

1. (b) $y = u_y t + \frac{1}{2} a_y t^2$

$$y = 5t + \frac{1}{2}(d)t^2$$

$$y = 5t + 2t^2$$

$$\text{and } x = 0 + \frac{1}{2}(10)(t^2) = 20$$

$$t = 2 \text{ s}$$

$$\Rightarrow y = 10 + 8 = 18 \text{ m}$$

2. (a) $\tau = MB \sin 30^\circ$

$$0.018 = MB \left(\frac{1}{2} \right)$$

$$MB = 0.036$$

$$W = \Delta U = 2MB = 0.072 \text{ J}$$

3. (a) By property of electromagnetic wave spectrum.

4. (b) $E_x = \frac{Ax}{(x^2 + a^2)^{3/2}}$

$$\frac{-dv}{dx} = \frac{Ax}{(x^2 + a^2)^{3/2}}$$

$$\int_0^v dv = - \int_\infty^x \frac{Ax}{(x^2 + a^2)^{3/2}} dx$$

$$v = \frac{A}{(x^2 + a^2)^{1/2}}$$

5. (a)

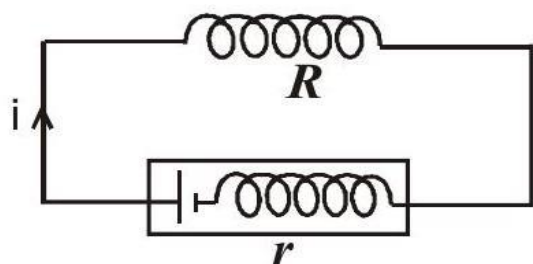
6. (b)

$$P_0 = 0.5 \text{ W}$$

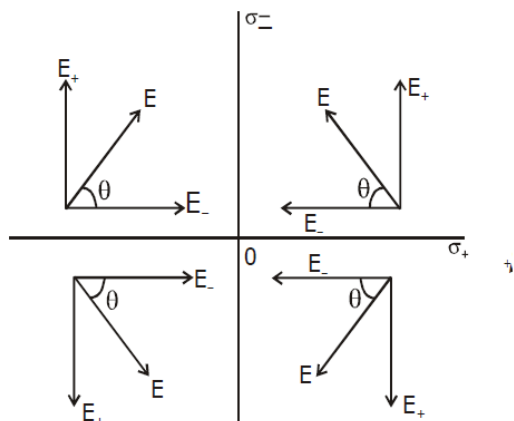
$$i(2.5) = 0.5$$

$$i = 1/5 \text{ A}$$

$$P_r = \left(\frac{1}{5} \right) (0.5) = 0.1 \text{ W}$$



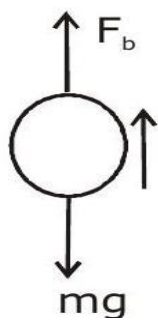
7. (a)



$$|\vec{E}_+| > |\vec{E}_-|$$

$$\theta > 45^\circ$$

8. (d)



$$F_b - mg = ma \Rightarrow m = \frac{F_b}{g + a}$$

$$m = \frac{v \cdot \rho_w g}{g + a}$$

$$m = \frac{(4/3)\pi r^3 \cdot \rho_w \cdot g}{g + a} = 4.15 \text{ gm}$$

9. (b)

$$B_A = \frac{\mu(b) \left(\frac{3\pi}{2}\right)}{2(a)(2\pi)} = \frac{3\mu}{4a}$$

$$B_B = \frac{\mu(c) \left(\frac{5\pi}{3}\right)}{2(2a)(2\pi)} = \frac{5\mu}{8a}$$

$$\frac{B_A}{B_B} = \frac{3\mu}{4a} \times \frac{8a}{5\mu} = 6:5$$

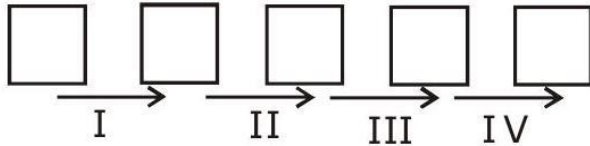
$$10. (c) V_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$

$$V_1 = \frac{\frac{m}{2} - m/3}{\frac{m}{2} + m/3} V_0 = V_0/5$$

$$\lambda' = \frac{h}{\frac{m}{2} \cdot \frac{V_0}{5}} = 5 \cdot \frac{h}{\frac{m}{2} \cdot V_0} = 5\lambda_0$$

$$\Delta\lambda = 4\lambda_0$$

11. (a)

