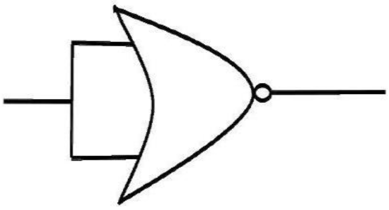
1. (c)



Since, there is only one input hence the operation is reversible.

2. (a) Friction will oppose the motion

Net force $= 2000g + 4000 = 24000N$

Power of lift $= 60\text{HP}$

Power $=$ Force $\times$ Velocity

$$v = \frac{P}{F} = \frac{60 \times 746}{24000}$$

$v = 1.86$ m/s

3. (a) For damped oscillator by Newton's second law

$$-kx - bv = ma$$
$$kx + bv + ma = 0$$
$$kx + b\frac{dx}{dt} + m\frac{d^2x}{dt^2} = 0$$

For LCR circuit by KVL

$$-IR - L\frac{dI}{dt} - \frac{q}{c} = 0$$
$$\Rightarrow IR + L\frac{dI}{dt} + \frac{q}{c} = 0$$
$$\Rightarrow \frac{q}{c} + R\frac{dq}{dt} + L\frac{d^2q}{dt^2} = 0$$

By comparing

$$R \Rightarrow b$$
$$c \Rightarrow \frac{1}{k}$$
$$m \Rightarrow L$$

4. (a) By energy conservation,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$\Rightarrow gh = \frac{v^2}{2} + \frac{\omega^2 R^2}{4}$$

Since the rope is inextensible and also it is not slipping,

$\therefore v = R\omega$

from eq. (1) and (2)

$$gh = \frac{\omega^2 R^2}{2} + \frac{\omega^2 R^2}{4}$$

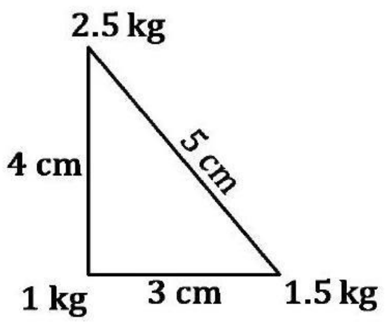
$$\Rightarrow gh = \frac{3}{4} R^2 \omega^2$$

$$\Rightarrow \omega^2 = \frac{4gh}{3R^2}$$

$$\Rightarrow \omega = \frac{1}{R}\sqrt{\frac{4gh}{3}}$$

5. (b) Taking 1 kg as the origin

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$



$$x_{\text{com}} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5}$$

$$x_{\text{com}} = 0.9$$

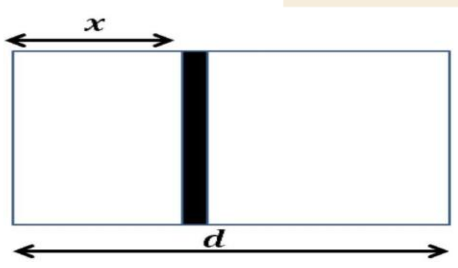
$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$y_{\text{com}} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5}$$

$$y_{\text{com}} = 2$$

Centre of mass is at (0.9,2)

6. (d)



Given, $k(x) = k(1 + \alpha x)$

$$\text{dC} = \frac{A\varepsilon_0 k}{dx}$$

Since all capacitance are in series, we can apply

$$\frac{1}{Ceq}=\int \frac{1}{dC}=\int_0^d \frac{dx}{k(1+\alpha x)\epsilon_0 A}$$

$$\frac{1}{Ceq}=\left[\frac{\ln(1+\alpha x)}{k\epsilon_0 A\alpha}\right]_0^d$$

On putting the limits from 0 to d

$$=\frac{\ln(1+\alpha d)}{k\epsilon_0 A\alpha}$$

Using expression $\ln(1+x)=x-\frac{x^2}{2}+\cdots$

And putting $x=\alpha d$ where, x approaches to 0 .

$$\frac{1}{C}=\frac{d}{k\epsilon_0 A d\alpha}\left[\alpha d-\frac{(\alpha d)^2}{2}\right]$$

$$\frac{1}{C}=\frac{d}{k\epsilon_0 A}\left[1-\frac{\alpha d}{2}\right]$$

$$C=\frac{k\epsilon_0 A}{d}\left[1+\frac{\alpha d}{2}\right]$$

7. (a) Time period is proportional to $\frac{n^3}{Z^2}$.

Let T_1 be the time period in ground state and T_2 be the time period in it's first excited state.

$$T_1=\frac{n^3}{2^2}$$

(Where, n = excitation level and 2 is atomic no.)

$$\frac{T_1}{T_2}=\left(\frac{n_1}{n_2}\right)^3$$

Given,

$T_1=1.6\times 10^{-16}s$

So,

$$\frac{1.6\times 10^{-16}}{T2}=\left(\frac{1}{2}\right)^3$$

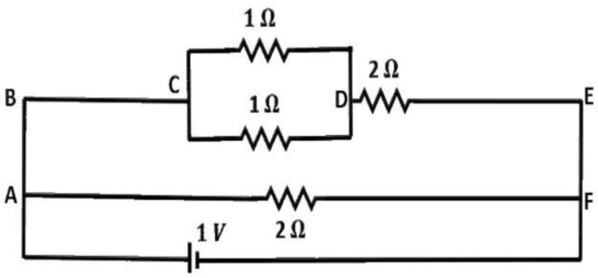
$$T_2=12.8\times 10^{-16}\text{ s}$$

Frequency is given by $f=\frac{1}{T}$

$$f=\frac{1}{12.8}\times 10^{16}\text{ Hz}$$

$$f=7.8128\times 10^{14}\text{ Hz}$$

8. (a)



Net resistance across CD $= \frac{1}{2}\Omega$

Net resistance across BE $= 2 + \frac{1}{2} = \frac{5}{2}\Omega$

Net resistance across BE $= \frac{\frac{5}{2}\times 2}{\frac{5}{2}+2} = \frac{10}{9}\Omega.$

Total current in circuit $= \frac{V}{R} = \frac{9}{10}A$

In the given circuit,voltage across BE $=$ voltage across BF $= 1$ V

Current across BE $= \frac{V_{BE}}{R} = \frac{2}{5}$ A

Current across CD and DE will be same which is $\frac{2}{5}A$.

