1. (a)

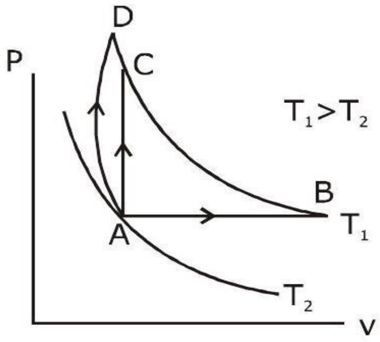
$E_{AB} = E_{AC} = E_{AD}$

$dU = \frac{nfR}{2}(T_f - T_i)$

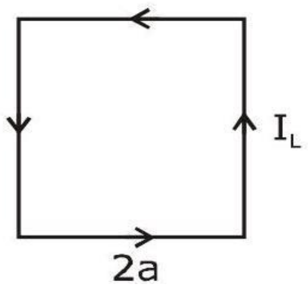
$w_{AB} > 0(+)V \uparrow$

$W_{AC} = 0$ V const.

$W_{AD} < 0(-)V \downarrow$



2. (a)
3. (c)

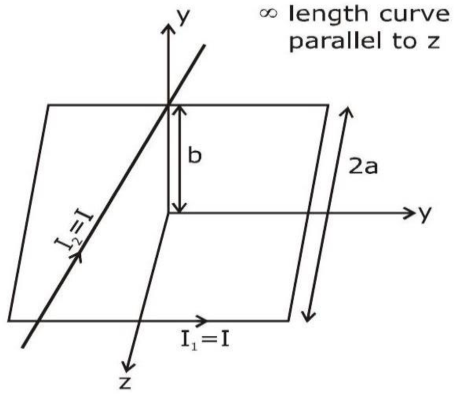


$M = I_2(2a)^2 = 4a^2 I_2$

(magnetic moment)

$B = \frac{\mu_0 I_2}{2\pi b}$

$\tau = \text{MB}\sin\theta$



θ angle between B and M$[\theta = 90°]$

$$\tau = 4(a^2 I_2)\frac{\mu_0 I_1}{2\pi b}$$

$$\tau = \frac{2\mu_0 I_1 I_2 a^2}{\pi b} = \frac{2\mu_0 I^2 a^2}{\pi b}$$

4. (d) $S_0 = \frac{\Delta P}{\beta k} = \frac{\Delta P}{\rho v^2 \frac{\omega}{V}} = \frac{\Delta P}{\rho V \omega} = \frac{\Delta P}{\rho v 2\pi f}$

$\therefore$ Proportionally constant $= 1$

$$S_0 = \frac{\Delta P}{\rho v f}$$

$$= \frac{10}{1 \times 300 \times 1000}\ \mathrm{m}$$

$$= \frac{10}{300}\ \mathrm{mm}$$

$$= \frac{3}{90}$$

$$\simeq \frac{3}{100}\ \mathrm{mm}$$

5. (d) $+V_C^V\ U_i = \frac{1}{2} C\ V^2 + \frac{1}{2}(2C)(2\ V)^2$

$$\underset{2\ V}{\overset{+2CC^-}{\rightarrow}} = \frac{9}{2}Cv^2$$

$\ddagger$

$$q_1 + q_2 = q_1{}' + q_2{}'$$

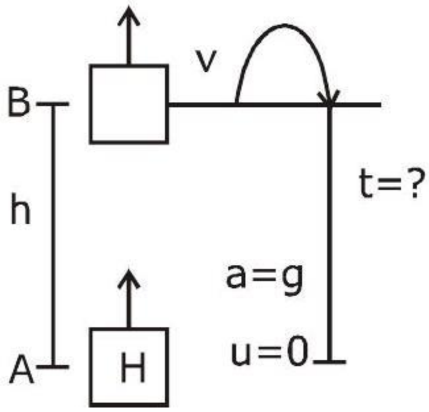
$$-CV + (2C(2V) = (C + 2C)V'$$

$$V' = \frac{3CV}{3C} = V$$

$$U_f = \frac{1}{2}CV^2 + \frac{1}{2}(2C)V^2$$

$$U_f = \frac{3}{2}CV^2$$

6. (a)



$V_B^2 = 0^2 + 2gh$

$V_B = \sqrt{2gh}$

$-h = (V_B)t - \frac{1}{2}gt^2$

$-h = \sqrt{2gh}t - \frac{1}{2}gt^2$

$gt^2 - 2\sqrt{2ght} - 2h = 0$

$t = \frac{\sqrt{2ght} \pm \sqrt{8gh + 8gh}}{2\ g} = \frac{2\sqrt{2gh} \pm \sqrt{16}gh}{2\ g} = \frac{\sqrt{2gh} + 2\sqrt{gh}}{g}$

$t = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{h}{g}} = \sqrt{\frac{h}{g}}(\sqrt{2} + 2) = 3.4\sqrt{\frac{h}{g}}$

7. (b)

$\left(\frac{1}{2}mv^2\right) \times \frac{1}{2} = ms\Delta T\ s = 0.03\text{cal/y°C}$

$\frac{v^2}{4} = 126 \times \Delta T \qquad = \frac{0.03 \times 4.2\ \text{J}}{10^{-3}\text{kgC}}$

$v^2 = 4 \times 126 \times \Delta T \qquad = 126\ \text{J/kgC}$

$(210)^2 = 4 \times 126 \times \Delta T$

$210 \times 210 = 4 \times 126 \times \Delta T$

$44100 = 504 \times \Delta T$

$\Delta T = \frac{44100}{504} = 87.5°\text{C}$

8. (a) $k_i = \frac{1}{2}I\omega^2$

$k_f = \frac{1}{2}(4I)(\omega')^2$

$= 2I\left(\frac{\omega}{4}\right)^2 = \frac{1}{8}I\omega^2$

A.M.C

$I\omega = (I + 3I)\omega'$

$\omega' = \frac{I\omega}{4I} = \frac{\omega}{4}$

$= \frac{K_i - K_f}{K_i} \Rightarrow \frac{\frac{1}{2}I\omega^2 - \frac{1}{8}I\omega^2}{\frac{1}{2}I\omega^2}$