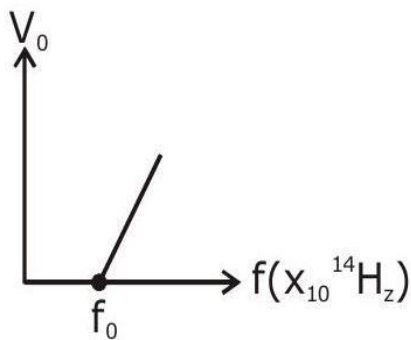


There will be total 4 collisions in each collision K.E. decreasing by 50%

$$E_f = \frac{1}{2^4} E_i = \frac{E_i}{16} = 6.25\%$$

i.e. 93.75% loss

12. (b)



$$\phi = hf_0 = 6.62 \times 10^{-34} \times 5.5 \times 10^{14}$$

$$= 36.41 \times 10^{-20} \text{ J} = 2.27 \text{ eV}$$

13. (c) Heat loss by water

$$Q = m_w s \Delta\theta$$

$$= \left(\frac{200}{1000} \right) \cdot (4200)(25) = 21000 \text{ J}$$

$$\text{and } \Delta m_i L = 21000$$

$$\Delta m_i = \frac{21000}{3.4 \times 10^5} \times 10^3 \text{ gm} = 61.7 \text{ grams}$$

$$14. (a) p = p_0 \cos^2 \omega t$$

$$E_{\text{avg}} = \langle p \rangle \cdot T = \frac{p_0}{2} T$$

$$E_{\text{avg}} = \langle P \rangle \cdot T = \frac{p_0}{2} \cdot \frac{2\pi}{\omega} = \frac{10^{-3} \times 3.14}{31.4} = 10^{-4} \text{ J}$$

15. (d)

$$1.5 = (2n_1 + 1)\lambda/2$$

$$5 = n_2\lambda$$

n_1 & n_2 are integer

$$n_1 = 1, n_2 = 5$$

$$n_1 = 2, n_2 \text{ is not integer}$$

$$n_1 = 3, n_2 \text{ is not integer}$$

$$n_1 = 4, n_2 = 15, \lambda = 1/3$$

16. (d)

17. (b)

$$\Delta q \cdot q / \frac{q}{+q}$$

$$\Delta U = \frac{1}{4\pi\epsilon_0} \cdot \frac{4q \cdot q}{(3d/2)} - \frac{1}{4\pi\epsilon_0} \cdot \frac{4q \cdot q}{(d/2)}$$

$$= \frac{4q^2}{4\pi\epsilon_0} \left(\frac{2}{d} \right) \left(-\frac{2}{3} \right)$$

$$= (-) \frac{4q^2}{3\pi\epsilon_0 \cdot d}$$

$$= \text{decrease by } (-)$$

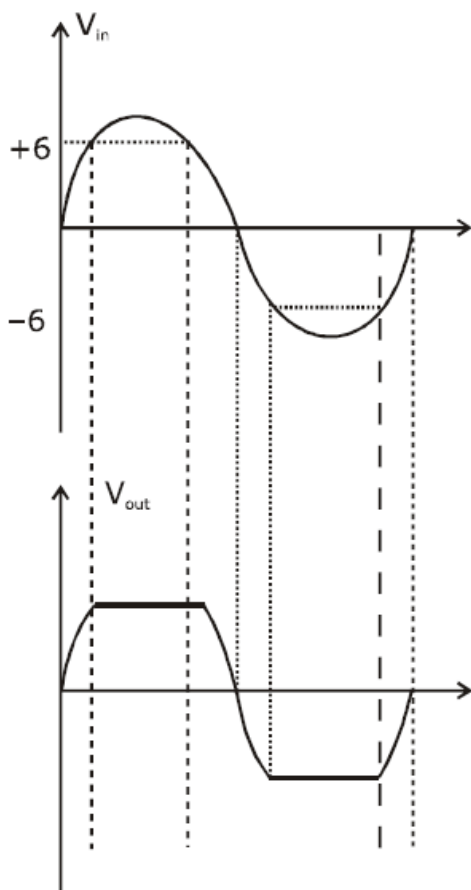
18. (a) $\rightarrow V, h$ curve will be parabolic

$\rightarrow$ downward velocity is negative and upward is positive

$\rightarrow$ when ball is coming down graph will be in IV quadrant and when going up graph will be in I quadrant

19. (a) $\frac{dQ}{dt} = \frac{Kl\Delta T}{A}$

20. (c)



21. (10553) $\frac{1}{R} = 912 \text{ \AA}$

in Paschen series

$$\frac{1}{\lambda_s} = R \left(\frac{1}{3^2} \right) = \frac{R}{9}$$

$$\frac{1}{\lambda_l} = R \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = \frac{7R}{144}$$

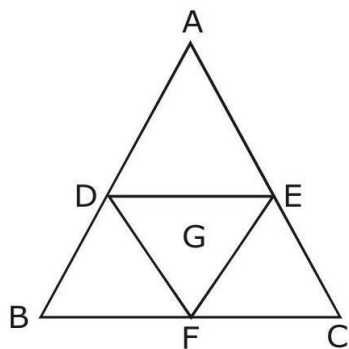
$$(\lambda, -\lambda_s) = \left(\frac{144}{7} - 9 \right) R = 10553 \text{ \AA}$$