Now, current across any 1Ω resistor will be same and given by $= I = \frac{1}{2}\times\frac{2}{5} = \frac{1}{5} = 0.20$ A

9. **(c)** As we know,

$$T.E_{\text{ground}} = T.E_R$$
$$\frac{1}{2}mu^2 + \left(\frac{-GMm}{R}\right) = \frac{1}{2}mv^2 + \left(\frac{-GMm}{2R}\right)$$
$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \left(\frac{-GMm}{2R}\right)$$
$$v^2 = u^2 + \left(\frac{-GMm}{R}\right)$$
$$\Rightarrow v = \sqrt{u^2 + \left(\frac{-GMm}{R}\right)}$$

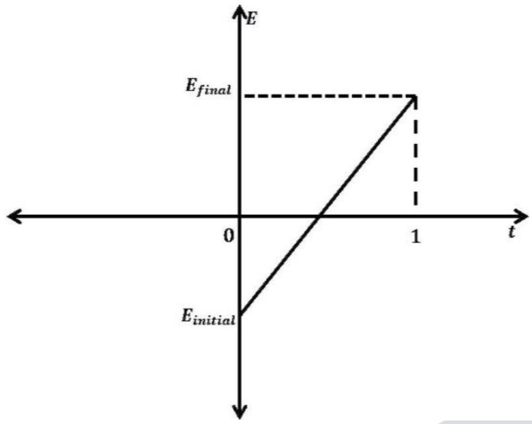
The rocket splits at height R. Since, sepration of rocket is impulsive therefore conservation of momentum in both radial and tangential direction can be applied.

$$\frac{m}{10}V_T = \frac{9m}{10}\sqrt{\frac{GM}{2R}}$$
$$\frac{m}{10}V_r = m\sqrt{u^2 - \frac{GM}{R}}$$

Kinetic energy of satellite $= \frac{1}{2}\times\frac{m}{10}(V_T^2 + V_R^2) = \frac{m}{20}\left(81\frac{GM}{2R} + 100u^2 - 100\frac{GM}{R}\right)$

$$= \frac{m}{20}\left(100u^2 - \frac{119GM}{2R}\right)$$
$$= 5m\left(u^2 - \frac{119GM}{200R}\right)$$

10. (d)



Field due to solenoid near the middle $= \mu_o NI$

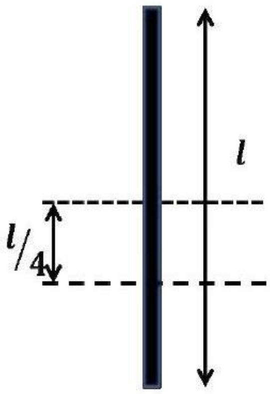
Flux, $\phi = BA$ where $(A = \pi(R)^2)$
$= \mu_o NI_o t(1-t)\pi R^2$
$E = -\frac{d\phi}{dt}$ [By Lenz's law]
$E = -\frac{d}{dt}(\mu_o NI_o t(1-t)^2)$
$E = -\mu_o NI_o \pi R^2 \frac{d}{dt}[t(1-t)]$
$E = -\pi\mu_o I_o NR^2(1-2t)$

Current will change its direction when EMF will be zero

$\Rightarrow (1-2t) = 0$

So, $t = 0.5$sec

11. (a)



Moment of inertia of rod about axis perpendicular to it passing through its centre is given by

$$I = \frac{Ml^2}{12} + M\left(\frac{l}{4}\right)^2$$
$$= \frac{3Ml^2 + 4Ml^2}{48}$$
$$= \frac{7Ml^2}{48}$$

Now, comparing with $I = Mk^2$ where k is the radius of gyration

$$k = \sqrt{\frac{7l^2}{48}}$$
$$k = l\sqrt{\frac{7}{48}}$$

12. (b) For first gas having $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$

Using formula $C_p = \frac{R\gamma}{\gamma-1}$

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p = \frac{5R}{2}\ C_v = \frac{3R}{2}$$

Similarly for 2^{nd} gas having $\gamma = \frac{C_p}{C_v} = \frac{4}{3}$

$$C_p = 4R\ C_v = 3R$$

Now γ of mixture $= \frac{n_1C_{p_1}+n_2C_{p_2}}{n_1C_{v_1}+n_2C_{v_2}}$

Given that $n_1 = 2$ and $n_2 = 3$

$$\gamma = \frac{2 \times \frac{5R}{2} + 3 \times 4R}{2 \times \frac{3R}{2} + 3 \times 3R} = \frac{17}{12} = 1.42$$

13. (b) Given, $P_1 = 1$ atm, $T_1 = 273K$ (At STP)

In adiabatic process,

$$P_1V_1^\gamma = P_2V_2^\gamma$$
$$P_2 = P_1\left[\frac{V_1}{V_2}\right]^\gamma$$
$$P_2 = 1 \times \left[\frac{1}{3}\right]^{1.4}$$
$$P_2 = 0.2164 \text{ atm}$$

Work done in adiabatic process is given by

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$
$$W = \frac{1 \times 1 - 3 \times .2164}{0.4} \times 101.325$$

