1. (d)

If $u = -10$ cm

$v = +10$ cm

$\Rightarrow f = 5$ cm

Glass plate shift $= t\left(1-\frac{1}{\mu}\right) = 1.5\left(1-\frac{2}{3}\right) = 0.5$ cm

So, new $u = 10 - 0.5 = 9.5$ cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{-9.5}=\frac{1}{5}$$

After solving we get,

$v = \frac{47.5}{4.5}$. Hence, shift $\frac{47.5}{4.5} - 10 = \left(\frac{2.5}{4.5}\right) = 0.55$ cm

2. (b)

3. (a) $i = neAV_d$

$1.5 = 9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times Vd$

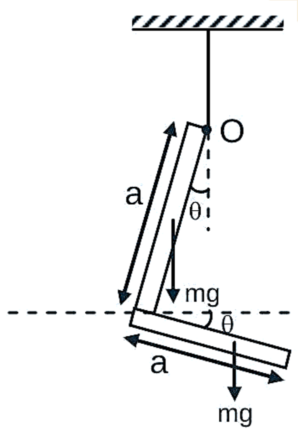
$Vd = 0.02$

4. (d) Lets considered mass of each rod is m for stable equilibrium the torque about point O should be zero. Torque balance about O

$$mg\frac{a}{2}\sin\ \theta = mg\left(\frac{a}{2}\cos\ \theta - a\sin\ \theta\right)$$

$$\tan\ \theta = \frac{1}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{3}\right)$$



5. (c) $\frac{dx}{dt} = y; \frac{dy}{dt} = x$

$$\frac{dx}{dy}=\frac{y}{x}$$

$$\Rightarrow y^2=x^2+c$$

6. (a)

$$\frac{(V_{RMS})_{He}}{(V_{RMS})_{Ar}}=\sqrt{\frac{M_{Ar}}{M_{He}}}$$

$$=\sqrt{\frac{40}{4}}=\sqrt{10}=3.16$$

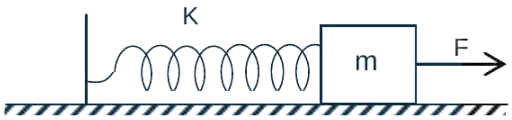
7. (d) Magnetic field at centre of an area subtending angle θ at the centre $\frac{\mu_0 I}{4\pi r}\theta$.

$$B=\left(\frac{\mu_0}{4\pi}\right)\times 10\left(\frac{1}{3\times10^{-2}}-\frac{1}{5\times10^{-2}}\right)\frac{\pi}{4}$$

$$=B=\frac{\pi}{30}\times10^{-4}=\frac{\pi}{3}\times10^{-5}\approx10^{-5}$$

8. (d) When $V_{max}\Rightarrow$ acceleration $=0$

$$\Rightarrow x=\frac{F}{K}$$



Apply work energy theorem

$$W_{sp}+W_F=\Delta K.E.$$

$$-\frac{1}{2}Kx^2+F.x=\Delta K.E.\ ;\ -\frac{1}{2}K\frac{F^2}{K^2}+\frac{F^2}{K}=\frac{1}{2}mu_{max}^2$$

$$\frac{F^2}{2K}=\frac{1}{2}mu_{max}^2\ ;\ \frac{F}{\sqrt{mK}}=V_{max}$$

9. (b) Electric field

$$E=\frac{kQx}{(x^2+R^2)^{3/2}}$$

For maxima $\frac{dE}{dx}=0$

After solving we get, $\left(x\pm\frac{R}{\sqrt{2}}\right)$

10. (b) $\left(\frac{\sqrt{l_1}+\sqrt{l_2}}{\sqrt{l_1}-\sqrt{l_2}}\right)=\frac{16}{1};\frac{\sqrt{l_1}+\sqrt{l_2}}{\sqrt{l_1}-\sqrt{l_2}}=\frac{4}{1}$

$$\Rightarrow 3\sqrt{l_1}=5\sqrt{l_2}$$

$$\Rightarrow \frac{l_1}{l_2}=\frac{25}{9}$$

11. (a) $\frac{1240}{350} - \phi = (KE)_1 = 4x$

$\frac{1240}{540} - \phi = (KE)_{\parallel} = x$

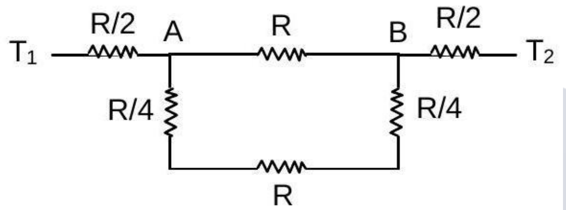
(1) - (2)

$\frac{1240}{350} - \frac{1240}{540} = 3.542 - 2.296 = 3x$

$1.246 = 3x\,;\ x = 0.41$

$\phi = 2.296 - 0.41 = 1.886$

12. (a) $T_A - T_B = \frac{T_1 - T_2}{\frac{8R}{5}} \times \frac{3R}{5} = \frac{3}{8} \times 120 = 45$



13. (a) As temperature at point A and C is same.