22. (266)

$$(0.1) \left(\frac{3}{2} R \right) (T - 200) = (0.05) \left(\frac{3}{2} R \right) (400 - T)$$

$$T = 266.6 \text{ K}$$

23. (11)



Let $I_0 = kmc^2$

$$I_{DEF} = K \left(\frac{m}{\ell} \right) \left(\frac{\ell}{2} \right)^2 = \left(\frac{I_0}{16} \right)$$

and $I_{ADE} = I_{BDE} = I_{EFC} = I$

$$3I = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$$

$$\Rightarrow I = \frac{5I_0}{16}$$

$$I_{\text{remaining}} = 2I + \frac{I_0}{16} = \frac{11I_0}{16}$$

24. (6.25) $L = 20, f_0 = 1 \text{ cm}, M = 100$

$$M = \frac{v_0}{u_0} \left(1 + \frac{D}{f_e} \right)$$

$$M = \frac{L}{f_0} \left(1 + \frac{D}{f_e} \right) [v_0 \approx L, u_0 \approx f_0]$$

on solving we get

$$f_e = 6.25 \text{ cm}$$

25. (20) $I_f \omega_f = I_i \omega_i$

$$I_i = \frac{MR^2}{2}$$

$$I_f = \frac{MR^2}{2} + \frac{M(R/2)^2}{2}$$

$$= \frac{5}{4} \cdot \frac{MR^2}{2}$$

$$\left[\frac{MR^2}{2} + \frac{M}{2} \left(\frac{R}{2} \right)^2 \right] \omega' = \left(\frac{MR^2}{2} \right) \cdot \omega$$

$$\left[\frac{MR^2}{2} \cdot \left(\frac{5}{4} \right) \right] \omega' = \frac{MR^2}{2} \omega$$

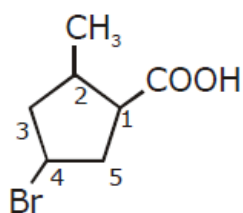
$$\omega' = \frac{4}{5} \omega$$

$$\text{loss of K.E.} = \frac{\text{Loss}}{K_i} \times 100 = \frac{\omega^2 - \omega'^2 (5/4)}{\omega^2} \times 100$$

$$\frac{\omega^2 - \frac{16}{25} \omega^2 \left(\frac{5}{4}\right)}{\omega^2} = \left(1 - \frac{80}{100}\right) \times 100$$

$$= 20\%$$

26. (b)

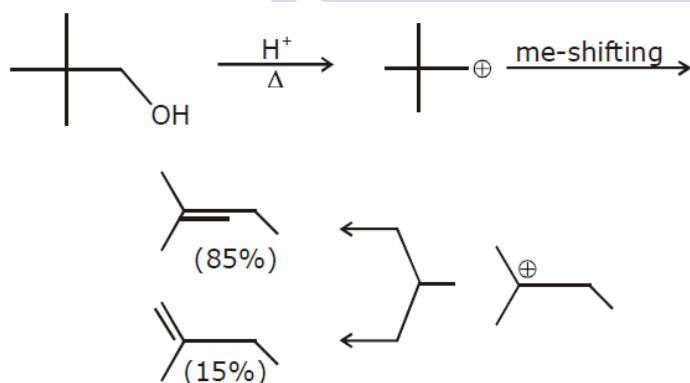


4-Bromo-2-methylcyclopentane carboxylic acid

27. (a)

28. (c) $O^{2-} > F^- > Na^+ > Mg^{2+}$

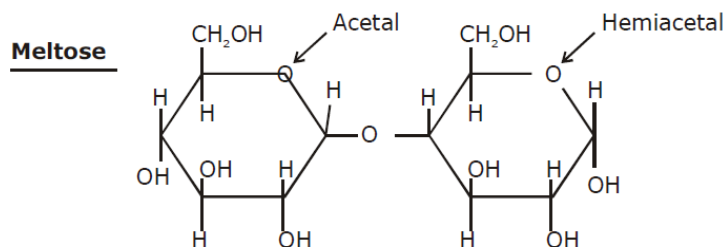
29. (c)



30. (d)

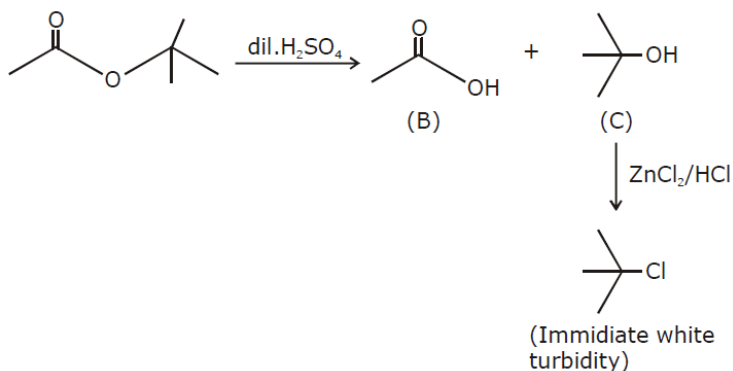
31. (b)