Since, 1 atm $= 101.325$kPa and 1 Liter $= 10^{-3}$ m^3

$W = 89.87J$

14. (b) We know that,

$$\left|\frac{E_0}{B_0}\right| = c$$
$$B_0 = 3 \times 10^{-8}$$
$$\Longrightarrow E_0 = B_0 \times c = 3 \times 10^{-8} \times 3 \times 10^8$$
$$= 9N/C$$
$$\therefore E = E_o \sin(\omega t - kx + \phi)\hat{k} = 9\sin(\omega t - kx + \phi)\hat{k}$$

15. (d)

So, $0.1I_o = I_o \cos^2\phi$
$\Rightarrow \cos\phi = \sqrt{0.1}$
$\Rightarrow \cos\phi = 0.316$

Intensity after polarisation through polaroid $= I_o \cos^2\phi$

Since, $\cos\phi < \cos 45^\circ$ therefore, $\phi > 45^\circ$ If the light is passing at 90° from the plane of polaroid, than its intensity will be zero.

Then, $\theta = 90^\circ - \phi$ therfore, θ will be less than 45°. So, the only option matching is option d which is 18.4°

16. (a) Given, $M = 6$ grams $= 6 \times 10^{-3}$ kg

$L = 60$ cm $= 0.6$ m
$A = 1$ mm$^2 = 1 \times 10^{-6}$ m^2
Using the relation, $v^2 = \frac{T}{\mu}$

$$\Rightarrow T = \mu v^2 = V^2 \times \frac{M}{L}$$

As Young's modulus, $Y = \frac{\text{stress}}{\text{strain}}$

$$\text{Strain} = \frac{\text{Stress}}{Y} = \frac{T}{AY}$$
$$\text{Strain} = \frac{\Delta L}{L} = \frac{V^2\frac{M}{L}}{AY} = V^2\frac{M}{AYL}$$
$$\Rightarrow \Delta L = \frac{V^2 M}{AY}$$
$$\Delta L = \frac{8100 \times 6 \times 10^{-3}}{1 \times 10^{-6} \times 1.6 \times 10^{11}}$$
$$\Delta L = 0.3 \text{ mm}$$

17. (a) Field due to single plate $= \frac{\sigma}{2\epsilon_o} = [\vec{E}_1] = [\vec{E}_2]$

Net electric field $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$= \frac{\sigma}{2\epsilon_0}\cos 30^0(-\hat{j}) + \frac{\sigma}{2\epsilon_0}\sin 30°(-\hat{\imath}) + \frac{\sigma}{2\epsilon_0}(\hat{j})$$

$$= \frac{\sigma}{2\epsilon_0}\left(1 - \frac{\sqrt{3}}{2}\right)(\hat{j}) - \frac{\sigma}{4\epsilon_0}(\hat{\imath})$$

$$= \frac{\sigma}{2\epsilon_0}\left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{y} - \frac{1}{2}\hat{x}\right]$$

18. (c) Magnification of compound microscope for least distance of distinct vision setting (strained eye)

$$M = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

where L is the tube length

f_0 is the focal length of objective

D is the least distance of distinct vision $= 25$ cm

i.e. $375 = \frac{150 \times 10^{-3}}{5 \times 10^{-2}}\left(1 + \frac{25 \times 10^{-2}}{f_e}\right)$

i.e. $125 = 1 + \frac{25 \times 10^{-2}}{f_e}$

i.e. $\frac{25 \times 10^{-2}}{f_e} = 125$

$\therefore\ f_e \approx 2 \times 10^{-3}$ m $= 2$ mm

19. (a) For single slit diffraction experiment:

Angle of minima are given by

$$\sin\theta_n = \frac{n\lambda}{d} \quad (\sin\theta_n \neq \theta_n \text{ as } \theta \text{ is large })$$

$$\sin\theta_2 = \sin 60° = \frac{\sqrt{3}}{2} = \frac{2\lambda}{d} = \frac{2 \times 6000 \times 10^{-10}}{d}$$

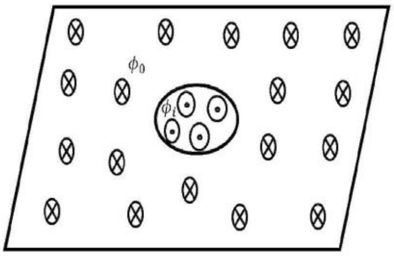
$$\sin\theta_1 = \frac{\lambda}{d} = \frac{6000 \times 10^{-10}}{d}$$

Dividing (1) and (2)

$$\Rightarrow \frac{\sqrt{3}}{2\sin\theta_1} = 2 \Rightarrow \sin\theta_1 = \frac{\sqrt{3}}{4} = 0.43$$

As, the value is coming less than 30° the only available option are 20° and 25° but by using approximation we get $\theta_1 = 25°$

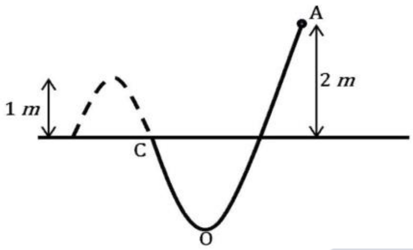
20. (a)



As magnetic field line of ring will form close loop therefore all outgoing from circular hole passing through the infinite plate.

$\therefore \phi_1 = -\phi_0$ (because the magnetic field lines going inside is equal to the magnetic field lines coming out.)

