1. (a) $\vec{F} = m\vec{a} = m[-a\omega_1^2\cos\omega, t\hat{i} - b\omega_2^2\sin\omega_2 t\hat{j}$

$\vec{t}_{t=0} = -ma\omega_1^2\hat{\imath}$

$\vec{r}_{t=0} = (x_0 + a)\hat{\imath} + y\hat{\jmath}$

$\vec{\tau} = \vec{r} \times \vec{F} = my_0 a\omega_1^2\hat{k}$

2. (d)

$$v = \frac{v + v_o}{v} v_0$$

$$\Rightarrow v_o = \left(\frac{v}{v_0} - 1\right) v$$

$$v_o = \left(\frac{530}{500} - 1\right) 300 = 18 \text{ m/s}$$

$$v_0 = \left|\left(\frac{480}{500} - 1\right) 300\right| = 12 \text{ m/s}$$

3. (b)

$$2MV, \cos 30^\circ + Mv_2 \cos 45^\circ = 10M\cos 30^\circ + 10\cos 45^\circ$$

$$\Rightarrow v_1\sqrt{3} + \frac{v_2}{\sqrt{2}} = 5\sqrt{3} + 5\sqrt{2}$$

$$2MV, \sin 30^\circ - MV_2 \sin 45^\circ = -10M\sin 30^\circ + 10M\sin 45^\circ$$

$$V_1 - \frac{V_2}{\sqrt{2}} = -5 + 5\sqrt{2}$$

$$V_1 = \frac{5(\sqrt{3} - 1) + 10\sqrt{2}}{\sqrt{3} + 1} = \frac{17.5}{2.7} = 6.5 \text{ m/s}$$

$$V_2 = 6.3 \text{ m/s}$$

4. (b)

5. (d)

$$\frac{\frac{1}{T^2}}{\frac{1}{T_0^2}} + \frac{\ln R(T)}{\ln R(T_0)} = 1$$

$$\Rightarrow \ln R(T) = \ln R(T_0)\left(1 - \frac{T_0^2}{T^2}\right)$$

$$R(T) = R_0 e^{-\left(\frac{T_0^2}{T^2}\right)}$$

6. (c)

$$\Delta Q = nCC_v\Delta T = n\frac{3}{2}R\Delta T$$

$$= \left(\frac{67.2}{22.4}\right)\left(\frac{3}{2} \times 8.31\right)(20)$$

$$\approx 748 \text{ J}$$

7. (b) $-(g+\gamma v^2)=\frac{dv}{dt}$

$$-gdt=\frac{g}{\gamma}\left(\frac{dv}{\frac{g}{\gamma}+v^2}\right)$$

Integrating $0 \to t$ and $V_0 \to 0$:

$$-gt=-\sqrt{\frac{g}{\gamma}}\tan^{-1}\left(\frac{V_0}{\sqrt{\frac{g}{\gamma}}}\right)$$

$$t=\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$$

8. (d)

$$I_{Disc}=\int_0^R (dm)r^2 \Rightarrow I_{Disc}=\int_0^R (\sigma 2\pi r dr)r^2$$

$I_{Disc}=\int_0^R (kr^2 2\pi r dr)r^2$ Mass of disc

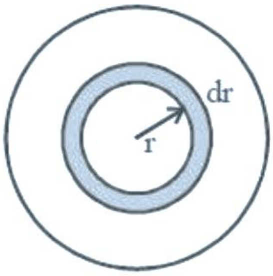
$$I_{Disc}=2\pi k\int_0^R r^2 dr \quad M=\int_0^R 2\pi r dr k r^2$$

$$I_{Disc}=2\pi k\left(\frac{r^6}{6}\right)_0^R \quad M=2\pi k\int_0^R r^3 dr$$

$$I_{Disc}=2\pi k\frac{R^6}{6} \quad M=2\pi k\left.\frac{r^4}{4}\right|_0^R$$

$$I_{Disc}=\frac{\pi k R^6}{3}=\left(\frac{\pi k R^4}{2}\right)\frac{R^2 2}{3} \quad M=2\pi k\left.\frac{r^4}{4}\right|_0^R$$

$$I_{Disc}=\frac{M2R^2}{3} \quad ; \quad I_{Disc}=\frac{2}{3}MR^2$$



9. (d)

$$E_i=\frac{1}{2}I_1\times\omega_1^2+\frac{1}{2}\frac{I}{2}\times\frac{\omega_1^2}{4}$$

$$=\frac{I_1\omega_1^2}{2}\left(\frac{9}{8}\right)=\frac{9}{16}I_1\omega_1^2$$

$$I_1\omega_1+\frac{I_1\omega_1}{4}=\frac{3I_1}{2}\omega\ ;\ \frac{5}{4}I_1\omega_1=\frac{3I_1}{2}\omega$$