32. (a)

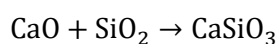
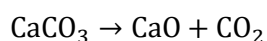


33. (A)

34. (d)



35. (c) Lime stone finally goes to slag formation



slag

36. (a) For ideal gas

$$\left. \frac{\delta v}{\delta v} \right|_T = 0 \text{ \& } \left. \frac{\delta H}{\delta v} \right|_T = 0$$

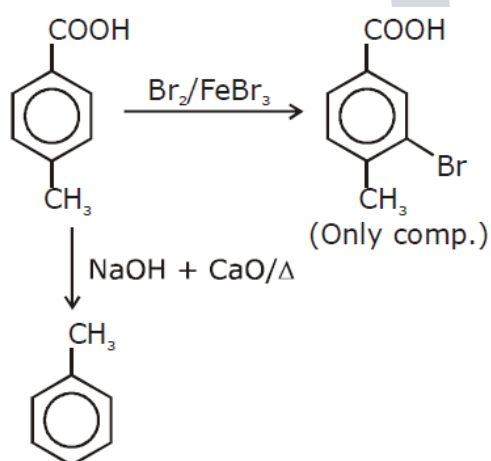
(a) Hence function of temp. only.

(b) Compressibility factor (z) = 1 Always

$$(c) C_{p,m} - C_{v,m} = R$$

$$(d) dv = nC_{v,m}^m dT \text{ for all process}$$

37. (b)

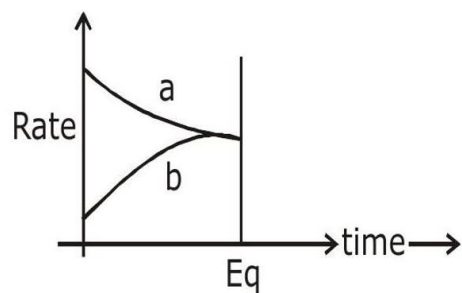


38. (b) At equilibrium

Rate of forward = Rate of backward

$$a = b$$

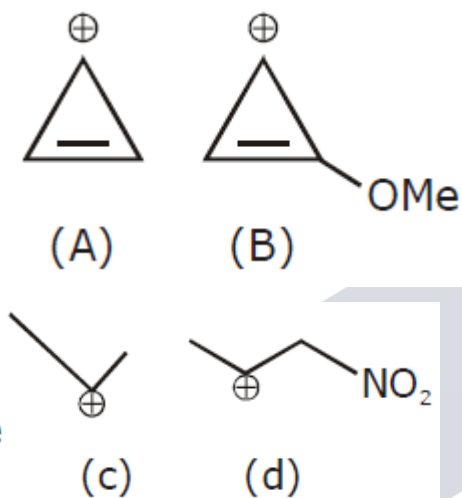
Hence



39. (d)

40. (b) Three linkage isomer NO_2^- ; ONO^-

41. (a)



Order or stability

$(B) > (A) > (C) > (D)$

42. (d)

43. (c)

44. (d) $Z = 101 \rightarrow [R_n]^{86}7s^25f^{13}$

Actinoids

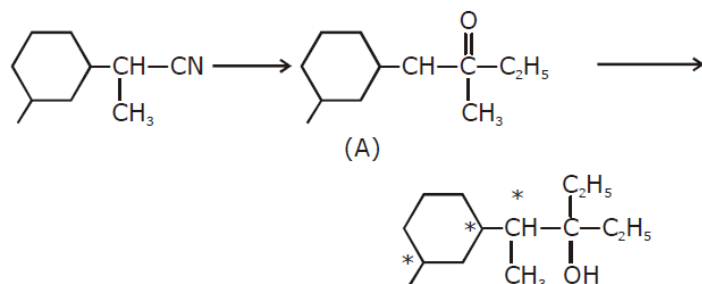
$Z = 104 \rightarrow [R_n]^{86}7s^25f^{14}6d^2$

4th group element

Ans Actinoids & 4th group

45. (b) Li_2O , Na_2O_2 , K_2O_2

46. (3)



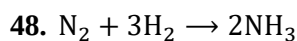
3 chiral center is present in final products

$$47. 550 = \frac{1}{4} \times p_{C_6H_{14}}^0 + \frac{3}{4} \times p_{C_7H_{16}}^0$$

$$560 = \frac{1}{5} \times p_{C_6H_{14}}^0 + \frac{4}{5} \times p_{C_7H_{16}}^0$$

$$p_{C_7H_{16}}^0 = [560 \times 5 - 550 \times 4]$$

$$= 550 + 50 = 600 \text{ mm of Hg}$$



2800 g 1000 g

100 mol 500 mol

L.R.