21. (10j)



As the particle starts from rest the total energy at point $A = mgh = T \cdot E_A$(where $h = 2m$)

After reaching point P

$T.E_c = K.E.+mgh$

By conservation of energy

$$T \cdot E_A = T \cdot E_p$$
$$\Longrightarrow K.E \cdot= mgh = 10 \text{ J}$$

22. (600j)

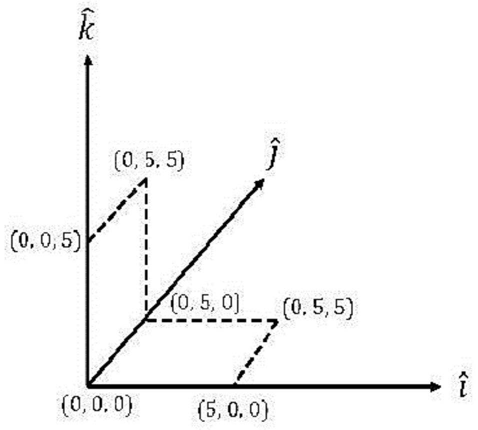
$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = \frac{2}{3}$$

Given, $W = 1200J$

From conservation of energy

$$Q_1 - Q_2 = W$$
$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1} \Longrightarrow Q_1 = 1800 \text{ J}$$
$$\Rightarrow Q_2 = Q_1 - W = 600 \text{ J}$$

23. (175)



As we know, magnetic flux $= \vec{B} \cdot \vec{A}$

$$\Rightarrow (B_x + B_z) \cdot (A_x + A_z)$$
$$\Rightarrow (3\hat{\imath} + 4\hat{k}) \cdot (25\hat{\imath} + 25\hat{k})$$
$$\Rightarrow (75 + 100)Wb$$
$$\Rightarrow 175 \text{ Wb}$$

24. (11)

$$P = \text{ Intensity } \times \text{ Area}$$
$$= 6.4 \times 10^{-5} \text{ W} - \text{cm}^{-2} \times 1 \text{ cm}^2$$
$$= 6.4 \times 10^{-5} \text{ W}$$

For photoelectric effect to take place, energy should be greater than work function Now,

$$E = \frac{1240}{310} = 4\text{eV} > 2\text{eV}$$

Therefore, photoelectric effect takes place

Here n is the number of photons emitted.

$$n \times E = I \times A$$
$$\Rightarrow n = \frac{IA}{E} = \frac{6.4 \times 10^{-5}}{6.4 \times 10^{-19}} = 10^{14}$$

Where, n is number of incident photon

Since, 1 out of every 1000 photons are sucessfull in ejecting 1 photoelectron

Therefore, the number of photoelectrons emitted is

$$= \frac{10^{14}}{10^3}$$
$$\therefore x = 11$$

25. (60) We know that, $V = xyz$

$$\frac{\Delta v}{v}=\frac{\Delta x}{x}+\frac{\Delta y}{y}+\frac{\Delta z}{\Delta z}$$
$$\frac{1}{T}\frac{\Delta v}{v}=\frac{1}{T}\frac{\Delta x}{x}+\frac{1}{T}\frac{\Delta y}{y}+\frac{1}{T}\frac{\Delta z}{z}$$
$$y=\alpha_n+\alpha_y+\alpha_z$$
$$y=50\times10^{-6}/{}^\circ\mathrm{C}+5\times10^{-6}/{}^\circ\mathrm{C}+5\times10^{-6}/{}^\circ\mathrm{C}$$
$$y=60\times10^{-6}/{}^\circ\mathrm{C}$$
$$\therefore n=60$$

26. **(c)** Ion-ion interactions are stronger because they have stronger electrostatic forces of attraction whereas dipoles have partial charges and hence the electrostatic forces in their case would be relatively weak

27. **(c)** Alkali metals always possess a $+1$ oxidation state, whereas oxygen present in $\mathrm{K_2O}$ (oxide) is -2 , and in $\mathrm{K_2O_2}$ (peroxide) is -1 and in $\mathrm{KO_2}$ (superoxide) is $-\frac{1}{2}$.

28. **(d)** $\mathrm{P_{Total}}=\mathrm{P_T}=\mathrm{P_A^0X_A}+\mathrm{P_B^0X_B}$

The maximum value $\mathrm{X_A}$ can hold is 1 , and hence the maximum value of $\mathrm{P_T}$ should come out to be 512 mm of Hg, which is less than the value of $\mathrm{P_T}$ observed (600 mm of Hg). Therefore, positive deviation from Raoult's law is observed. This implies that A-A interactions and B-B interactions are stronger than A-B interactions.

As we know, for a system not obeying Raoult's law and showing positive deviation,

$\Delta\mathrm{V_{mix}}>0, \Delta\mathrm{H_{mix}}>0$

29. **(c)** Because of Lanthanide contraction, an increase in $\mathrm{Z_{eff}}$ is observed and so, the size of Au instead of being greater, as is expected, turns out to be similar to that of Ag.

30. **(c)** All the three compounds possess a tetrahedral geometry. In both $\mathrm{CCl_4}$ and $\mathrm{CH_4}, \mu_{\mathrm{net}}=0$, whereas in $\mathrm{CHCl_3}, \mu_{\mathrm{net}}>0$.

31. **(d)**

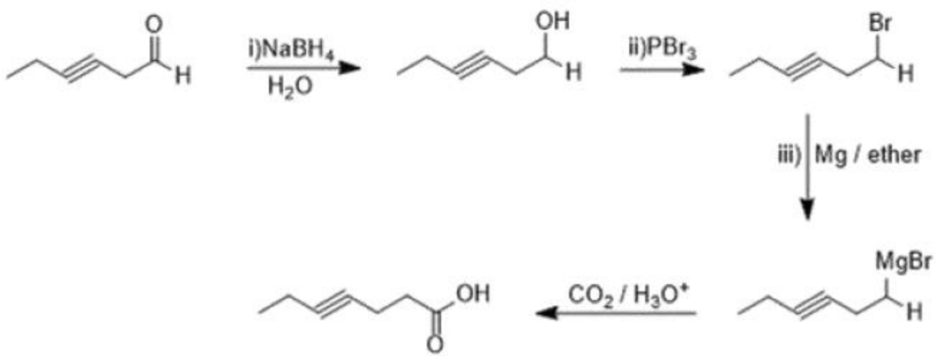
32. **(b)** When gases combine or react in a chemical reaction they do so in a simple ratio by volume provided all gases are maintained at the same temperature and pressure - Gay-Lussac's law.