$\frac{\frac{3}{8}I\omega^2}{\frac{1}{2}I\omega^2} = \frac{3}{4}$

9. (b) $\triangle ABD$

$\tan 45 = \frac{h_1}{d}$

$\Rightarrow 1 = \frac{h_1}{d} \Rightarrow h_1 = d$

$\triangle ACE$

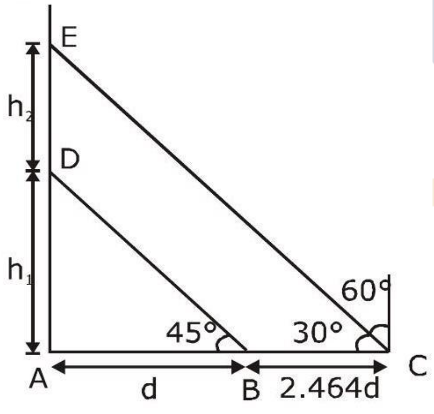
$\tan 30 = \frac{h_1 + h_2}{d + 2.464d}$

$0.5774 = \frac{d + h_2}{3.464\ d}$

$d + h_2 = 0.5774 \times 3.464 \times d$

$h_2 = 2.0001136\ d - d$

$h_2 = 2.000\ d - d = d$



10. (d) $\eta = \frac{P_{out}}{(P_{out} + P_{loss})} \times 100$

$I = \frac{P}{V}$

$= \frac{100}{220} = \frac{5}{11}$ A

$P_{loss} = I^2R$

$= \left(\frac{50}{11}\right)^2 \times 2 = 41.322$

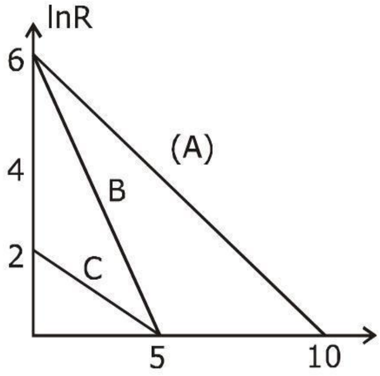
$\eta = \frac{1000}{1000 + 41.322} \times 100$

$\eta = 96\%$

11. (d)

$$R_A = R_0 A e^{-\frac{\ln 2}{T}(t)}$$
$$\ln(R_A) = \ln(R_0)$$
$$\lambda_A = \frac{6}{10} = \frac{\ln 2}{T_A}$$
$$\lambda_B = \frac{6}{5} = \frac{\ln 2}{T_B}$$
$$\lambda_C = \frac{2}{5} = \frac{\ln 2}{T_C}$$

$\ln(R_A) = \ln(R_0 A) - \lambda t$ ($\lambda =$ slope of graph)



$T_A = \frac{5}{3}\ln 2$

$I_B = \frac{5}{6}\ln 2\Big\} \Rightarrow T_A : T_B : T_C = \frac{1}{3} : \frac{1}{6} : \frac{1}{2}$

$T_C = \frac{5}{2}\ln 2$

$= 2 : 1 : 3$

12. (c) $g_{\text{at high}} = g_{\text{at depth}}$

$g_{\text{surface}} = \frac{GM}{R^2}$

$g\left(1 - \frac{d}{R}\right) = \frac{GMe}{(R+h)^2}$

$$g\left(1-\frac{d}{R}\right)=\frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}=\frac{g}{\left(1+\frac{R}{2R}\right)^2}=\frac{4g}{9}$$

$$\frac{d}{R}=1-\frac{4}{9}=\frac{5}{9}$$

13. (b)

$F_B = mg$

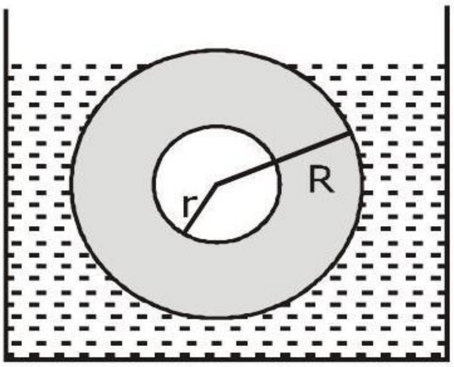
$\rho_\ell V_{body}\, g$ (displaced water) $=$ $\rho_b V_b g$ where mater present

$$\frac{4}{3}\pi R^3=\frac{\rho_b}{\rho_\ell}\left(\frac{4}{3}\pi R^3-\frac{4}{3}\pi r^3\right)$$

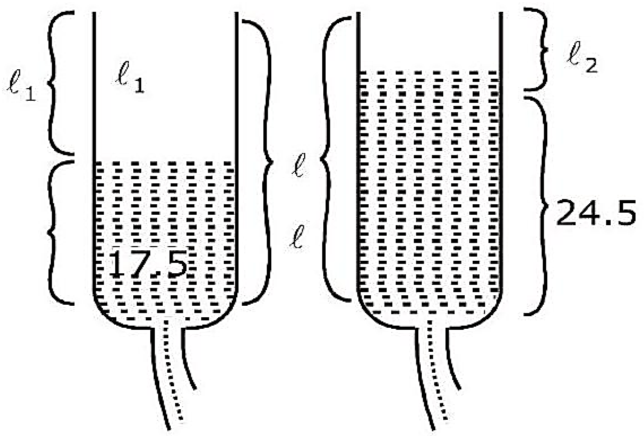
$$R^3=\frac{27}{8}(R^3-r^3)$$

$$\frac{8}{27}R^3=R^3-r^3\Rightarrow r^3=R^3-\frac{8R^3}{27}=\frac{19}{27}R^3$$

$$r=\frac{(19)^{1/3}}{3}\mathrm{R}\approx 0.88\approx\frac{8}{9}\mathrm{R}$$



14. (a)



$$\ell_1=\ell-17$$

$\ell_2 = \ell - 24.5$

$v = 2f(\ell_1 - \ell_2)$

$330 = 2 \times \mathrm{f} \times [(\mathrm{f} \times [(\ell - 17) - (\ell - 24.5)] \times 10^{-2}$