∴ Internal energy change will be same.

$$Q - W = Q' - W'$$
$$60 - 30 = Q' - 10$$
$$Q' = 40J$$

14. (b)

$\sin 90^\circ = \mu \sin\theta$

$\Rightarrow \sin\theta = \frac{1}{\mu}$

$\mu \sin\theta = 1.5 \sin r$

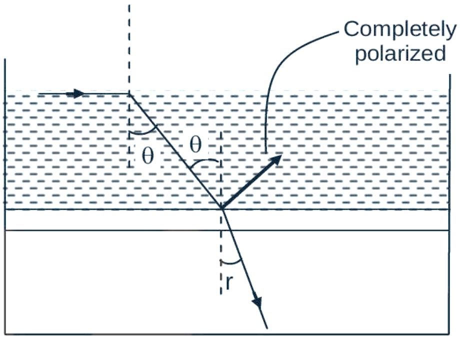
$\mu \tan\theta = 1.5$

$\Rightarrow \tan\theta = \frac{1.5}{\mu}$

$\sin\theta = \frac{3}{\sqrt{9 + 4\mu^2}} = \frac{1}{\mu}$

$9\mu^2 = 9 + 4\mu^2$

$\Rightarrow \mu = \frac{3}{\sqrt{5}}$



15. (c)

16. (b)

17. (a)

18. (c) $R = \frac{\rho\ell}{A}$

$R = \frac{\rho\ell^2}{A\ell} = \frac{\rho\ell^2}{V}$

$\frac{dR}{R} = \frac{2\,d\ell}{\ell}$

Hence, $\frac{dR}{R} = 1\%$.

19. (c)

20. (b) $v = \sqrt{T/\mu} = \sqrt{M\frac{g}{\mu}}$

$\frac{\sqrt{g^2+a^2}}{g} = \left(\frac{60.5}{60}\right)^2$

$1 + \frac{1}{2}\frac{a^2}{g^2} = 1 + \frac{1}{60}$ using by binomial approximation.

$\Rightarrow a = \frac{g}{\sqrt{30}}$

21. (a)

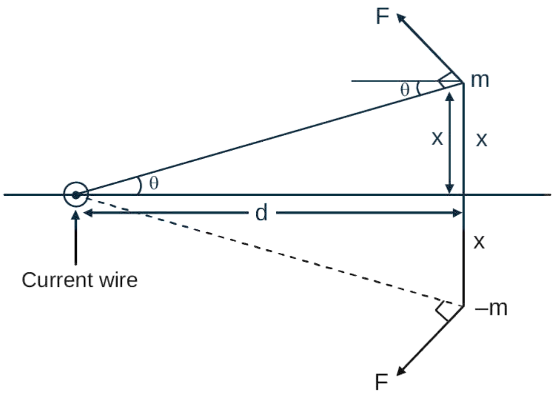
22. (d) Force on one pole

$F = \frac{m\mu_o l}{2\pi\sqrt{d^2+x^2}}$ $m =$ pole

Total force $= 2\,F\sin\theta$

$= \frac{2\times\mu_0 \mathrm{Im}\times x}{2\pi\sqrt{d^2+a^2}\sqrt{d^2+a^2}} = \frac{\mu_0 Imx}{\pi[d^2+a^2]}$

$= m2x = M = I\pi a^2$



Total force $= \frac{\mu_0 I a^2}{2(d^2+a^2)}$

$\approx \frac{\mu_0 I a^2}{2\, d^2}$ [$\because$ d >> a]

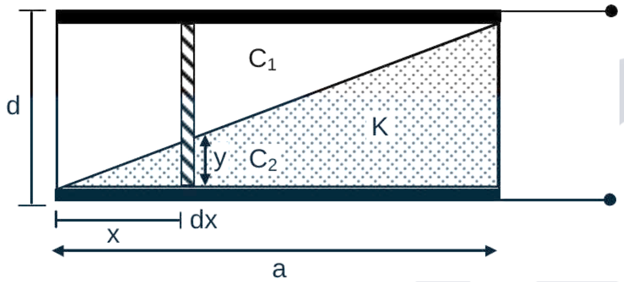
23. (a) For equilibrium of the block net force should be zero. Hence we can write. $\mathrm{mgsin}\ \theta + 3 = \mathrm{P} +$ friction

$\mathrm{mgsin}\ \theta + 3 = \mathrm{P} + \mu\mathrm{mgcos}\ \theta$. After solving, we get, $P = 32\ \mathrm{N}$.

