1. (BONUS) $\rho = \frac{m}{v}$

Maximum % error in ρ will be given by

$$\frac{\Delta p}{p} \times 100\% = \left(\frac{\Delta m}{m}\right) \times 100\% + 3\left(\frac{\Delta L}{L}\right) \times 100\%$$

This is not applicable as error is big.

$$\rho_{min} = \frac{m_{min}}{v_{max}} = \frac{9.9}{(0.11)^3} = 7438 \text{ kg/m}^3$$

& $\rho_{max} = \frac{m_{max}}{v_{min}} = \frac{10.1}{(0.09)^3} = 13854.6 \text{ kg/m}^3$

$\Delta p = 6416.6 \text{ kg/m}^3$

2. (a) $\phi = NBA = LI$

$N\mu_0 n I \pi R^2 = LI$

$N\mu_0 \frac{N}{\ell} I \pi R^2 = LI$

N and R constant

Self inductance $(L) \propto \frac{1}{\ell} \propto \frac{1}{\text{length}}$

3. Correct option should be (Bonus)

Path difference at central maxima $\Delta x = (\mu - 1)t$, whole pattern will shift by same amount which will be given by $(\mu - 1)t\frac{D}{d} = n\frac{\lambda D}{d}$, according to eh question $t = \frac{n\lambda}{(\mu - 1)}$

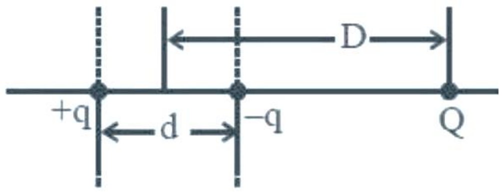
No option is matching, therefore question should be award bonus.

4. (a)

$U_{total} = U_{\text{self of dipole}} + U_{\text{interaction}}$

$= -\frac{kq^2}{d} - \left(\frac{kQ}{D^2}\right) q\, d$

$= -k\left[\frac{q^2}{d} + \frac{qQd}{D^2}\right]$



5. (b) $G = 50\Omega$

$S = 5000\Omega$

$I_g = 4 \times 10^{-3}$

$V = i_g(G + S)$

$V = 4 \times 10^{-3}(50 + 5000)$

$= 4 \times 10^{-3}(5050) = 20.2\text{volt}$

6. (d) Height of liquid rise in capillary tube $h = \frac{2T\cos\theta_C}{\rho rg}$

$\Rightarrow h \propto \frac{1}{r}$

When radius becomes double height become half

$\therefore h' = \frac{h}{2}$

Now, $M = \pi r^2 h \times \rho$ and $M' = \pi(2r)^2(h/2) \times \rho = 2M$.

7. (a) Input current $= 15 \times 10^{-6}$

Output current $= 3 \times 10^{-3}$

Resistance out put $= 1000$

$V_{\text{input}} = 10 \times 10^{-3}$

Now $V_{\text{input}} = r_{\text{input}} \times i_{\text{input}}$

$10 \times 10^{-3} = r_{\text{input}} \times 15 \times 10^{-6}$

$r_{\text{input}} = \frac{2000}{3} = 0.67\ \text{K}\Omega.$

Voltage gain $= \frac{V_{\text{output}}}{V_{\text{input}}} = \frac{1000 \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$

8. (a) According to equipartion energy theorem

$\frac{1}{2}\ m(v_{rms}^2) = 3 \times \frac{1}{2}\ K_b T$

$T = \frac{m\bar{v}_{rms}^2}{3k}$

9. (a) $\omega = 6 \times 10^{14} \times 2\pi$

$f = 6 \times 10^{14}$

$C = f\lambda$

$\lambda = \frac{C}{f} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5000\text{Å}$

Energy of photon $\Rightarrow \frac{12375}{5000} = 2.475\text{eV}$

From Einstein's equation

$KE_{max} = E - \phi$

$eVV_s = E - \phi$

$eV_s = 2.475 - 2$

$eV_o = 0.475eV$

$V_o = 0.48\ V$

10. (d) Initial and final states for both the processes are same,

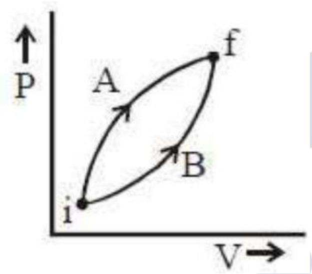
$\therefore \Delta U_A = \Delta U_B$

Work done during process A is greater than in process B. Because area is more

By First law of thermodynamics

$\Delta Q = \Delta U + W$

$\Rightarrow \Delta Q_A > \Delta Q_B$



11. (b) Mass of the hanging part $= \frac{M}{n}$

$h_{COM} = \frac{L}{2n}$

Work done $W = mgh_{Com} = \left(\frac{M}{n}\right) g \left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$

12. (b) $\frac{1}{2}\left(m + \frac{1}{R^2}\right)v^2 = mgh$

If radius of gyration is k, then

$h = \frac{\left(1 + \frac{k^2}{R^2}\right)v^2}{2g}; \frac{k_{ring}}{R_{ring}} = 1, \frac{k_{solid\ cylinder}}{R_{solid\ cylinder}} = \frac{1}{\sqrt{2}}$