$165 = \mathrm{f} \times 7.5 \times 10^{-2}$

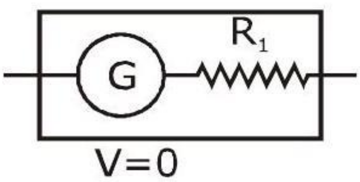
$$f = \frac{165 \times 1000}{7.5}$$

$\mathrm{f} = 2200\ \mathrm{Hz}$

15. (a)

16. (d) $V = I(R_1 + G)$

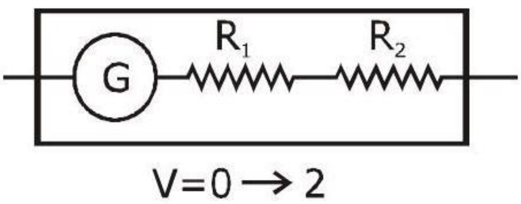
$$\frac{1}{2} = \frac{I(R_1 + G) \ldots .(i)}{I(R_1 + R_2 + G) \ldots ..(ii)}$$



$$\frac{1}{2} = \frac{R_1 + G}{R_1 + R_2 + G}$$

$R_1 + R_2 + G = 2R_1 + 2G$

$R_2 = R_1 + G$



17. (a) $\mathrm{KE} = \frac{3}{2}\mathrm{kT} \Rightarrow \left(\mathrm{T} = \frac{2\mathrm{E}}{3\mathrm{k}}\right), \mathrm{PV} = \mathrm{NkT}$

$\mathrm{P} = \rho \mathrm{gh}, \mathrm{V} = 4\ \mathrm{cm}^3$

$13.6 \times 10^3 \times 9.8 \times 2 \times 10^{-2} \times 4 \times 10^{-6}$

$$= \mathrm{Nk} \times \frac{2\mathrm{E}}{3\mathrm{k}} = \frac{\mathrm{N} \times 2}{3} \times 4 \times 10^{-14} \times 10^7$$

$$N = \frac{13.6 \times 19.6 \times 4 \times 10^{-5} \times 3 \times 10}{8}$$

$\mathrm{N} = 399.84 \times 10^{16}$

$= 3.99 \times 10^{18}$

$N = 4 \times 10^{18}$

18. **(b)** $v_e = 0.1C$ along y-axis direction of emwave - along (x)

$\vec{E} = \vec{V} \times \vec{B}$

$E = CB \Rightarrow B = E/C$

$\therefore$ force on e^- will be max.

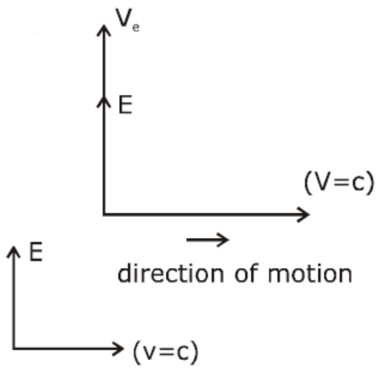
If B is $\perp$ to y-along z-axis

[$\because$ E also $\perp$ B, B also $\perp$ to direction of motion of wave]

$\therefore$ B $\rightarrow$ along Bz(−z) as

$$B = \frac{30}{C}\sin(1.5 \times 10^7 t - 5 \times 10^{-2} x)$$

$$B_{\max} = \frac{30}{3 \times 10^8} = 10^{-7}\ \text{T}$$



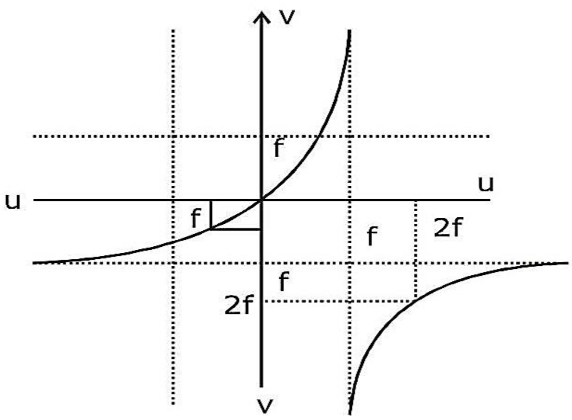
$\theta = 90$ between v_e & B so $F_{max} = qvB$

$$F_{\max} = e \times (0.1 \times C) \times \frac{30}{C}$$

$$= 1.6 \times 10^{-19} \times 3$$

$$F_{max} = 4.8 \times 10^{-19}\ \text{N}$$

19. **(d)** Concave lens graph u v/s v by u-v graph theory



20. **(d)** $z = a^2 b^{2/3} c^{-1/2} d^{-3}$

$$100 \times \frac{dz}{z} = \left(2\frac{da}{a} + \frac{2}{3}\frac{db}{b} + \frac{1}{2}\frac{dc}{c} + 3\frac{d(d)}{(d)}\right) \times 100$$