24. (d) Let's consider a strip of thickness dx at a distance of x from the left end as shown in the figure.

$$\frac{y}{x} = \frac{d}{a}$$

$$\Rightarrow y = \left(\frac{d}{a}\right)x$$



$$C_1 = \frac{\varepsilon_0 a dx}{(d-y)}\ ;\ C_2 = \frac{k\varepsilon_0 a dx}{y}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{k\varepsilon_0 a dx}{kd + (1-k)y}$$

Now integrating it from 0 to a

$$\int_0^a \frac{k\varepsilon_0 a dx}{kd + (1-k)\frac{d}{a}x} = \frac{k\varepsilon_0 a^2 \ell nk}{d(k-1)}$$

25. (a) $\Delta L = L \propto \Delta T$

Strain $= \frac{\Delta L}{L} = \alpha\Delta T\ ;\ Y = \frac{F}{A\alpha\Delta T}$

26. (b) $B = \mu_0 H; \mu_0 ni = \mu_0 H$

$$\frac{100}{0.2} \times 5.2 = H$$

H = 2600 A/m

27. (c) Apply LMC (Linear Momentum Conservation)

$mv = (2m + M)v'$

$$v' = \frac{mV}{2m + M}$$

Initial energy

$$\frac{1}{2}mv^2$$

Final energy

$$\frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2$$

Initial kinetic energy - Final kinetic energy $= \frac{5}{6}$ of initial kinetic energy.

After solving, we get, $\frac{M}{m} = 4$.

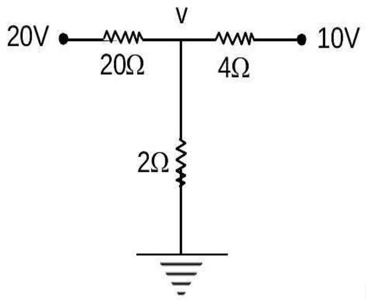
28. (b) $i_3 + i_2 = i_1$

$$\frac{20 - v}{2} + \frac{10 - v}{4} = \frac{v}{2}$$

$v = 10v$

$$\Rightarrow\ i_1 = \frac{10}{2}$$

$= 5$amp.



29. (c) Based on Kepler's law.

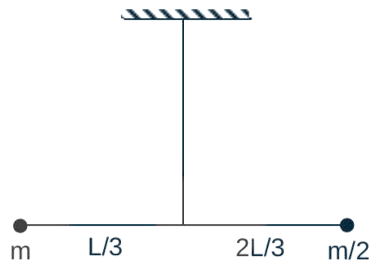
$$\frac{\text{dA}}{\text{dt}} = \frac{\text{L}}{2\text{ m}}$$

30. (c) $\Omega = \sqrt{\frac{k}{I}};\ \omega = \theta_0 \times \Omega$

$$\text{T} = \text{m}\omega^2\frac{\ell}{3}$$

$\text{T} = \text{m}\omega^2\frac{\ell}{3}\theta_0\frac{\text{k}}{\text{I}}$ where $\text{I} = \text{m}\frac{\ell^2}{3}$

$$= \frac{\theta_0^2\text{k}}{\ell}$$

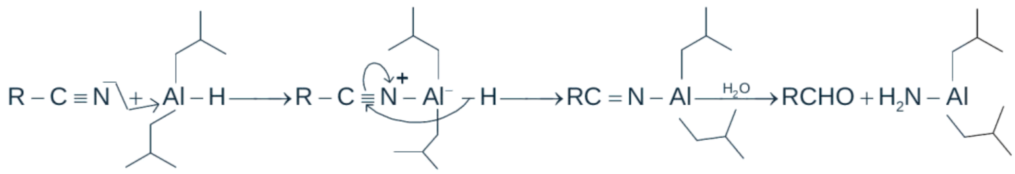


31. (a) Δ_0 is calculated from the energies of absorbed radiation not from emitted radiation (complementary colour).

32. (d) Acid strength in this case varies directly with the electron withdrawing power of the groups attached to the α-carbon of CH_3COOH. The order of electron withdrawing tendency is

$NO_2 > CN > F > Cl$

33. (c)



34. (a) Maximum number of unpaired electron of metal or metal ion in complexes $= n = 5$

$\therefore \mu_s = \sqrt{n(n+2)} = \sqrt{35} = 5.916 \approx 5.92$

35. (d) PV = nRT

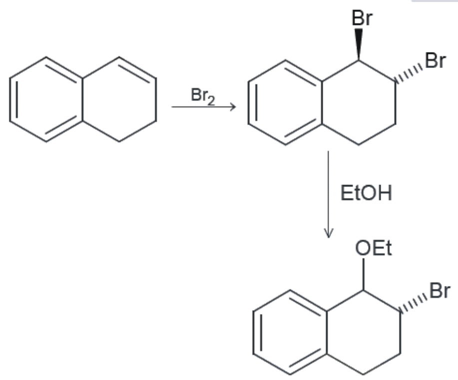
$200 \times 10 = (0.5 + x)R \times 1000$

On solving $x = \frac{4-R}{2R}$

36. (c) Materials those produce electric current when they are put under mechanical stress are called piezoelectric materials.