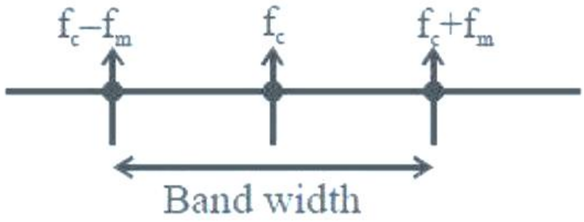
$\frac{k_{solid\ sphere}}{R_{solid\ sphere}} = \sqrt{\frac{2}{5}}$

$H_1 : h_2 : h_3 :: (1 + 1) : \left(1 + \frac{1}{2}\right) : \left(1 + \frac{2}{5}\right) :: 20 : 15 : 14$

Therefore most appropriate option is (B)

Although which is not in correct sequence.

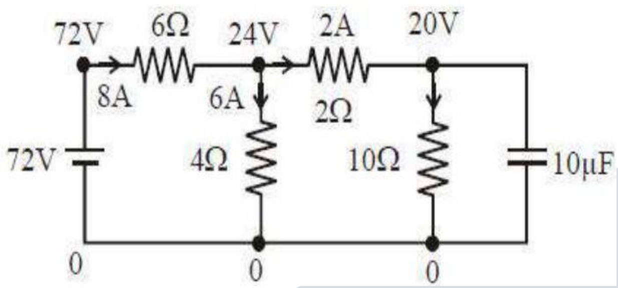
13. (b)



14. (d) $m = \frac{f}{f-u}$

$5 = \frac{-40}{-40-u}; u = -32$ cm

15. (a) Different potential is shown at different points.



$q = eV$

$q = 10\mu F \times 20 = 200\mu C$

16. (c) $\vec{t}| = |\bar{M} \times \bar{B}|$

$\tau = NI \times A \times B \times \sin 45^\circ$

$\tau = 0.27 Nm$

17. (d) Kinetic energy $KE = \frac{1}{2} I \omega^2 = k\theta^2$

$\Rightarrow \omega^2 = \frac{2k\theta^2}{I} \Rightarrow \omega = \sqrt{\frac{2k}{I}}\theta$

Differentiate (A) wrt time →

$\frac{d\omega}{dt} = \alpha = \sqrt{\frac{2k}{I}}\left(\frac{d\theta}{dt}\right)$

$\Rightarrow \alpha = \sqrt{\frac{2k}{I}} \cdot \sqrt{\frac{2k}{I}}\theta\{\text{ by } (1)\}$

$\Rightarrow \alpha = \frac{2k}{I}\theta$

18. (c) For a simple pendulum $T = 2\pi\sqrt{\frac{L}{g_{err}}}$

Situation 1: when pendulum is in air $\rightarrow g_{eff} = g$

Situation 2:when pendulum is in liquid

$\rightarrow g_{\text{eff}} = g\left(1 - \frac{\rho_{\text{liquid}}}{\rho_{\text{body}}}\right) = g\left(1 - \frac{1}{16}\right) = \frac{15\text{ g}}{16}$

So, $\frac{T'}{T} = \frac{2\pi\sqrt{\frac{L}{15g/16}}}{2\pi\sqrt{\frac{L}{g}}}$

$\Rightarrow \mathrm{T}' = \frac{4\text{ T}}{\sqrt{15}}$

19. (d) $\mathrm{V_{rms}} = \sqrt{\frac{3RT}{\mathrm{M_w}}}$

$\Rightarrow \mathrm{v_{rms}} \propto \sqrt{\mathrm{T}}$

Now, $\frac{v}{200} = \sqrt{\frac{500}{400}} \Rightarrow \frac{v}{200} = \frac{\sqrt{5}}{2}$

$\Rightarrow v = 100\sqrt{5}\text{ m/s}$

20. (b) Maximum electric field $E = (B)(C)$

$\vec{\mathrm{E}}_0 = (3 \times 10^{-5})\mathrm{c}(-\hat{\jmath})$

$\vec{\mathrm{E}}_1 = (2 \times 10^{-6})\mathrm{c}(-\hat{\imath})$

Maximum force

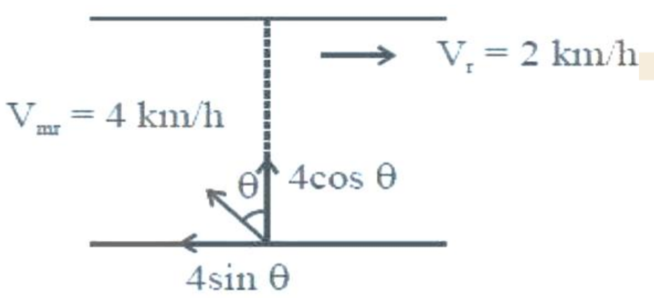
$\vec{\mathrm{F}}_{\text{net}} = 10^{-4} \times 3 \times 10^8\sqrt{(3 \times 10^{-5})^2 + (2 \times 10^{-6})^2} = 0.9\text{ N}$

$\mathrm{F_{rms}} = \frac{\mathrm{F_0}}{\sqrt{2}} = 0.6\text{ N (approx)}$

21. (c) For swimmer to cross the river straight

$\Rightarrow 4\sin\theta = 2$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$



So, angle with direction of river flow $= 90° + \theta = 120°$.