$$\omega=\frac{5}{6}\omega_1\ ;\ E_f=\frac{1}{2}\times\frac{3I_1}{2}\times\frac{25}{36}\omega_1^2$$

$$=\frac{25}{48}l_1\omega_1^2$$

$$\Rightarrow E_f-E_i=I_1\omega_1^2\frac{25}{49}-\frac{-2}{48}I_2\omega_1^2$$

$$=\frac{25}{48}l_1\omega_1^2$$

$$\Rightarrow E_f-E_i=I_1\omega_1^2\left(\frac{25}{48}-\frac{9}{16}\right)=\frac{-2}{48}I_1\omega_1^2$$

$$=\frac{-l_1\omega_1^2}{24}$$

10. (d) $h=\frac{2\,S_1\cos\theta}{r_1\rho_1\,g}$

$$h=\frac{2\,s_2\cos\theta_2}{r_2\rho_2\,g}$$

$$\Rightarrow\frac{r_1}{r_2}=\frac{2}{5}$$

11. (a) As $q=CV$

Hence slope of graph will give capacitance. Slope will be more in parallel combination. Hence capacitance in parallel should be $50\mu F$ and in series combination must be $8\mu F$. Only in option $40\mu f$ and $10\mu F$.

$$C_{\text{parallel}}=40+10=50\mu F$$

$$C_{\text{series}}=\frac{40\times10}{40+10}=8\mu F$$

12. (b)

13. (d) $r=\frac{mv}{qB}=\frac{\sqrt{2mK}}{qB}$

$$r_{He}=r_p>r_e$$

14. (c) $\frac{GM}{(R+h)^2}=\frac{GM}{2R^2}$

$$R+h=\sqrt{2}R$$

$$h=(\sqrt{2}-1)R$$

$$\simeq 2.6\times10^6\ \text{m}$$

15. (b)

$$R_{eq}=\frac{15\times10}{25}+2+2r$$

$= 8 + 2r$

$i = \dfrac{3}{8 + 2r}$

$2 = iR_{\text{eq}} = \dfrac{3}{8 + 2r} \times 6$

$16 + 4r = 18$

$\Rightarrow r = 0.5\Omega$

16. (a) $\because \vec{E} \times \vec{B} \parallel \vec{V}$

Given that wave is propagating along positive z-axis and $\vec{E}$ along positive x-axis. Hence $\vec{B}$ along y-axis.

From Maxwell equation

$\vec{\nabla} \times \vec{E} = -\dfrac{\partial B}{\partial t}$

i.e. $\frac{\partial E}{\partial z} = -\frac{\partial B}{dt}$ and $B_0 = \frac{E_0}{c}$

17. (b) $A_v \times \beta = P_{\text{gain}}$

$60 = 10\log_{10}\left(\dfrac{P}{P_0}\right)$

$P = 10^6 = \beta^2 \times \dfrac{R_{\text{out}}}{R_{\text{in}}}$

$= \beta^2 \times \dfrac{10^4}{100}$

$\beta^2 = 10^4; \beta = 100$

18. (a)

as $x = \frac{R(100-\ell)}{\ell}$

for (A) $x = \frac{1000\times(100-60)}{40} \approx 667$

for (b) $x = \frac{100\times(100-13)}{13} \approx 669$

for (c) $x = \frac{10\times(100-1.5)}{98.5} \approx 656$

for (d) $x = \frac{1\times(100-1)}{1} \approx 95$

So reading in serial no. (4) is completely different hence correct answer (a).

19. (a) Given $N_P = 300, N_s = 150, P_0 = 2200$ W

$I_S = 10$ A

$P_0 = V_0 I_0 \Rightarrow 2200 = V_0 \times 10 \Rightarrow V_0 = 220\text{ V}$

$\because \frac{V_i}{V_0} = \frac{N_p}{N_s} \Rightarrow V_i = 2 \times 220 = 440\text{ V}$

Also, $P_0 = V_i i_i$

$\Rightarrow I_i = \frac{2200}{440} = 5\text{ A}$

20. (d) $U_i + K_i = U_f + K_f$

$$\frac{kq^2}{\sqrt{16a^2+9a^2}} + \frac{1}{2}mv^2 = \frac{kq^2}{3a}$$
$$\frac{1}{2}mv^2 = \frac{kq^2}{a}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2kq^2}{15a}$$
$$v = \sqrt{\frac{4kq^2}{15ma}}$$