33. **(b)** Weaker the conjugate acid, stronger the base. (i) is the most basic as it has a guanidine like structure. It has a high tendency of accepting a proton, giving rise to a very stable conjugate acid and hence, is a very strong base.

In compound (iii), the N is $\mathrm{sp^2}$ hybridised and its electronegativity is higher as compared to the compound (ii) which is a 2^0 amine ($\mathrm{sp^3}$ hybridise(d). So compound (ii) is more basic compared to compound (iii).

So, the order of basicity is i > ii > iii and thus the order of $\mathrm{pK_b}$ value will be iii > ii > i

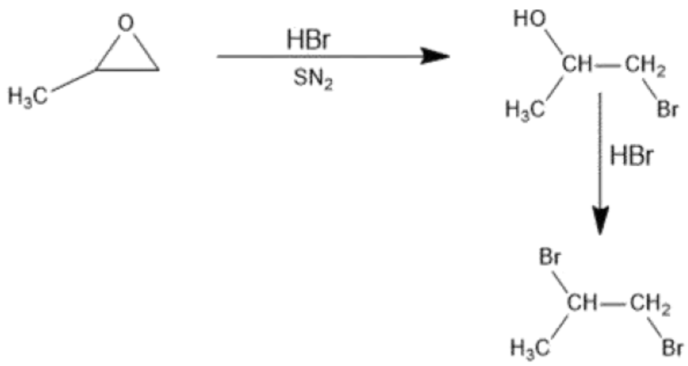
34. **(a)**



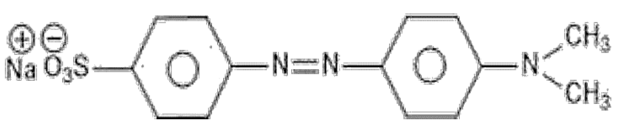
35. **(c)** $\mathrm{n}=5; l=(\mathrm{n}-1)=4$; hence the possible sub-shells for $\mathrm{n}=5$ are: 5 s, 5p, 5 d, 5f and 5 g.

The number of orbitals in each would be 1,3,5,7 and 9, respectively and summing them up gives the answer as 25

36. (b)
37. (c)
38. (a)
39. (a)



40. (d)



X formed is methyl orange.

41. (b)

Vitamins	Deficiency diseases
i) Riboflavin (Vitamin B_2)	Cheilosis
ii) Thiamine (Vitamin B_1)	Beri beri
iii) Ascorbic acid (Vitamin C)	Scurvy
iv) Pyridoxine (Vitamin B_6)	Convulsions

42. (a)

$Cu^{2+} + 2e^- \rightarrow Cu \; E^\circ = 0.340 \text{ V}$

$Cu \rightarrow Cu^+ + e^- \; E^\circ = -0.522 \text{ V}$

$Cu^{2+} + e^- \rightarrow Cu^+ \; E^\circ = ?$

Applying $\Delta G = -nFE^\circ$

We get,

$(-1 \times FxE^\circ) = (-2 \times F \times 0.340) + (-1 \times Fx - 0.522)$

Solving, we get, $E^\circ = 0.158$ V

43. (a) m-chlorobenzoic acid being the most acidic can be separated by a weak base like $NaHCO_3$ and hence will be labelled fraction A.

m-chlorophenol is not as acidic as m-chlorobenzoic acid, and can be separated by a stronger base like NaOH, and hence can be labelled as fraction B. m-chloroaniline being a base, does not react with either of the bases and hence would be labelled as fraction C.

44. (c) $Cl > F > Br > I$

45. (c)

46. (10.6)

Molarity of NaOH(4 g in 100 L) $= 10^{-3}M$

Molarity of H_2SO_4(9.8 g in 100 L) $= 10^{-3}M$

Equivalents of NaOH $= M \times V \times n_f = 10^{-3} \times 40 \times 1 = 0.04$

Equivalents of $H_2SO_4 = MxVxn_f = 10^{-3} \times 10 \times 2 = 0.02$

$M_{NaOH} \cdot V_{NaOH} \cdot (n_f)_{NaOH} - M_{H2SO4\cdot} \cdot V_{H2SO4\cdot}(n_f)_{H2SO4} = M \cdot V_{total}$

$10^{-3} \times 40 \times 1 - 10^{-3} \times 10 \times 2 = M.50$

$M = 4 \times 10^{-4}$

$pOH = -\log M$

$= 4 - 2\log 2$

$= 3.4$

$pH = 14 - 3.4 = 10.6$

47. (23.03) All nuclear processes follow first order kinetics, and hence

$t_{1/2} = \frac{0.693}{\lambda}$

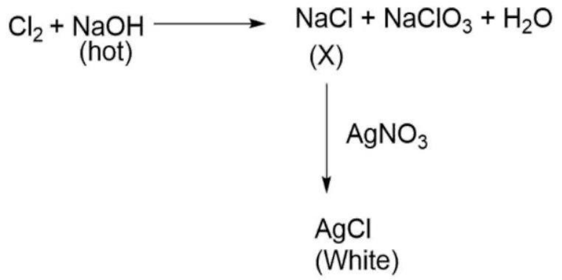
$\lambda = 0.1\ (\text{year})^{-1}$

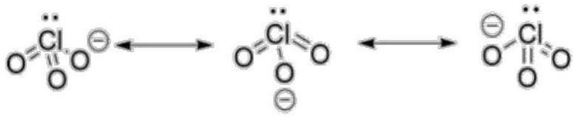
$t = \frac{2.303}{\lambda}\log\left(\frac{a_0}{a_t}\right)$