37. (a)

38. (a)

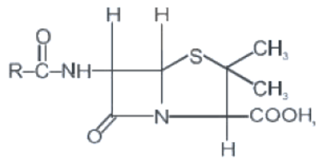


39. (a)

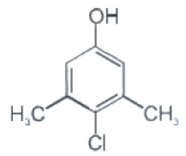
40. (c) Molality of $Na^+ = \left(\frac{W}{M} \times \frac{1000}{W}\right) \times 2(Na_2SO_4$ contains two Na^+ions)

$$= \left[\left(\frac{92}{23} \times \frac{1000}{1000}\right)\right] \times 2 = 8$$

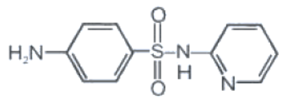
41. (c)



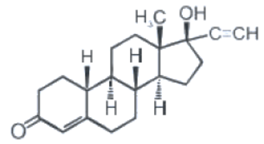
Penicillin



Chloroxylenol



Sulphapyridine



Norethindrone

42. (b) The permissible level in ppm unit is

$Fe = 0.2$

$Mn = 0.05$

$Cu = 3$

$Zn = 5$

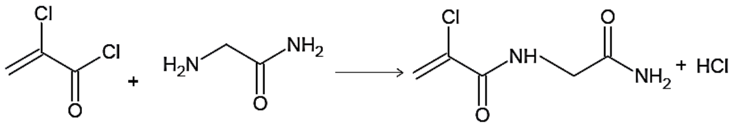
Mn is higher

43. (c) $Pb(s) + SO_4^{2-} \longrightarrow PbSO_4 + 2e^-$

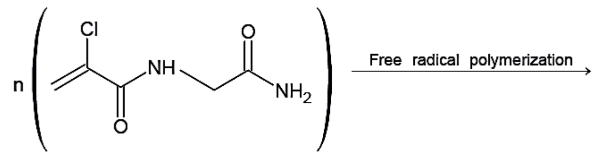
For 2 F current passed, $PbSO_4$ deposited $= 303$ g/mol

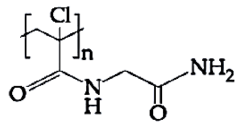
For 0.05 F: $PbSO_4$ deposited $= \frac{0.05\times303}{2} = 7.6$ g

44. (a)
45. (a)
46. (d)



$Et_3N + HCl \longrightarrow Et_3\overset{+}{N}H\overset{-}{Cl}$





47. (d) Due to larger size of Ba^{2+} ion, $Ba(NO_3)_2$ can-not hold water molecules during crystallization.
48. (b)
49. (c) $\frac{x}{m} = KP^{1/n}$

or $\log \frac{x}{m} = \log K + \frac{1}{n}\log P$

Slope $= \frac{1}{n} = \frac{2}{4} = \frac{1}{2}$

$\therefore \frac{x}{m} \alpha P^{1/2}$

50. (c) $CH(CN)_3 \rightleftharpoons \bar{C}(CN)_3 + H^+$

Negative charge of the conjugate base $\bar{C}(CN)_3$ in extensively delocalized through the $C \equiv N$ group.

51. (a) Copper pyrites is $CuFeS_2$

Malachite: $CuCO_3 . Cu(OH)_2$

Azurite : $2Cu(CO)_3 \cdot Cu(OH)_2$

Dolomite: $CaCO_3 \cdot MgCO_3$

52. (d) For emission line

$n_f < n_i$

$\therefore \bar{\nu} = RZ^2\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right] = R\left[\frac{1}{8^2} - \frac{1}{n^2}\right]$

or, $\bar{\nu} = R_H\left(\frac{1}{64} - \frac{1}{n^2}\right)$

$= \frac{R_H}{64} - \frac{R_H}{n^2}$

$\bar{\nu} = -R_H\left(\frac{1}{n^2}\right) + \frac{R_H}{64}$

$\therefore y = mx + c$

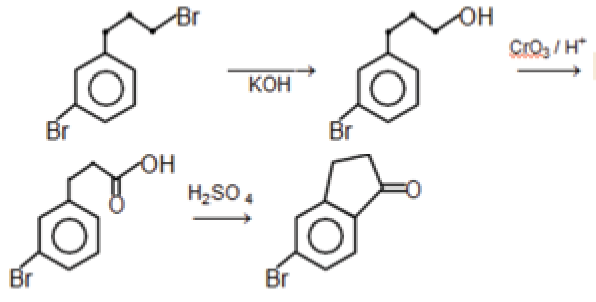
Slope $= -R_H$

53. (c) The isotopes are:

${}_1^1H$, ${}_1^2H$ and ${}_1^3H \equiv P, D, T$

54. (c) Both Li_2^+ and Li_2^- have same bond order. But the number of antibonding electrons is less in Li_2^+ than in Li_2^-

55. (a)



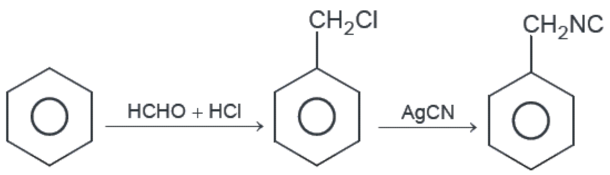
56. (a)

57. (c)

58. (b)

59. (b)

60. (d)



61. (b) $\int_0^{\frac{\pi}{2}}(|\cos x|^3 + |\cos(\pi - x)|^3)dx$

$\Rightarrow 2\int_0^{\frac{\pi}{2}}|\cos x|^3 \mathrm{dx}$

$\Rightarrow 2\int_0^{\frac{\pi}{2}}(\cos x)^3 \mathrm{dx}$

$\Rightarrow 2\left(\frac{2}{3}\right) = \frac{4}{3}$ (By wallis formula)

62. (d) $\ell = 3$

$\mathrm{r}^2 + \mathrm{h}^2 = 9$

Volume of cone is $= \frac{1}{3}\pi r^2 h$

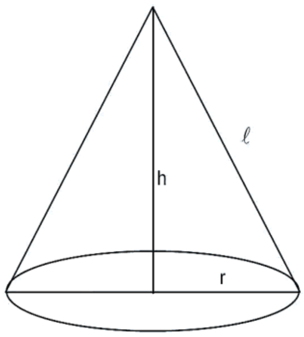
$\mathrm{V} = \frac{1}{3}\pi\mathrm{h}(9 - \mathrm{h}^2)$

$\frac{\mathrm{dv}}{\mathrm{dh}} = \frac{1}{3}\pi(9 - 3\ \mathrm{h}^2) = 0$

$9 - 3h^2 = 0$

$h^2 = 3, h = \sqrt{3}$

$V = \frac{1}{3}(\pi)(6)\sqrt{3} = 2\sqrt{3}\pi$



63. (d) $\int x\sqrt{\frac{2\sin(x^2-1)(1-\cos(x^2-1))}{2\sin(x^2-1)(1+\cos(x^2-1))}}$

$= \int x\frac{\sin\left(\frac{x^2-1}{2}\right)}{\cos\left(\frac{x^2-1}{2}\right)}dx$

$= \int x\tan\left(\frac{x^2-1}{2}\right)dx$

Let $\frac{x^2-1}{2} = t \Rightarrow 2xdx = 2dt$

$= \int \tan(t)dt = \ln|\sec t| + c$

$$= \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$$

64. (c) $x\frac{dy}{dx} + 2y = x^2$

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

This is linear differential equation in $\frac{dy}{dx}$

Integrating factor $= e^{\int \frac{2}{x}dx} = x^2$

Solution of differential equation is $yx^2 = \int x^3 dx$

$$yx^2 = \frac{x^4}{4} + c$$

Curve passes through (1,1)

then $c = \frac{3}{4}$

$$yx^2 = \frac{x^4+3}{4}$$

Put $x = \frac{1}{2}$

$$y\left(\frac{1}{4}\right) = \frac{\left(\frac{1}{2}\right)^4 + 3}{4}$$

$$y = \frac{49}{16}$$

65. (b) Vertex is (2,0)

$a = 2$

Any general point on given parabola can be taken as $(2 + 2t^2, 4t)\forall t \in R$.

(8,6) does not lie on this.

66. (a) $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$

$\because$ e > 2 (given)

$$e^2 > 4 \Rightarrow 1 + \frac{\sin^2\theta}{\cos^2\theta} > 4$$

$\Rightarrow 1 + \tan^2\theta > 4$

$\Rightarrow \tan^2\theta > 3$

$\because \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Latus rectum $= 2\frac{\sin^2\theta}{\cos\theta} = 2\tan\theta\sin\theta$

$\because$ for $\theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$, $2\tan\theta\sin\theta$ is increasing function