22. (d) $\mathrm{F_3}$ & $\mathrm{F_4}$ cancel each other.

Force on PQ will be $F_1 = 2_B 1a$

$= I_2 \frac{\mu_0 l_1}{2\pi a} a$

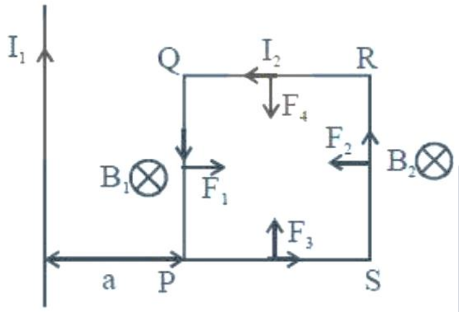
$= \frac{\mu_0 l_i}{2\pi a} a =$

$\frac{\mu_0 I_1 I_2}{2\pi}$

Force on RS will be $F_2 = I_2 B_2$ a

$= I_2 \frac{\mu_0 I_1}{2\pi 2a} a$

$= \frac{\mu_0 l_1 I_2}{4\pi}$



Net force $= F_1 - F_2 = \frac{\mu_0 l_1 l_2}{4\pi}$ repulsion

23. (c) $\frac{1}{660} = \mathrm{R}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5\mathrm{R}}{36}$

$\frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3\mathrm{R}}{16}$

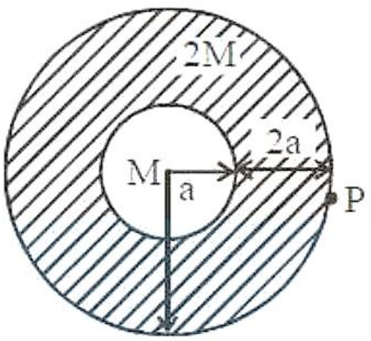
Divide equation (1) with (B)

$\frac{\lambda}{660} = \frac{5 \times 16}{36 \times 3}$

$\lambda = \frac{4400}{9} = 488.88 = 488.9$ nm

24. (d) We use Gauss's Law for gravitation g. $4\pi r^2$ = (Mass enclosed) $4\pi G$

$$g = \frac{3M4\pi G}{4\pi(3a)^2} = \frac{GM}{3a^2}$$



25. (a) 4^{th} harmonic

$4\frac{\lambda}{2}=\ell; 2\lambda=\ell$

From equation $\frac{2\pi}{\lambda}=0.157$

$\lambda=40;\ \ell=2\lambda=80$ m

26. (c) Momentum $p=mv$

and for motion under gravity $h=\frac{u^2-v^2}{2g}$

$$h=\frac{u^2-p^2/m}{2\,g}$$

27. (a) Work done $=\Delta U$

$$=U_f-U_i$$
$$=\frac{q^2}{2C_r}-\frac{q^2}{2C_i}$$
$$=\frac{(5\times10^{-6})^2}{2}\left(\frac{1}{2\times10^{-6}}-\frac{1}{5\times10^{-6}}\right)$$
$$=\frac{15}{4}\times10^{-6}$$
$$=3.75\times10^{-6}\ \mathrm{J}$$

28. (b) By conservation of linear momentum:

$$2v_0=2\left(\frac{v_0}{4}\right)+mv\Rightarrow 2v_0=\frac{v_0}{2}+mv2$$
$$\Rightarrow\ \frac{3v_0}{2}=mv\ \ldots(1)$$

Since collision is elastic →

$V_{separation}=V_{approch}$

$$\Rightarrow\ v-\frac{v_0}{4}=v_0\Rightarrow m=\frac{6}{5}=1.2\ \mathrm{kg}$$

29. (d) Speed of wave from wave equation

$$v=-\frac{(\text{coeffecient of }t)}{(\text{coeffecient of }x)}$$

$$v=-\frac{1000}{(-3)}=\frac{1000}{3}$$

Since speed of wave $\propto\sqrt{T}$

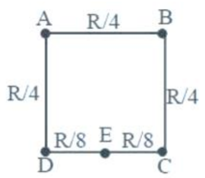
So $=\frac{1000}{3}=\sqrt{\frac{273}{T}}$

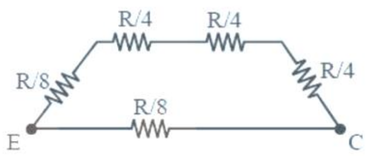
336

$\Rightarrow T = 277.41\ K$

$T = 4.41^{\circ}C$

30. (b)

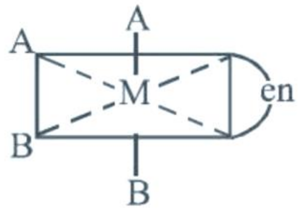




$$\frac{1}{R_{eq}} = \frac{8}{7R} + \frac{8}{R}$$

$$\frac{1}{R_{eq}} = \frac{8+56}{7R}; R_{eql} = \frac{7R}{64}$$

31. (c)
32. (a) $[Cu(H_2O)_4]SO_4 \cdot H_2O$
33. (a)