$t = \frac{2.303}{0.1}\log\left(\frac{a_0}{0.1a_0}\right)$

On solving, $t = 23.03$ years

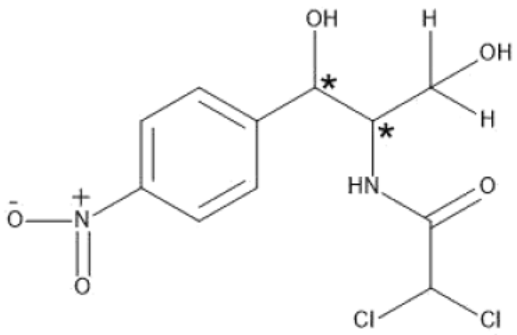
48. (1.67)





Bond order $= \frac{\text{Total no of bonds}}{\text{Total resonating structures}} = \frac{5}{3} = 1.67$

49. (2)



50. (-2.7)

$\Delta H = \Delta U + \Delta n_g RT$

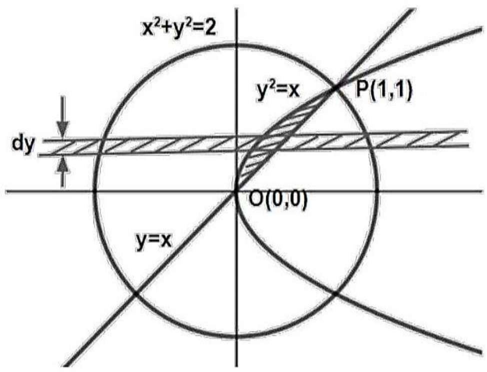
$\Delta H = 2100 + (2 \times 2 \times 300)\ (R = 2\text{calK}^{-1}\ \text{mol}^{-1})$

$= 3300\text{cal}$

$\Delta G = \Delta H - T\Delta S$

$\Delta G = 3300 - (300 \times 20) = -2700\text{cal} = -2.7\text{kcal}$

51. (b) Required area = area of the circle - area bounded by given line and parabola



Required area $= \pi r^2 - \int_0^1 (y - y^2)dy$

Area $= 2\pi - \left(\frac{y^2}{2} - \frac{y^3}{3}\right)_0^1 = 2\pi - \frac{1}{6}$ sq. units

52. (b) $g(x) = x^2 + x - 1$

$g \circ f(x) = 4x^2 - 10x + 5$

$g(f(x)) = 4x^2 - 10x + 5$

$f^2(x) + f(x) - 1 = 4x^2 - 10x + 5$

Putting $x = \frac{5}{4}$ & $f\left(\frac{5}{4}\right) = t$

$$t^2 + t + \frac{1}{4} = 0$$

$$t = -\frac{1}{2}$$

$$f\left(\frac{5}{4}\right) = -\frac{1}{2}$$

53. (c) $e^y(y' - 1) = e^x$

$$\Rightarrow \frac{dy}{dx} = e^{x-y} + 1$$

Let $x - y = t$

$$1 - \frac{dy}{dx} = \frac{dt}{dx}$$

So, we can write

$$\Rightarrow 1 - \frac{dt}{dx} = e^t + 1$$

$$\Rightarrow -e^{-t}dt = dx$$

$$\Rightarrow e^{-t} = x + c$$

$$\Rightarrow e^{y-x} = x + c$$

$$1 = 0 + c$$

$$\Rightarrow e^{y-x} = x + 1$$

at $x = 1$

$$\Rightarrow e^{y-1} = 2$$

$$\Rightarrow y = 1 + \ln 2$$

54. (c) Any tangent to the parabola $y^2 = 4x$ is $y = mx + \frac{a}{m}$

Comparing it with $y = mx + 4$, we get $\frac{1}{m} = 4 \Rightarrow m = \frac{1}{4}$

Equation of tangent becomes $y = \frac{x}{4} + 4$

$y = \frac{x}{4} + 4$ is a tangent to $x^2 = 2by$

$\Rightarrow x^2 = 2b\left(\frac{x}{4} + 4\right)$

Or $2x^2 - bx - 16b = 0$,

$D = 0$

$b^2 + 128b = 0$,

$\Rightarrow b = 0$ (not possible),

$b = -128$

55. (a) $(k+1)\tan^2 x - \sqrt{2}\lambda \tan x = 1 - k$

$\tan^2(\alpha + \beta) = 50$

Now,

$$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1},\ \tan\alpha\tan\beta = \frac{k-1}{k+1}$$

$$\Rightarrow \left(\frac{\frac{\sqrt{2}\lambda}{k+1}}{1 - \frac{k-1}{k+1}}\right)^2 = 50$$

$$\Rightarrow \frac{2\lambda^2}{4} = 50$$

$\Rightarrow \lambda^2 = 100$

$\Rightarrow \lambda = 10$

56. (b) Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$

Now $2ae = 6 \& \frac{2a}{e} = 12$

$$\Rightarrow ae = 3 \& \frac{a}{e} = 6$$

$\Rightarrow a^2 = 18$

$\Rightarrow a^2e^2 = c^2 = a^2 - b^2 = 9$

$\Rightarrow b^2 = 9$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 9}{\sqrt{18}} = 3\sqrt{2}$

57. (a)

p	q	$p \to q$	$\sim p$	$q \to \sim p$	$(p \to q) \wedge (q \to \sim p)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p \to q) \wedge (q \to \sim p)$ is equivalent to $\sim p$

58. (c)