mole of NH_3 produced = 200 mol

mass = 3400 g

49. (60)

$$t_{75\%} = 90 \text{ min} = 2 \times t_{1/2}$$

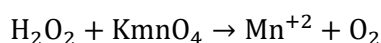
$$t_{1/2} = 45 \text{ min}$$

$$\frac{\ln(b)}{45} \times t_{60\%} = \ln\left\{\frac{100}{40}\right\}$$

$$t_{60\%} = 45 \times \frac{0.4}{0.3}$$

$$t_{60\%} = 60 \text{ min}$$

50. (85)



$$[\text{moles of } H_2O_2] \times 2 = \frac{0.316}{158} \times 5$$

$$\text{moles of } H_2O_2 = 5 \times 10^{-3}$$

$$\text{mass of } H_2O_2 = 170 \times 10^{-3} \text{ g}$$

$$\% \text{ purity} = \frac{170 \times 10^{-3}}{0.2} \times 100 = 85\%$$

51. (b)

$$xy' - y = x^2(x \cos x + \sin x)x > 0, y(\pi) = \pi$$

$$y' - \frac{1}{x}y = x\{x \cos x + \sin x\}$$

$$\text{I.F.} = e^{-\int \frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$$

$$\therefore y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot x(x \cos x + \sin x) dx$$

$$\frac{y}{x} = \int (x \cos x + \sin x) dx$$

$$\frac{y}{x} = \int \frac{d}{dx} (x \sin x) dx$$

$$\frac{y}{x} = x \sin x + C$$

$$\Rightarrow y = x^2 \sin x + cx$$

$$x = \pi, y = \pi$$

$$\pi = \pi C \Rightarrow C = 1$$

$$y = x^2 \sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

$$y' = 2x \sin x + x^2 \cos x + 1$$

$$y'' = 2 \sin x + 2x \cos x + 2x \cos x - x^2 \sin x$$

$$y''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4} \Rightarrow y\left(\frac{\pi}{2}\right) + y''\left(\frac{\pi}{2}\right) = 2 + \frac{\pi}{2}$$

52. (a) $\sum_{r=0}^{20} {}^{50-r}C_6$

$$\Rightarrow {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + \dots + {}^{31}C_6 + {}^{30}C_6$$

add and subtract ${}^{30}C_7$

Using

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \Rightarrow {}^{30}C_6 + {}^{30}C_7 = {}^{31}C_7$$

$${}^{31}C_6 + {}^{31}C_7 = {}^{32}C_7$$

Similarly solving

$${}^{51}C_7 - {}^{30}C_7$$

53. (b)

$$[x]^2 + 2[x + 2] - 7 = 0$$

$$[x]^2 + 2[x] - 3 = 0$$

$$\text{let } [x] = y$$

$$y^2 + 3y - y - 3 = 0$$

$$(y - 1)(y + 3) = 0$$

$$[x] = 1 \text{ or } [x] = -3$$

$$x \in [1, 2) \& x \in [-3, -2)$$

54. (c) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{9}{a^2} - \frac{9}{b^2} = 1$$

$$\text{Equation of normal} \Rightarrow \frac{a^2x}{3} + \frac{b^2y}{3} = a^2e^2$$

$$\text{at } x - \text{axis} \Rightarrow y = 0$$

$$\frac{a^2x}{3} = a^2e^2 \Rightarrow x = 3e^2 = 9$$

$$e^2 = 3$$

$$e = \sqrt{3}$$

$$e^2 = 1 + \frac{b^2}{a^2} = 3$$

$$b^2 = 2a^2$$

$$\text{put in equation 1}$$

$$\frac{9}{a^2} - \frac{9}{2a^2} = 1 \Rightarrow \frac{9}{2a^2} = 1 \Rightarrow a^2 = \frac{9}{2}$$

$$\therefore (a^2, e^2) = \left(\frac{9}{2}, 3\right)$$

55. (c) $L. R = \frac{2b^2}{a} = 10$

$$\phi(t) = \frac{5}{12} - \left(t - \frac{1}{2}\right)^2 + \frac{1}{4} = \frac{8}{12} - \left(t - \frac{1}{2}\right)^2$$

$$\therefore \phi(t)_{\max} = \frac{2}{3} = e$$

$$e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9} \Rightarrow \frac{b^2}{a^2} = \frac{5}{9}$$

$$\frac{b^2}{a \cdot a} = \frac{5}{9} \text{ from (a)}$$

$$\frac{5}{a} = \frac{5}{9} \Rightarrow a = 9$$

$$\therefore b^2 = 45$$

$$a^2 + b^2 = 45 + 81 = 126$$

$$56. (d) f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$$

$$x = \tan^2 t$$

$$dx = 2 \tan t \sec^2 t dt$$

$$f(x) = \int \frac{\tan t \cdot 2 \tan t \sec^2 t dt}{\sec^4 t}$$

$$= 2 \int \sin^2 t dt$$

$$x = 3 \Rightarrow t = \frac{\pi}{3}$$

$$x = 1 \Rightarrow t = \frac{\pi}{4}$$

$$\therefore f(c) - f(a) = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 - \cos 2t) dt \Rightarrow \left(t - \frac{1}{2} \sin 2t \right)_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