No plane of symmetry

34. (c) $T_C = \frac{8a}{27Rb}$
35. (b) C_2 configuration

$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \,{}_{\pi 2p_z}^{2}\,{}^{2}\sigma 2Px$

C_2^- configuration

$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \,\frac{\pi 2p_y{}^2}{\pi 2p_z}\, \sigma 2P_x^1$

36. (a) $P_M^{\circ} = 450$
$P_N^{\circ} = 700$

$P_M = x_M 450$

$P_N = x_N 700$

$Y_M P_T = P_M$

$Y_N P_T = P_N$

$$\frac{Y_M P_T}{Y_N P_T} = \frac{x_M}{x_N} \frac{450}{700}$$

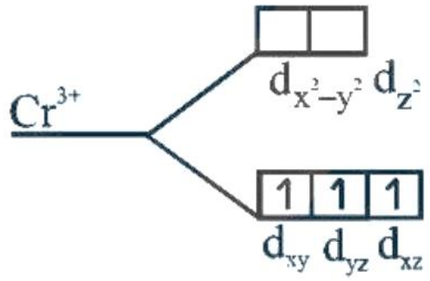
$\frac{Y_M}{Y_N} = \frac{x_M}{x_N}(0:64)$

$\frac{x_M}{x_N} > \frac{Y_M}{Y_N}$

37. (a) $\Delta U = q + w$

$\Delta G = \Delta H - T\Delta S$

38. (a) Degenerate orbitals of $[Cr(H_2O)_6]^{3+}$



39. (a)
40. (d) For first order reaction

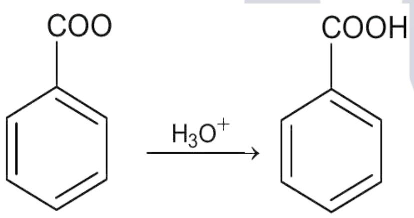
$\ell n[R]_t = -Kt + \ell n\,[R]_0$

For zero order reaction

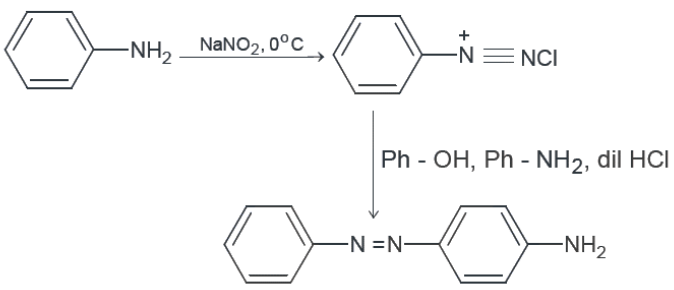
$[R]_t = -Kt + [R]_0$

Where ' R ' is reactant.

41. (c)



42. (b)
43. (c)



44. (c)
45. (d) Cryolite is $Na_3[AlF_6]$
46. (d)
47. (b) $LiAlH_4$ does not reduce double bond it reduces ester in alcohol.

48. (b) After losing one electron 'K' acquires noble gas configuration.

49. (c) $56\ \mathrm{g}$ of N_2 means 2 mole

2 mole of N_2 requires 6 mole of H_2 for complete reaction but available H_2 is $10\ \mathrm{g}$ i.e. 5 mole hence H_2 is limiting reagent.

50. (b)

51. (b) $\pi_{xy} = 4\pi_{BaCl_2}$

$\pi_{xy} = \mathrm{iCRT}$

$\pi_{xy} = 2\mathrm{CRT}$

$\pi_{BaCl_2} = 3 \times 0.01\mathrm{RT}$

$\mathrm{RT} = 12 \times 0.01\mathrm{RT}$

$\frac{12 \times 0.01}{2} = 0.06$

52. (c) $\Delta\overline{V}_{Lymann} = \frac{R_H}{4}$

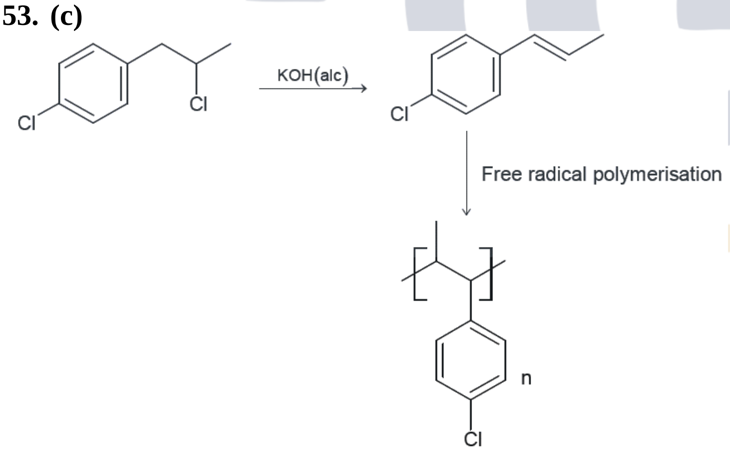
$\Delta\overline{V}_{Balmer} = \frac{R_H}{9}$