21. (b) $\mu = \frac{V_d}{E}$ $E = \rho J$

$$= \frac{1.1 \times 10^{-3}}{1.7 \times 10^{-8} \times \frac{5}{\pi \times 25 \times 10^{-6}}}$$
$$= \frac{1.1 \times 10^{-3} \times \pi \times 25 \times 10^{-6}}{1.7 \times 10^{-8} \times 5} \approx 1.01\ \text{m}^2/\text{Vs}$$

22. (b)

$= nR\Delta T$

$\Delta H = (C_v + nR)\Delta T$

$\frac{\omega}{\Delta H} = \frac{nR}{C_v + nR}$

23. (d) $f_m = 100\text{MHz} = 10^8\text{ Hz}, (V_m)_0 = 100\text{ V}$

$f_c = 300\text{GHz}, (V_c)_0 = 400\text{ V}$

$\therefore\ UBF - LBF = 2f_m = 2 \times 10^8\text{ Hz}$

24. (d)

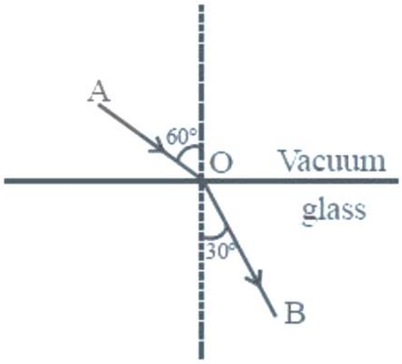
From Snell's law

$1\sin 60^\circ = \mu\sin 30^\circ$

$\Rightarrow \mu = \sqrt{3}$

Optical path $= AO + \mu(OB)$

$$= \frac{a}{\cos 60^\circ} + \sqrt{3}\frac{b}{\cos 30^\circ}$$
$$= 2a + 2b$$



25. (d) $A = A_0 e^{-0.1t} = \frac{A_0}{2}$

$\ln 2 = 0.1t$

$t = 10\ln 2 = 6.93 \approx 7sec.$

26. (c) $N_1 = N_0 e^{-10\lambda t}$; $N_2 = N_0 e^{-\lambda t}$

$\frac{1}{e} = \frac{N_1}{N_2} = e^{-9\lambda t}$

$\Rightarrow 9\lambda t = 1$

$\Rightarrow t = \frac{1}{9\lambda}$

27. (a) $v = \frac{V_{av}}{\lambda}$

$\lambda = \frac{RT}{\sqrt{2}\pi\sigma^2 N_A P}$

$\sigma = 2 \times 0.3 \times 10^{-9}$

$P = \frac{RT}{V}$

$\Rightarrow \quad = \frac{V}{\sqrt{2}\pi\sigma^2 N_A}$

$V_{av} = \sqrt{\frac{8}{3\pi}} \times V_{rms}$

$\therefore \ v = \frac{200 \times \sqrt{2}\pi \times \sigma^2 N_A}{25 \times 10^{-3}} \times \sqrt{\frac{8}{3\pi}}$

$= 17.68 \times 10^8/sec$

$= 0.1768 \times 10^{10}/sec \sim 10^{10}$

28. (b) $K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_o}$

$\Rightarrow K_{max} = hc\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$

$\Rightarrow K_{max} = (1237)\left(\frac{380 - 260}{380 \times 260}\right)$

$= 1.5eV$

29. (c) For 1st lens $\frac{1}{f_1}=\left(\frac{\mu_1-1}{1}\right)\left(\frac{1}{\infty}-\frac{1}{-R}\right)=\frac{\mu_1-1}{R}$

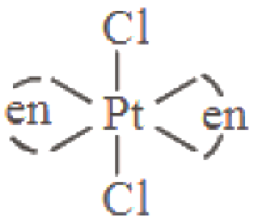
For 2nd lens $\frac{1}{f_2}=\left(\frac{\mu_2-1}{1}\right)\left(\frac{1}{-R}-0\right)=-\frac{\mu_2-1}{R}$

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\frac{1}{f_{eq}}=\frac{\mu_1-1}{R}+\frac{-(\mu_2-1)}{R}\Rightarrow f_{eq}=\frac{R}{\mu_1-\mu_2}$$

30. (BONUS)

31. (b) The trans-isomer of $[Pt(en)_2Cl_2]^{2+}$



32. (d)

33. (a) The synonoyms for water gas is syn gas.