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$
$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$
$$\Rightarrow \begin{vmatrix} 2 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$
$$\Rightarrow (3b-2a)(c-a) - (4c-2a)(b-a) = 0$$
$$\Rightarrow 3bc - 2ac - 3ab + 2a^2 - [4bc - 4ac - 2ab + 2a^2] = 0$$
$$\Rightarrow -bc + 2ac - ab = 0$$
$$\Rightarrow ab + bc = 2ac$$
$$\Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$$

59. (b) Let 5 numbers be $a-2d, a-d, a, a+d, a+2d$

$5a = 25$

$a = 5$

$(a-2d)(a-d)a(a+d)(a+2d) = 2520$

$(25-4d^2)(25-d^2) = 504$

$4d^4 - 125d^2 + 121 = 0$

$4d^4 - 4d^2 - 121d^2 + 121 = 0$

$d^2 = 1$ or $d^2 = \frac{121}{4}$

$d = \pm\frac{11}{2}$

For $d = \frac{11}{2}, a + 2d$ is the greatest term, $a + 2d = 5 + 11 = 16$

60. (c) The roots of equation $x^2 + x + 1 = 0$ are complex cube roots of unity.

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

$$A^2 = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & \alpha & \alpha^2\\1 & \alpha^2 & \alpha\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\1 & \alpha & \alpha^2\\1 & \alpha^2 & \alpha\end{bmatrix}$$

$$A^2 = \frac{1}{3}\begin{bmatrix}3 & 0 & 0\\0 & 0 & 3\\0 & 3 & 0\end{bmatrix}$$

$$A^2 = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$$

$$A^4 = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$$

$$A^4 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = I$$

$A^4 = I$

$A^{28} = I$

Therefore, we get

$A^{31} = A^{28}A^3$

$A^{31} = IA^3$

$A^{31} = A^3$

61. (b) $x^k + y^k = a^k$

$$kx^{k-1} + ky^{k-1}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{y}{x}\right)^{1-k} = 0$$

$$\Rightarrow 1 - k = \frac{1}{3}$$

$$\Rightarrow k = \frac{2}{3}$$

62. (c) $z = x + iy$

$$\frac{x + iy - 1}{2x + 2iy + i} = \frac{(x-1) + iy}{2x + i(2y+1)}\left(\frac{2x - i(2y+1)}{2x - i(2y+1)}\right) = 1$$

$$\frac{2x(x-1) + y(2y+1)}{4x^2 + (2y+1)^2} = 1$$

$$2x^2 + 2y^2 - 2x + y = 4x^2 + 4y^2 + 4y + 1$$

$$x^2+y^2+x+\frac{3}{2}y+\frac{1}{2}=0$$

Circle's centre will be $\left(-\frac{1}{2},-\frac{3}{4}\right)$

Radius $=\sqrt{\frac{1}{4}+\frac{9}{16}-\frac{1}{2}}=\frac{\sqrt{5}}{4}$

Diameter $=\frac{\sqrt{5}}{2}$

63. (a) $y(\alpha)=\sqrt{2\frac{\tan\alpha+\cot\alpha}{1+\tan^2\alpha}+\frac{1}{\sin^2\alpha}}$

$$y(\alpha)=\sqrt{2\frac{1}{\sin\alpha\cos\alpha\times\frac{1}{\cos^2\alpha}}+\frac{1}{\sin^2\alpha}}$$

$$y(\alpha)=\sqrt{2\cot\alpha+\operatorname{cosec}^2\alpha}$$

$$y(\alpha)=\sqrt{(1+\cot\alpha)^2}$$

$$y(\alpha)=-1-\cot\alpha$$

$$\frac{dy}{d\alpha}=0+\operatorname{cosec}^2\alpha\Big|_{\alpha=\frac{5\pi}{6}}$$

$$\frac{dy}{d\alpha}=\operatorname{cosec}^2\frac{5\pi}{6}$$

$$\frac{dy}{d\alpha}=4$$

64. (a) $1+49+49^2+\cdots+49^{125}=\frac{49^{126}-1}{49-1}$

$$=\frac{(49^{63}+1)(49^{63}-1)}{48}$$

$$=\frac{(49^{63}+1)((1+48)^{63}-1)}{48}$$

$=\frac{(49^{63}+1)(1+48I-1)}{48}$; Where I is an integer

$=(49^{63}+1)I$

Greatest positive integer is $k=63$

65. (d) P is the centroid which is $\equiv\left(\frac{1+6+\frac{3}{2}}{3},\frac{1+5+2}{3}\right)$

$$\text{P}=\left(\frac{17}{6},\frac{8}{3}\right)$$

$Q = \left(-\frac{7}{6}, -\frac{1}{3}\right)$

$PQ = \sqrt{(4)^2 + (3)^2} = 5$

66. (yourself)

67. (c) $f(-7) = -3$ and $f'(x) \leq 2$

Applying LMVT in $[-7,0]$, we get

$$\frac{f(-7) - f(0)}{-7} = f'(c) \leq 2$$

$$\frac{-3 - f(0)}{-7} \leq 2$$

$f(0) + 3 \leq 14$

$f(0) \leq 11$

Applying LMVT in $[-7,-1]$, we get

$$\frac{f(-7) - f(-1)}{-7+1} = f'(c) \leq 2$$

$$\frac{-3 - f(-1)}{-6} \leq 2$$

$f(-1) + 3 \leq 12$

$f(-1) \leq 9$

Therefore, $f(-1) + f(0) \leq 20$

68. (a) Points $A(2,1,0), B(6,3,3) C(5,2,2)$

$\overrightarrow{AB} = (4,2,3)$

$\overrightarrow{AC} = (3,1,2)$

$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = (1,1,-2)$

Equation of the plane is $x + y - 2z = 3$.