% error in z

$$= \left(2 \times 2 + \frac{2}{3} \times 1.5 + \frac{1}{2} \times 4 + 3 \times 2.5\right)\%$$

$$= 4 + 1 + 2 + 7.5$$

$$= 14.5\%$$

21. (51)

$m_H = 1\text{GeVC}^2 = 1000\text{MeV}/C^2,\ m_{\text{particle}} = 200\text{meV}/c^2 = m$

$\therefore mv_0 + 0 = 0 + 5m' \Rightarrow v' = \frac{v_0}{5}$

loss in KE

$$= \frac{1}{2}mv_0^2 - \frac{1}{2}(5m)\left(\frac{v_0}{5}\right)^2$$

$$= \frac{4}{5}\left(\frac{mv_0^2}{2}\right) = \frac{4}{5}\text{k}$$

$$\frac{4}{5}\text{k} = 10.2$$

$$k = 12.75\text{eV} = \frac{12.75}{100} = \frac{51}{4}$$

so $=$ n $=$ 51

22. (5) $B_2 = \frac{\mu_0 I_2\ N_2}{2R_2}$

$$\phi = N_1\ B_2 \pi R_1^2 = N_1\ N_2 \frac{\mu_0 I}{2R_2}\pi R_1^2$$

$$e = \frac{d\phi}{dt}$$

$$\phi = \frac{500 \times 200 \times 4\pi \times 10^{-7} \times (5t^2 - 2t - 3)\pi(10^{-2})^2}{2 \times 20 \times 10^{-2}}$$

$$\frac{10^5 \times 4\pi^2 \times 10^{-7}(5t^2 - 2t + 3) \times 10^{-4}}{40 \times 10^{-2}}$$

$$\phi = (5t^2 - 2t + 3) \times 10^{-4}$$

$$e = \left|\frac{d\phi}{dt}\right| = (10t - 2) \times 10^{-4}$$

$t = 1\text{sec}$

$e = 8 \times 10^{-4} = 0.8\text{mV} = \frac{0.8}{10} = \frac{4}{5}$

$x = 5$

23. (50) (Planck constant $h = 6.64 \times 10^{-34}$Js, $1\text{eV} = 1.6 \times 10^{-19}$ J, electron mass $m = 9.1 \times 10^{-31}$ kg)

$$2\ \text{d} \sin\theta = n\lambda = 1 \times \sqrt{\frac{150}{v}} \times 10^{-10}, \theta = 90 - \frac{\phi}{2}$$

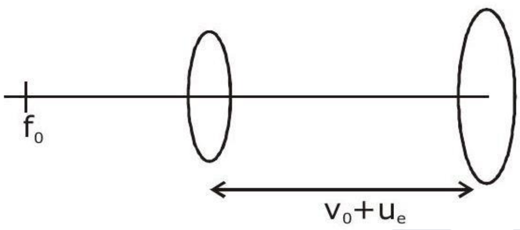
$$2 \times 10^{-10} \times \sin 60 = \sqrt{\frac{150}{V}} \times 10^{-10}, \theta = 90 - \frac{60}{2} = 60$$

$$2 \times \frac{\sqrt{3}}{2} = \sqrt{\frac{150}{V}}$$

$V = \frac{150}{3} = 50$ volt

$E = ev = 50ev$

24. (50)



$f_0 = 1$ cm, $f_e = 5$ cm, $u_0 = ?$
final image at (∞)
$(v_e = \infty)$
$v_0 + u_e = 10$ cm

$v_0 + 5 = 10$

$v_0 = 5$ cm

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$
$$\frac{1}{5} - \frac{1}{u_0} = \frac{1}{1}$$

$$\frac{1}{u_0} = \frac{1}{5} - 1 = -\frac{4}{5} \Rightarrow u_0 = -\frac{4}{5}$$

$$|u_0| = \frac{5}{4} = \frac{50}{40} = \frac{n}{40}$$

$\therefore n = 50$

$L = v_0 + u_e = 10$ cm
$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$
$$\frac{1}{\infty} - \frac{1}{u_e} = \frac{1}{5}$$

$u_e = -5$ cm

$|u_e| = 5$

25. (195)

$\vec{\tau} = \vec{r} \times F = (3\hat{\imath} + \hat{\jmath} - 2\hat{k}) \times (\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$

$= \begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$

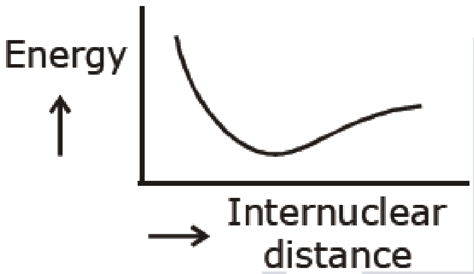
$\hat{\imath}(3+4) - \hat{\jmath}(9+2) + \hat{k}(6-1)$

$\vec{\tau} = 7\hat{\jmath} - 11\hat{\jmath} + 5\hat{k}$

$|\vec{\tau}| = \sqrt{49+121+25} = \sqrt{195}$

$x = 195$

26. (b)



27. (d)

$CH_3CH_2CN \xrightarrow{LiAlH_4} CH_3CH_2CH_2NH_2$

28. (b) Tyrosine in not an essential amino acid
29. (a) Hydrolysis of ester occurs in basic medium.
30. (b) In solid state PCl_5 exist in Ionpair i.e. $(PCl_4^+)$and (PCl_6^-)

PCl_4^+ (sp^3 - tetrahedral)

PCl_6^- ($sp^3\ d^2$) - octahedral)

31. (d)

$X_2 \rightarrow BCC$

$a = 300pm$

$d = 6.17\ g/cm^3 = \dfrac{2 \times GMM}{6 \times 10^{23} \times (300 \times 10^{-10})^3}$