34. (b)

35. (b) $Ti^{+2}=1s^2 2s^2 2p^6 3s^2 3p^6 3d^2$

unpaired electrons $=2$.

spin only magnetic moment $(\mu)=\sqrt{2(2+2)}=\sqrt{8}$ B.M

$Ti^{+3}=1s^2 2s^2 2p^6 3s^2 3p^6 3d^1$

unpaired electrons $=1$

$\mu=\sqrt{1(1+2)}=\sqrt{3}$ B.M

$V^{+2}=1s^2 2s^2 2p^6 3s^2 3p^6 3d^3$

$\mu=\sqrt{3(3+2)}=\sqrt{15}$ B.M

$Sc^{+3}=1s^2 2s^2 2p^6 3s^2 3p^6$

$\mu=0$

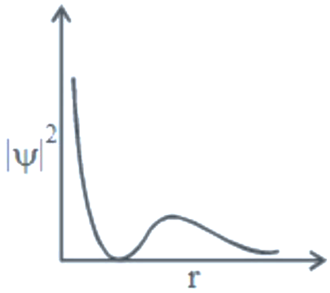
36. (c) Amylopetcin is a homopolymer of a-D-glucose where C_1-C_4 linkage and C_1-C_6 linkage are present.

37. (a) Increase in pressure leads to the increase in adsorption capacity and the physical adsorption is an exothermic process with the increase in temperature adsorption decrease.

38. (d)

39. (c) As we know that for s-orbital graph starts from top and no. of radial node $=n-\ell-1$ For 2 s orbital it will $=2-0-1=1$.

∴ The graph



is of $2s$

40. (d) Gas A and C have same value of ' b ' but different value of ' a ' so gas having higher value of 'a' have more force of attraction so molecules will be more closer hence occupy less volume. Gas B and D have same value of 'a' but different value of 'b' so gas having lesser value of 'b' will be more compressible.

41. (b)

42. (a) The principle of column chromatography is differential adsorption of substance

43. (b) $C_xH_y + \left(x + \frac{y}{4}\right) O_2 \longrightarrow xCO_2 + \frac{y}{2} H_2O$

$1010\left(x + \frac{y}{4}\right)$

10x

By given data, $10\left(x + \frac{y}{4}\right) = 55$

$10x = 40$

$\therefore x = 4, y = 6 \Rightarrow C_4H_6$

44. (b) As we known that

Strong ligand α CFSE $\alpha E_{\text{absorbed}}\ \alpha \frac{1}{\lambda_{\text{absorbed}}}$

We have

$[CoCl(NH_3)_5]^{2+}, [Co(NH_3)_5H_2O]^{3+}$ and $[CO(NH_3)_6]^{3+}$

(I)

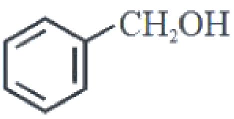
$\therefore$ III $>$ II $>$ I (as per Eabsorbed)

(III)

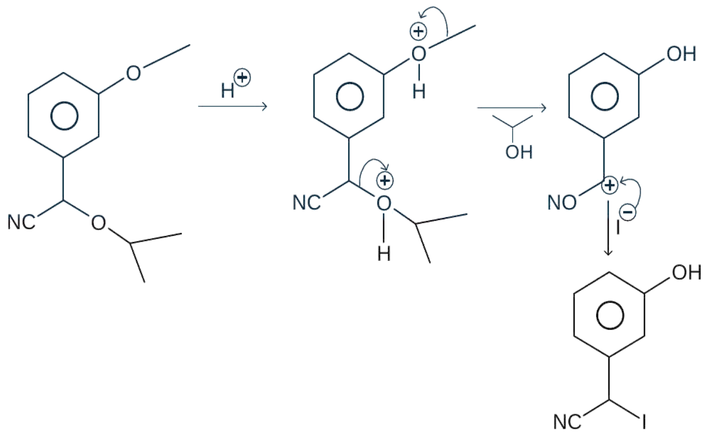
$\therefore \lambda_{\text{absorbed}}$

I $>$ II $>$ III

45. (b)

This is cross cannizaro reaction so more reactive carbonyl compound is oxidized and less reactive is reduced so answer is  $+ HCO_2H$

46. (c)



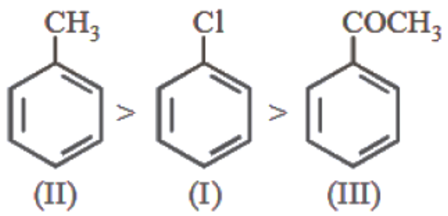
47. (d) Liquation is used for Sn.

Zone refining is used for Ga.

Mond's process is used for Ni.

Van arkel process is used for Zr.

48. (b) Rate of aromatic electrophilic substitution is



49. (c) At constant P and T and for the process to be spontaneous we should have $\Delta G = -ve$ and we know that

$\Delta G = \Delta H - T\Delta S$

If $\Delta H = -$ ve and $\Delta S = +ve$ then at all the temperature the process will be spontaneous.