$\Rightarrow \frac{\Delta\overline{V}_{Lymann}}{\Delta\overline{V}_{Balmer}} = \frac{9}{4}$

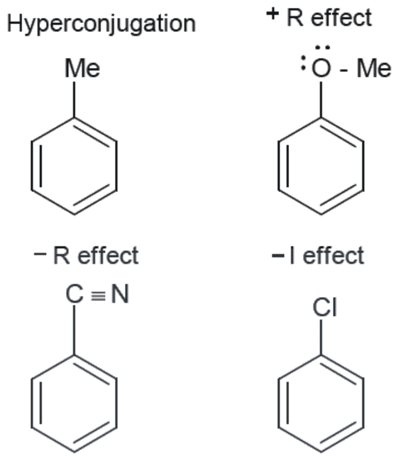
Formula

$\overline{V} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

53. (c)



54. (c)



55. (a)

56. (a) $\overset{+2}{NO}, \overset{+1}{NO}, \overset{+4}{NO_2}, \overset{+3}{NO_3}$

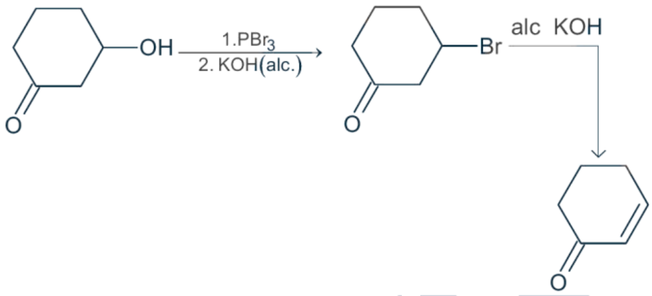
57. (b) $\Delta G = -2 \times 96000 \times 2$

$\Delta G = -nFE^\circ$

$\Delta G = -2 \times 96000 \times 2 \text{ J}$

$\Delta G = -384 \text{ kJ mol}^{-1}$

58. (b)



59. (a) C_1 of α-glucose and C_2 of β-fructose

60. (d)

61. (a) $\frac{x^2}{1-x^2}$

Range of y: $R - [-1,0)$ for surjective function, A must be same as above range.

62. (b) In given question $p, q \in R$. If we take other root as any real number α, then quadratic equation will be $x^2 - (\alpha + 2 - \sqrt{3})x + \alpha \cdot (2 - \sqrt{3}) = 0$

Now, we can have none or any of the options can be correct depending upon ' α '.

Instead of $p, q \in R$ it should be $p, q \in Q$ then other root will be $2 + \sqrt{3}$

$\Rightarrow p = -(2 + \sqrt{3} - 2 - \sqrt{3}) - -4$ and $q = (2 + \sqrt{3})(2 - \sqrt{3}) = 1$

$\Rightarrow p^2 - 4q - 12 = (-4)^2 - 4 - 12$

$= 16 - 16 = 0$

63. (a) $\vec{\alpha} = 3\hat{\imath} + \hat{\jmath}$

$\vec{\beta} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$

$\vec{\beta} = \overrightarrow{\beta_1} - \overrightarrow{\beta_2}$

$\overrightarrow{\beta_1} = \lambda(3\hat{\imath} + \hat{\jmath}), \overrightarrow{\beta_2} = \lambda(3\hat{\imath} + \hat{\jmath}) - 2\hat{\imath} + j - 3\hat{k}$

$\vec{\beta}_2 \cdot \vec{\alpha} = 0$

$(3\lambda - 2) \cdot 3 + (\lambda + 1) = 0$

$9\lambda - 6 + \lambda + 1 = 0$

$\lambda = \frac{1}{2}$

$\Rightarrow \vec{\beta}_1 = \frac{3}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$

$\Rightarrow \vec{\beta}_2 = -\frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} - 3\hat{k}$

Now, $\vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -3 \end{vmatrix}$

$= \hat{\imath}\left(-\frac{3}{2} - 0\right) - \hat{\jmath}\left(-\frac{9}{2} - 0\right) + \hat{k}\left(\frac{9}{4} + \frac{1}{4}\right)$

$= \frac{3}{2}\hat{\imath} + \frac{9}{2}\hat{\jmath} + \frac{5}{2}\hat{k}$

$= \frac{1}{2}(-3\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

64. (d)

$I = \int \frac{dx}{(\sin x)^{4/3} \cdot (\cos x)^{2/3}}$

$I = \int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3} \cdot \cos^2 x}$

$\Rightarrow I = \int \frac{\sec^2 x}{(\tan x)^{4/3}} dx$

put $\tan x = t \Rightarrow \sec^2 xdx = dt$

$\therefore I = \int \frac{dt}{t^{4/3}} \Rightarrow I = \frac{-3}{t^{1/3}} + C$

$\Rightarrow I = \frac{-3}{(\tan x)^{1/3}} + C$

65. (a)

