 0$

$\Rightarrow \mathrm{x} + 2\mathrm{y} + 11 = 0$

distance of the point $(0,0,256)$ from this plane.

$$= \left|\frac{0 + 0 + 11}{\sqrt{1 + 4}}\right| = \frac{11}{\sqrt{5}}$$

84. (b) $p \Rightarrow (q \vee r)$ is false

$(\because T \Rightarrow F = F)$

So, $p = T, q = F$ and $r = F$

85. (c)

$$f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$$

Let $\mathrm{f}_1 = \frac{1}{4-\mathrm{x}^2}$ and $\mathrm{f}_2 = \log_{10}(\mathrm{x}^3 - \mathrm{x})$

$\Rightarrow 4 - x^2 \neq 0\ x^3 - x > 0$

$\Rightarrow \mathrm{x} \neq \pm 2\ \Rightarrow \mathrm{x}(\mathrm{x} + 1)(\mathrm{x} + 1) > 0$

$x \in (-1,0) \cup (1, \infty) - \{2\}$

$x \in (-1,0) \cup (1,2) \cup (2,\infty)$

86. (b) $S = 1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \cdots \ldots. +$ upto 11 terms

n^{th} term of the series is $T_n = n(2n-1)$

$\Rightarrow S = \sum_{n=1}^{11} T_n = \sum_{n=1}^{11} (2n^2 - n)$

$\Rightarrow S_n = \frac{2n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$

Put $n = 1$

$\Rightarrow S_{11} = \frac{2(11)(12)(23)}{6} - \frac{11(12)}{2}$

$\Rightarrow S_{11} = 946$

87. (a) mean $= 42$

$\Rightarrow \frac{10 + 22 + 26 + 29 + 34 + x + 42 + 67 + 70 + y}{10} = 45$

$\Rightarrow x + y = 120$

and median $= 35$

$\Rightarrow \frac{34 + x}{2} = 35 \Rightarrow x = 36$

from (i) $y = 84$

$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

88. (b) $y^2 = 2x$

and $x - y - 4 = 0$

solving (1) and (2)

$(x-y)^2 = 2x$

$\Rightarrow x^2 - 10x + 16 = 0$

$\Rightarrow x = 8,2$ and $y = 4, -2$

$A = \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2}\right) dy$

$A = \left(\frac{y^2}{4} + 4y - \frac{y^3}{6}\right)_{-2}^{4}$

$A = \left(4 + 16 - \frac{64}{6}\right) - \left(1 - 8 + \frac{8}{6}\right) = 18$

89. (c) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \gamma = 1$

$\Rightarrow \cos^2 \gamma = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos \gamma = \pm \frac{1}{2}$

$\Rightarrow \gamma = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

90. (b) Let vertex C is $(6, k)$ then centre of circle $\left(-1, \frac{5+k}{2}\right)$

It lies on diameter $3y = x + 7$

$\Rightarrow 3\left(\frac{5+k}{2}\right) = -1 + 7$

$\Rightarrow k = -1$

So, $AB = 14$ and $BC = 6$

Area $= 14 \times 6 = 84$