50. (c)

(a) $H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$

$400 \times .1 = 40400 \times 1 = 40$

20

0

$\therefore [H^+] = \dfrac{20 \times 2}{800} = \dfrac{1}{20} \Rightarrow pH = -\log\left(\dfrac{1}{20}\right)$

$\therefore$ pH $= 1.3$ so (a) is correct

(b) $\log\left(\frac{Kw_2}{Kw_1}\right) = \frac{\Delta H}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

so ionic product of water is temp. dependent

hence (b) is correct.

(c) $K_a = 10^{-5}, pH = 5 \Rightarrow [H^+] = 10^{-5}$

$$K_a = \frac{c\alpha^2}{(1-\alpha)} \Rightarrow K_a = \frac{[H^+]\cdot\alpha}{(1-\alpha)}$$

$$\therefore 10^{-5} = \frac{10^{-5}\cdot\alpha}{(1-\alpha)} \Rightarrow 1-\alpha = \alpha \Rightarrow \alpha = \frac{1}{2} = 50\%$$

so (c) is correct.

(d) Le-chatelier's principle is applicable to common -Ion effect so option (d) is wrong

51. **(d)** It is the final step of Gabriel pthalimide synthesis reaction.

52. **(b)** In this order of catenation is asked. Catenation is a self-linking property here and for group 14: Self-linking is through covalent bonding

$C > Si > Ge \approx Sn$

In C there is $2p - 2p$ overlapping further $3p - 3p, 4p - 4p$ and so on and the extent of overlapping is more in $2p - 2p > 3p - 3p > 4p - 4p \approx 5p - 5p$

53. **(a)**

54. **(a)** Nylon-6,6 is a condensation polymer of hexamethylene diamine and adipic acid. Buna-S, Teflon and Neoprene are addition polymer.

55. **(a)** We know that

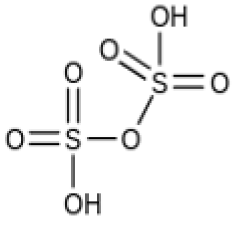
$\lambda_m = \frac{K}{c}$ Here $\lambda_m =$ molar conductivity

$K =$ conductivity
$C =$ concentration

When concentration increase conductivity always increases. The molar conductivity always increase with the decrease in the concentration.

56. **(d)** Rate of S_N1 reaction α stability of $C^{\oplus} - I.M$

57. **(c)** $H_2\,S_2O_7$



58. **(c)** In this we have to choose isoelectronic set of ions:

Isoelectronic species are those which have same no. of electron in total,

59. **(c)** For aircraft construction aluminium and its alloys are used, because they are lighter.

60. **(d)** Lowering of vapour pressure $= p^0 - p = p^0 \cdot x_{solute}$

$$\therefore \Delta p = 35 \times \frac{0.6/60}{\frac{0.6}{60} + \frac{360}{18}} = 35 \times \frac{.01}{.01 + 20} = 35 \times \frac{.01}{20.01} = .017 \text{ mmHg}$$

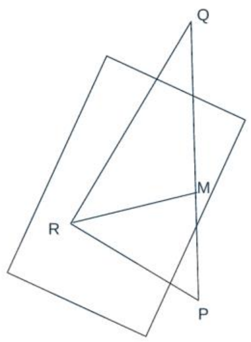
61. **(a)** $MQ = \frac{|1-12-2|}{\sqrt{9+1+16}} = \frac{13}{\sqrt{26}} = \sqrt{\frac{13}{2}}$

$PM = \sqrt{26}$

$RQ = \sqrt{9+1} = \sqrt{10}$

$RM = \sqrt{10 - \frac{13}{2}} = \sqrt{\frac{7}{2}}$

$Ar\ (\Delta PQR) = \frac{1}{2} \times \sqrt{26} \times \sqrt{\frac{7}{2}} = \frac{\sqrt{91}}{2}$



62. **(c)** Coefficient of $x^2 = {}^{15}C_2 \times 9 - 3a({}^{15}C_1) + b = 0$

$\Rightarrow {}^{15}C_2 \times 9 - 45a + b = 0$

Coefficient of $x^3 = -27 \times {}^{15}C_3 + 9a \times {}^{15}C_2 - 3b \times {}^{15}C_1 = 0$