Hence latus rectum $\in (3, \infty)$

67. (a) $x \in R - \{0,1\}$

$f_1(x) = \frac{1}{x}, f_2(x) = 1 - x, f_3(x) = \frac{1}{1-x}$

Given $\mathrm{f_2(\,J(f_1(x)))} = \mathrm{f_3(x)}$

$1 - \mathrm{J(f_1(x))} = \mathrm{f_3(x)}$

$J(f_1(x)) = 1 - f_3(x) = 1 - \frac{1}{1-x}$

$\mathrm{J(f_1(x))} = \frac{\mathrm{x}}{\mathrm{x}-1}$

$J\left(\frac{1}{x}\right) = \frac{x}{x-1} = \frac{1}{1-\frac{1}{x}}$

$J(x) = \frac{1}{1-x} = f_3(x)$

68. (a) $a = \hat{\imath} - \hat{\jmath}, b = \hat{\imath} + \hat{\jmath} + \hat{k}, c = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\vec{a} \times \vec{c} + \vec{b} = 0$

$$\Rightarrow \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 0 \\ x & y & z \end{vmatrix} + (\hat{\imath} + \hat{\jmath} + \hat{k}) = 0$$

$\hat{\imath}(-z) - \hat{\jmath}(z) + \hat{k}(y + x)$

$\Rightarrow 1 - z = 0 \Rightarrow z = 1,$

Also $x + y = -1$, and $\vec{a} \cdot \vec{c} = 4 \Rightarrow x - y = 4$

$\Rightarrow \mathrm{x} = \frac{3}{2}, \mathrm{y} = \frac{5}{2}$

$\therefore |\vec{c}|^2 = x^2 + y^2 + z^2 = \frac{9}{4} + \frac{25}{4} + 1 = \frac{38}{4} = \frac{19}{2}$

69. (d) $a + ar + ar^2 = xar$

since $a \neq 0$ so $\frac{r^2+r+1}{r} = x;\ 1 + r + \frac{1}{r} = x$

$\because r + \frac{1}{r} \in (-\infty, -2] \cup [2, \infty) \Rightarrow x \in (-\infty, -1] \cup [3, \infty)$

70. (a) $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$

$$\cos^{-1}\left(\frac{2}{3x} \times \frac{3}{4x} - \sqrt{1 - \frac{4}{9x^2}}\sqrt{1 - \frac{9}{16x^2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{1}{2x^2} = \frac{\sqrt{9x^2 - 4}\sqrt{16x^2 - 9}}{12x^2}$$

$$\Rightarrow 6 = \sqrt{9x^2 - 4}\sqrt{16x^2 - 9}$$

Square both side

$$36 = 144x^4 - 81x^2 - 64x^2 + 36$$

$$\Rightarrow 144x^4 = 145x^2$$

$$\Rightarrow x^4 = \frac{145x^2}{144} \Rightarrow x = \pm\frac{\sqrt{145}}{12}, 0$$

$\therefore x > \frac{3}{4}$ hence $x = \frac{\sqrt{145}}{12}$

71. (b) $ty = x + t^2$

$$\left|\frac{3 + t^2}{\sqrt{1 + t^2}}\right| = 3$$

$$\Rightarrow t = \sqrt{3}$$

$$\Rightarrow \sqrt{3}y = x + 3$$

72. (d) By applying Crammer's Rule

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix}$$

$$= 3(a^2 - 1) - 6 - 2(a^2 - 1) + 4$$

$$= a^2 - 1 - 2 = a^2 - 3$$

If $|a| \neq \pm\sqrt{3} \Rightarrow$ system has unique solution

If $|a| = \sqrt{3}$

$$x + y + z = 1$$

$$2x + 3y + 2z = 1$$

$$2x + 3y + 2z = \pm\sqrt{3} + 1$$

Hence system is inconsistent for $|a| = \sqrt{3}$

73. (b) $\frac{2^{403}}{15}=\frac{2^3\cdot 2^{400}}{15}=\frac{8\cdot(1+15)^{100}}{15}$

$$=\frac{8\left({}^{100}C_0+{}^{100}C_1(15)+{}^{100}C_2(15)^2+\cdots\ldots..\right)}{15}$$

$$\frac{8}{15}+8\left({}^{100}C_1(15)+{}^{100}C_2(15)^2+\cdots\ldots.\right)$$

Remainder is 8.

74. (c)

Let the line L be $\frac{x+4}{a}=\frac{y-3}{b}=\frac{z-1}{c}$

L ∥ x + 2y − z − 5 = 0

L intersects $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$

$$\Rightarrow\begin{vmatrix}3 & 0 & 1\\ a & b & c\\ -3 & 2 & -1\end{vmatrix}=0$$

$$\Rightarrow 2a+0b-6c=0$$

Also $a+2b-c=0$

$$\therefore \frac{a}{3}=\frac{b}{-1}=\frac{c}{1}$$

$$\therefore L \text{ is } \frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$$

75. (a) $px+qy+r=0$

$$px+qy+\left(\frac{-3p-2q}{4}\right)=0$$

$$p\left(x-\frac{3}{4}\right)+q\left(y-\frac{2}{4}\right)=0$$

$x=\frac{3}{4}$ and $y=\frac{1}{2}$.