Let $ax + by + cz = 1$ be the equation of the plane

$\Rightarrow 0 - b + 0 = 1$

$\Rightarrow \mathrm{b} = -1$

$0 + 0 + c = 1$

$\Rightarrow \mathrm{c} = 1$

$\cos\theta = \left|\frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}\right|$

$\frac{1}{\sqrt{2}} = \frac{|0-1-1|}{\sqrt{(a^2+1+1)}\sqrt{0+1+1}}$

$\Rightarrow a^2 + 2 = 4$

$\Rightarrow \mathrm{a} = \pm\sqrt{2}$

$\Rightarrow \pm\sqrt{2}\mathrm{x} - \mathrm{y} + \mathrm{z} = 1$

Now for - sign

$-\sqrt{2}, \sqrt{2} - 1 + 4 = 1$

66. (a) $\mathrm{y} = \mathrm{x}^3 + \mathrm{ax} - \mathrm{b}$

$(1, -5)$ lies on the curve

$\Rightarrow -5 = 1 + a - b \Rightarrow a - b = -6$

Also, $y' = 3x^2 + a$

$y'(1, -5) = 3 + a$ (slope of tangent)

$\because$ this tangent is $\perp$ to $-\mathrm{x} + \mathrm{y} + 4 = 0$

$\Rightarrow (3 + a)(1) = -1$

$\Rightarrow \mathrm{a} = -4$

By (i) and (ii): $a = -4, b = 2$

$\therefore \mathrm{y} = \mathrm{x}^3 - 4\mathrm{x} - 2, (2, -2)$ lies on this curve

67. (c) S.D. $= \sqrt{\frac{\sum(\mathrm{x}-\bar{\mathrm{x}})^2}{\mathrm{n}}}$

$\bar{x} = \frac{\sum x}{4} = \frac{-1+0+1+k}{4} = \frac{k}{4}$

Now $\sqrt{5} = \sqrt{\frac{\left(-1-\frac{k}{4}\right)^2 + \left(0-\frac{k}{4}\right)^2 + \left(1-\frac{k}{4}\right)^2 + \left(k-\frac{k}{4}\right)^2}{4}}$

$\Rightarrow 5 \times 4 = 2\left(1 + \frac{k}{16}\right)^2 + \frac{5k^2}{8}$

$\Rightarrow 18 = \frac{3k^2}{4}$

$\Rightarrow k^2 = 24$

$\Rightarrow k = 2\sqrt{6}$

68. (a) $f(x) = 15 - |x - 10|, x \in R$

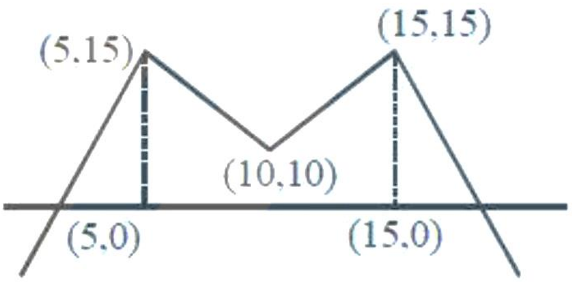
$f(f(x)) = 15 - |f(x) - 10|$

$= 15 - |15 - |x - 10| - 10|$

$= 15 - |5 - |x - 10||$

$x = 5,10,15$ are point of non

differentiability



69. (d) $T_4 = T_{3+1} = \binom{6}{3}\left(\frac{2}{x}\right)^3 \cdot \left(x^{\log_8 x}\right)^3$

$20 \times 8^7 = \frac{160}{x^3} \cdot x^{3\log_8 x}$

$8^6 = x^{\log_2 x} - 3$

$2^{18} = x^{\log_2 x - 3}$

$\Rightarrow 18 = (\log_2 x - 3)(\log_2 x)$

Let $\log_2 x = t$

$\Rightarrow t^2 - 3t - 18 = 0$

$\Rightarrow (t - 6)(t + 3) = 0$

$\Rightarrow t = 6, -3$

$\log_2 x = 6 \Rightarrow x = 2^6 = 8^2$

$\log_2 x = -3 \Rightarrow x = 2^{-3} = 8^{-1}$

70. (c) $f'(x) = \lambda(x + 1)(x - 0)(x - 1) = \lambda(x^3 - x)$

$\Rightarrow f(x) = \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + \mu$

Now $f(x) = f(0)$

$\Rightarrow \lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+\mu=\mu$

$\Rightarrow x=0,0,\pm\sqrt{2}$

Two irrational and one rational number

71. (c) $\sim(p \vee(\sim p \wedge q))$

$=\sim p \wedge \sim(\sim p \wedge q)$

$=\sim p \wedge(p \vee \sim q)$

$=(\sim p \wedge p) \vee(\sim p \wedge \sim q)$

$=c \vee(\sim p \wedge \sim q)$

$=(\sim p \wedge \sim q)$

72. (c) Since there are 8 males and 5 females. Out of these 13 , if we select 11 persons, then there will be at least 6 males and at least 3 females in the selection.