$\Rightarrow -273 + 21a - b = 0$

(1) + (2) give

$-24a + 672 = 0 \Rightarrow a = 28, b = 315$

63. **(b)**

$$T_n = \frac{(3 + (n-1) \times 2)(1^3 + 2^3 + \cdots .. + n^3)}{(1 + 2^2 + \cdots . + n^2)}$$

$$= \frac{3}{2} n(n+1)$$

$S_n = \sum T_n$

$$= \sum \frac{3}{2} \cdot n \cdot (n+1)$$

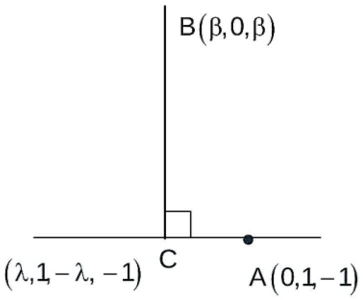
On solving $S_n = \frac{n(n+1)(n+2)}{2} \Rightarrow S_{10} = 660$

64. (b) $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}=\lambda$

A point on this line is $A(0,1,-1)$

$\overrightarrow{AB}\cdot\overrightarrow{BC}=0$

We get $\lambda=\frac{-1}{2}$



$\therefore C\equiv\left(-\frac{1}{2},1,\frac{-1}{2}\right)$

$|\overrightarrow{BC}|=\sqrt{\frac{2}{3}}$

$$\sqrt{\left(\beta+\frac{1}{2}\right)^2+\left(1^2+\left(\beta+\frac{1}{2}\right)\right)^2}=\sqrt{\frac{2}{3}}$$

$(\lambda,1,-\lambda,-1)$

$A(0,1,-1)$

$\therefore\beta=0,-1$

$\beta=-1\ (\beta\neq0)$

65. (d) $RHL=\lim_{x\to0^+}\frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}$

$=\lim_{x\to0^+}\frac{\sqrt{1+x}-1}{x}=\frac{1}{2}$

$\text{LHL}=\lim_{x\to0^-}\frac{\sin(\beta+1)x+\sin x}{x}$

$=(p+1)+1$

$=p+2$

For function to be continuous

$LHL=RHL=f(0)$

$\Rightarrow(\mathrm{p},\mathrm{q})=\left(\frac{-3}{2},\frac{1}{2}\right)$

66. (d) From options

$(p \vee q) \wedge (\sim p \vee \sim q) = (p \vee q) \wedge \sim (p \wedge q) \rightarrow$ Not a tautology

$(p \vee q) \vee (p \vee \sim q) = p \vee (q \vee \sim q) \rightarrow$ tautology

$(p \wedge q) \vee (p \wedge \sim q) \equiv p \wedge (q \vee \sim q) \rightarrow$ Not a tautology

$(p \vee q) \wedge (p \vee \sim q) \equiv p \vee (q \wedge \sim q) \rightarrow$ Not a tautology

67. (a)

$x^2 + x\sin\theta - 2\sin\theta = 0$

$\alpha + \beta = -\sin\theta$

$\alpha\beta = -2\sin\theta$

Now, $\frac{\alpha^{12}+\beta^{12}}{\left(\frac{1}{\alpha^{12}}+\frac{1}{\beta^{12}}\right)(\alpha-\beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$

$$= \frac{(\alpha\beta)^{12}}{((\alpha+\beta)^2 - 4\alpha\beta)^{12}}$$
$$= \left[\frac{\alpha\beta}{(\alpha+\beta)^2 - 4\alpha\beta}\right]^{12}$$
$$= \left(\frac{-2\sin\theta}{\sin^2\theta + 8\sin\theta}\right)^{12}$$
$$= \frac{2^{12}}{(\sin\theta + 8)^{12}}$$

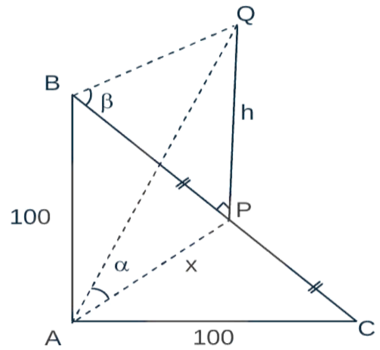
68. (d) $\operatorname{cosec}\beta = 2\sqrt{2}$

$\cot\alpha = 3\sqrt{2}$

$\frac{\mathrm{x}}{\mathrm{h}} = 3\sqrt{2}$

So $\frac{\alpha}{\sqrt{10^4 - \mathrm{x}^2}} = \frac{1}{\sqrt{7}}$

From (i) and (ii) h = 20



69. (b) $z = \frac{(1+i)^2}{a-i} = \frac{2i(a+i)}{a^2+1}$

$|z| = \frac{2}{\sqrt{a^2+1}} = \sqrt{\frac{2}{5}} \Rightarrow a = 3$

$$\therefore \bar{z} = \frac{-2i(3-i)}{10}$$

$$\Rightarrow \frac{-1-3i}{5}$$

70. (d) $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$= x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$\Rightarrow -x^3$