$GMM = \dfrac{6.17 \times 6 \times 9 \times 3 \times 10^{-1}}{2}$

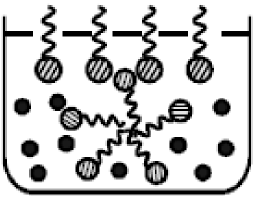
$GMM = 81 \times 6.17 \times 10^{-1}$

$= 49.97\ g/mol$

No. of molecules $= \frac{200\ \text{g}}{49.97\ \text{g/mol}} = 4$ mol

$= 4\ N_A$

32. (a)
33. (a)
34. (d)



35. (b) If nor-adrenaline is low, person may suffer from depression. Hence, anti -depressant drug is suggested.
36. (d) d^6 (octahedral) → high spin complex

$= t_{2g^4}\ eg^{\ 2}$

$\text{CFSE} = \left(-\frac{2}{5} \times 4 + \frac{3}{5} \times 2\right) \Delta_0$

$= \left(\frac{-8+6}{5}\right) \Delta_0$

$= -0.4\Delta_0$

d^6(tetrahedral) → high spin complex

$= eg^{\ 3} t_{2\ g^3}$

$\text{CFSE} = \left(-\frac{3}{5} \times 3 + \frac{2}{5} \times 3\right) \Delta_t$

$= -0.6\Delta_t$

37. (d) $A_t = A_0 \cdot e^{-k_1 t}$

$B_t = B_0 \cdot e^{-k_2 t}$

$k_1 = \frac{\ln 2}{300}$

$k_2 = \frac{\ln 2}{180}$

A_t and B_t are related as $[A] = 4[B]$

$A_0 \cdot e^{-k_1 t} = 4 \times B_0 \cdot e^{-k_2 t}$

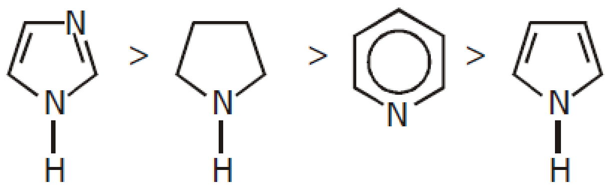
$\frac{t}{180} - \frac{t}{300} = 2$

$\frac{t}{3} - \frac{t}{5} = 120$

$\frac{2t}{15} = 120$

$t = 900\text{sec}$

38. (d) Correct order of basicity



39. (d) Eutrophication is the condition in which excessive richness of nutrients in a lake or water body, which causes dense growth of plant life and BOD increases.

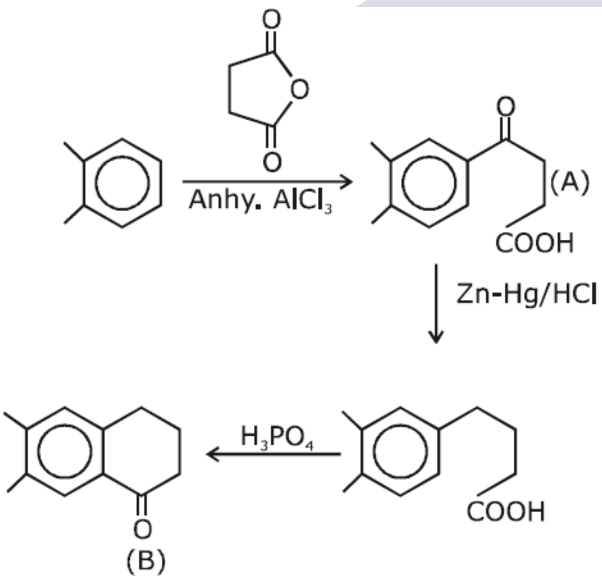
40. (d)

41. (d) $(R_4 - R_3)_{L^{+2}} = \frac{0.529}{3}\{4^2 - 3^2\} = \Delta R_1$

$$(R_4 - R_3)_{\text{He}^{+2}} = \frac{0.529}{2}\{4^2 - 3^2\} = \Delta R_2$$

$$\frac{\Delta R_1}{\Delta R_2} = \frac{1/3}{1/2} = \frac{2}{3}$$

42. (d)



43. (b)

Gd $\rightarrow [\text{Xe}]^{54} 4\text{f}^7 5\ \text{d}^1 6\ \text{s}^2$

$Z = 64$

$$\Big\downarrow -3e^{\ominus}$$

$\text{Gd}^{+3} = [\text{Xe}]^{54} 4\text{f}^7$

$\mu = \sqrt{7(7+2)} = \sqrt{63}$

$= 7.9\text{BM}$

44. (d)
45. (b)
46. (18)

$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

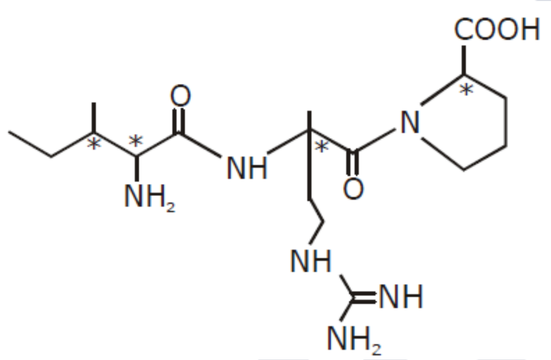
1 mol5 mol

$C_4H_{10} + \frac{13}{2}O_2 \rightarrow 4CO_2 + 5H_2O$

2 mol13 mol

Total required mol of $O_2 = 5 + 13 = 18$

47. (d)



48. (37) $CO_2 + H_2O \rightarrow H_2CO_3$

30 bar .... $\rightarrow$ 1 m/ lit.