Let the image of point $(2,1,6)$ is (l, m, n)

$$\frac{l-2}{1} = \frac{m-1}{1} = \frac{n-6}{-2} = \frac{-2(-12)}{6} = 4$$

$\Rightarrow l = 6, m = 5, n = -2$

Hence the image of R in the plane P is $(6,5,-2)$

69. (a)

Let $(1 + x + x^2 + \cdots + x^{2n})(1 - x + x^2 + \cdots + x^{2n}) = a_o + a_1 x + a_2 x^2 + \cdots$

Put $x = 1$

$2n + 1 = a_o + a_1 + a_2 + a_3 + \cdots \ldots \ldots \ldots$

Put $x = -1$

$2n + 1 = a_o - a_1 + a_2 - a_3 + \cdots \ldots \ldots \ldots.$

Add (1) and (2)

$2(2n + 1) = 2(a_o + a_2 + a_4 + \cdots \ldots \ldots ..$

$2n + 1 = 61$

$n = 30$

70. (a) $k =$ no. of consecutive heads

$P(k = 3) = \frac{5}{32}$ (HHHTH, HHHTT, THHHT, HTHHH, TTHHH)

$P(k = 4) = \frac{2}{32}$ (HHHHT, HHHHT)

$P(k = 5) = \dfrac{1}{32}$ (HHHHH)

$$P(\bar{3} \cap \bar{4} \cap \bar{5}) = 1 - \left(\frac{5}{32} + \frac{2}{32} + \frac{1}{32}\right) = \frac{24}{32}$$

$$\sum XP(X) = \left(-1 \times \frac{24}{32}\right) + \left(3 \times \frac{5}{32}\right) + \left(4 \times \frac{2}{32}\right) + \left(5 \times \frac{1}{32}\right) = \frac{1}{8}$$

71. (d) $f(a + b + 1 - x) = f(x)$

$x \to x + 1$

$f(a + b - x) = f(x + 1)$

$$I = \frac{1}{a + b}\int_a^b x(f(x) + f(x + 1))dx$$

From (1) and (2)

$$I = \frac{1}{a + b}\int_a^b (a + b - x)(f(x + 1) + f(x))dx$$

Adding (3) and (4)

$$2I = \int_a^b (f(x) + f(x + 1))dx$$

$$2I = \int_a^b f(x + 1)dx + \int_a^b f(x)dx$$

$$2I = \int_a^b f(a + b - x + 1)dx + \int_a^b f(x)dx$$

$$2I = 2\int_a^b f(x)dx$$

$I = \int_a^b f(x)dx\, ; x = t + 1, dx = dt$

$I = \int_{a-1}^{b-1} f(t+1)dt$

$I = \int_{a-1}^{b-1} f(x+1)dx$

72. (1800)

Selecting all 5 digits $= {}^5C_5 = 1$ way

Now, we need to select one more digit to make it a 6 digit number $= {}^5C_1 = 5$ ways

Total number of permutations $= \frac{6!}{2!}$

Total numbers $= {}^5C_5 \times {}^5C_1 \times \frac{6!}{2!} = 1800$

73. (72)

$$\lim_{x\to 2} \frac{3^x + \frac{3^x}{3} - 12}{\frac{1}{3^{\frac{x}{2}} + \frac{3}{3^x}}}$$

$$\lim_{x\to 2} \frac{\frac{4}{3}3^x - 12}{\frac{1}{3^{\frac{x}{2}} + \frac{3}{3^x}}}$$

Put $3^{\frac{x}{2}} = t$

$$\lim_{t\to 3} \frac{\frac{4t^2}{3} - 12}{-\frac{3}{t^2} + \frac{1}{t}} = \lim_{t\to 3} \frac{4(t^2-9)t^2}{3(-3+t)} = \lim_{t\to 3} \frac{4t^2(3+t)}{3} = \frac{4\times 9\times 6}{3} = 72$$

74. (18) For N Natural number variance

$$\sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$$

$$\frac{\sum x_i^2}{N} = \frac{1^2 + 2^2 + 3^2 + \cdots N \text{ term}}{N} = \frac{N(N+1)(2N+1)}{6N}$$

$$\frac{\sum x_i}{N} = \frac{1 + 2 + 3 + \cdots N \text{ terms}}{N} = \frac{N(N+1)}{2N}$$

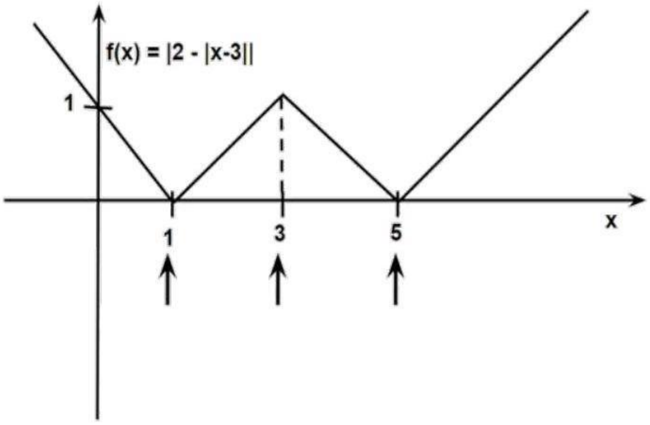
$\sigma^2 = \frac{N^2-1}{12} = 10$ (given)

$\Rightarrow N = 11$

Variance of $(2,4,6 \ldots) = 4 \times$ variance of $(1,2,3,4 \ldots) = 4 \times \frac{M^2-1}{12} = \frac{M^2-1}{3} = 16$ (given) $\Rightarrow M = 7$

Therefore, $N + M = 11 + 7 = 18$

75. (3)



There will be three points $x = 1,3,5$ at which $f(x)$ is non-differentiable.

So $f(f(1)) + f(f(3)) + f(f(5))$

$= f(0) + f(2) + f(0)$

$= 1 + 1 + 1$

$= 3$