76. (a) $(1+x)^n\cong 1+nx($ when $x\to 0)$

So, $\sqrt{1+y^4}=1+\frac{y^4}{2}$

$$\lim_{y\to 0}\frac{\sqrt{2+\frac{y^4}{2}}-\sqrt{2}}{y^4}$$

$$=\frac{\sqrt{2}\left(1+\frac{y^4}{8}-1\right)}{y^4}=\frac{\sqrt{2}}{8}=\frac{1}{4\sqrt{2}}$$

77. (c) Equation of required plane is

$$(x+y+z-1)+\lambda(2x+3y-z+4)=0$$

$$\Rightarrow(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)=0$$

since given plane is parallel to $y-$ axis $\Rightarrow 3\lambda+1=0 \Rightarrow = -\frac{1}{3}$

Hence equation of plane is $x+4z-7=0$

78. (b) $y=x^2+2$ and $y=10-x^2$

Meet at $(\pm 2,6)$

$\Rightarrow m_1 = 4$ and $m_2 = -4$

$|\tan\theta| = \frac{8}{15}$

79. (c) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

By using symmetry

$A^{-50} = \begin{bmatrix} \cos(-50\theta) & -\sin(-50\theta) \\ \sin(-50\theta) & \cos(-50\theta) \end{bmatrix}$

At $\theta = \frac{\pi}{12}$

$$A^{-50} = \begin{bmatrix} \cos\frac{25\pi}{6} & \sin\frac{25\pi}{6} \\ -\sin\frac{25\pi}{6} & \cos\frac{25\pi}{6} \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{6} & \sin\frac{\pi}{6} \\ -\sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

80. (a) Check all option repeatedly

(i) $(A \wedge B) \wedge (\sim A \vee B) \equiv A \wedge (B \wedge (\sim A \vee B))$

$\equiv A \wedge (B) \equiv A \wedge B$

$\Rightarrow$ (i) is correct

(ii) $(A \wedge B) \wedge (\sim A \wedge B) \equiv (A \wedge \sim A) \wedge B$

$\equiv f \wedge B \equiv f$

(iii) $(A \vee B) \wedge (\sim A \vee B) \equiv B$

(iv) $(A \vee B)(\sim A \vee B)$

$\equiv B \vee (A \wedge \sim A) = B \vee f \equiv f$

81. (c) Let 5 students are x_1, x_2, x_3, x_4, x_5

Given $\bar{x} = \frac{\sum x_i}{5} = 150 \Rightarrow \sum_{i=1}^{5} = 750$

$\frac{\sum x_i^2}{5} - (\bar{x})^2 = 18 \Rightarrow \frac{\sum x_i^2}{5} - (150)^2 = 18$

$\Rightarrow \sum x_i^2 = (22500 + 18) \times 5 \Rightarrow \sum_{i=1}^{5} x_i^2 = 112590$

Height of new student $= 156$ (Let x_6)

Then $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 750 + 156$

$$\bar{x}_{\text{new}} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \frac{906}{6} = 151$$

Variance (new) $= \frac{\sum x_i^2}{6} - (\bar{x}_{\text{new}})^2$

from equation (2) and (3)

variance (new) $= \frac{112590+(156)^2}{6} - (151)^2 = 22821 - 22801 = 20$

82. (b) $3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6 \theta$

$= 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6 \theta$

$= 9 + 3\sin^2 2\theta + 4\sin^6 \theta$

$= 9 + 12\sin^2 \theta\cos^2 \theta + 4(1 - \cos^2 \theta)^3$

$= 9 + 12(1 - \cos^2 \theta)\cos^2 \theta + 4(1 - 3\cos^2 \theta + 3\cos^4 \theta - \cos^6 \theta)$

$= 13 + 12\cos^2 \theta - 12\cos^4 \theta - 12\cos^2 \theta + 12\cos^4 \theta - 4\cos^6 \theta$

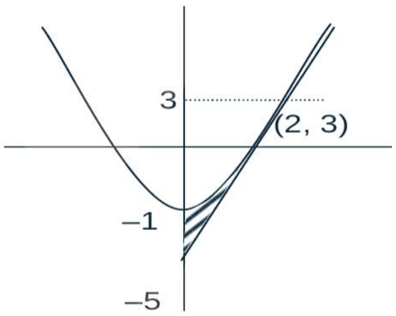
$= 13 - 4\cos^6 \theta$

83. (a) Area $= \int_{-5}^{3} x dy - \int_{-1}^{3} x dy$

$= \int_{-5}^{3} \left(\frac{y+5}{4}\right) - \int_{-1}^{3} \sqrt{y+1} dy$

$$= \left|\frac{\frac{y^2}{2} + 5y}{4}\right|_{-5}^{3} - \left|\frac{2}{3}(y+1)^{3/2}\right|_{-1}^{3}$$

$$= \left|\frac{\left(\frac{9}{2} + 15\right) - \left(\frac{25}{2} - 25\right)}{4}\right| = \left|\frac{16}{3}\right| = \frac{8}{3}$$