$$57. (b) T_n = 1 - (2n)^2(2n - 1)$$

$$= 1 - 4n^2(2n - 1)$$

$$= 1 - 8n^3 + 4n^2$$

$$S_n = \sum_{n=1}^{10} T_n = n - \sum 8n^3 + \sum 4n^2$$

$$= n - 8 \times \frac{n^2(n+1)^2}{4} + \frac{4n(n+1)(2n+1)}{6}$$

$$= 10 - 2 \times 100 \times 121 + \frac{2}{3} \times 10 \times 11 \times 21$$

$$= 10 - 24200 + 1540$$

$$= 10 - 22660$$

$$\therefore \text{Sum of series} = 11 - 22660 = \alpha - 220\beta$$

$$\alpha = 11, \beta = 103$$

$$58. (a)$$

$$\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$$

$$\int \underbrace{x \sec x}_I \cdot \underbrace{\frac{x \cos x}{(x \sin x + \cos x)^2}}_{II} dx$$

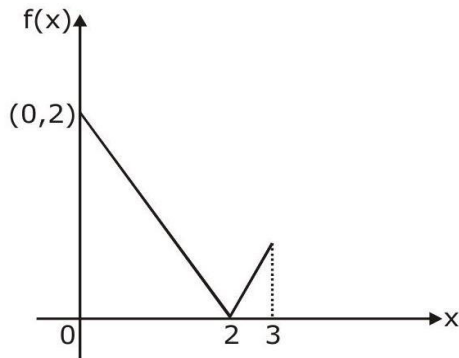
$$x \sec x \left(\frac{-1}{x \sin x + \cos x} \right) + \int \frac{\sec x + x \sec x \tan x}{(x \sin x + \cos x)} dx$$

$$\Rightarrow \frac{-x \sec x}{x \sin x + \cos x} + \int \frac{(\cos x + x \sin x)}{\cos^2 x (x \sin x + \cos x)} dx \Rightarrow \frac{-x \sec x}{x \sin x + \cos x} + \tan x + C$$

59. (c) $f(x) = |x - 2|$

$$g(x) = ||x - 2| - 2| = \begin{cases} \text{if } x \geq 2 & \Rightarrow |x - 4| \\ \text{if } x < 2 & \Rightarrow |-x| \end{cases}$$

$$\therefore \int_0^3 (g(x) - f(x)) dx$$



$$= \int_0^3 g(x) dx - \int_0^3 f(x) dx$$

$$= \int_0^2 x dx + \int_2^3 (4 - x) dx - \int_0^2 (2 - x) dx - \int_2^3 (x - 2) dx$$

$$\Rightarrow \left(\frac{x^2}{2} \right)_0^2 + \left(4x - \frac{x^2}{2} \right)_2^3 - \left(\frac{x^2}{2} - 2x \right)_0^2 - \left(\frac{x^2}{2} - 2x \right)_2^3$$

$$\Rightarrow 2 + \left\{ 12 - \frac{9}{2} - 8 + 2 \right\} + \{ 2 - 4 \} - \left(\frac{9}{2} - 6 - 2 + 4 \right)$$

$$= 2 + \left\{ 6 - \frac{9}{2} \right\} - 2 - \left\{ \frac{9}{2} - 4 \right\} = 2 + \frac{3}{2} - \left(2 + \frac{1}{2} \right) = \frac{7}{2} - \frac{5}{2} = 1$$

60. (a)

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$$

$$\Rightarrow x\{x^2 - 2\} + 2\{-2x + 7\} + 3\{4 - 7x\}$$

$$= x^3 - 2x - 4x + 14 + 12 - 21x$$

$$f(x) = x^3 - 27x + 26$$

$$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$$

$$\text{Max at } x_0 = -3$$

$$\therefore \vec{a} = (-3, -2, 3), \vec{b} = (-2, -3, -1), \vec{c} = (7, -2, -3)$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 6 + 6 - 3 - 14 + 6 + 3 - 21 + 4 - 9 = 25 - 47 = -22$$

61. (b) $AB = \sqrt{4 + 1} = \sqrt{5}$

$$\frac{1}{2} \times \sqrt{5} \times x = 5\sqrt{5}$$

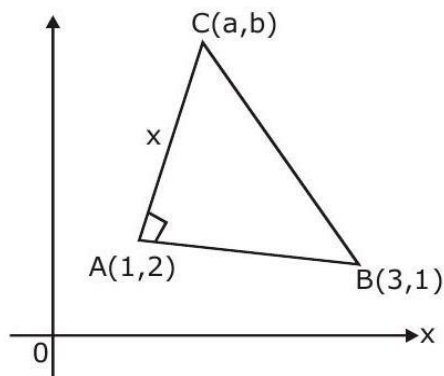
$$x = 10$$

$$m_{AB} = \frac{1}{-2}$$

$$m_{AC} = 2 = \tan \theta$$

$$\therefore \sin \theta = \frac{2}{\sqrt{5}}, \cos \theta = \frac{1}{\sqrt{5}}$$

by parametric co - ordinates $a = 1 + 10 \times \frac{1}{\sqrt{5}} = 1 + 2\sqrt{5}$