3 bar .... $\rightarrow$ 0.1 m/lit

$t = 0 \; H_2CO_3 \quad 0.1 \quad H^{\oplus} + HCO_3^-$
0 0

Eq. $0.1(1-\alpha)\ 0.1\alpha\ 0.1\alpha$

$4 \times 10^{-7} = \frac{0.1\alpha^2}{1-\alpha}$

$(1-\alpha) \simeq 1$

$\alpha^2 = 4 \times 10^{-6}$

$\alpha = 2 \times 10^{-3}$

$[H^+] = 2 \times 10^{-4}\text{M}$

$\text{pH} = -[-4 \times \log(b)] = 3.7 = 37 \times 10^{-1}$

49. (6)

$\Delta G^{\circ} = -nFE^{\circ}$

$$17.37 \times 1000 = -3 \times 96500 \times E^{\circ}$$
$$E^{\circ} = \frac{17370}{3 \times 96500}$$
$$E^{\circ} = \frac{579}{9650} \text{ volt}$$
$$= 0.06 = 6 \times 10^{-2} \text{volt}$$

50. (6) EDTA $^{4-}$ is hexadentate ligand

51. (b)

$$\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$$

$(12 + n^2) - (6 + n) + n(2n - 4) = 158$

$3n^2 - 5n + 6 - 158 = 0$

$3n^2 - 5n - 152 = 0$

$3n^2 - 24n + 19n - 152 = 0$

$(3n + 19)(n - 8) = 0$

$\Rightarrow \mathrm{n} = 8$

$\Rightarrow \vec{b} \cdot \vec{c} = 10$

52. (d)

$n(\text{ coffee }) = \frac{73}{100}$

$n(\text{ tea }) = \frac{65}{100}$

$n(T \cap C) = \frac{x}{100}$

$n(C \cup T) = \mathrm{n(C)} + \mathrm{n(T)} - \mathrm{x} \leq 100$

$= 73 + 65 - x \leq 100$

$\Rightarrow \mathrm{x} \geq 38$

53. (d)

$$\operatorname{Var}(x) = \sum \frac{x_i^2}{n} - (\bar{x})^2$$
$$16 = \frac{x_1^2 + x_2^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2}{7} - 64$$

$80 \times 7 = \mathrm{x}_1^2 + \mathrm{x}_2^2 + \mathrm{x}_3^2 + \cdots .. + \mathrm{x}_7^2$

Now, $x_2^6 + x_7^2 = 560 - \left(x_1^2 + \cdots \ldots . x_5^2\right)$

$x_6^2 + x_7^2 = 560 - (4 + 16 + 100 + 144 + 196)$

$x_6^2 + x_7^2 = 100$

Now, $\frac{x_1+x_2+\cdots+x_7}{7} = 8$

$x_6 + x_7 = 14$

from (a) & (b)

$(x_6 + x_7)^2 - 2x_6x_7 = 100$

$2x_6x_7 = 96 \Rightarrow x_6x_7 = 48$

Now, $|x_6 - x_7| = \sqrt{(x_6 + x_7)^7 - 4x_6x_7}$

$= \sqrt{196 - 192} = 2$

54. (a) let

$S' = 2^{10} + 2^9 3^1 + 2^8 3^2 + - \cdot - \cdot - + 2.3^9 + 3^{10}$

$$\frac{3 \times S'}{2} = 2^9 \times 3^1 + 2^8 \cdot 3^2 + - \cdot - \cdot +3^{10} + \frac{3^{11}}{2}$$

$$\frac{-S'}{2} = 2^{10} - \frac{3^{11}}{2}$$

$S' = 3^{11} - 2^{11}$

Now $S' = S - 2^{11}$

$S = 3^{11}$

55. (d) $28 = 3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha}$

$$28 = \frac{9^{\sin 2\alpha}}{3} + \frac{81}{9^{\sin 2\alpha}}$$

Let $9^{\sin 2\alpha} = t$

$$28 = \frac{t}{3} + \frac{81}{t}$$

$t^2 - 84t + 243 = 0$

$t = 81, 3$

$9^{\sin 2\alpha} = 9^2$ or 3

$\sin 2\alpha = 2$ or $\sin 2\alpha = 1/2$

(Not possible)

Now three terms in A.P. are

1,14,27

Next term are

40,53,66

56. (c) $y = mx + \frac{1}{m}$

$$x^2 = 4\left(mx + \frac{1}{m}\right)$$

$$x^2 - 4mx - \frac{4}{m} = 0$$

$$D = 0$$

$$16m^2 + \frac{16}{m} = 0$$

$$16\left(\frac{m^3+1}{m}\right) = 0$$

$$m = -1$$

$$\Rightarrow y + x = -1$$

Now, $\left|\frac{-1}{\sqrt{2}}\right| = c$

$$c = \frac{1}{\sqrt{2}}$$

57. (b)

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

$C_1 \to C_1 - C_2, C_3 \to C_3 + C_2$

$$\begin{vmatrix} 1 & -1-\sin^2\theta & -\sin^2\theta \\ 1 & -1-\cos^2\theta & -\cos^2\theta \\ 2 & 10 & 8 \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix} 1 & -1 & -\sin^2\theta \\ 1 & -1 & -\cos^2\theta \\ 2 & 2 & 8 \end{vmatrix}$$

$$1(2\cos^2\theta - 8) + (8 + 2\cos^2\theta) - 4\sin^2\theta$$

$$f(\theta) = 4\cos 2\theta$$

58. (b)

$$D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0$$

$$2(-14) + 4(4-\lambda) + \lambda(6\lambda - 10) = 0$$

$-28+16-4\lambda+6\lambda^2-10\lambda=0$

$6\lambda^2-14\lambda-12=0$

$3\lambda^2-7\lambda-6=0$

$3\lambda^2-9\lambda+2\lambda-6=0$

$(3\lambda+2)(\lambda-3)=0$

$\lambda=-2/3,3$

$\mathrm{D}_1=\begin{vmatrix}1 & -4 & \lambda\\ 2 & -6 & 1\\ 3 & -10 & 4\end{vmatrix}$