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$

$\Rightarrow -x^3$

$\Delta_1 + \Delta_2 = -2x^3$

71. (c) $\frac{dy}{dx} = (\tan x - y)\sec^2 x$

$\frac{dy}{dx} + y\sec^2 x = \tan x\sec^2 x$

Let $\tan x = t \Rightarrow \sec^2 x = \frac{dt}{dx}$

$\therefore \frac{dy}{dt} = (t-y)$

$\frac{dy}{dt} + y = t$ (Linear differential equation)

After solving we get

$ye^t = e^t(t-1) + c$

$\Rightarrow y = (\tan x - 1) + ce^{-\tan x}$

$y(0) = 0 \Rightarrow c = 1$

$y = \tan x - 1 + e^{-\tan x}$

So, $y\left(\frac{-\pi}{4}\right) = e - 2$

72. (a) $g'(c) = \lim_{h\to 0} \frac{|f(c+h)| - |f(c)|}{h}$ $\because f(c) = 0$

$= \lim_{h\to 0} \frac{|f(c+h)|}{h}$

$= \lim_{h\to 0} \left|\frac{f(c+h) - f(c)}{h}\right| \cdot \frac{|h|}{h}$

$= \lim_{h\to 0} |f'(c)| \frac{h}{h} |= 0$ if $f'(c) = 0$

i.e. $g(x)$ is differentiable at $x=c$ if $f'(c)=0$

73. (c) $(s)=[-2,2]$

$f(g(s))=[0,4]=5$

$f(S)=[0,16] \Rightarrow f(g(s)) \neq f(s)$

$\mathrm{g(f(s))}=[-4,4] \neq \mathrm{g(s)}$

therefore, $\mathrm{g(f(s))} \neq \mathrm{S}$

74. (c)

$x+3y+\lambda z-u=a(x+y+z-5)+b(x+2y+2z-6)$

Comparing coefficients we get

$\mathrm{a}+\mathrm{b}=1$ and $\mathrm{a}+2\,\mathrm{b}=3$

$(a,b)=(-1,2)$

So, $x+3y+\lambda z-u=x+3y+3z-\lambda$

$\Rightarrow \mathrm{u}=7, \lambda=3$

75. (b) Let the six digit number be abcdef for this number to be divisible by 11

$|(a+c+e)-(b+d+f)|$ must be multiple of 11

$\therefore$ possibility is a +c + c = b + d + f = 12

Case I $\{a,c,e\}=\{7,5,0\}$ and $\{b,d,f\}=\{9,2,1\}$

So, number of numbers $=2\times 2!\times 3!=24$

Case II $\{a,c,e\}=\{3,2,1\}$ $\{b,d,f\}=\{7,5,0\}$

So number of numbers $=3!\times 3!=36$

Total $\Rightarrow 24+36=60$

76. (d) Equation of circle is given as $S+\lambda\mathrm{L}=0$

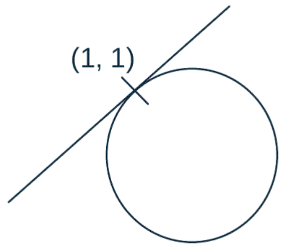
$(x-1)^2+(y-1)^2+\lambda(x-y)=0$ passes

through $(1,-3)$

$16+\lambda\times 4=0 \Rightarrow \lambda=-4$

$\therefore (\mathrm{x}-1)^2+(\mathrm{y}-1)^2-4(\mathrm{x}-\mathrm{y})=0$

$r=2\sqrt{2}$



77. (a) $h(x) = f(g(x))$

$\therefore h'(x) = f'(g(x))g'(x)$ and $f'(x) = e^x - 1$

$h'(x) = \left(e^{g(x)} - 1\right)g'(x)$

$h'(x) = \left(e^{x^2-x} - 1\right)(2x-1) \geq 0$

Case : 1

$e^{x^2-x} \leq 1$ and $2x - 1 \leq 0$

$\Rightarrow x \in \left[0, \frac{1}{2}\right]$

Case : 2

$e^{x^2-x} \geq 1$ and $2x - 1 \geq 0$

$\Rightarrow x \in [1, \infty)$

from (i) and (ii)

$x \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$

78. (d) $I = \int_0^{2\pi} [\sin 2x(1 + \cos 3x)]dx$

$21 = \int_0^{2\pi} ([\sin 2x(1 + \cos 3x)] + [-\sin 2x - \sin 2x \cos 3x])dx$

$2I = \int_0^{2\pi} - dx$

$2I = 2\int_0^{\pi} - dx$

$I = \int_0^{\pi} - dx \Rightarrow -\pi$

79. (b) Equation of common chord $4kx + \frac{1}{2}y + k + \frac{1}{2} = 0$ and given line $4x + 5y - k = 0$

on comparing (i) and (ii) we get $k = \frac{1}{10} = \frac{k+\frac{1}{2}}{-k}$

$\Rightarrow$ No real value of k exist.