62. (a)

$$f(b) = 8, f'(b) = 5, f'(x) \geq 1, f''(x) \geq 4$$

$$x \in (1,6)$$

$$\int_2^5 f'(x) \geq \int_2^5 1 dx$$

$$f(5) - f(b) \geq 3$$

$$f(5) \geq 11$$

$$\text{also } \int_2^5 f''(x) dx \geq \int_2^5 4 dx$$

$$f(5) + f'(5) \geq 28$$

$$f'(5) - f'(b) \geq 12$$

$$f'(5) \geq 17$$

63. (b)

64. (b) $u = \frac{2z+i}{z-ki}, z = x + iy$

$$= \frac{2x + i(2y + 1)}{x + i(y - k)} \times \frac{x - i(y - k)}{x - i(y - k)}$$

$$\Rightarrow \frac{2x^2 + (2y + 1)(y - k) + i\{2xy + x - 2xy + 2xk\}}{x^2 + (y - k)^2}$$

$$\text{Re}(u) + \text{Im}(u) = 1$$

$$2x^2 + (2y + 1)(y - k) + x + 2xk = x^2 + (y - k)^2$$

at y - axis, $x = 0$

$$(2y + 1)(y - k) = (y - k)^2$$

$$2y^2 + y - 2yk - k = y^2 + k^2 - 2yk$$

$$y^2 + y - (k + k^2) = 0(y_1, y_2)$$

diff. of roots = 5

$$\sqrt{1 + 4k + 4k^2} = 5$$

$$4k^2 + 4k = 24$$

$$k^2 + k - 6 = 0$$

$$(k + 3)(k - 2) = 0$$

$$k = 2$$

65. (d)

$$\begin{bmatrix} c & is \\ is & c \end{bmatrix} \begin{bmatrix} c & is \\ is & c \end{bmatrix} = z \begin{bmatrix} c^2 - s^2 & 2ics \\ 2ics & c^2 - s^2 \end{bmatrix} = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix} \text{ (where } c = \cos \theta, s = \sin \theta \text{)}$$

$$A^5 = \begin{bmatrix} \cos(2^4\theta) & i\sin(2^4\theta) \\ i\sin(2^4\theta) & \cos(2^4\theta) \end{bmatrix}$$

$$a = d = \cos(16\theta)$$

$$b = c = i\sin(16\theta)$$

$$a^2 - b^2 = \cos^2(16\theta) + \sin^2 16\theta = 1$$

66. (c)

$$\frac{5 + 7 + 10 + 12 + 14 + 15 + x + y}{8} = 10$$

$$x + y = 17$$

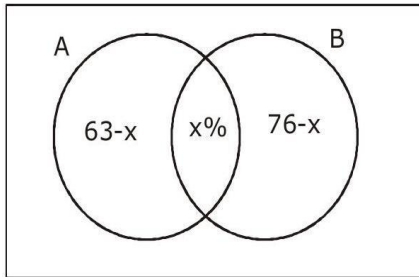
$$\text{variance} = \frac{739 + x^2 + y^2}{8} - 100 = 13.5$$

$$x^2 + y^2 = 169$$

$$\therefore x = 12, y = 5$$

$$|x - y| = 7$$

67. (d)



$$A \cup B = 13 - x \leq 100$$

$$x \geq 39$$

$$\text{also } x \leq 63$$

68. (d)

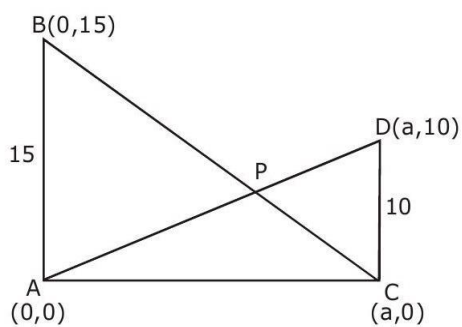
	p	q	$\sim q$	$q \vee p$	$p \leftrightarrow \sim q$	$(q \vee p) \rightarrow (p \leftrightarrow \sim q)$
$S_1 =$	T	T	F	T	F	F
	T	F	T	T	T	T
	F	T	F	T	T	T
	F	F	T	F	F	T

S_1 is not correct

p	q	$\sim q$	$\sim p$	$\sim p \leftrightarrow q$	$\sim q \wedge (\sim p \leftrightarrow q)$
T	T	F	F	F	F
T	F	T	F	T	T
F	T	F	T	T	F
F	F	T	T	F	F