$\Rightarrow -14+4(5)+\lambda(-2)$

$\Rightarrow -2\lambda+6$

$D_2=\begin{vmatrix}2 & 1 & \lambda\\ 1 & 2 & 1\\ \lambda & 3 & 4\end{vmatrix}$

$\Rightarrow 2(5)-1(4-\lambda)+\lambda(3-2\lambda)$

$\Rightarrow 10-4+\lambda+3\lambda-2\lambda^2$

$\Rightarrow -2\lambda^2+4\lambda+6$

$\Rightarrow -2(\lambda^2-2\lambda-3)$

$\Rightarrow -2[\lambda^2-3\lambda+\lambda-3]$

$\Rightarrow -2(\lambda-3)(\lambda+1)$

$D_3=\begin{vmatrix}2 & -4 & 1\\ 1 & -6 & 2\\ \lambda & -10 & 3\end{vmatrix}\Rightarrow 2(-18+20)+4(3-2\lambda)+1(-10+6\lambda)$

$=4+12-8\lambda-10+6\lambda$

$=-2\lambda+6$

$\Rightarrow \lambda=-2/3$

59. (d) Let P be $(\sqrt{5}\cos\theta, 2\sin\theta)$

Now, $PQ=\sqrt{(\sqrt{5}\cos\theta)^2+(2\sin\theta+4)^2}$

$PQ=\sqrt{5\cos^2\theta+(2\sin\theta+4)^2}$

$\frac{\mathrm{d(PQ)}}{\mathrm{d}\theta}=0\Rightarrow -10\sin\theta\cos\theta+(4\sin\theta+8)\cos\theta=0$

$\Rightarrow -6\sin\theta\cos\theta+8\cos\theta=0$

$\cos\theta = 0$ or $\sin\theta = \frac{4}{3}$

Not possible

So P is either $(0,2)$ or $(0,-2)$

$PQ^2 = 36$

60. (a) $9t^2 - 18t + 5 = 0$

$9t^2 - 15t - 3t + 5 = 0$

$(3t-5)(3t-1) = 0$

$$|x| = \frac{5}{3}, \frac{1}{3}$$

$$\Rightarrow x = \frac{5}{3}, \frac{-5}{3}, \frac{1}{3}, \frac{-1}{3}$$

$$\Rightarrow P = \frac{25}{81}$$

61. (d) $\frac{dy}{dx} + \left(e^x \times \frac{y+2}{e^x+5}\right) = 0$

$$\frac{dy}{dx} + \left(\frac{e^x}{e^x+5}\right)y = \frac{-2e^x}{e^x+5}$$

$\text{I.F.} = e^{\int \frac{e^x}{e^x+5}dx}$

$= e^{\int \left(1-\frac{5}{e^x+5}\right)dx}$

$= e^{\int \left(1-\frac{5e^{-x}}{1+5e^{-x}}\right)dx}$

$= e^{x+\ln(1+5e-x)}$

$= e^x \cdot (1+5e^{-x}) \Rightarrow (e^x+5)$

$y(e^x+5) = -\int 2e^x dx$

$y(e^x+5) = -2e^x + C$

$\Downarrow x = 0$

$(6) = -2 + C \Rightarrow C = 8$

$$y(\ln 13) = \frac{8 - 2\times 13}{13+5} = \frac{-18}{18} = -1$$

62. (b)

$$S = \tan^{-1}\left(\frac{1}{1+1\times 2}\right) + \tan^{-1}\left(\frac{1}{1+2\times 3}\right) + \cdots$$

$T_r = \tan^{-1}\left(\frac{1}{1+r(r+1)}\right)$

$T_r = \tan^{-1}(r+1) - \tan^{-1} r$

$T_1 = \tan^{-1} 2 - \tan^{-1} 1$

$T_2 = \tan^{-1} 3 - \tan^{-1} 2$

$T_3 = \tan^{-1} 4 - \tan^{-1} 3$

$T_{10}^{3} = \tan^{-1} 11 - \tan^{-1} 10$

$\Rightarrow S = \tan^{-1} 11 - \tan^{-1} 1$

$\Rightarrow \tan S = \frac{10}{12} = \frac{5}{6}$

63. (a) $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$

$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{e^{\sin x}}{1+e^{\sin x}} dx \;\Rightarrow 2I = \pi$

$I = \frac{\pi}{2}$

64. (a)

$\overrightarrow{PM} \perp (2\hat{\imath} - 2\hat{\jmath} - \hat{k})$

$\Rightarrow (2\lambda - 2)\cdot 2 + (1 - 2\lambda)(-2) + (3 - \lambda)(-1) = 0$

$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$

$\Rightarrow 9\lambda = 9 \Rightarrow \lambda = 1$

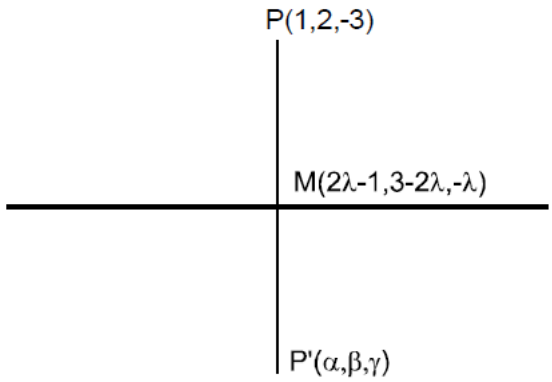
$\Rightarrow m(1,1,-1)$

Now, $p' = 2M - P$

$= 2(1,1,-1) - (1,2,-3)$

$= (1,0,1)$

$a + b + c = 2$



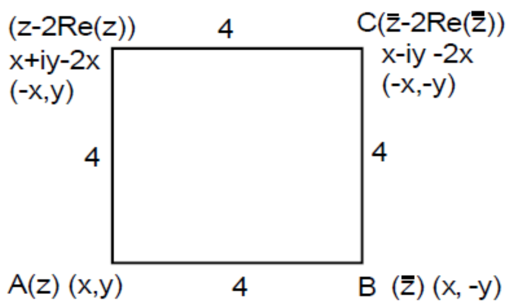
65. (d)

$$f(x) = \begin{cases} 2k_1(x-\pi); & x \le \pi \\ -k_2 \sin x & ; x > \pi \end{cases}$$

$$f''(x) = \begin{cases} 2k_1 & ; x \le \pi \\ -k_2 \cos x; & x > \pi \end{cases}$$