$m=n=\binom{13}{11}=\binom{13}{2}=\frac{13 \times 12}{2}=78$

73. (c) Any point on the given line can be

$(1+2\lambda,-1+3\lambda, 2+4\lambda) ; \lambda \in \mathrm{R}$

Put in plane

$1+2\lambda+(-2+6\lambda)+(6+12\lambda)=15$

$20\lambda+5=15$

$20\lambda=10$

$\lambda=\frac{1}{2}$

$\therefore$ Point $\left(2, \frac{1}{2}, 4\right)$

Distance from origin $=\sqrt{4+\frac{1}{4}+16}=\frac{\sqrt{16+1+64}}{2}=\frac{\sqrt{81}}{2}$

$=\frac{9}{2}$

74. (b) $\frac{1}{2}\left(2 \cos ^2 10^{\circ}-2 \cos 10^{\circ} \cos 50^{\circ}+2 \cos ^2 50^{\circ}\right)$

$\Rightarrow \frac{1}{2}\left(1+\cos 20^{\circ}-\left(\cos 60^{\circ}+\cos 40^{\circ}\right)+1+\cos 100^{\circ}\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2}+\cos 20^{\circ}+2 \sin 70^{\circ} \sin \left(-30^{\circ}\right)\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20° - \sin 70°\right)$

$\Rightarrow \frac{3}{4}$

75. (d)

$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\ldots\ldots.\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 1+2+3+\cdots\ldots+n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \frac{n(n-1)}{2} = 78 \Rightarrow n = 13, -12$ (reject)

$\therefore$ we have to find inverse of $\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}$

$\therefore \begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

76. (d) Let $\frac{\alpha+i}{\alpha-1} = z$

$\Rightarrow \left|\frac{\alpha+i}{\alpha-i}\right| = |z|$

$\Rightarrow 1 = |z|$

$\Rightarrow$ circle of radius 1

77. (a) $S_n = 50n + \frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$= 50n + \frac{n(n-7)}{2}A - 50(n-1) - \frac{(n-1)(n-8)}{2}A$

$= 50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8]$

$= 50 + A(n-4)$

$d = T_n - T_{n-1}$

$= 50 + A(n-4) - 50 - A(n-5)$

$= A$

$T_{50} = 50 + 46\text{ A}$

$(d, A_{50}) = (A, 50 + 46A)$

78. (c)

$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}dx$

$\Rightarrow I = \int_0^{\pi/4} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}dx$

$= \int_0^{\pi/4} (1 - \sin\ x \cos x) dx$

$= \left(x - \frac{\sin^2\ x}{2} \right)_0^{\pi/4}$

$= \frac{\pi}{4} - \frac{1}{4}$

$= \frac{\pi - 1}{4}$

79. (a) $\frac{x^2}{24} - \frac{y^2}{18} = 1 \Rightarrow a = \sqrt{24}; b = \sqrt{18}$

Parametric normal:

$\sqrt{24} \cos\theta \cdot x + \sqrt{18} \cdot y \cot\theta = 42$

At $x = 0; y = \frac{42}{\sqrt{18}} \tan\theta = 7\sqrt{3}$ (from given equation)

$\Rightarrow \tan\theta = \sqrt{\frac{3}{2}} \Rightarrow \sin\theta = \pm\sqrt{\frac{3}{5}}$

slope of parametric normal $= \frac{-\sqrt{24}\cos\theta}{\sqrt{18}\cot\theta} = m$

$\Rightarrow \mathrm{m} = -\sqrt{\frac{4}{3}} \sin\theta = -\frac{2}{\sqrt{5}}$ or $\frac{2}{\sqrt{5}}$

80. (b) $x\frac{dy}{dx} + 2y = x^2 : y(1) = 1$

$\frac{dy}{dx} + \left(\frac{2}{x}\right) y = x(\text{ LDE in } y)$

$\mathrm{IF} = \mathrm{e}^{\int \frac{2}{x} \mathrm{dx}} = \mathrm{e}^{2 \ln x} = \mathrm{x}^2$

$y \cdot (x^2) = \int x \cdot x^2 dx = \frac{x^4}{4} + C$

$y(1) = 1$

$1 = \frac{1}{4} + C \Rightarrow C = 1 - \frac{1}{4} = \frac{3}{4}$

$yx^2 = \frac{x^4}{4} + \frac{3}{4}$

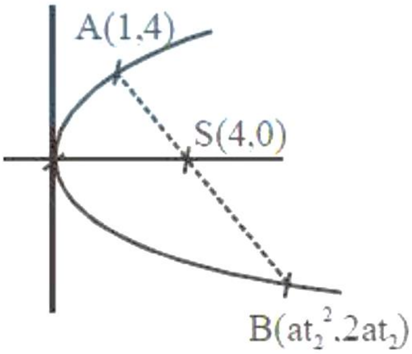
$y = \frac{x^2}{4} + \frac{3}{4x^2}$

81. (a) $y^2 = 4ax = 16x \Rightarrow a = 4$

$A(1,4) \Rightarrow 2 \cdot 4 \cdot t_1 = 4 \Rightarrow t_1 = \frac{1}{2}$

$\therefore$ length of focal chord $= a\left(t+\frac{1}{t}\right)^2$

$$= 4\left(\frac{1}{2}+2\right)^2 = 4\cdot\frac{25}{4} = 25$$



82. (d)