S_2 is false

69. (d)



$$\text{equation of } AD: y = \frac{10x}{a}$$

$$\text{equation of } BC: \frac{x}{a} + \frac{y}{15} = 1$$

$$\Rightarrow \frac{a \cdot y}{10a} + \frac{y}{15} = 1 \Rightarrow \frac{3y + 2y}{30} = 1$$

$$5y = 30 \Rightarrow y = 6$$

70. (a) $(a + \sqrt{2}b \cos x)(a - \sqrt{2}b \cos y) = a^2 - b^2$

diff both sides w.r.t y

$$-\sqrt{2}b \sin x \cdot \frac{dx}{dy} (a - \sqrt{2}b \cos y) + (a + \sqrt{2}b \cos x)(\sqrt{2}b \sin y) = 0$$

$$x = y = \frac{\pi}{4} \Rightarrow \frac{-b dx}{dy} (a - b) + (a + b)(b) = 0$$

$$\frac{dx}{dy} = \frac{a + b}{a - b}$$

$$f(x + y) = f(x) + f(y) + xy^2 + x^2y$$

71. $x = y = 0$

$$f(0) = 2f(0) \Rightarrow f(0) = 0$$

Partially diff. w.r.t. x

$$f(x + y) = f(x) + y^2 + 2xy$$

$$x = 0, y = x$$

$$f'(x) = f'(0) + x^2$$

$$f'(x) = 1 + x^2$$

given $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$

$$\therefore f(x) = x + \frac{x^3}{3} + c$$

by L' hospital

put $x = 0 \Rightarrow c = 0$

$$\lim_{x \rightarrow 0} \frac{f'(x)}{1} = 1$$

$$f'(3) = 10$$

72. $p_1 + \lambda p_2 = 0$

$$(x + 4y - z + 7) + \lambda(3x + y + 5z - 8) = ax + by + 6z - 15$$

$$\frac{1 - 3\lambda}{a} = \frac{4 + \lambda}{b} = \frac{-1 + 5\lambda}{6} = \frac{7 - 8\lambda}{-15}$$

$$\therefore 15 - 75\lambda = 42 - 48\lambda$$

$$-27 = 27\lambda$$

$$\lambda = -1$$

$$\therefore \text{plane is } -2x + 3y - 6z + 15 = 0$$

$$d = \left| \frac{-6 + 6 + 6 + 15}{\sqrt{4 + 9 + 36}} \right| = 3$$

73.

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$$1(a + 7) + 2(2a - 1) + 3(-14 - 1) = 0$$

$$a + 7 + 4a - 2 - 45 = 0$$

$$5a = 40$$

$$a = 8$$

$$D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0$$

$$\Rightarrow 9(8 + 7) + 2(8b - 24) + 3(-7b - 24) = 0$$

$$\Rightarrow 135 + 16b - 48 - 21b - 72 = 0$$

$$15 = 5b \Rightarrow b = 3$$

$$a - b = 5$$

74. (8)

$$(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$$

$$a_7 = \text{coeff of } x^7$$

$$a_{13} = \text{coeff of } x^{13}$$

$$\frac{10!}{p!q!r!} (2x^2)^p (3x)^q (4)^r$$

$$\text{for } x^7$$

$$p \quad q \quad r$$

$$3 \quad 1 \quad 6$$

$$2 \quad 3 \quad 5$$

$$1 \quad 5 \quad 4$$

$$0 \quad 7 \quad 3$$

$$a_7 = \frac{2^3 \cdot 3 \cdot 10!}{3!6!} + \frac{10! 2^2 \cdot 3^3}{2!3!5!} + \frac{10! 2 \cdot 3^5}{5!4!} + \frac{10! \cdot 3^7}{7!3!}$$

$$\text{for } x^{13}$$

$$p \quad q \quad r$$

$$6 \quad 1 \quad 3$$

$$5 \quad 3 \quad 2$$

$$4 \quad 5 \quad 1$$

$$3 \quad 7 \quad 0$$

$$a_{13} = \frac{2^6 \cdot 3 \cdot 10!}{6!3!} + \frac{2^5 \cdot 3^3 \cdot 10!}{5!3!2!} + \frac{2^4 \cdot 3^5 \cdot 10!}{4!5!} + \frac{2^3 \cdot 10!}{3!7!} \therefore \frac{a_7}{a_{13}} = 8$$

75. (3)

$$P(H) = \frac{1}{10}; P(M) = \frac{9}{10}$$

$$P(H) + P(M) \cdot P(H) + P(M) \cdot P(M) \cdot P(H) + \cdots \dots \dots$$

$$= 1 - P(M)^n \geq \frac{1}{4}$$

$$= 1 - \left(\frac{9}{10}\right)^n \geq \frac{1}{4}$$

$$\left(\frac{9}{10}\right)^n \leq \frac{3}{4}; n \geq 3$$