$2k_1 = k_2$

66. (d)



Let $z = x + iy$

$CA^2 = AB^2 + BC^2$

$2^2x^2 + 2^2y^2 = 32$

$x^2 + y^2 = 8$

$\sqrt{x^2+y^2} = 2\sqrt{2}$

67. (a)

$$\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x+e^{-x})}dx$$

$$\int (e^{2x} + e^x - 1)e^{(e^x+e^{-x})}dx + \int (e^x - e^{-x})e^{(e^x+e^{-x})}dx$$

$$\int (e^x + 1 - e^{-x})e^{(e^x+e^{-x}+x)}dx + \int (e^x - e^{-x})e^{(e^x+e^{-x})}dx$$

$e^{(e^x+e^{-x}+x)} + e^{e^x+e^{-x}} + C$

$\left(e^{e^x+e^{-x}}\right)[e^x+1] + C$

$\Downarrow$

$g(x)$

$\Rightarrow g(0) = 2$

68. (b) As we know

$\sim(p \leftrightarrow q) = (p \wedge \sim q) \vee (\sim p \wedge q)$
$\Rightarrow$ so, $\sim(x \leftrightarrow \sim y) = (x \wedge y) \vee (\sim x \wedge \sim y)$

69. (b)

$f(x) = x^2 - x - 2\langle {}^{2}_{-1} = \alpha$

$$\lim_{x\to 2^+} \frac{\sqrt{1-\cos(x-2)(x+1)}}{x+\alpha-4}$$

$$\lim_{x\to 2+} \frac{\sqrt{1-\cos(x-2)(x+1)}}{(x-2)}$$

$$\lim_{h\to 0} \frac{\sqrt{1-\cos(h\times(h+3))}}{h}$$

$$\lim_{h\to 0} \sqrt{\frac{1-\cos(h(h+3))}{h^2\times(h+3)^2}\times(h+3)^2} \Rightarrow \sqrt{\frac{1}{2}\times 9} = \frac{3}{\sqrt{2}}$$

70. (d) $\frac{x^2}{16}+\frac{y^2}{9}=1$

$$e = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$$

$F_1(\sqrt{7},0), F_2(-\sqrt{7},0)$

$PF_1 + PF_2 = 2a$

$PA + PB = 2\times 4 = 8$

71. (13) $T_{r+1} = {}^{22}C_r(x^m)^{22-r}\left(\frac{1}{x^2}\right)^r$

$= {}^{22}C_r(x)^{22m-mr-2r}$

Given ${}^{22}C_r = 1540 = {}^{22}C_{19} \Rightarrow r = 19$

$\because 22m - rm - 2r = 1$

$$\Rightarrow m = \frac{2r+1}{22-r}$$

$m = 13$ (At $r = 19$)

72. (11)

(atteat 2 or 3) = ${}^4C_2\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right)^1 + {}^4C_4\left(\frac{2}{6}\right)^4$

$$= 6\times\frac{1}{9}\times\frac{4}{9}+4\times\frac{1}{27}\times\frac{2}{3}+\frac{1}{81}$$

$= \frac{33}{81} = \frac{11}{27} \Rightarrow \text{nP} \Rightarrow 11$

73. (8)

$f(x) = x\left[\frac{x}{2}\right], -10 < x < 10$

$-5 < \frac{x}{2} < 5$

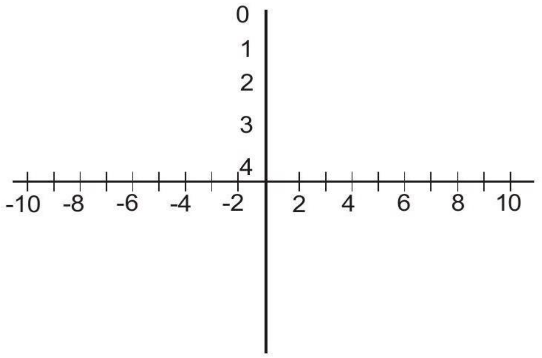
$-5x \; -5 < \frac{x}{2} < -4$

$-4x \; -4 < \frac{x}{2} < 3$

$-3\text{x} \; -3 < \text{x}/2 < -2$

$-2x \; -2 < x/2 < -1$

$-x \; -1 < x/2 < 0$



Number of point of discontinuity = 8

74. (240)

75. (30) $L_1: 2x - y + 3 = 0$

$L_2: 4x - 2y + \alpha = 0$

$L_3: 6x - 3y + \beta = 0$

$\frac{\left|\frac{\alpha}{2} - 3\right|}{\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\Rightarrow \frac{\alpha}{2} - 3 = 1, -1$

$\Rightarrow \alpha = 8,4$

$\frac{\left|\frac{\beta}{3} - 3\right|}{\sqrt{5}} = \frac{2}{\sqrt{5}}$

$\Rightarrow \frac{\beta}{3} - 3 = 2, -2$

$\Rightarrow \beta = 15{,}3$