$f(1) = 1-1-2 = -2$

$f(-1) = -1-1+2 = 0$

$$m = \frac{f(1)-f(-1)}{1+1} = \frac{-2-0}{2} = -1$$

$$\frac{dy}{dx} = 3x^2-2x-2$$

$3x^2-2x-2 = -1$

$\Rightarrow 3x^2-2x-1 = 0$

$\Rightarrow (x-1)(3x+1) = 0$

$\Rightarrow x = 1, -\frac{1}{3}$

83. (d) $x = 2+r\cos\theta$

$y = 3+r\sin\theta$

$\Rightarrow 2+r\cos\theta+3+r\sin\theta = 7$

$\Rightarrow r(\cos\theta+\sin\theta) = 2$

$\Rightarrow \sin\theta+\cos\theta = \frac{2}{r} = \frac{2}{\pm 4} = \pm\frac{1}{2}$

$\Rightarrow 1+\sin 2\theta = \frac{1}{4}$

$\Rightarrow \sin 2\theta = -\frac{3}{4}$

$\Rightarrow \frac{2m}{1+m^2} = -\frac{3}{4}$

$\Rightarrow 3\,m^2 + 8\,m + 3 = 0$

$\Rightarrow m = \dfrac{-4 \pm \sqrt{7}}{1-7}$

$\dfrac{1-\sqrt{7}}{1+\sqrt{7}} = \dfrac{(1-\sqrt{7})^2}{1-7} = \dfrac{8-2\sqrt{7}}{-6} = \dfrac{-4+\sqrt{7}}{3}$

84. (d) From the given functional equation:

$$f(x) = 2^x \forall x \in N$$
$$2^{a+1} + 2^{a+2} + \cdots . + 2^{a+10} = 16(2^{10} - 1)$$
$$2^a(2 + 2^2 + \cdots .. + 2^{10}) = 16(2^{10} - 1)$$
$$2^a \cdot \frac{2\cdot(2^{10}-1)}{1} = 16(2^{10}-1)$$
$$2^{a+1} = 16 = 2^4$$
$$a = 3$$

85. (b) $2(1-\sin^2\theta) + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}; \sin\theta = 2$ (reject)

roots : $\pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, -\frac{\pi}{6}, -\pi + \frac{\pi}{6}$

$\Rightarrow$ sum of values $= 2\pi$

86. (c) Let the mid -point be $S(h,k)$

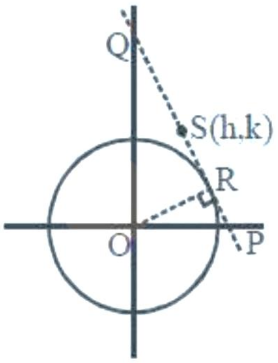
$\therefore$ $P(2\,h, 0)$ and $Q(0, 2k)$ equation of

$PQ: \dfrac{x}{2h} + \dfrac{y}{2k} = 1$

$\because$ PQ is tangent to circle at R (say)

$\therefore OR = 1 \Rightarrow \left|\dfrac{-1}{\sqrt{\left(\frac{1}{2\,h}\right)^2 + \left(\frac{1}{2k}\right)^2}}\right| = 1$

$\Rightarrow \dfrac{1}{4\,h^2} + \dfrac{1}{4k^2} = 1$



$\Rightarrow x^2 + y^2 - 4x^2y^2 = 0$

87. (a) Let persons be A, B, C, D

P (Hit) $= 1 - P$ (none of them hits)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$= 1 - P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C}) \cdot P(\bar{D})$

$= 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{7}{8}$

$= \frac{25}{32}$

88. (c)

$x^2 \leq y \leq x + 2$

$x^2 = y; y = x + 2$

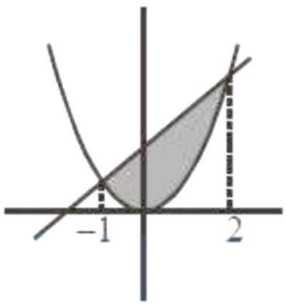
$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 2, -1$

Area $= \int_{-1}^{2} (x + 2) - x^2 dx = \frac{9}{2}$



89. (c) Roots of the equation $x^2 + x + 1 = 0$ are $\alpha = \omega$ and $\beta = \omega^2$ where ω, ω^2 are complex cube roots of unity

$\therefore \Delta = \begin{vmatrix} y + 1 & \omega & \omega^2 \\ \omega & y + \omega^2 & 1 \\ \omega^2 & 1 & y + \omega \end{vmatrix}$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \Delta = y\begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

Expanding along R_1, we get

$\Delta = y \cdot y^2 \Rightarrow D = y^3$

Or

If $\alpha = \omega^2, \beta = \omega$ we get same value or on expansion using $\alpha + \beta = -1, \alpha\beta = 1$ we get value y^3.

90. (c) function should be continuous at $x = \frac{\pi}{4}$

$\therefore \lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$$

$\Rightarrow \lim_{x\to\frac{\pi}{4}} \frac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x} = k$ (Using L Hospital Rule)

$\lim_{x\to\pi} \sqrt{2}\sin^3 x = k$

$$\Rightarrow k = \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2}$$