80. (b) Tangent at $\left(3, \frac{-9}{2}\right)$

$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$

Comparing with $x-2y=12$

$$\frac{3}{a^2}=\frac{9}{4\,b^2}=\frac{1}{12}$$

$\Rightarrow a=6$ and $b=3\sqrt{3}$

Length of latus rectum $=\frac{2b^2}{a}=9$

81. (a) $a_1+a_4+a_7+a_{10}+a_{13}+a_{16}=114$

$$\Rightarrow \frac{6}{2}(a_1+a_{16})=114$$

$a_1+a_{16}=38$

So $a_1+a_6+a_{11}+a_{16}=\frac{4}{2}(a_1+a_{16})$

$=2\times 38\Rightarrow 76$

82. (a) Mean $\bar{x}=\frac{\sum x_i f_i}{\sum f_i}$

$\because \sum f_i=(x+1)^2+(2x-5)+(x^2-3x)+x=20$

$\Rightarrow x=3,-4$ (rejected)

$$\therefore \bar{x}=\frac{\sum x_i f_i}{\sum f_i}=2.8$$

83. (b) $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to 1}(x+1)(x^2+1)$

$$\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}=\frac{k^2+k^2+k^2}{2k}$$

$(1)=(2)$

$$\Rightarrow k=\frac{8}{3}$$

84. (b)

$$2^{\sqrt{\sin^2 x-2\sin x+5}}\cdot 4^{-\sin^2 y}\le 1$$

$$\Rightarrow 2^{\sqrt{(\sin x-1)^2+4}}\le 4^{\sin^2 y}$$

$$\underbrace{\sqrt{\underbrace{(\sin x-1)^2}_{\ge 0}+4}}_{\ge 2}\le \underbrace{2\sin^2 y}_{\le 2}$$

this is possible only if $\sin x=1$ and $|\sin y|=1$

85. (a)

86. (b) Let hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and passes through $(4,-2\sqrt{3})$ therefore $\frac{16}{a^2}-\frac{12}{b^2}=1$

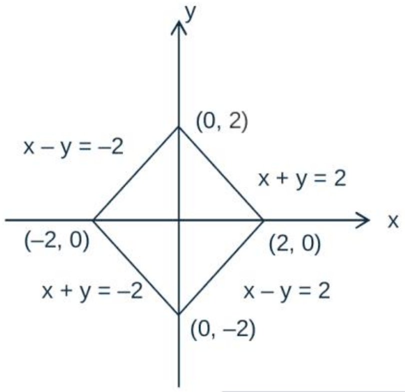
$\because b^2=a^2(e^2-1)$

$$x=\frac{4\sqrt{5}}{5}=\frac{a}{e}\Rightarrow a^2=\frac{16}{5}e^2$$

On solving (i) and (ii)

$\Rightarrow 4e^4-24e^2+35=0$

87. (d) Shown figure is square with side length $2\sqrt{2}$



88. (d) $P(\text{ Boy })=P(\text{ girl })=\frac{1}{2}$

Required probability $=\frac{\text{all four girls}}{\text{Atleast two girls}}$

$$=\frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4+{}^4C_3\left(\frac{1}{2}\right)^4+{}^4C_2\left(\frac{1}{2}\right)^4}$$
$$=\frac{1}{11}$$

89. (d) G will be centroid of $\triangle$ ABC

$G\equiv(2,4,2)$

$\overrightarrow{OG}=2\hat{i}+4\hat{j}+2\hat{k}$

$\overrightarrow{OA}=3i-\hat{k}$

$$\cos(\angle GOA)=\frac{\overrightarrow{OG}\cdot\overrightarrow{OA}}{|\overrightarrow{OG}||\overrightarrow{OA}|}=\frac{1}{\sqrt{15}}$$

90. (c)

$$\int\frac{dx}{(x^2-2x+10)^2}=\int\frac{dx}{((x-1)^{21}+9)^2}$$

Let $x-1=3\tan\theta$

$dx=3\sec^2\theta d\theta$

$$\therefore\frac{1}{27}\int\cos^2\theta d\theta=\frac{1}{54}\int(1+\cos 2\theta)d\theta=\frac{1}{54}\left(\theta+\frac{\sin 2\theta}{2}\right)$$

$$= \frac{1}{54}\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right) + C$$