84. (a) $S = \sum_{i=1}^{30} a_i, T = \sum_{i=1}^{15} a_{2i-1}, a_5 = 27, S - 2T = 75$

Let $a_i = a + (i - 1)D$

$\mathrm{S}=\mathrm{a}_1+\mathrm{a}_2+\mathrm{a}_3++\mathrm{a}_{30}$

$\mathrm{T}=\mathrm{a}_1+\mathrm{a}_3+\mathrm{a}_5++a_{29}$

$\therefore 2\mathrm{~T}=2\mathrm{a}_1+2\mathrm{a}_3+2\mathrm{a}_5+\cdots\ldots\ldots\ldots\ldots\ldots+2\mathrm{a}_{29}$

$\mathrm{S}-2\mathrm{~T}=(\mathrm{a}_2-\mathrm{a}_1)+(\mathrm{a}_4-\mathrm{a}_3)+(\mathrm{a}_6-\mathrm{a}_5)++(\mathrm{a}_{30}-\mathrm{a}_{29})=75$

$=15\mathrm{D}$

But $\mathrm{S}-2\mathrm{~T}=75 \Rightarrow 15\mathrm{D}=75 \Rightarrow \mathrm{D}=5$

Now $\mathrm{a}_5=27 \Rightarrow \mathrm{a}+4\mathrm{D}=27$

$\therefore\ a=27-20 \Rightarrow a=7$

now $\mathrm{a}_{10}=\mathrm{a}+9\mathrm{D}$

$=7+45=52$

85. (b)

86. (d) $\mathrm{z}=\frac{3+2\mathrm{isin}\,\theta}{1-2\mathrm{isin}\,\theta}\times\frac{1+2\mathrm{isin}\,\theta}{1+2\mathrm{isin}\,\theta}$

$$z=\frac{(3-4\sin^2\theta)+8i\sin\theta}{1+4\sin^2\theta}$$

For purely imaginary real part should be zero.

i.e. $3-4\sin^2\theta=0$.

i.e. $\sin\theta=\pm\frac{\sqrt{3}}{2}$

$\theta=-\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}$, Sum of all values is $-\frac{\pi}{3}+\frac{\pi}{3}+\frac{2\pi}{3}=\frac{2\pi}{3}$

87. (c) Number of ways $=$ Total number of ways without restriction - When two specific boys are in team without any restriction, total number of ways of forming team is ${}^7C_3\times{}^5C_2=350$ If two specific boys B_1, B_2 are in same team then total number of ways of forming team equals to ${}^5C_1\times{}^5C_2=50$ ways total ways $=350-50=300$ ways

88. (a) $x^2+2x+2=0 \Rightarrow (x+1)^2=-1$

$x=-1\pm i=\sqrt{2}e^{i\left(\pm\frac{3\pi}{4}\right)}$

$\therefore \alpha^{15},\beta^{15}=(\sqrt{2})^{15}\times 2\cos\left(15.\frac{3\pi}{4}\right)$

$=2^8\sqrt{2}\times\left(-\frac{1}{\sqrt{2}}\right)=-256$

89. (a) Length of direct common tangent for circle C_1 and C_2 is

$AB=\sqrt{(a+b)^2-(a-b)^2}$

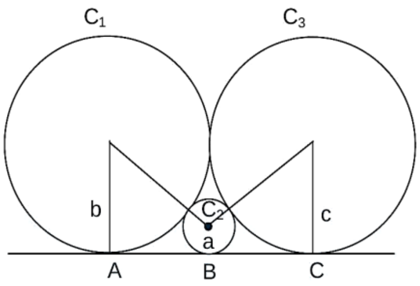
For C_2 and C_3

Length of direct common tangent is

$BC = \sqrt{(a+c)^2 - (a-c)^2}$

For C_1 and C_3

Length of direct common tangent is



$AC = \sqrt{(b+c)^2 - (b-c)^2}$

$AB + BC = AC$

$\sqrt{(a+b)^2 - (a-b)^2} + \sqrt{(a+c)^2 - (a-c)^2}$

$= \sqrt{(b+c)^2 - (b-c)^2}$

$\sqrt{\text{ab}} + \sqrt{\text{ac}} = \sqrt{\text{bc}}$

$\frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}}$

90. (d) $\text{P}(\text{x} = 1) = \frac{4}{52} \times \frac{48}{52} \times 2 = \frac{24}{169}$

$\text{P}(\text{x} = 2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

$\Rightarrow \text{P}(\text{x} = 1) + \text{P}(\text{x} + 2) = \frac{25}{169}$